

Synthesis of the phenomenology of the underdoped cuprates

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The underdoped cuprates have a number of interesting and unusual properties that often seem hard to reconcile with one another. In this paper we show how many of these diverse phenomena can be synthesized into a single coherent theoretical picture. Specifically we present a description where a pseudogap and gapless Fermi arcs exist in the normal state above the superconducting transition temperature (T_c) but give way to the observed quantum oscillations and other phenomena at low temperature when the superconductivity is suppressed by a magnetic field. We show the consistency between these phenomena and observations of enhanced Nernst and diamagnetic signals above T_c . We also develop a description of the vortex core inside the superconducting state and discuss its relation with the high-field phenomena.

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I. INTRODUCTION

In the last several years, a large number of experiments have studied the properties of the underdoped cuprate materials. The phenomenology of these materials is rather unusual, and a unified theoretical picture of the diverse phenomena is yet to emerge. First below a temperature scale T^* there is the onset of the pseudogap in various probes. Remarkably angle resolved photoemission spectroscopy (ARPES) studies of the electron spectral function reveal the existence of gapless Fermi arcs in the pseudogap region above the superconducting (SC) transition.¹ The length of these Fermi arcs decreases with decreasing temperature (T), possibly extrapolating to zero² in the limit $T \rightarrow 0$. Second in a fairly wide range of temperatures above T_c , but below the T^* line, there is an onset of local superconducting order without long-range phase coherence.^{3,4} This is most strikingly evidenced by an enhancement of the Nernst effect and diamagnetic response in this region. A third piece of the phenomenology comes from recent quantum oscillation studies of the state obtained by suppressing superconductivity in a high field at low temperature.^{5,6} Remarkably sharp oscillation frequencies are seen indicating the presence of a sharp Fermi surface with charge carriers that are fermionic Landau quasiparticles. The detailed behavior of the oscillation frequencies and other experiments have led to a suggestion that the oscillations are associated with gapless electron pockets centered near the antinodal region of the Brillouin zone.⁷⁻¹⁰

Currently there is considerable confusion about how these various pieces of information about the normal (i.e., nonsuperconducting) state of the underdoped cuprates fit together. The ARPES results at low field and high T seem to be in conflict with the existence of a closed Fermi surface at high fields and low T . Further the suggested electron pocket interpretation of the oscillation experiments also has a difficulty with ARPES (and other probes at low field above T_c); it is precisely along the antinodal region that the pseudogap is seen in the electron spectrum. It is puzzling how a relatively modest field of 40 T can close the large antinodal gap. Finally the high-field experiments (in particular studies of the Hall effect⁷) have questioned the correctness of the conclusions of Ong and co-workers⁴ based on their Nernst/magnetization experiments.

In this paper we show how these various seemingly conflicting results may be reconciled into a single coherent theoretical picture. We are cognizant of other interesting phenomena that have attracted attention recently—notably signatures of time-reversal symmetry breaking.¹¹⁻¹³ The role they play in determining the electronic structure is not understood, and we will not discuss them in this paper. Our theory is based on a small set of assumptions that we justify on various theoretical and empirical grounds. We then obtain a description of the underdoped cuprates where a pseudogap and gapless Fermi arcs occur above T_c but give way to the observed quantum oscillation and other phenomena at high fields. Further we show that there is no real conflict between the conclusions of the Nernst/diamagnetism experiments and the high-field oscillation experiments. As a bonus we obtain a description of the vortex core structure inside the superconducting phase and discuss its relation with the phenomena described above. In a companion paper¹⁴ we explore a specific microscopic model that captures a number of aspects of the overall picture that emerges from the present one.

We begin by stating and justifying our central assumptions.

(1) *At zero field there is a coherence scale T_{coh} below which the single-particle electronic excitations become well defined in the sense of Landau; i.e., the inverse lifetime is less than $k_B T$. T_{coh} increases with increasing hole concentration x and is roughly of order T_c .*

The existence of such a coherence scale is a fairly general feature of metals near a Mott transition and is supported by many theoretical calculations of a doped Mott insulator, for instance, in slave boson theory¹⁵ or through dynamical mean-field theory and their extensions.¹⁶ Empirically ARPES experiments see such a fairly sharp onset of coherent quasiparticles upon cooling into the superconducting state,¹⁷ at least at the antinodal region of the Brillouin zone. The other indirect evidence comes from microwave transport¹⁸ and thermal Hall conductivity¹⁹ experiments where the scattering rate drops rapidly for T below $\sim T_c$. A similar drop in the scattering rate of the nodal excitations below T_c has also been reported in ARPES.²⁰

(2) *The true H_{c2} where local superconducting order is suppressed is large, possibly ≈ 150 T or higher for moderately underdoped materials. The resistive transition out of*

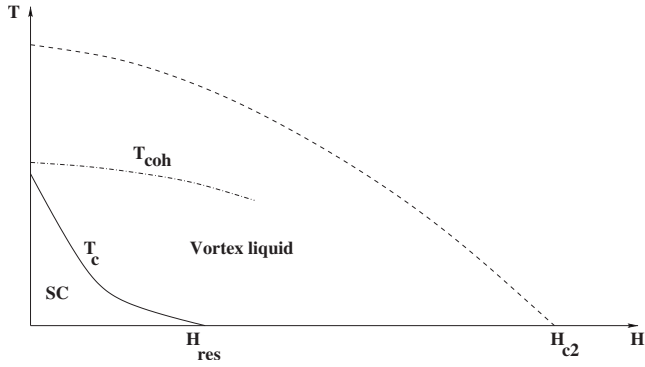


FIG. 1. Schematic field (H)-temperature (T) phase diagram showing the evolution of the superconducting properties. The full line is the resistive superconducting phase transition, while the dashed line corresponds to the onset temperature of the crossover to local superconductivity. The coherence crossover temperature T_{coh} is however not affected much by fields on the order of H_{res} . Not shown is the field-induced magnetic ordering.

the superconductor however happens at a much lower field H_{res} . (For instance, for $H_{\text{res}} \sim 40$ T).

The large value of H_{c2} is a central conclusion of the Ong Nernst/magnetization experiments. Thus we simply assume that this is correct despite the recent criticism from Taillefer and co-workers.⁷ The H - T phase diagram thus has the schematic form shown in Fig. 1. At low temperature and fields between the resistive transition and H_{c2} the system may be dubbed as a ‘‘vortex liquid.’’ A discussion of this assumption in terms of vortex structure will be given at the end of the paper. We emphasize that the quantum oscillation experiments are done at very low T at fields near H_{res} and not near H_{c2} .

(3) *Magnetic fields of order H_{res} only destroy superconducting phase coherence but not the local superconducting pairing or the electron ‘‘coherence.’’*

In other words H of order H_{res} is efficient in suppressing T_c but not T_{coh} or the pairing scale Δ (see Fig. 1). Not destroying the pairing gap is obvious from the previous assumption. Not destroying the electron coherence is an additional assumption. We note that in the simplest slave boson mean-field description of the doped Mott insulator, T_c and T_{coh} are necessarily tied together. Thus the present assumption goes beyond the simplest slave boson mean-field theory and must be justified theoretically in a more sophisticated theory. Some empirical evidence in support of this assumption comes from scanning tunnel microscope (STM) studies of the vortex core in the superconducting state and will be discussed further at the end of the paper.

(4) *For magnetic fields $H > H_{\text{res}}$ but $\ll H_{c2}$, the Cooper pair phase has a finite memory time $\tau_\phi(H)$ which we assume is large compared to the inverse pairing gap Δ_0 , i.e., $\frac{\hbar}{\tau_\phi(H)} \ll \Delta_0$. There is an associated memory length $\xi_\phi = v_F \tau_\phi$ where v_F is the Fermi velocity.*

Thus ξ_ϕ is much larger than the ‘‘bare’’ coherence length $\frac{\hbar v_F}{\Delta_0}$ for $H \ll H_{c2}$. The scale τ_ϕ is infinite for $H < H_{\text{res}}$ and decreases as H increases toward H_{c2} . We expect that $\frac{\hbar}{\tau_\phi(H)} \sim \Delta_0$ for fields of the order H_{c2} . Further in the vortex liquid regime we expect that ξ_ϕ may be roughly estimated as the

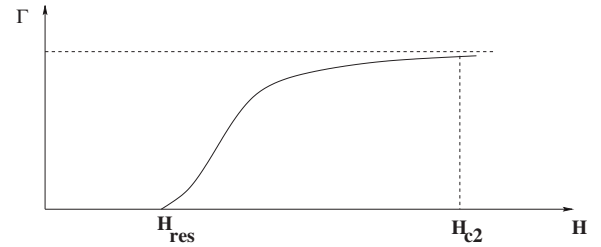


FIG. 2. Sketch of the Cooper pair phase decay rate as a function of field at $T=0$. The saturated value at fields near H_{c2} is of order $1/\Delta_0$.

intervortex spacing, i.e., $\xi_\phi \sim \sqrt{\frac{\hbar c}{eH}}$. In Fig. 2 we show a sketch of the expected form of the Cooper pair phase decay rate $\Gamma = \frac{1}{\tau_\phi}$ as a function of field H .

(5) *Finally we assume that fields of the order of H_{res} induce a freezing of the dynamic incommensurate magnetic fluctuations known to exist in zero field.*

Field-induced magnetic ordering has been previously reported in the $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ (LSCO) family of cuprates²¹ and has been studied theoretically.²² Very recently direct evidence for field-induced magnetic ordering has been obtained²³ in $\text{YBCO}_{6.45}$. Further in $\text{YBCO}_{6.5}$ recent studies of quantum oscillations have shown that there is no Zeeman splitting of the oscillation frequencies,²⁴ as expected if there is magnetic ordering. However the magnetic ordering appears as a low energy phenomenon that competes with the long-range superconducting order. It can therefore be treated separately once the underlying electronic structure (including phenomena such as the pseudogap) is established. With these assumptions we now examine the properties of the system in various regimes of temperature and field.

II. LOW T AND FIELDS $H \sim H_{\text{res}}$: EMERGENCE OF LARGE FERMI SURFACE

First consider low temperature $T \ll T_{\text{coh}}$ and fields in the range $H_{\text{res}} \sim H \ll H_{c2}$. To begin with we will also ignore the field-induced magnetic ordering and incorporate it later. In this regime we may think in terms of an effective model of a superconductor disordered by quantum fluctuations of the order-parameter phase. Previous studies of this model have suggested various possible ground states; the simplest to contemplate perhaps is a Cooper pair insulator. More exotic insulating states with fractionalized quasiparticles (such as holons or spinons) have also been proposed.^{25,26} Here we argue for the possibility of a particularly simple metallic state, namely, just a Fermi liquid with a large Fermi surface. To appreciate this it is instructive to first consider the simpler problem of a clean s -wave Bardeen-Cooper-Schrieffer superconductor in $d=2$ with short coherence length. Then the (Kosterlitz-Thouless) T_c will be much smaller than the pairing gap Δ_0 and can be readily suppressed in a magnetic field without suppressing the pairing. The resulting nonsuperconducting quantum ground state might of course be an insulator of Cooper pairs that breaks translational symmetry. However if it stays metallic the simplest option is to regain the large Fermi surface albeit with a reduced quasiparticle weight Z at

the Fermi surface. Below we demonstrate this directly for the d -wave case relevant to the cuprates through a simplified model.

Let q_α^\dagger create a low energy electron quasiparticle with spin $\alpha = \uparrow, \downarrow$ that exists below the coherence scale T_{coh} . At zero field in the SC state an appropriate low energy Hamiltonian is simply the BCS one,

$$H_{\text{eff}} = \sum_{\mathbf{K}} \epsilon_{\mathbf{K}} q_{\mathbf{K}\alpha}^\dagger q_{\mathbf{K}\alpha} + \sum_{\mathbf{K}} \Delta_{\mathbf{K}} (q_{\mathbf{K}\uparrow}^\dagger q_{-\mathbf{K}\downarrow}^\dagger - q_{\mathbf{K}\downarrow}^\dagger q_{-\mathbf{K}\uparrow}^\dagger) + \text{h.c.} \quad (1)$$

Here $\epsilon_{\mathbf{K}}$ is the fully renormalized quasiparticle dispersion, and the pairing gap $\Delta_{\mathbf{K}} \sim \Delta_0 [\cos(K_x) - \cos(K_y)]$. The operator q^\dagger creates a “renormalized” low energy quasiparticle that has a nonzero overlap $\sqrt{Z_0}$ with the bare electron. It is expected that Z_0 goes to zero as the hole concentration x goes to zero. (For instance, in the slave boson mean-field theory, $Z_0 \sim x$.)

Now consider a situation where Δ maintains its phase only over a finite time scale τ_ϕ and length $\xi_\phi = v_F \tau_\phi$. Then the pairing term in the Hamiltonian is modified to

$$H = \sum_{\mathbf{R}} \hat{\Delta}(\mathbf{R}) \sum_{\mathbf{r}} \eta(\mathbf{r}) (q_{\mathbf{R}\uparrow}^\dagger q_{\mathbf{R}+\mathbf{r}\downarrow}^\dagger - q_{\mathbf{R}\downarrow}^\dagger q_{\mathbf{R}+\mathbf{r}\uparrow}^\dagger) + \text{h.c.}, \quad (2)$$

where \mathbf{R} is a site of the lattice and the sum over \mathbf{r} extends over the four nearest neighbors. The constant $\eta(\pm \mathbf{x}) = +1$ and $\eta(\pm \mathbf{y}) = -1$. The amplitude $\hat{\Delta}(\mathbf{R})$ is written as

$$\hat{\Delta}(\mathbf{R}) = \Delta_0 e^{i\hat{\phi}(\mathbf{R})}, \quad (3)$$

where the phase $\hat{\phi}$ is to be regarded as a quantum operator-valued field.

At zero temperature, in space and imaginary time τ we take the correlators of $\Delta(\mathbf{R}, \tau)$ to have the form

$$\langle \Delta^*(\mathbf{R}, \tau) \Delta(0, 0) \rangle = \Delta_0^2 F(\mathbf{R}, \tau) \quad (4)$$

such that

$$F(\mathbf{0}, 0) = 1, \quad (5)$$

$$F(|\mathbf{R}| \rightarrow \infty, \tau) \simeq e^{-|\mathbf{R}|/\xi_\phi}, \quad (6)$$

$$F(|\mathbf{R}|, \tau \rightarrow \infty) \simeq e^{-|\tau|/\tau_\phi}. \quad (7)$$

As a concrete and illustrative example we consider the specific function

$$F(\mathbf{R}, \tau) = e^{-|\mathbf{R}|/\xi_\phi} e^{-|\tau|/\tau_\phi}. \quad (8)$$

This has the Fourier transform

$$F(\mathbf{p}, \omega) = \frac{4\pi\xi_\phi^2}{(p^2\xi_\phi^2 + 1)^{3/2}} \frac{\tau_\phi}{\tau_\phi^2\omega^2 + 1}. \quad (9)$$

This form of the correlator describes a “propagating” pair fluctuation mode with a constant frequency Γ . Alternately we may consider a dissipative form of the correlator where the frequency dependent part is modified (at low imaginary frequency) to $\frac{1}{|\omega| + \Gamma}$. The essential features of the results described below are not sensitive to such a modification. Henceforth we will therefore simply use Eq. (9) to illustrate the main physics.

In the limit that the amplitude of the pairing gap Δ_0 is small compared to the Fermi energy E_F (which is not unreasonable for the cuprates), we may calculate the quasiparticle self-energy from scattering off the fluctuating pair order parameter in second-order perturbation theory. This gives

$$\Sigma(\mathbf{K}, i\omega) = \Delta_{0\mathbf{K}}^2 \int_{\mathbf{p}, \Omega} \frac{F(\mathbf{p}, \Omega)}{i(\Omega - \omega) - \epsilon_{\mathbf{p}-\mathbf{K}}}, \quad (10)$$

with $\Delta_{0\mathbf{K}} = \frac{\Delta_0}{2} (\cos K_x - \cos K_y)$. For the specific functional form chosen for F in Eq. (9) the integral can be evaluated readily both for small and large $|\omega|$ near the Fermi surface. For small ω and \mathbf{K} close to the Fermi surface we find

$$\Sigma(\mathbf{K}, i\omega) \simeq \frac{\Delta_{0\mathbf{K}}^2}{\pi\Gamma^2} (i\omega - v_F k_\parallel), \quad (11)$$

where k_\parallel is the deviation of \mathbf{K} from the Fermi surface at the point of closest approach. For large $|\omega| \gg \Gamma$, we find

$$\Sigma(\mathbf{K}, i\omega) \simeq -\frac{\Delta_{0\mathbf{K}}^2}{i\omega + \epsilon_{\mathbf{K}}}. \quad (12)$$

For the electron Green’s function this implies that for small $|\omega|$,

$$\mathcal{G}(\mathbf{K}, i\omega) \simeq \frac{1}{\left(1 + \frac{\Delta_{0\mathbf{K}}^2}{\pi\Gamma^2}\right) (i\omega - v_F k_\parallel)}. \quad (13)$$

Thus the quasiparticle Green’s function has a quasiparticle pole everywhere on the original large Fermi surface. However the quasiparticle weight has a reduction factor $Z_\Delta(\mathbf{K})$ due to the scattering off the pair field,

$$Z_\Delta(\mathbf{K}) = \frac{1}{\left(1 + \frac{\Delta_{0\mathbf{K}}^2}{\pi\Gamma^2}\right)}. \quad (14)$$

We emphasize that this reduction is over and above any reduction Z_0 in the quasiparticle weight that is already present in defining the low energy effective model. The total quasiparticle weight therefore is $Z = Z_0 Z_\Delta$. Note that Z_Δ is strongly angle dependent on the Fermi surface. Near the nodal direction $Z_\Delta \approx 1$, while near the antinodal direction $Z_\Delta \approx \frac{\pi\Gamma^2}{\Delta_0^2} \ll 1$. Thus the emergent large Fermi surface at low temperature has a highly anisotropic quasiparticle weight. Despite this strongly reduced Z for the antinodal quasiparticle the scattering off the pair fluctuations does not cause a mass enhancement due to a compensation between the momentum and frequency-dependent parts of the self-energy of the q field [at least within the model defined by the correlation function in Eq. (8)].

For large $|\omega|$ (i.e., $|\omega| \gg \Gamma$) on the other hand the form of the self-energy implies that the quasiparticle Green’s function takes the form

$$\mathcal{G}(\mathbf{K}, i\omega) \simeq \frac{-(i\omega + v_F k_\parallel)}{\omega^2 + v_F^2 k_\parallel^2 + \Delta_{0\mathbf{K}}^2}, \quad (15)$$

which is the same as that inside the d -wave superconducting phase. Physically this just reflects the picture that the system looks like a superconductor at short time scales $< \tau_\phi$ and

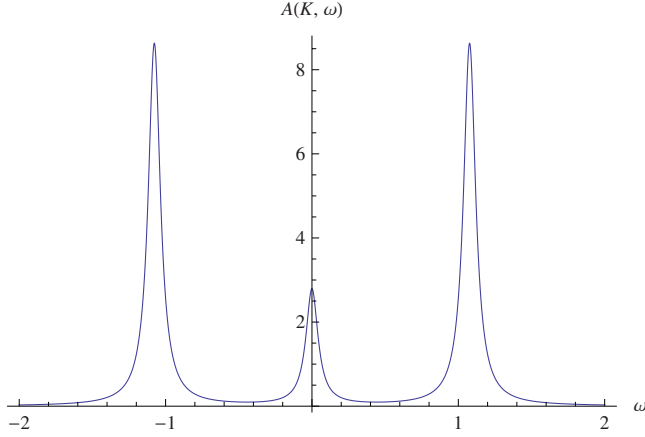


FIG. 3. (Color online) Spectral function $A(\mathbf{K}, \omega)$ at the antinodal point of the Fermi surface at low T if the superconductivity is suppressed. We chose units where $\Delta_0=1$ and the parameter $\Gamma=0.4$. A small broadening of the single-particle energies $\gamma=0.05$ [see Eq. (20)] was used in generating this plot.

length scales $< \xi_\phi$. However at longer length and time scales the large Fermi surface emerges but with a reduced angle-dependent spectral weight Z . A simple expression for $\Sigma(\mathbf{K}, i\omega)$ that interpolates between the small and large $|\omega|$ limits is

$$\Sigma(\mathbf{K}, i\omega) = \Delta_{0\mathbf{K}}^2 \frac{(-i\omega + \epsilon_{\mathbf{K}})}{\omega^2 + \epsilon_{\mathbf{K}}^2 + \pi\Gamma^2}. \quad (16)$$

Then the quasiparticle Green's function takes the form

$$\mathcal{G}(\mathbf{K}, i\omega) = \frac{1}{(i\omega - \epsilon_{\mathbf{K}}) \left(1 + \frac{\Delta_{0\mathbf{K}}^2}{\omega^2 + \epsilon_{\mathbf{K}}^2 + \pi\Gamma^2} \right)}. \quad (17)$$

This Green's function has a pole at zero frequency at the location of the Fermi surface and no zeroes anywhere in the Brillouin zone. The electron spectral function $A(\mathbf{K}, \omega)$ may be readily extracted from this Green's function. In Fig. 3 we show a plot of $A(\mathbf{K}, \omega)$ as a function of ω for \mathbf{K} at the antinodal portion of the Fermi surface.

Thus far we have ignored both the field-induced incommensurate magnetic ordering (i.e., assumption 5 above) and the direct modification of the electron spectrum due to the magnetic field. As the emergent gapless excitations are Landau quasiparticles with definite electric charge (unlike the Bogoliubov quasiparticles inside the superconductor), the latter leads to the usual Landau quantization of orbits. The former can be incorporated in the low energy theory as a Hartree term,

$$H_m \sim g N_{\mathbf{Q}} \sum_{\mathbf{K}} (q_{\mathbf{K}+\mathbf{Q}}^\dagger \sigma^z q_{\mathbf{K}} + \text{h.c.}), \quad (18)$$

where $\vec{N}_{\mathbf{Q}} = N_{\mathbf{Q}} \hat{z}$ is the magnetic order parameter. Here the ordering wave vector $\mathbf{Q} = 2\pi(1/2 \pm \delta, 1/2)$ which is where soft fluctuations are seen in zero field. (g is a coupling constant). This term will lead to a reconstruction of the Fermi surface into hole pockets centered near the nodal direction

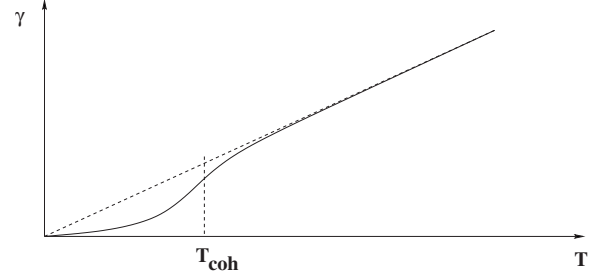


FIG. 4. Sketch of the single-particle scattering rate γ as a function of temperature.

and electron pockets centered near the antinodal direction (see, for instance, Ref. 8).

A number of previous papers⁶⁻⁸ have described how the resultant electron pockets can explain the frequency and other details of the quantum oscillation experiments, and we have nothing to add here. However our main point is that in the absence of the expected field-induced magnetism the phase disordered d -wave superconductor has a large Fermi surface with an anisotropic quasiparticle weight as described above. The full Fermi surface is restored not by closing the pairing gap but by the appearance of quasiparticles with small weight in the antinodal region. We emphasize that the small quasiparticle weight does not directly affect the observability of the electron pocket in magnetization or resistivity oscillation experiments. *It is the emergence of this gapless large Fermi surface that makes the reconstruction into electron and hole pockets possible in the presence of magnetic order.* The crucial conceptual question then is to reconcile the emergence of this large gapless Fermi surface at low temperature with the pseudogap and other phenomena observed above T_c at low fields. We turn now to this question.

III. $T > T_c$ AND SMALL H : PSEUDOGAP AND FERMI ARCS

For $H=0$, as T increases above T_c , two things happen: (a) the superconducting phase coherence is lost and (b) the electron quasiparticle coherence is also lost. Thus the superconducting transition is not just a phase disordering transition but also a ‘‘coherence’’ transition for the electronic quasiparticle. We may model the loss of the superconducting phase coherence just as above with a fluctuating order parameter with phase coherence time τ_ϕ and associated length scale ξ_ϕ . We expect ξ_ϕ to be roughly of order the spacing between thermally excited vortices. The Nernst/diamagnetism experiments imply that $\xi_\phi(T) \gg \xi_0 = \frac{\hbar v_F}{\Delta_0}$ at least for T up to about $T_{\text{onset}} \approx 130$ K. Then the phase decay rate $\Gamma(T) \ll \Delta_0$ up to at least T_{onset} , though we may reasonably expect that it actually holds even up to T^* . The loss of single-particle coherence may be modeled by introducing a scattering rate γ which drops sharply for $T < T_{\text{coh}}$ and is large and roughly $\propto T$ for $T > T_{\text{coh}}$. This is depicted in Fig. 4. The linear temperature dependence is not significant for the qualitative physics but affects details of the results below. In any case this is supported by experiments for T above T_{coh} . Under these condi-

tions the retarded Green's function for the q field may be written as

$$G_R(\mathbf{K}, \omega) = \frac{1}{\omega - \epsilon_{\mathbf{K}} + i\gamma - \Sigma(\mathbf{K}, \omega)}. \quad (19)$$

(We emphasize once again that the measured electron Green's function will differ from the one above by a factor of $Z_0 \sim o(x)$.)

The electron self-energy comes from scattering off the phase fluctuations of the superconducting order parameter. This may again be calculated as above. Taking the simple expression for Σ used earlier in Eq. (16) to capture both low- and high-frequency regimes, we now find (in real frequency)

$$\Sigma(\mathbf{K}, \omega) = \frac{-\Delta_{0\mathbf{K}}^2(\omega - \epsilon_{\mathbf{K}} + i\gamma)}{-(\omega + i\gamma)^2 + \epsilon_k^2 + \Gamma^2}. \quad (20)$$

Let us examine this self-energy in various regimes of interest. At low T and zero fields $H=0$, we have $\gamma=\Gamma=0$. Then Σ reduces to the usual d -wave BCS self-energy. At low T , if the superconductivity is suppressed (in a field), we have $\gamma=0$ but $\Gamma \neq 0$. Then the self-energy reduces to the one described above in Eq. (16) and yields a large Fermi surface. At high $T > T_{\text{coh}}$ and fields $H=0$, we have $\gamma \gg \Gamma$. This is the regime where the pseudogap and the Fermi arc phenomena are seen in ARPES experiments. Writing Eq. (20) as

$$\Sigma(\mathbf{K}, \omega) = \frac{\Delta_{0\mathbf{K}}^2}{\omega + \epsilon_{\mathbf{K}} + i\gamma - \frac{\Gamma^2}{\omega - \epsilon_{\mathbf{K}} + i\gamma}}, \quad (21)$$

we see that the term involving Γ in the denominator is small in this regime. Thus to leading order in the small number Γ/E_0 [where $E_0 = \max(\gamma, |\omega|, |\epsilon_{\mathbf{K}}|)$], we simply have

$$\Sigma(\mathbf{K}, \omega) \simeq \frac{\Delta_{0\mathbf{K}}^2}{\omega + \epsilon_{\mathbf{K}} + i\gamma}. \quad (22)$$

This is exactly the model self-energy introduced by Norman *et al.*²⁷ and Chubukov *et al.*²⁸ to describe ARPES data in the underdoped cuprates. In particular this captures well the essential features of the measured spectra. It leads to a pseudogap in the antinodal direction and a gapless Fermi arc near the nodal direction. The key point, as emphasized in Ref. 28, is that so long as $\gamma > \sqrt{3}\Delta_{0\mathbf{K}}$, the spectral function has a peak at zero frequency while if $\gamma < \sqrt{3}\Delta_{0\mathbf{K}}$ it has particle-hole symmetric peaks away from zero energy. The former is the defining criterion for gaplessness in the experiments; near the nodal region the condition $\gamma > \sqrt{3}\Delta_{0\mathbf{K}}$ is always satisfied above T_{coh} in a finite segment of the Fermi surface. A gapless Fermi arc (as defined in the experiments) then follows. Furthermore the length of the Fermi arc scales linearly with the temperature.

IV. RELATION TO VORTEX STRUCTURE

It is useful to view the phenomenology described in the paper from the point of view of the structure of the vortex inside the superconducting state. It is interesting that STM experiments²⁹⁻³¹ find a large region (order 70Å) around each vortex where the coherence peak is visible, but low energy

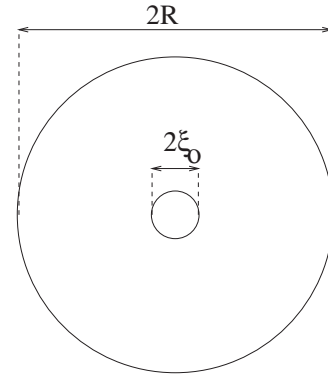


FIG. 5. Sketch of the vortex core showing the small inner core and the larger halo.

“core” states appear inside the gap. The existence of the intermediate scale “halo³²” where superconducting long-range order is weakened but quasiparticle coherence remains is empirical support of our assumption 3.

From a theoretical point of view, quite generally we know that there are two distinct energy scales T_c and Δ_0 in the underdoped cuprates. We may then naturally expect two corresponding length scales $R = v_F/T_c$ and $\xi_0 = v_F/\Delta_0$ that may be visible in many properties including the structure of the vortex cores (see Fig. 5). Specifically there is a core of size ξ_0 and a “halo” of size $R \gg \xi_0$. Inside the halo the pairing suffers from strong phase fluctuations. This was first pointed out by Lee and Wen³³ who suggested that the suppression of the superfluid density due to the excitations of nodal quasiparticles is responsible for the enhanced phase fluctuation in the halo region. However the presence of these two length scales for the vortex structure is fairly generic to superconductivity in a doped Mott insulator (see, for instance, Ref. 34).

The separation of scale between H_{c2} and H_{res} can then be understood by recognizing that H_{c2} and H_{res} are the fields where the cores and the halos overlap, respectively. Clearly the quantum oscillation is probing the low-temperature properties of overlapping halos. In the case of Bi-2212 the tunneling spectrum exhibits peaks at 8 meV which shows roughly $4a \times 4a$ periodicity. These peak structures are very naturally understood as being smoothly connected to the quasiparticle states of the full Fermi surface when the halos overlap above H_{res} . Specifically we may model the halo region as simply a disk of radius R of the high-field state embedded inside the bulk superconductor. The antinodal quasiparticle states of the high-field state will then lead to peaks in the tunneling conductivity at a small gap that is induced by the nonzero superconducting order parameter that will exist in this halo region. This picture explains qualitatively an old empirical observation³⁵ that the position of the low energy peak scales linearly with the maximum gap Δ_0 (rather than with Δ_0^2 as expected for core states in a BCS vortex).

Now consider $H < H_{\text{res}}$. As the temperature is raised, vortices and anti vortices are created and T_c is reached when their halos overlap. Above T_c the overlapping halos form the vortex liquid smoothly connected to the $H > H_{\text{res}}$ region in Fig. 1. It follows from our assumption 3 that the quasiparti-

cles will exhibit coherence over a range of temperature $T_c < T < T_{\text{coh}}$.

V. DISCUSSION

In conclusion, we see that a unified picture of the diverse phenomena in the underdoped cuprates can be constructed. Here we briefly reiterate and discuss some key points of this paper. First we discussed quantum oscillation phenomena in high fields and low T within the context of the assumption that local superconductivity is not killed in fields of order H_{res} . We showed that quasiparticle scattering off the pair field fluctuations resurrects a gapless large Fermi surface but with strongly anisotropic quasiparticle weight. Field-induced incommensurate magnetic ordering can then reconstruct this large Fermi surface to give electron/hole pockets postulated in the literature. We reiterate that the emergence for the gapless large Fermi surface is a precondition for the magnetic ordering to be able to produce the antinodal electron pockets. Next we showed how the assumption of a coherence crossover scale T_{coh} that is unaffected by fields $\sim H_{\text{res}}$ for the single-particle excitations allows us to understand the pseudogap and Fermi arc physics above T_c at low fields. Finally we discussed the structure of the vortex inside the superconducting state and related it to various other phenomena.

A key aspect of our discussion is that the pseudogap above T_c should be thought of *primarily* as being caused by pairing of *incoherent* electronic excitations. Some direct support for this picture comes from interesting recent ARPES results on underdoped Bi-2212 materials.³⁶ The spectra are found to show broad particle-hole symmetric peaks around a nonzero gap near the antinodal portion of the Brillouin zone. A particle-hole symmetric peak naturally arises if the gap is due to pair formation. As the pseudogap primarily resides at the antinodal region this is clearly supportive of the picture that it is due to pairing of incoherent particles. The data also show that particle-hole symmetry is not present close to the tip of the arc region. We suggest that this is due to some less dominant phenomenon that modifies the primary particle-hole symmetric pairing that gives rise to the pseudogap. We however leave for the future an elucidation of the physics of this subdominant effect.

In the future it should be extremely interesting for a variety of experiments to study the regime $T_c < T < T_{\text{coh}}$ at not

too high fields. In this regime it may be possible to expose the emergence of the underlying large Fermi surface without magnetic ordering as described in Sec. II. For instance STM studies may be possible in this region and should see an enhanced zero-bias conduction peak corresponding to the emergence of coherent quasiparticles even away from the Fermi arc region upon cooling through T_{coh} . Transport experiments may also be a useful probe of this region though their interpretation will be complicated by the interplay between quasiparticle and vortex physics. Another obviously important experiment is to study the vortex core structure through STM at low T in underdoped samples and then follow the evolution of the halo and the core with decreasing doping.

It is interesting that we have been able to obtain a synthesis of the phenomenology without invoking broken-symmetry orders such as staggered flux/ d -density wave order^{9,37} or more exotic non-Fermi-liquid states (such as algebraic charge liquids^{38–40}). Both of these possibilities have been considered in the literature. Indeed we have only invoked the incommensurate spin density wave induced by a field. There is good experimental support for such field-induced magnetic order. We emphasize however that even this field-induced ordering is a low energy phenomenon. In particular in our theory it plays very little role in electronic phenomena such as the pseudogap at high T .

A key challenge for microscopic theory is to describe the physics of the low coherence scale T_{coh} for single-particle excitations. In particular it is important to develop a microscopic theory for the incoherent scattering above T_{coh} , the development of coherence at low temperatures below this scale, and the interplay with the pairing into a spin singlet Cooper pair. In a companion paper¹⁴ we present results from a particular model of a doped Mott insulator which captures these aspects of the physics.

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