## **Identification of the ±***s***-wave pairing state in iron-pnictide superconductors using the Riedel anomaly**

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We theoretically propose a method to identify  $\pm s$ -wave order parameter in recently discovered Fe-pnictide superconductors. Our idea uses the Riedel anomaly in ac-Josephson current through a  $SI(\pm S)$  (single-band *s*-wave superconductor/insulator/ $\pm$ *s*-wave two-band superconductor) junction. We show that the Riedel peak effect leads to vanishing ac-Josephson current at some values of biased voltage. This phenomenon does not occur in the case when the  $\pm s$ -wave superconductor is replaced by a conventional *s*-wave one so that the observation of this vanishing Josephson current would be a clear signature of  $\pm s$ -wave pairing state in Fe-pnictide superconductors.

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The pairing symmetry is one of the most important issues in Fe-pnictide superconductors. $1-5$  Since the discovery of LaFeAsO<sub>[1](#page-3-0)−*x*</sub>F<sub>*x*</sub>,<sup>1</sup> various key properties of these materials have been clarified. FeAs layers form a quasi-twodimensional electron system, consisting of hole and electron pockets around the  $\Gamma$  point and *M* point, respectively.<sup>6[–13](#page-3-3)</sup> An antiferromagnetic (AF) phase exists without carrier doping,  $14$ so that the possibility of pairing mechanism associated with AF spin fluctuations has been discussed[.15](#page-3-5)[–18](#page-3-6) The decrease in Knight shift<sup>19</sup> below the superconducting phase-transition temperature  $T_c$  indicates a singlet pairing state. A tunneling experiment, $20$  as well as angle-resolved photoemission spectroscopy  $(ARPES),$ <sup>[11](#page-3-9)[,12](#page-3-10)</sup> have shown that Fe-pnictides are multigap superconductors. The ARPES experiment also reports that the order parameter in each band may have a nodeless *s*-wave symmetry[.11,](#page-3-9)[12](#page-3-10) While this is consistent with the exponential temperature dependence of the penetration depth far below  $T_c$ <sup>[21](#page-3-11)</sup>, it seems contradicting with the  $T<sup>3</sup>$  behavior of NMR  $T_1^{-1}$ ,  $2^{2}$ ,  $2^{3}$  implying the existence of nodes.

As a candidate for the pairing symmetry, a  $\pm s$ -wave state has been proposed[.15–](#page-3-5)[18](#page-3-6) In this pairing state, nodeless *s*-wave order parameters in electron and hole bands have opposite sign to each other. This unconventional superconductivity can consistently explain the observed superconducting properties mentioned above,  $11,12,19,20$  $11,12,19,20$  $11,12,19,20$  $11,12,19,20$  except for the power-law behavior of NMR  $T_1^{-1}$ .<sup>[22,](#page-3-12)[23](#page-3-13)</sup> However, some theory groups have shown that the NMR result can be also explained within the framework of  $\pm s$ -wave scenario, when one includes impurity scattering<sup>24</sup> and/or anisotropic Fermi surfaces.<sup>25</sup> It has been also reported that the enhancement of inelastic neutron-scattering rate at a finite momentum transfer observed in superconducting  $Ba_{0.6}K_{0.4}Fe_2As_2$  is consistent with the  $\pm s$ -wave scenario.<sup>18[,26](#page-3-16)</sup>

In this paper, we theoretically propose a method to confirm the  $\pm s$ -wave order parameter in Fe-pnictide superconductors. In identifying the pairing symmetry, phase-sensitive experiments are very powerful. For example, the  $\pi$ -junction superconducting quantum interference device (SQUID) played crucial roles to identify the  $d_{x^2-y^2}$ -wave order parameter in high  $T_c$  cuprates.<sup>27</sup> Our idea uses the ac-Josephson current  $I_J$  through a  $SI(\pm S)$  (single-band *s*-wave supercon-

ductor/insulator/ $\pm s$ -wave superconductor) junction shown in Fig. [1.](#page-0-0) In this case,  $I_J$  consists of two components associated with two bands in the  $\pm s$ -wave superconductor. Because of the sign difference of two order parameters in the  $\pm s$ -wave state, these two current components are found to flow in the opposite direction to each other. In addition, as in the case of ordinary ac-Josephson current, each current component shows the Riedel anomaly,  $28,29$  $28,29$  where the Josephson current diverges at a certain value of biased voltage *V* across the junction. These two phenomena are shown to give vanishing total ac-Josephson current  $I_J$  at some values of *V*. This vanishing *IJ* does *not* occur when the order parameters in the two-band superconductor have the same sign. Since the ARPES experiment reports a nodeless *s*-wave order parameter in each band, $11,12$  $11,12$  the observation of the vanishing ac-Josephson current would be a clear signature of  $\pm s$ -wave state in Fe-pnictides.

To explain the details of our idea, we explicitly calculate the ac-Josephson current through the  $SI(\pm S)$  junction in Fig. [1.](#page-0-0) The Hamiltonian is given by  $H = H_s + H_{\pm s} + H_T$ , where  $H_s$ and  $H_{\pm s}$ , respectively, describe the single-band *s*-wave superconductor on the left of the junction and  $\pm s$ -wave superconductor on the right of the junction. Tunneling effects are described by  $H_T$ . In the BCS approximation,  $H_s$  is given by  $H_s = \sum_{\mathbf{p}, \sigma} \varepsilon_{\mathbf{p}}^s a_{\mathbf{p}\sigma}^{\sigma \dagger} a_{\mathbf{p}\sigma}^s + \sum_{\mathbf{p}} [\Delta_s a_{\mathbf{p}\dagger}^{\sigma \dagger} a_{-\mathbf{p}\dagger}^{\sigma \dagger} + \text{h.c.}].$  Here,  $a_{\mathbf{p}\sigma}^{\sigma \dagger}$  is the creation operator of an electron in the *s* band, with the kinetic energy  $\varepsilon_p^s$  measured from the Fermi energy.  $\Delta$ <sub>*s*</sub> =  $U_s \sum_{p} (a_{p}^s \cdot p) a_{p}^s$ <sup>*s*</sup> is the order parameter in the *s* band, where  $U_s < 0$  is a pairing interaction.

For  $H_{\pm s}$ , we simply assume a two-band system as a minimal model to describe  $\pm s$ -wave superconductivity (although band calculations,  $6-10,15$  $6-10,15$  as well as ARPES experiment,<sup>11</sup> indicate the existence of more than two bands).

<span id="page-0-0"></span>

s-wave superconductor <b>±s-wave superconductor</b>	
s-band: $\Lambda$ e	hole band: Лh
	electron band: $\Lambda$ e

FIG. 1. Model  $SI(\pm S)$  junction considered in this paper.

<span id="page-1-0"></span>

FIG. 2. Temperature dependence of the order parameter  $|\Delta_h|$  and  $|\Delta_e|$ , normalized by the value  $\Delta_h(T=0)$ . The temperature is normalized by the superconducting phase-transition temperature  $T_c = \frac{2\gamma\omega_c}{\pi}e^{-1/|U_{he}|\sqrt{N_e(0)N_h(0)}}$ , where  $\gamma = 1.78$ , and  $\omega_c$  is the ordinary cutoff energy in the BCS theory.  $N_\alpha(0)$  ( $\alpha = e, h$ ) is the density of states at the Fermi level in the normal state of the  $\alpha$  band. We set  $U_{hh} = U_{ee} = 0$ ,  $|U_{he}|(N_h(0) + N_e(0)) = 1.0$ , and  $N_e(0)/N_h(0) = 0.4$ . We note that  $\Delta_h$  and  $\Delta_e$  have opposite sign to each other when  $U_{he}$  > 0, while they have the same sign when  $U_{he}$  < 0.

In the mean-field approximation,  $H_{\pm s}$  is given by<sup>17[,30](#page-3-22)</sup>  $H_{\pm s} = \sum_{\mathbf{p},\sigma,a} \varepsilon_{\mathbf{p}}^{\alpha} c_{\mathbf{p}\sigma}^{\alpha \dagger} c_{\mathbf{p}\sigma}^{\alpha} + \sum_{\mathbf{p},a} [\Delta_{\alpha} c_{\mathbf{p}}^{\alpha \dagger} c_{-\mathbf{p}\downarrow}^{\alpha \dagger} + \text{h.c.}],$  where  $c_{\mathbf{p}\sigma}^{\alpha \dagger}$ is the creation operator of an electron in the  $\alpha (=e,h)$  band, with the kinetic energy  $\epsilon_p^{\alpha}$  measured from the Fermi level.  $\Delta_h \equiv U_{hh} \Sigma_p \langle c_{-\mathbf{p}\downarrow}^h c_{\mathbf{p}\uparrow}^h \rangle + U_{he} \Sigma_p \langle c_{-\mathbf{p}\downarrow}^e c_{\mathbf{p}\uparrow}^e \rangle$  and  $\Delta_e$  $\equiv U_{ee}\Sigma_{\bf p}\langle c^e_{-\bf p\downarrow}c^e_{\bf p}\rangle + U_{eh}\Sigma_{\bf p}\langle c^h_{-\bf p\downarrow}c^h_{\bf p}\rangle$  are, respectively, the order parameters in the *h* and *e* band,<sup>30</sup> where  $U_{\alpha\alpha}$  is an intraband interaction in the  $\alpha$  band and  $U_{\alpha\alpha'}$  ( $\alpha \neq \alpha'$ ) describes a pair tunneling between the *e* band and *h* band. We take  $U_{eh} = U_{he}$ .) In this model, the  $\pm s$ -wave state is easily obtained by setting  $U_{he} > 0$  and  $U_{ee} = U_{hh} = 0$ . We note that  $\Delta_h$ and  $\Delta_e$  have the same sign when  $U_{he}$  < 0. In Fig. [2,](#page-1-0) we show the calculated  $\Delta_h$  and  $\Delta_e$ . We will use these results in evaluating the ac-Josephson current. We briefly note that although we take  $U_{ee} = U_{hh} = 0$  to realize  $\pm s$ -wave superconductivity in a simple manner, the following discussions are not affected by detailed values of  $U_{\alpha\alpha'}$ , as far as  $\pm s$ -wave state is realized.

The tunneling Hamiltonian has the form  $H_T = A + A^{\dagger}$ , where  $A = \sum_{\mathbf{p},\mathbf{k},\sigma,\alpha=h,e} T_{\mathbf{p},\mathbf{k}}^{\alpha} a_{\mathbf{p}\sigma}^{\beta \dagger} c_{\mathbf{k}\sigma}^{\alpha} \equiv \sum_{\alpha} A_{\alpha}$ . Here,  $T_{\mathbf{p},\mathbf{k}}^{\alpha}$  is the tunneling-matrix element between the  $s$  band and the  $\alpha$  band, which satisfies the time-reversal symmetry, as  $T_{p,k}^{\alpha} = T_{-p,-k}^{\alpha*}$ . Assuming a weak junction, we calculate the tunneling current  $I(t) \equiv -e\langle N(t)_{s} \rangle = ie\langle A(t) - A^{\dagger}(t) \rangle$  within the lowest order in terms of  $T_{p,k}^{\alpha}$  (where  $N_s = \sum_{p,\sigma} a_p^{s\dagger} a_{p\sigma}^s$  is the total number operator of electrons on the left of the junction in Fig. [1](#page-0-0) and  $A(t) \equiv e^{i(H_s + H_{\pm s})t} A e^{-i(H_s + H_{\pm s})t}$ . Effects of finite voltage *V* across the junction are conveniently incorporated by replacing *A*(*t*) by  $e^{-ieVt}A(t)$ .

The total current *I* involves both the Josephson current  $I_J$ and the quasiparticle current  $I_q$ . Extracting the former component, we find

$$
I_J = -2e \sum_{\alpha=h,e} \text{Im}[e^{-2ieVt} \Pi_\alpha(\omega = eV)] \equiv I_J^h + I_J^e. \tag{1}
$$

<span id="page-1-1"></span>Here,  $\Pi_{\alpha}(\omega) = -i \int_{-\infty}^{t} dt e^{i\omega t} \langle [A_{\alpha}(t), A_{\alpha}(0)] \rangle_{0}$ , where the average  $\langle \cdots \rangle_0$  is taken in the absence of  $H_T$ . The key of our idea is that Eq.  $(1)$  $(1)$  $(1)$  is given by the sum of the contribution

from the *h* band  $(\equiv I_j^h)$  and that from the *e* band  $(\equiv I_j^e)$ . In the  $\pm s$ -wave superconductor, when the Josephson current component between the *s* band and the *h* band has the form  $J_h$  sin  $\Phi$  (where  $\Phi$  is the phase difference between  $\Delta_s$ and  $\Delta_h$  across the junction), the Josephson current component between the *s* band and the *e* band behaves as  $J_e$  sin( $\Phi + \pi$ )= $-J_e$  sin  $\Phi$ , so that they flow toward the opposite direction to each other. (Note that the phase  $\pi$  comes from the phase difference between  $\Delta_e$  and  $\Delta_h$ .)

This phenomenon is similar to the suppression of the Josephson current in a *d*-wave superconductor/insulator/*s*-wave superconductor (DIS) junction. As shown in Ref. [33,](#page-4-0) the Josephson current in the DIS junction vanishes within the second-order tunneling process because the Josephson current component, coming from the momentum region where the *d*-wave order parameter is positive, is cancelled out by the component coming from the region where the *d*-wave order parameter is negative. However, in contrast to the DIS junction, we will find that the total Josephson current  $J = J_h \sin \Phi + J_e \sin(\Phi + \pi)$  in our case does not vanish except at some values of *V*.

We now evaluate the Josephson current in Eq.  $(1)$  $(1)$  $(1)$  from the analytic continuation of

<span id="page-1-2"></span>
$$
\Pi_{\alpha}(i\nu_n) = -\int_0^{1/T} d\tau e^{i\nu_n \tau} \langle T_{\tau} \{ A_{\alpha}(\tau) A_{\alpha}(0) \} \rangle_0
$$
  
=  $-2T \sum_{\mathbf{p}, \mathbf{k}} |T^{\alpha}_{\mathbf{p}, \mathbf{k}}|^2 \sum_{\omega_m} G^s_{21}(\mathbf{p}, i\omega_m) G^{\alpha}_{12}(\mathbf{k}, i\omega_m + i\nu_n),$  (2)

where  $\nu_n$  and  $\omega_m$  are the boson and fermion Matsubara frequencies, respectively, and  $A(\tau) \equiv e^{\tau(H_s + H_{\pm s})} A e^{-\tau(H_s + H_{\pm s})}$ .  $G_{12}^{\lambda}(\mathbf{p}, i\omega_m) \equiv -\Delta_{\lambda}/(\omega_m^2 + \varepsilon_p^{\lambda 2} + |\Delta_{\lambda}|^2)(\lambda = s, \alpha)$  is the offdiagonal Green's function, which satisfies  $G_{21}^{\lambda} = G_{12}^{\lambda^*}$ .

For simplicity, we approximate the tunneling-matrix element  $T_{p,k}^{\alpha}$  to the value averaged over the Fermi surface  $(\equiv \langle T^{\alpha}_{\mathbf{p},\mathbf{k}} \rangle)$ . Executing the momentum summations in Eq. ([2](#page-1-2)), we obtain  $\Pi_{\alpha}(i\bar{\nu}_n) = 2\pi^2 |\langle T^{\alpha}_{\mathbf{p},\mathbf{k}} \rangle|^2 N_s(0) N_{\alpha}(0) \Delta_s^* \Delta_{\alpha} \Lambda_{\alpha}(i\bar{\nu}_n)$ , where

$$
\Lambda_{\alpha}(i\nu_n) = T \sum_{\omega_m} \frac{1}{\sqrt{\omega_m^2 + |\Delta_s|^2}} \frac{1}{\sqrt{(\omega_m + \nu_n)^2 + |\Delta_\alpha|^2}}.
$$
 (3)

<span id="page-1-3"></span>Here,  $N<sub>s</sub>(0)$  is the density of states at the Fermi level in the normal state of the *s* band. We evaluate the  $\omega_m$  summation in Eq. ([3](#page-1-3)) by transforming it into the complex integration. Changing the integration path so as to be able to carry out the analytic continuation, we execute  $i\nu_n \rightarrow \omega + i\delta$ . Substituting the result into Eq.  $(1)$  $(1)$  $(1)$ , we find that the sine component of ac-Josephson current  $(\equiv \bar{I}_j)$  can be written as  $\bar{I}_j$  $= J_h \sin(2eVt + \phi_s - \phi_h) + J_e \sin(2eVt + \phi_s - \phi_e)$ , where  $\phi_s$ ,  $\phi_h$ , and  $\phi_e$  are the phases of the order parameter  $\Delta_s$ ,  $\Delta_h$ , and  $\Delta_e$ , respectively. When the phase difference between  $\Delta_h$  and  $\Delta_e$ is  $\pi$  or 0 (i.e.,  $\phi_e = \phi_h + \pi$  or  $\phi_h$ ), this expression can be written as  $\overline{I}_J = J \sin(2eVt + \phi_s - \phi_h)$ , where

<span id="page-2-1"></span>

FIG. 3. Calculated ac-Josephson current *J* , normalized by  $J_0 = \sqrt{G_h G_e} [\Delta_h (T=0) + |\Delta_e (T=0)|]/e$ , as a function of biased voltage *V*. (a)  $T=0$ , (b)  $T/T_c^s=0.9$ , where  $T_c^s$  is  $T_c$  of the superconductor on the left of the junction in Fig. [1.](#page-0-0) The Riedel anomaly can be seen at  $eV/(\vert\Delta_{s}\vert+\vert\Delta_{h}\vert)=1.0$  and 1.48 in panel (a), and at 0.80 and 1.24 in panel (b). In addition to these peaks, we also find weak singularities at  $eV = |\Delta_{\alpha}| - |\Delta_{s}|$ , for example,  $eV/(|\Delta_{s}| + |\Delta_{h}|) \approx 0.37$  in panel (b). We take  $\langle T_{\mathbf{p},\mathbf{k}}^h \rangle = \langle T_{\mathbf{p},\mathbf{k}}^e \rangle$  for simplicity. For the values of  $\Delta_h(T)$  and  $\Delta_e(T)$ , the results in Fig. [2](#page-1-0) are used. Values of the interaction  $U_s$  is chosen to realize  $\Delta_h(T=0)/\Delta_s(T=0)=1.5$ .

$$
J = J_h + \eta J_e. \tag{4}
$$

<span id="page-2-0"></span> $J_h$  and  $J_e$  describe the Josephson current component between the *s* band and the *h* band and that between the *s* band and the *e* band, respectively. They are given by $31$ 

$$
J_{\alpha} = \frac{G_{\alpha}}{e} \frac{|\Delta_{s}| |\Delta_{\alpha}|}{2} \int_{-\infty}^{\infty} dz \tanh \frac{|z|}{2T}
$$
  
 
$$
\times \left[ \frac{\theta(|\Delta_{s}| - |z - eV|) \theta(|z| - |\Delta_{\alpha}|)}{\sqrt{|\Delta_{s}|^{2} - (z - eV)^{2}} \sqrt{z^{2} - |\Delta_{\alpha}|^{2}}} + \frac{\theta(|z| - |\Delta_{s}|) \theta(|\Delta_{\alpha}| - |z + eV|)}{\sqrt{z^{2} - |\Delta_{s}|^{2}} \sqrt{|\Delta_{\alpha}|^{2} - (z + eV)^{2}}} \right],
$$
 (5)

where  $G_{\alpha} = 4\pi e^2 N_s(0) N_{\alpha}(0) |\langle T^{\alpha}_{p,k} \rangle|^2$  $G_{\alpha} = 4\pi e^2 N_s(0) N_{\alpha}(0) |\langle T^{\alpha}_{p,k} \rangle|^2$  $G_{\alpha} = 4\pi e^2 N_s(0) N_{\alpha}(0) |\langle T^{\alpha}_{p,k} \rangle|^2$ . In Eq. (4),  $\eta$  involves useful information about the phase difference between  $\Delta_h = |\Delta_h|^{i\phi_h}$  and  $\Delta_e = |\Delta_e| e^{i\phi_e}$ , as

$$
\eta = \begin{cases}\n-1 & \left[\phi_e = \phi_h + \pi\right], \\
+1 & \left[\phi_e = \phi_h\right].\n\end{cases} \tag{6}
$$

When  $\eta = -1$  [SI( $\pm$ S) junction], the phase difference between  $\Delta_h$  and  $\Delta_e$  equals  $\pi$ . In this case, the current  $J_e$  flows in the opposite direction to  $J<sub>h</sub>$ . This leads to the suppression of the total Josephson current as  $J = J_h - J_e$ . We note that although this is the same mechanism as the suppression of the Josephson current in the DIS junction mentioned previously,  $33$  in the present case, *J* remains finite (except at some values of biased voltages, as shown in Fig. [3](#page-2-1)). When

<span id="page-2-3"></span>

FIG. 4. Step height  $I_1(\omega_{rf})$  of the *N*=1 Shapiro step.

 $\phi_e = \phi_h$  in the case of  $\eta = +1$ , the Josephson current is simply given by  $J = J_h + J_e$ .

Figure [3](#page-2-1)(a) shows |*J*| at *T*=0. Each  $J_h$  and  $J_e$  has a peak at  $eV = |\Delta_s| + |\Delta_{\alpha}| (\alpha = e, h)$  (Riedel anomaly). The ac-Josephson current *J*( $\eta$ =−1)=*J<sub>h</sub>*−*J<sub>e</sub>* vanishes when the voltage *V* satisfies  $J_h(eV) = J_e(eV)$  [see the solid line in Fig. [3](#page-2-1)(a)]. The Riedel peaks at  $eV = |\Delta_s| + |\Delta_h|$  and  $eV = |\Delta_s| + |\Delta_e|$  guarantee that this condition always satisfies at a voltage  $(\equiv V_0)$  in the region, $32$ 

$$
|\Delta_s| + \text{Min}[|\Delta_h|, |\Delta_e|] < eV_0 < |\Delta_s| + \text{Max}[|\Delta_h|, |\Delta_e|]. \tag{7}
$$

<span id="page-2-4"></span>When the phase difference between  $\Delta_h$  and  $\Delta_e$  is absent  $(\eta=+1)$ , *J* is always finite, as shown in Fig. [3](#page-2-1)(a). Thus, the vanishing ac-Josephson current would be a clear signature of  $\pm$ *s*-wave state in Fe-pnictides.

The vanishing Josephson current can be also seen at finite temperatures, as shown in Fig.  $3(b)$  $3(b)$ . On the other hand, when  $|\Delta_h| = |\Delta_e|$  and  $J_h(eV) \neq J_e(eV)$  are accidentally satisfied, the Riedel peaks in  $J_h$  and  $J_e$  appear at the same value of *V*, so that  $J(\eta = -1) = J_h - J_e$  only has one Riedel peak at  $eV = |\Delta_h| + |\Delta_s|$ . In this special case, the vanishing *J* is not obtained even in the  $SI(\pm S)$  junction. However, since two different energy gaps have been observed in Fe-pnictides,  $\frac{11,12,20}{1}$  $\frac{11,12,20}{1}$  $\frac{11,12,20}{1}$  $\frac{11,12,20}{1}$  we can determine the relative sign of the two order parameters corresponding to the observed two energy gaps by our method.

We expect that the vanishing ac-Josephson current discussed above may be observed by using the conventional method to observe the Riedel peak and the amplitude of ac-Josephson current by measuring the step height of Sha-piro step.<sup>29[,35,](#page-4-2)[36](#page-4-3)</sup> When the ac voltage  $V(t) = V_{rf} \cos \omega_{rf}t$  is applied to the junction by radiating radio-frequency field with angular frequency  $\omega_{rf}$ , the height of the *N*th Shapiro step is given by<sup>35</sup>  $I_N = |\sum_{n=-\infty}^{\infty} \tilde{J}_n(\alpha) \tilde{J}_{N-n}(\alpha) J(|n-\frac{N}{2}|\omega_{rf})|$  (where  $\tilde{J}_n$  is the *n*th Bessel function and  $\alpha = eV_{rf}/\hbar \omega_{rf}$ ). In particular, for the first Shapiro step  $(N=1)$ , we have

$$
I_1 = \left[ 2 \sum_{n=0}^{\infty} \widetilde{J}_n(\alpha) \widetilde{J}_{1-n}(\alpha) J \left( \left| n - \frac{1}{2} \right| \omega_{rf} \right) \right].
$$
 (8)

<span id="page-2-2"></span>When one sets  $\omega_{rf} \approx 2(|\Delta_s| + |\Delta_h|)$ , the term with  $n=0$ dominantly contributes to the summation in Eq.  $(8)$  $(8)$  $(8)$  because  $J(\omega)$  rapidly decreases above  $\omega = |\Delta_s| + |\Delta_h|$  (see Fig. [3](#page-2-1)). Thus, in this case, one may approximate Eq.  $(8)$  $(8)$  $(8)$  to  $I_1 \approx 2\tilde{J}_0(\alpha)\tilde{J}_1(\alpha)J(\frac{\omega_f}{2})$  when  $\omega_{rf} \sim 2eV_0$ . Indeed, as shown in Fig. [4,](#page-2-3) we clearly see the remarkable suppression of  $I_1$  at  $\omega_{rf} = 2eV_0$  when  $\eta = -1$ , reflecting the vanishing *J* at  $V = V_0$ . Thus, the observation of step height of the *N*=1 Shapiro step would be useful for the confirmation of  $\pm s$ -wave pairing state based on our idea.

One can immediately extend our idea to the case with more than two order parameters. In this case, when all the order parameters do not have the same sign, we again obtain the vanishing ac-Josephson current due to the same mechanism discussed in this paper.

Multiband superconductivity is affected by even nonmagnetic impurities,  $3<sup>4</sup>$  so that the Riedel anomaly may be weakened by impurity effects. The suppression of the Riedel anomaly is also expected when one includes anisotropic Fermi surfaces. When the Riedel peak in  $J_e$  is broadened and the peak height becomes smaller than the value of  $J<sub>h</sub>$  at  $eV = |\Delta_s| + |\Delta_e|$  in Fig. [3](#page-2-1)(a), the vanishing ac-Josephson current is no longer obtained. In this case, however, unless the Riedel peak becomes very broad, *J* would show a dip (peak)

structure at  $eV = |\Delta_s| + |\Delta_e|$  when  $\eta = -1$  ( $\eta = +1$ ), which may still be useful to confirm  $\pm s$ -wave superconductivity. Since any real superconductor more or less has impurities, as well as anisotropic band structure, it is an interesting problem how our idea discussed in this paper is modified when more realistic situations are taken into account. We will separately discuss this problem in our future paper.

To conclude, we have proposed a possible method to confirm the  $\pm s$ -wave pairing symmetry in Fe-pnictides. Since the symmetry of order parameter is deeply related to the mechanism of superconductivity, our method discussed in this paper would be also helpful in clarifying the mechanism of superconductivity in Fe-pnictides.

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