Anomalous transition temperature oscillations in the Larkin-Ovchinnikov-Fulde-Ferrell state

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We consider Aharonov-Bohm (AB) effect at normal-metal-inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrell superconducting state transition. It is shown that magnetic flux can increase the transition temperature and AB oscillations can have the double-peak structure at one period. Expressions for fluctuational heat capacity and persistent current are calculated for a thin ring and a cylinder. We also discuss the effect of fluctuations interaction in the nonuniform states in the vicinity of the superconducting transition.

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I. INTRODUCTION

Spin polarization of the Cooper pairs in magnetic field destroys the superconducting state at Chandrasekhar-Clogstone limit when paramagnetic energy coincides with the superconducting condensation energy. Paramagnetic limit is attained at critical field $H_p=\sqrt{2}\Delta/2\ \mu_B$,[1] where Δ is the superconducting gap and μ_B is the Bohr magneton. Orbital pair breaking effect usually dominates over the paramagnetic limit. However, orbital effect could be suppressed in low-dimensional systems (thin wires) or by applying magnetic field parallel to the conductive planes of quasi-two-dimensional (2D) systems.

Spin polarization in external magnetic field or intrinsic exchange fields could lead to the formation of inhomogeneous superconductivity. Larkin and Ovchinnikov,[2] Fulde, and Ferrel[3] predicted the existence of the nonuniform superconducting state in ferromagnetic superconductors at low temperatures in magnetic field larger than critical H_p (see for a review Refs. 4 and 5).

LOFF state is formed by Cooper pairs with nonzero momentum $\sim 2\mu_B H/v_F$, where v_F is the Fermi velocity and at fields higher than the paramagnetic limit has lower energy compared to the uniform superconducting state. This finite momentum of Cooper pairs results in a spatial modulation of the superconducting order parameter.

Mathematically, LOFF state appears due to the change in sign of coefficient β at the gradient term of the Ginzburg-Landau (GL) free-energy functional $\beta|\nabla\Psi|^2$, where Ψ is the order parameter. Coefficient β is a function of temperature and Zeeman energy $\mu_B H$. In BCS model it becomes negative at high-magnetic fields $H>1.07T_c(0)/\mu_B$ and low temperatures $T<0.56T_c(0)$, where $T_c(0)$ is the temperature of transition to superconducting state at zero magnetic field, signaling of the formation of nonuniform LOFF state. As a result one has to take into account higher terms in the GL functional expansion $|\nabla^2\Psi|^2$.

The theoretical research of nonuniform LOFF state includes, for example, the study of impurities effect[6] that suppresses the region of nonuniform superconductivity, the LOFF-type proximity effect at the ferromagnetic-superconductor boundary,[7] interplay of orbital and paramagnetic effects,[8–10] the study of phase transitions in the vicinity of the tricritical point,[11] and intrinsic pinning of vortices in layered superconductors.[12]

Heavy-fermion compound $CeCoIn_5$ was found to show the signatures of LOFF phase. The existence of the LOFF state in heavy-fermion superconductor was experimentally investigated by specific-heat measurements[13–15] and nuclear magnetic resonance.[16,17]

The phase transition between possibly the LOFF state and the homogenous superconducting state was reported for organic superconductors such as $\lambda-(BETS)_2FeCl_4$ (Refs. 18–21) with quasi-2D electronic structures and organic $(TMTSF)_2ClO_4$ (Ref. 22) with quasi-one-dimensional electronic structure.

These experiments were focused on the identification of the phase transition inferred from a kink of thermal conductivity,[18] observation of peculiar properties-dip structures in the resistance,[19] and changes in the rigidity of the vortex system.[20] The thermodynamic evidence of the existence of narrow intermediate state attributed to LOFF state, which separates the uniform superconducting state and normal state based on specific-heat measurements was presented in paper.[21] Finally, the transition temperature dependence on the strength and direction of the magnetic field was studied in a paper.[22]

Nonuniform state of condensate was uncovered in systems of ultracold atoms in optical lattices.[23] Experimentally optical lattices could be formed by standing-wave laser which provides a periodical potential for ultracold atomic gas. Recently, the observation of phase transition between normal and nonuniform superfluid state was reported for systems of strongly interacting Fermi gas with imbalanced spin population.[24]

Crossovers between different fluctuational regimes of paraconductivity and specific heat in the vicinity of the LOFF transition were discussed theoretically in paper.[25] Authors showed that these fluctuational contributions have specific temperature dependencies compared to the case of uniform superconductivity and could serve as an additional indicator of the LOFF state.

In the present paper we consider hollow superconducting cylinder/ring threaded by magnetic flux (Fig. 1). We calculate the expressions for the persistent current in thin superconducting ring and present numerical results for specific heat and persistent current for the cylinder. Magnetic-flux dependence of the current demonstrates the double-peak structure in Aharonov-Bohm (AB) oscillations. We also study the effect of fluctuations interaction on the nonuniform states in low-dimensional inhomogeneous superconductors.

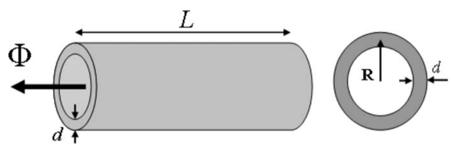

FIG. 1. Thin superconducting cylinder and rind.

This research is motivated by the fact that the fluctuation region in nonuniform systems is much larger than in the case of uniform states and requires separate theoretical study.

II. GL FREE ENERGY

We consider AB oscillations in quasi-one-dimensional ring and thin-walled cylinder which transverse size d is much smaller than the radius R; see Fig. 1. In case of second-order normal-metal LOFF transition the Ginzburg-Landau free-energy functional above transition temperature could be written as

$$F = \int d\mathbf{r}[a(T-\tilde{T}_c)|\Psi|^2 + \beta|\mathbf{D}\Psi|^2 + \delta|\mathbf{D}^2\Psi|^2], \qquad (1)$$

where $\beta=-|\beta|$ and $\mathbf{D}=-i\nabla-(2e/c)\mathbf{A}$, while tangent component of the vector potential is given as $A_\varphi=\Phi/2\pi R$. Representing the order parameter as

$$\Psi(\mathbf{r}) = \sum_{n,k} \Psi_n(k) e^{i\varphi n} e^{ikz}, \qquad (2)$$

where z is coordinate along the cylinder. We can write the free-energy functional as

$$F = V\sum_{n,k} E_n(k)|\Psi_n(k)|^2, \qquad (3)$$

where V is volume of the sample and

$$E_n(k) = a(T-T_c) + \frac{|\beta|}{2Q^2R^4}\left[\left(n-\frac{\Phi}{\Phi_0}\right)^2 + (kR)^2 - (QR)^2\right]^2. \qquad (4)$$

Here $\Phi_0=e/\pi c$ is the flux quantum,

$$Q = \sqrt{\frac{|\beta|}{2\delta}} \qquad (5)$$

is the modulus of superconducting modulation wave vector in inhomogeneous LOFF state, and

$$T_c = \tilde{T}_c + \frac{\beta^2}{4a\delta} \qquad (6)$$

is the transition temperature at $R\to\infty$. Recently,[26] we have examined the Aharonov-Bohm oscillations in thin ring near LOFF-metal transition. In contrast to the uniform superconductivity the applied magnetic flux can increase the transition temperature of LOFF state, and AB oscillations could have double-peak structure. The nature of these effects is the fluctuation energy spectrum of the inhomogeneous state. To see it in more detail let us consider the ring threaded by magnetic flux Φ. In this case spectrum $E_n(k)$ is given by Eq. (4) at $k=0$ as

$$E_n = a(T-T_c) + \frac{|\beta|}{2Q^2R^4}\left[\left(n-\frac{\Phi}{\Phi_0}\right)^2 - \phi^2\right]^2, \qquad (7)$$

where we introduce $\phi=QR$.

The independence of spectrum on k allows one to introduce flux-dependent transition temperature, corresponding to $E_n=0$ as

$$T_c(\Phi) = T_c - \frac{|\beta|}{2aQ^2R^4}\min\left[\left(n-\frac{\Phi}{\Phi_0}\right)^2 - \phi^2\right]^2. \qquad (8)$$

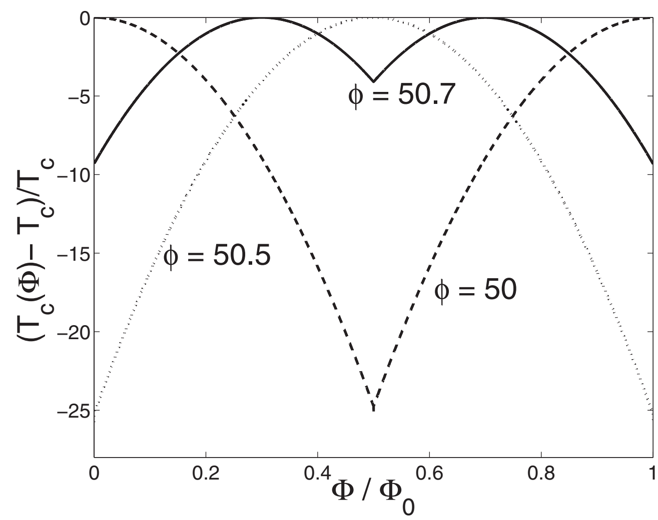

FIG. 2. One period of $[T_c(\Phi)-T_c]/T_c$ oscillations in magnetic flux for a set of $\phi=QR=(50,50.5,50.7)$, where $|\beta|/2aQ^2R^4=0.01$. See Eq. (8) in the text.

The transition temperature into the LOFF state of the superconducting ring is defined by $n_\pm$ which are nearest integers to the corresponding values $\Phi/\Phi_0\pm\phi$.

Generally, integers $n_\pm$ do not correspond to the minimum of energy (7) which provides the highest transition temperature (6) of the system. Thus, one can either increase or decrease E_n and correspondingly the transition temperature $T_c(\Phi)$ by changing the radius of the ring or by applying magnetic flux Φ. This is in contrast to the case of metal-uniform superconductor transition where transition temperature $T_c(\Phi)$ always decreases with applying magnetic flux. Moreover, the degeneracy of the energy spectrum allows the system to jump between E_{n_+} and E_{n_-} states leading to peculiar properties of AB effect such as double-peak structure per period of oscillations.

Let us discuss the possible temperature dependencies shown in Fig. 2 for the case of large $\phi\gg 1$. In this regime the transition temperature behavior is given by

$$[T_c(\Phi)-T_c]/T_c = -\frac{2|\beta|}{aT_cR^2}\max[f_+(\Phi)^2, f_-(\Phi)^2], \qquad (9)$$

where $f_\pm(\Phi)$ is the distance between $\Phi/\Phi_0\pm\phi$ and corresponding integer $n_\pm$.

If $0<\phi<0.5$ then for applied magnetic flux in the range $0<\Phi<\Phi_0/2$ the transition temperature behavior is governed by f_-. When Φ reaches $\Phi_0/2$ the crossover from f_- to f_+ takes place and the further magnetic-flux dependence is described by f_+ leading to the double-peak structure of oscillations. Note that if $0.5<\phi<1$, one has the opposite crossover from f_+ to f_-.

Suppose the value of phase ϕ equals to the half an integer number (dotted line in Fig. 2). Transition temperature first decreases as $[T_c(\Phi)-T_c]/T_c \propto -(\Phi/\Phi_0)^2$ with increasing applied magnetic flux. The maximal depression of $T_c(\Phi)$ occurs when $\Phi/\Phi_0=\frac{1}{2}$ and has a value of $[T_c(\Phi)-T_c]/T_c=-\frac{|\beta|}{2aT_cR^2}$. Further increase in Φ leads to the increase in transition temperature as $[T_c(\Phi)-T_c]/T_c \propto (\Phi/\Phi_0)^2-1$. Here both f_+ and f_- give equal dependence on Φ.

Finally, if the value of ϕ equals to the integer number then one has the opposite case (dashed line in Fig. 2). Transition temperature increases with applied magnetic flux starting from the value $[T_c(\Phi)-T_c]/T_c=-\frac{|\beta|}{2aT_cR^2}$. When Φ reaches $\Phi_0/2$, the crossover between f_+ and f_- leads to further decrease in $[T_c(\Phi)-T_c]/T_c$.

The variations in $T_c(\Phi)$ quantitatively explain flux dependence of physical quantities of quasi-one-dimensional ring. Interestingly, summation over momentum k in case of cylinder does not wash out the peculiarities of flux dependence. In this case $kR/2\pi$ plays a role of random phase and the cylinder could be considered as a set of rings with different phases ϕ. Superposition of different types of oscillations as we will show below leads to fact that the oscillation peculiarities appear at lower temperatures and/or smaller radii of the ring.

III. SPECIFIC HEAT

Carrying out the integral over the real and imaginary parts of order parameter $\Psi_n(k)$ one obtains the expression for fluctuational part of thermodynamical potential

$$\Omega = T\sum_{n,k} \ln\left(\frac{E_n(k)}{\pi T}\right). \tag{10}$$

We accept units where $k_B=1$. Fluctuational correction to specific heat is given as

$$C = -\frac{T}{V}\frac{\partial^2\Omega}{\partial T^2},$$

where V is the volume of the sample. Taking derivatives over the temperature dependence of $E_n(k)$ one obtains[27]

$$C = \frac{(aT_c)^2}{V}\sum_{n,k} E_n^{-2}(k). \tag{11}$$

Performing the Poisson transformation one obtains the fluctuation specific heat of the superconducting cylinder at temperatures $T>T_c$

$$C = \frac{q}{VR(T/T_c-1)^2}\Re\sum_{m,k}\int \frac{e^{2\pi i m(\Phi/\Phi_0+tq)}dt}{\{1+[t^2-z+(kR/q)^2]^2\}^2}, \tag{12}$$

where we introduce parameters

$$q = R\sqrt{\frac{\sqrt{2}Q}{\zeta}},$$

$$z = \frac{Q\zeta}{\sqrt{2}}. \tag{13}$$

Parameter z characterizes the LOFF inhomogeneity of the fluctuations and is proportional to the number of modulations of superconducting fluctuations of correlation length ζ.

Correlation length ζ measures the scale of superconducting fluctuations and is defined as

$$\zeta = \sqrt{\frac{|\beta|}{a(T-T_c)}}. \tag{14}$$

Now one can integrate over t and make summation over m. The expression for the specific heat is then

$$C = \frac{\pi q}{4VR(T/T_c-1)^2}\Re\sum_k [f(\Phi)+f(-\Phi)]. \tag{15}$$

Here

$$f(\Phi) = \left[\frac{1}{g}+\frac{i/2}{g^3}\right]\frac{1+e^{2\pi i\varphi}}{1-e^{2\pi i\varphi}} + \left[\frac{2\pi q}{g^2}\right]\frac{e^{2\pi i\varphi}}{1-e^{2\pi i\varphi}}, \tag{16}$$

while $\varphi=\Phi/\Phi_0+qg$ and $g=(z-(kR/q)^2+i)^{1/2}$.

Specific heat of the ring is determined by $k=0$ term in expression (15). The detailed analysis of the specific-heat magnetic-flux dependence for the case of thin superconducting ring was given in the paper.[26] There it was shown that magnetic flux can increase the critical temperature of LOFF state and AB oscillations could have double-peak structure.

Here in Fig. 3 we present the typical magnetic-flux dependencies of the specific heat of the ring. It is seen that for $Q\zeta \gg 1$, where the number of modulations of superconducting fluctuations is large, specific heat exhibits pronounced double-peak structure when $\sqrt{2}\pi R \sim \zeta$. In the case of small-scale superconducting fluctuations when $\sqrt{2}\pi R \gg \zeta$ one obtains the random sign behavior of specific heat on magnetic flux. In this regime magnetic flux can either increase or decrease the specific heat.

Summation over momentum in the case of cylinder is defined as

$$\sum_k = L\int \frac{dk}{2\pi},$$

where L is length of cylinder.

Numerical results for thin superconducting cylinder are shown on Fig. 4. It is seen from the Fig. 4 that magnetic flux can also either increase or decrease specific heat and leads to the double-peak structure in oscillations. However, the transition to the double-peak structure regime appears at smaller

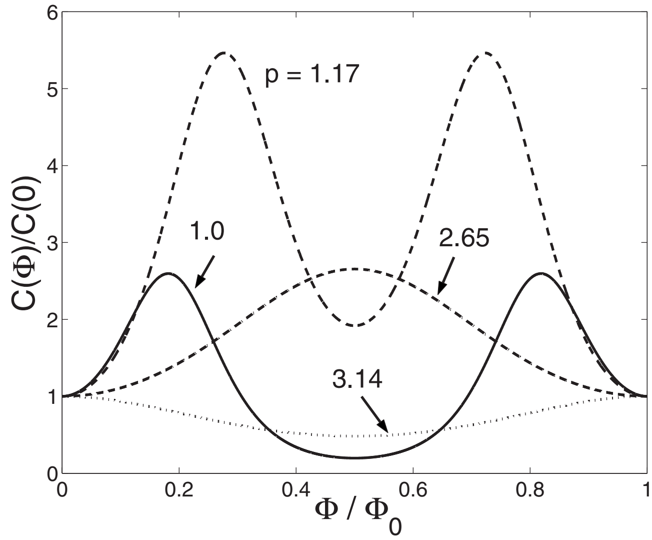

FIG. 3. Magnetic-flux dependence of specific heat of the ring for $Q\zeta/\sqrt{2}=10$. Parameter $p=\sqrt{2}\pi R/\zeta=[1.0,1.17,2.65,3.14]$ measures the ratio of rings' radius to the correlation length.

radii of the cylinder compared to the radius of the ring. This is the consequence of the averaging procedure over momentum k.

IV. PERSISTENT CURRENT

In this section we will discuss the persistent current in AB effect. Expression for the persistent current is given as

$$I=-\frac{\partial\Omega}{\partial\Phi}=-T\sum_{n,k}\frac{\partial E_n(k)/\partial\Phi}{E_n(k)}. \tag{17}$$

Using Eq. (10) and performing the Poisson transformation we obtain at $T>T_c$,

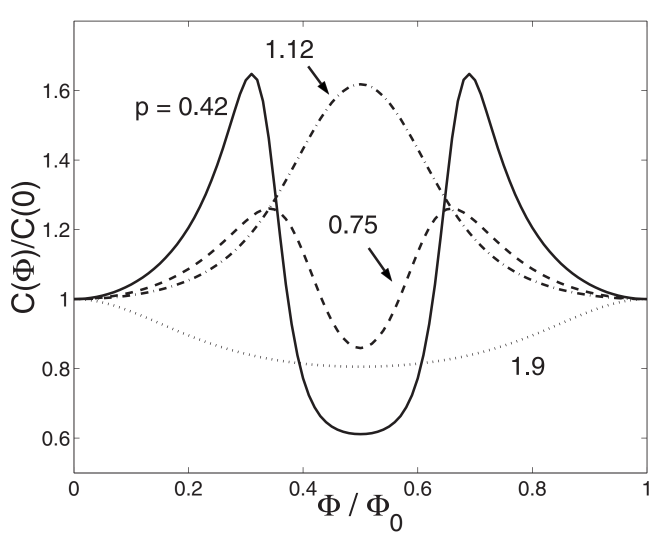

FIG. 4. Magnetic-flux dependence of specific heat of the cylinder for $Q\zeta/\sqrt{2}=10$. Parameter $p=\sqrt{2}\pi R/\zeta=[0.42,0.75,1.12,1.9]$ measures the ratio of rings' radius to the correlation length.

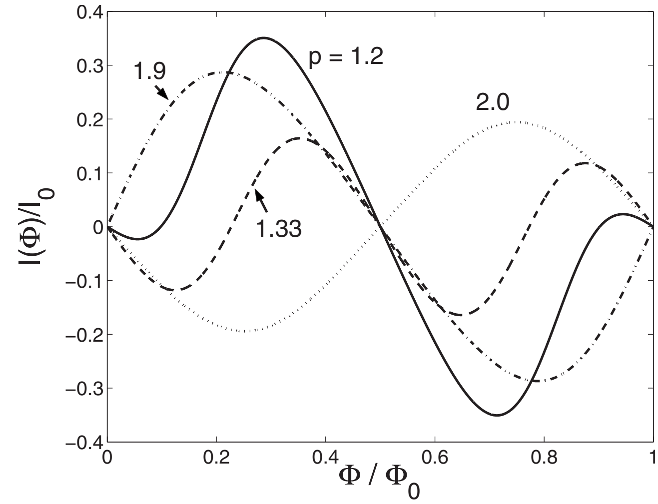

FIG. 5. Persistent current of the ring measured in units $I_0=4\pi T_c/\Phi_0$ for $Q\zeta/\sqrt{2}=10$ as a function of magnetic flux. Parameter $p=\sqrt{2}\pi R/\zeta=[1.2,1.2,1.33,2.0]$.

$$I=\frac{2T}{\Phi_0}\sum_{m,k}\int dt\frac{2t(t^2+y^2-z)}{1+(t^2+y^2-z)^2}e^{2\pi i m(\Phi/\Phi_0+tq)}. \tag{18}$$

Performing summation over m and integration over t we conclude with

$$I=-\frac{4\pi T}{\Phi_0}\Re\sum_k\frac{\sin(2\pi\Phi/\Phi_0)}{\cos[2\pi\sqrt{\phi^2-(Rk)^2+iq^2}]-\cos(2\pi\Phi/\Phi_0)}. \tag{19}$$

Persistent current of the ring is determined by term $k=0$ in expression (19).

A. Regime of strong inhomogeneity $Q\zeta \gg 1$

Let us first consider the temperature regime in the vicinity of the LOFF-metal transition which corresponds to the large number of modulations of superconducting fluctuations $Q\zeta \gg 1$. We first suggest the radius of the superconducting ring/cylinder to be larger than the correlation length

$$R \gg \zeta.$$

To calculate the persistent current for the superconducting ring in this regime one has to take into account mode $m=1$ in Eq. (18) since higher modes will be exponentially suppressed. As a result, the persistent current in the ring can be estimated as

$$I \simeq -\frac{8\pi T}{\Phi_0}\cos(2\pi\phi)\sin(2\pi\Phi/\Phi_0)e^{-\sqrt{2}\pi R/\zeta}. \tag{20}$$

Depending on the sign of the random-phase factor $\cos(2\pi\phi)$ persistent current could produce either diamagnetic or paramagnetic response at small flux: Fig. 5. That is in contrast to the case of homogenous superconductor-metal transition. The numerical result for the cylinder geometry is presented in the Fig. 6. One sees that the magnetic-flux dependence of the persistent current is also sensitive to the random phase.

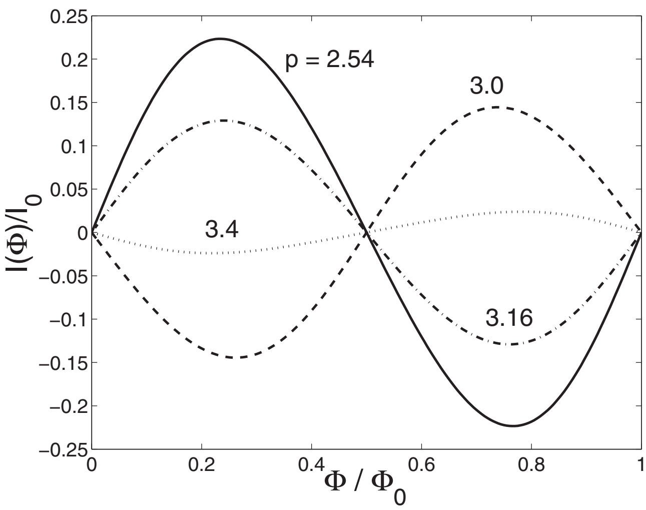

FIG. 6. Magnetic-flux dependence of persistent current of the cylinder measured in units $I_0=2T_cL/R\Phi_0$ for $Q\zeta/\sqrt{2}=10$ and large radius. Here $p=\sqrt{2}\pi R/\zeta=[2.54,3.0,3.16,3.4]$.

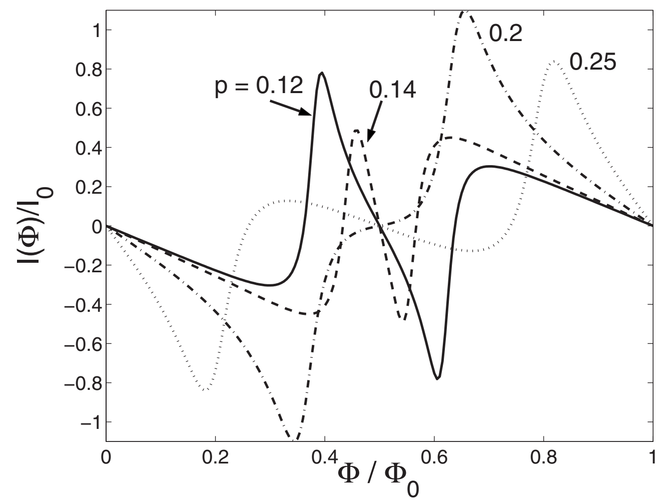

FIG. 7. Magnetic-flux dependence of persistent current of the cylinder measured in units $I_0=2T_cL/R\Phi_0$ for $Q\zeta/\sqrt{2}=10$ and small radius. Here $p=\sqrt{2}\pi R/\zeta=[0.12,0.14,0.2,0.25]$.

Now let us discuss the regime of nonuniform superconductivity in ring/cylinder with small radius

$$\zeta > R.$$

Again we concentrate on the temperatures in the vicinity of the LOFF-metal transition. One obtains for the current in thin superconducting ring

$$I = -\frac{2\pi T}{\Phi_0}[f(\Phi) - f(-\Phi)], \tag{21}$$

where

$$f(\Phi) \simeq \frac{2\sin[2\pi(\phi + \Phi/\Phi_0)]}{1 + (\pi R/\zeta)^2 - \cos[2\pi(\phi + \Phi/\Phi_0)]}. \tag{22}$$

In this case one observes the pronounced double-peak structure of the persistent current oscillations in thin superconducting ring: Fig. 5. The same result also holds for the superconducting cylinder and the numerical calculations of the current dependence on the magnetic flux are presented in the Fig. 7. Again, the double-peak oscillations structure exhibits at smaller radii of the cylinder compared to the ring due to summation over momentum k.

It is of interest to compare the result obtained above with the case of homogenous superconductor-normal-metal transition. In this regime the persistent current in the thin ring is given by the expression[28,29]

$$I = -\frac{2\pi T}{\Phi_0}\frac{\sin(2\pi\Phi/\Phi_0)}{\cosh(2\pi R/\zeta) - \cos(2\pi\Phi/\Phi_0)}. \tag{23}$$

Thus if the radius of the ring is larger than the coherence length then

$$I \simeq -\frac{4\pi T}{\Phi_0}e^{-2\pi R/\zeta}\sin(2\pi\Phi/\Phi_0). \tag{24}$$

Comparing expressions (20) and (21) with expressions (23) and (24) we see that the persistent current in nonuniform case is a result of the summation of two usual currents with phases shifted by $\pm\phi$.

B. Regime of weak inhomogeneity $Q\zeta<1$

Finally, we consider the oscillation regime in the vicinity of metal-LOFF transition where $Q\zeta<1$. This regime corresponds to weakly inhomogeneous superconducting fluctuations when $\beta\to 0$. The number of LOFF modulations per correlation length ζ is small. In Eq. (19) for persistent current we also consider the following condition

$$QR > \zeta/R. \tag{25}$$

This condition implies that if the radius of the ring is larger than the correlation length then the number of LOFF modulations per circumference of the ring QR should be large. This condition can be rewritten as

$$R > \xi, \tag{26}$$

where now the effective coherence length is given as

$$\xi = \left(\frac{\zeta}{\sqrt{2}Q}\right)^{1/2}. \tag{27}$$

With these assumptions one concludes with the following expression for the persistent current of the thin ring:

$$I = -\frac{16\pi T}{\Phi_0}\sin\frac{2\pi\Phi}{\Phi_0}\cos(\sqrt{2}\pi R/\xi)e^{-\sqrt{2}\pi R/\xi}. \tag{28}$$

This result is illustrated in the Fig. 8, where the persistent current dependence on the magnetic flux is presented. One sees that in this particular regime the amplitude of the oscillations decreases compared to the temperatures regime of strongly inhomogeneous fluctuations, where $Q\zeta>1$.

V. APPLICABILITY OF GAUSSIAN APPROXIMATION

Here we will examine applicability of Gaussian approximation in the vicinity of LOFF-metal transition in general.

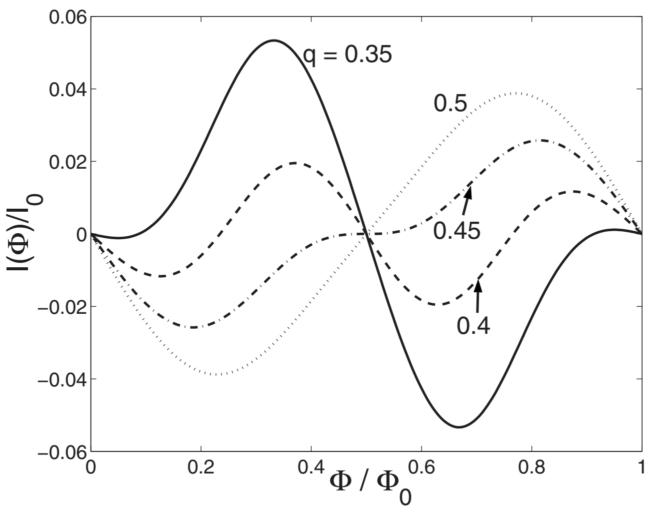

FIG. 8. Persistent current of the cylinder as a function of magnetic flux measured in units $I_0=2T_cL/R\Phi_0$. For the case of $Q\zeta/\sqrt{2}=0.1$ and $q=R\sqrt{\sqrt{2}Q/\zeta}=[0.35,0.4,0.45,0.5]$.

The first step of estimating the fluctuation interaction correction above T_c is to take into account the contribution of neglected $|\Psi|^4$ and $|\Psi|^6$ terms. The last term is important in case of first-order LOFF normal-metal transition. GL functional is then[11]

$$\mathcal{F}=F+\int d\mathbf{r}[\gamma|\Psi|^4+\nu|\Psi|^6], \quad (29)$$

where F is given by Eq. (1) and $\mathbf{D}=i\nabla$ since the magnetic-field orbital effects are neglected.

Here coefficient γ being a function of Zeeman energy and temperature could also change sign and become negative at high magnetic field and low temperatures. In clean superconductors with simple Fermi surface both coefficients β and γ change sign at the same point on the transition line—the so-called tricritical point. Brazovskii[30] showed that coupling of fluctuations in inhomogeneous LOFF-type systems is important and could lead to the first-order-type transition. This is why one has to keep term $\sim|\Psi|^6$ in GL functional.

The first-order correction in γ is given by the bubble containing fluctuation propagator (see Fig. 9). Fluctuations in LOFF-type systems are more singular near transition. Let us consider how they depend on dimensionality of the system.

FIG. 9. First-order fluctuation correction diagram.

First fluctuation correction, which determines transition temperature shift $a(T-T_c)\rightarrow a[T-T_c+\Delta T(D)]$, is given by expression[31]

$$a\Delta T(D)\equiv\gamma\frac{V_D}{V}\int\frac{d^D\mathbf{p}}{(2\pi)^D}\frac{T}{a(T-T_c)+\dfrac{|\beta|}{2Q^2}(p^2-Q^2)^2}, \quad (30)$$

where $V/V_3=1$ for three-dimensional system, $V/V_2=d$ for thin film with thickness $d<\zeta$, and $V/V_1=S$ for thin wire with cross-section area S and thickness less than ζ.

Calculating Eq. (30), we obtain at $Q\zeta>1$

$$a\Delta T(D)\sim\frac{\gamma T_c}{|\beta|}\frac{V_D}{V}\zeta Q^{D-1} \quad (31)$$

and in the regime of small $|\beta|$ when $Q\zeta=\sqrt{\frac{|\beta|}{2\delta}}\zeta<1$ we estimate

$$a\Delta T(D)\sim\frac{\gamma T_c}{\delta}\frac{V_D}{V}\left(\frac{\zeta}{Q}\right)^{4-D/2}. \quad (32)$$

Comparing temperature shift [Eqs. (31) and (32)] with $T-T_c$, we obtain Levanuk-Ginzburg parameter τ_{LG}, which determines the width of critical fluctuations region where Gaussian approximation fails.

In the case $Q\zeta>1$ the Levanuk-Ginzburg parameter in different dimensions can be estimated as

$$\tau_{\mathrm{LG}}\sim\left(\frac{T_c}{E_F}\right)^{4/3}G_3,\quad D=3,$$

$$\tau_{\mathrm{LG}}\sim\left(\frac{1}{dp_F}\frac{T_c}{E_F}\right)^{2/3}G_2,\quad D=2,$$

$$\tau_{\mathrm{LG}}\sim(Sp_F^2)^{-2/3}G_1,\quad D=1, \quad (33)$$

where p_F is Fermi momentum.

In expression (33) $G_D=[\frac{|\gamma|}{|\gamma_0|}]^{2/3}[\frac{|\beta|}{|\beta_0|}]^{(D-2)/3}$, $\beta_0\sim1/m$, and $\gamma_0\sim T_c^2/nE_F$ are the values of the coefficients β and γ far from the LOFF-metal transition. m and n are the electron mass and the density, correspondingly. Thus one always has $|\frac{\beta_0}{\beta}|>1$ and $|\frac{\gamma_0}{\gamma}|>1$ and thus G_D is a small parameter. In obtaining Eq. (33) we estimated $a\sim T_c/E_F$.

In the opposite case when $Q\zeta<1$ the Levanuk-Ginzburg parameter can be estimated as

$$\tau_{\mathrm{LG}}\sim\left(\frac{T_c}{E_F}\right)^{4/5}\tilde{G}_3,\quad D=3,$$

$$\tau_{\mathrm{LG}}\sim\left(\frac{1}{dp_F}\frac{T_c}{E_F}\right)^{2/3}\tilde{G}_2,\quad D=2,$$

$$\tau_{\mathrm{LG}}\sim(Sp_F^2)^{-4/7}\tilde{G}_1,\quad D=1, \quad (34)$$

where now $\tilde{G}_D=[\frac{\gamma}{\gamma_0}]^{4/(8-D)}$.

Indeed in three- and two-dimensional systems correction is much more singular and corresponding critical region is much larger than in case of uniform order parameter. In quasi-2D organic superconductors and in heavy-fermion $CeCoIn_5$ compound the critical fluctuations region is still very small provided $T_c/E_F \sim 10^{-2}-10^{-3}$ (Ref. 21) and $T_c/E_F \sim 0.15$ (Ref. 4), correspondingly.

However in one-dimensional case correction coincides with that for the case of uniform order parameter. In this sense the regimes of quasi-zero-dimensional ring ($R \sim \zeta$) and quasi-one-dimensional cylinder ($L \gg R, \zeta$) considered here are the same as in case of standard superconductors and do not deserve special discussion. The value, which determines the smallness of the Levanuk-Ginzburg parameter in quasi-one-dimensional wire, is $Sp_F^2 \gg 1$, i.e., large number of transverse modes.

If coefficient at term $|\Psi|^4$ changes sign and becomes negative then one should check if the first-order-type transition destroys the Gaussian approximation. Let us estimate the temperature width of first-order-type transition in case of supercooling.

Consider sample with sizeless or of the order of ζ. Varying Eq. (29) with respect to the amplitude of the order parameter written as $\Psi(\mathbf{r})=\Psi\cos(\mathbf{Qr})$ we obtain three solutions: $\Psi_0=0$ and

$$\Psi_{\pm} = \frac{|\gamma|}{3\nu} \pm \sqrt{\left(\frac{\gamma}{3\nu}\right)^2 - \frac{a(T-T_c)}{3\nu}}. \quad (35)$$

Temperature of first-order transition from $\Psi_0=0$ state to Ψ_+ state is determined by the condition

$$\mathcal{F}(\Psi_+) = \mathcal{F}(\Psi_0) = 0 \quad (36)$$

and is larger than T_c.

Solution Ψ_- corresponds to the maximum of functional (29)

$$\mathcal{F}(\Psi_-) \sim V\frac{[a(T-T_c)]^2}{4|\gamma|}. \quad (37)$$

The probability of thermal activation transition of the order parameter from the steady state Ψ_0 over the barrier of height $\mathcal{F}(\Psi_-)$ is proportional to the value

$$\sim \exp\left[-\frac{\mathcal{F}(\Psi_-)}{T}\right]. \quad (38)$$

In case of $\mathcal{F}(\Psi_-) \gg T_c$ system will stay in supercooled state. Corresponding temperature region can be estimated as

$$(T-T_c)/T_c > \frac{1}{aT_c}\left(\frac{T_c|\gamma|}{V}\right)^{1/2} \sim (N_0VT_c)^{-1/2}. \quad (39)$$

Here N_0 is the electron density of states at Fermi level. In case of not too small sample, when $N_0VT_c \gg 1$, supercooled $\Psi_0=0$ state can be extended at $(T-T_c)/T_c < 1$.

VI. SUMMARY

To summarize, we have shown that Aharonov-Bohm effect is very sensitive tool for studying the intrinsic properties of superconductors in the regime of inhomogeneous LOFF state.

Depending on the ratio of modulation period of superconducting order parameter and the radius of the ring/cylinder transition temperature in magnetic flux can be either increased or decreased, or even can have double-peak structure at one flux quantum. These effects arise due to the degeneracy of fluctuation energy spectrum of nonhomogeneous LOFF state.

We calculated the fluctuation contribution for the persistent current in thin superconducting ring and presented numerical results for fluctuation specific heat and persistent current for the cylinder. Flux dependencies of persistent current and specific heat qualitatively correspond to that of transition temperature.

We have shown that despite the enhancing of singularity due to fluctuation's interaction in higher dimensions, in quasi-one-dimensional system Levanuk-Ginzburg parameter coincides with that for homogeneous superconductor.

The most studied reason for inhomogeneity is Zeeman splitting due to external magnetic field or exchange splitting in ferromagnetic superconductor. In this case coefficients β, δ, and temperature $\tilde{T}_c$ itself depend on the magnetic field. Thus AB oscillations are superimposed by monotonous dependence on magnetic field. In ring geometry these two could be separated by measuring of AB oscillations in tilted magnetic field.

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