Influence of interfacial magnons on spin transfer torque in magnetic tunnel junctions

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(Received 4 December 2008; revised manuscript received 6 April 2009; published 1 May 2009)

The role of interfacial electron-magnon interaction in a noncollinear magnetic tunnel junction is investigated using the transfer Hamiltonian method. It is shown that the interfacial electron-magnon scattering modifies the bias dependence of both components of the spin-transfer torque. In particular, we find that at low temperature, the magnons emission adds a quadratic contribution and at finite temperature, both quadratic and linear terms appear. These contributions can explain recent experimental results on the bias dependence of spin-transfer torque.

DOI: 10.1103/PhysRevB.79.174401 PACS number(s): 72.25.−b, 73.43.Jn, 73.40.Rw, 73.43.Qt

I. INTRODUCTION

Spin-dependent transport in magnetic tunnel junctions[1] (MTJs) has attracted much attention for the past fifteen years from both experimental and theoretical points of view.[2] The recent observation of current-induced magnetization switching[3,4] in such devices[5–7] has enhanced the already important interest of MTJs. Up to now, a number of theories have been proposed to describe spin-transfer torque in tunnel junctions and especially its bias dependence.[8–13] In MTJs, the spin-transfer torque is on the form

$$\mathbf{T} = T_{\parallel}\mathbf{M} \times (\mathbf{M} \times \mathbf{P}) + T_{\perp}\mathbf{M} \times \mathbf{P} \tag{1}$$

where $\mathbf{M}$ ($\mathbf{P}$) refers to the magnetization of the free (fixed) layer and $T_{\parallel}$ ($T_{\perp}$) are usually referred to as in-plane (IP) and out-of-plane (OP) torque amplitudes. In a symmetric MTJ, in the absence of defects, impurities, or magnons, these two components possess well-defined bias dependence: $T_{\parallel}=a_1V+a_2V^2$, $T_{\perp}=b_0+b_2V^2$. These results have recently been confirmed by *ab initio* calculations in a MgO-based MTJ.[14] Note that the introduction of asymmetries in the MTJ itself[12] or at the interfaces[9] modifies this bias dependence.

Although recent "spin-diode" type experiments seem to agree with these theoretical predictions,[15–17] other experimental results underline the complexity of this bias dependence,[18–20] showing the presence of either a non-negligible linear or an antisymmetric quadratic component in the OP torque.

In Ref. 20, the authors interpret their data considering the presence of bulk magnons affecting the spin relaxation in the free layer. Indeed, electron-magnons interactions are known to influence the electron transport in bulk ferromagnets[21] and to affect the electron spin-diffusion length[22] as well as spin-transfer torque in metallic spin valves.[23] In MTJs, the interfacial electron-magnon scattering is responsible for the drop of tunneling magnetoresistance (TMR) at low-bias voltage and low temperature,[24–28] known as the zero-bias anomaly. This mechanism has been validated by a number of experiments on both AlOx-based[29] and MgO-based[30] magnetic tunnel junctions.

Levy and Fert[31] first considered the influence of electron-magnon scattering on spin-transfer torque. Although focusing on the IP component at zero temperature (only magnon emission is considered), this study clearly showed the potential influence of the magnons on the spin-transfer torque. The recent measurements of the spin-torque bias dependence[15–20] requires a more complete analysis of the role of the interfacial electron-magnon scattering in order to determine their contribution.

In this article, we study the interfacial electron-magnon interaction within the transfer Hamiltonian formalism, already used to derive the elastic[8] and zero temperature[31] in-plane torque. This method is very convenient since it gives straightforward formulae, easy to use to fit experimental data.[29,30]

We find that both components of the spin torque, in-plane and out-of-plane, are affected by the presence of magnons. At zero temperature, the magnon emission gives rise to a quadratic component $\propto JV$ at low bias and to a linear component at large bias. At finite temperature, both linear and quadratic contributions ($\propto JV$) arise from the electron-magnon interaction. In this case, the bias dependence of the spin torque becomes

$$T_{\parallel} = a_1(T)V + a_2V^2 + a_3J|V|,$$

$$T_{\perp} = b_0 + b_2V^2 + b_1(T)|V| + b_3JV$$

at low bias and

$$T_{\parallel} = a_1(T)V + a_2V^2,$$

$$T_{\perp} = b_0 + b_2V^2 + b_1(T)|V|$$

at large bias voltage.

The determination of these temperature-dependent coefficients (a_i, b_j) is our main focus. We organize the paper as follows: in Sec. II, the transfer Hamiltonian formalism extended to the spinor form is described. The calculations of the spin current and spin-transfer torques for elastic and magnon-assisted tunneling are carried in Sec. III. A brief comparison with experimental results is given in Sec. IV. Finally, we outline our results and conclusion in Sec. V.

II. TRANSFER HAMILTONIAN FORMALISM

A. Spin current in spinor form

The transfer Hamiltonian formalism has been widely used to describe the transport in tunnel junctions.[32] The Hamiltonian of a tunnel junction is described by

$$H = H_L + H_R + H_{\mathrm{tr}}, \tag{2}$$

where $H_{L(R)}=\Sigma_{\mathbf{k(p)}}\epsilon_{\mathbf{k(p)}}c^+_{\mathbf{k(p)}}c_{\mathbf{k(p)}}$ is the Hamiltonian of the left (right) electrode, $\epsilon_{\mathbf{k(p)}}$ is the conduction-electron energy, and $c^+_{\mathbf{k(p)}}[c_{\mathbf{k(p)}}]$ is the creation (annihilation) operator. $\mathbf{k}(\mathbf{p})$ refers to the electron wave vector within the left (right) electrode. The transfer Hamiltonian may be written as

$$H_{\mathrm{tr}} = \sum_{\mathbf{k},\mathbf{p}} [c^+_{\mathbf{k}}T_{\mathbf{kp}}c_{\mathbf{p}} + c^+_{\mathbf{p}}T_{\mathbf{pk}}c_{\mathbf{k}}], \tag{3}$$

where $T_{\mathbf{kp}}$ is the matrix element that transfers particles from the left electrode to the right electrode. The transfer matrix is assumed to only depend on the electron wave vectors $\mathbf{k}$ and $\mathbf{p}$, and $T_{\mathbf{pk}}=(T_{\mathbf{kp}})^+$. The corresponding tunneling current-density operator is related to the electron density, $J=-e\langle\dot{N}\rangle$, where $N=\Sigma_{\mathbf{p}}c^+_{\mathbf{p}}c_{\mathbf{p}}$,

$$J = -i\frac{e}{\hbar}\langle[H_{\mathrm{tr}},N]\rangle = -i\frac{e}{\hbar}\left\langle \sum_{\mathbf{k},\mathbf{p}} [c^+_{\mathbf{k}}T_{\mathbf{kp}}c_{\mathbf{p}} - c^+_{\mathbf{p}}(T_{\mathbf{kp}})^+c_{\mathbf{k}}]\right\rangle, \tag{4}$$

where the brackets $\langle\ldots\rangle$ refer to quantum-mechanical averaging. Then, assuming that the tunneling probability is small enough (the barrier is thick and high), the first-order perturbation gives

$$\langle c^+_{\mathbf{k}}T_{\mathbf{kp}}c_{\mathbf{p}}\rangle \approx \langle c^+_{\mathbf{k}}T_{\mathbf{kp}}H_{\mathrm{tr}}c_{\mathbf{p}}\rangle = \langle c^+_{\mathbf{k}}c_{\mathbf{k}}\rangle T_{\mathbf{kp}}\langle c^+_{\mathbf{p}}c_{\mathbf{p}}\rangle(T_{\mathbf{kp}})^+. \tag{5}$$

Consequently, the tunneling current can be expressed as a function of the unperturbed interfacial densities of states $\langle c^+_{\mathbf{k(p)}}c_{\mathbf{k(p)}}\rangle=2i\pi\rho_{\mathbf{k(p)}}$ (i.e., in the absence of tunneling) and yields

$$J = 2\pi\frac{e}{\hbar}\sum_{\mathbf{k},\mathbf{p}} [\rho_{\mathbf{k}}T_{\mathbf{kp}}\rho_{\mathbf{p}}(T_{\mathbf{kp}})^+f_L(1-f_R) - \rho_{\mathbf{p}}(T_{\mathbf{kp}})^+\rho_{\mathbf{k}}T_{\mathbf{kp}}f_R(1-f_L)], \tag{6}$$

where $f_{L(R)}$ is the Fermi-distribution function for electrons in reservoir $L(R)$. Note that the form $f_{L(R)}(1-f_{R(L)})$ accounts for the unoccupied electron states available after tunneling. This notation is important in the case of inelastic tunneling, where the energy of the incoming and outgoing electrons are different.

In the present study, we consider a magnetic-tunnel junction, composed of two ferromagnetic electrodes with magnetizations $\mathbf{S}^{L(R)}$, separated by a tunnel barrier (see Fig. 1). The left and right electrodes have different spin-quantization frames, (x',y,z') and (x,y,z), rotated around the y axis by an angle θ, y being perpendicular to the plane of the layers. The background magnetizations can be decomposed within a longitudinal and a transverse part $\mathbf{S}^{L(R)}=\mathbf{S}^{L(R)}_l+\mathbf{S}^{L(R)}_{\mathrm{tr}}$, with $|\mathbf{S}^{L(R)}_l|\gg|\mathbf{S}^{L(R)}_{\mathrm{tr}}|$. The longitudinal component $\mathbf{S}^{L(R)}_l$ lies along the $z'(z)$ axis whereas the transverse components are $\mathbf{S}^L_{\mathrm{tr}}=S_{x'}\mathbf{x}'+S_{y'}\mathbf{y}'$ and $\mathbf{S}^R_{\mathrm{tr}}=S_x\mathbf{x}+S_y\mathbf{y}$.

To account for spin-dependent transport, the transport quantities (current density, transfer matrix, and densities of states) are expressed in the *spinor* form, i.e., within the spin basis $(\hat{I},\boldsymbol{\sigma})$, where $\hat{I}$ is the 2×2 unity matrix and $\boldsymbol{\sigma}$ is the spin Pauli matrix. In the following, all the transport quantities will be expressed within the quantization frame of the *right* layer. The interfacial densities of states (DOS) $\hat{\rho}$ and the transfer-matrix $\hat{T}$ are now 2×2 matrices, describing the spin-dependent tunneling. The spin-dependent DOS are

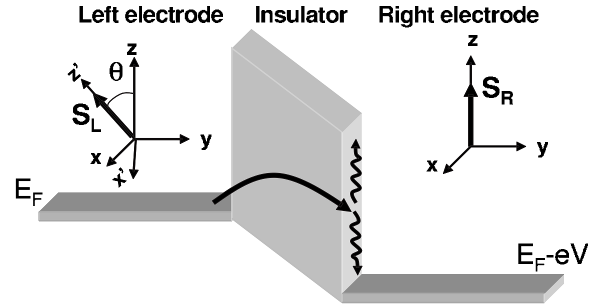

FIG. 1. Schematics of the potential profile of a magnetic-tunnel junction with noncollinear magnetizations. In our model, the tunneling of an electron across the barrier may be accompanied by the emission or absorption of an interfacial magnon (wavy arrows). Note that in this picture, the magnetizations are approximated by their longitudinal part $\mathbf{S}\approx\mathbf{S}_l$.

$$\hat{\rho}_{\mathbf{p}} = \rho_{\mathbf{p}}\hat{I} + \Delta\rho_{\mathbf{p}}\boldsymbol{\sigma}\,.\,\mathbf{z}, \tag{7}$$

$$\hat{\rho}_{\mathbf{k}} = \rho_{\mathbf{k}}\hat{I} + \Delta\rho_{\mathbf{k}}R^{-1}\boldsymbol{\sigma}\,.\,\mathbf{z}R, \tag{8}$$

where $\rho=(\rho_\uparrow+\rho_\downarrow)/2$ is spin-independent part of the DOS, $\Delta\rho=(\rho_\uparrow-\rho_\downarrow)/2$ and R is the unitary rotation matrix in the (x,z) plane

$$R = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{pmatrix}. \tag{9}$$

The extension of Eq. (6) to the spinor form should be done carefully since the transfer matrices $\hat{T}_{\mathbf{kp}}$ and $(\hat{T}_{\mathbf{kp}})^+$ must be written in the quantization axis of $\hat{\rho}_{\mathbf{k}}$ and $\hat{\rho}_{\mathbf{p}}$, respectively. Consequently, the spinor current takes the form

$$\hat{J} = 2\pi\frac{e}{\hbar}\sum_{\mathbf{k},\mathbf{p}} [\hat{\rho}_{\mathbf{k}}\hat{T}_{\mathbf{kp}}\hat{\rho}_{\mathbf{p}}(\hat{T}_{\mathbf{kp}})^+f_L(1-f_R) - \hat{\rho}_{\mathbf{p}}(\hat{T}_{\mathbf{kp}})^+\hat{\rho}_{\mathbf{k}}\hat{T}_{\mathbf{kp}}f_R(1-f_L)]. \tag{10}$$

In the case of a magnetic-tunnel junction in the presence of interfacial magnons, the spin-dependent transfer matrix $\hat{T}_{\mathbf{kp}}$ accounts for both elastic and inelastic tunneling

$$\hat{T}_{\mathbf{kp}} = \hat{T}^d_{\mathbf{kp}}\left[\hat{I} + \sqrt{\frac{Q}{N}}(\sigma\,.\,\mathbf{S}^R_{\mathrm{tr}} + \sigma\,.\,\mathbf{S}^L_{\mathrm{tr}})\right] \tag{11}$$

where $\hat{T}^d_{\mathbf{kp}}$ is the direct-tunneling matrix whose matrix elements will be determined later, Q is the phenomenological electron-magnon efficiency (see below), N is the number of atoms per cell, $\boldsymbol{\sigma}$ is the vector of Pauli-spin matrices, and $\mathbf{S}^{L(R)}_{\mathrm{tr}}$ are the transverse part of the magnetizations of the left and right electrodes. In Eq. (11), we have assumed that the transmission probability of the electrons through the barrier is small enough so that elastic and inelastic tunneling are additive.

The magnons excitations at the interfaces are contained in the transverse components $S_x(S_{x'})$ and $S_y(S_{y'})$ and can be described using a linearized Holstein-Primakoff transformation[33]

$$S_{+}^{R(L)} = S_{x(')} + iS_{y(')} = \sqrt{2S_{R(L)}}\sum_{\mathbf{q}} e^{-i\mathbf{q}.\mathbf{r}} a_{\mathbf{q}}^{R(L)}, \tag{12}$$

$$S_{-}^{R(L)} = S_{x(')} - iS_{y(')} = \sqrt{2S_{R(L)}}\sum_{\mathbf{q}} e^{i\mathbf{q}.\mathbf{r}} a_{\mathbf{q}}^{R(L)+}. \tag{13}$$

Note that the magnons coordinates are different for the left and right interfaces due to the rotation of the spin-quantization frames. Here, only one-magnon processes are considered and higher-order interactions are neglected. We also neglect the influence of the magnon density $n_{\mathbf{q}}$ on the longitudinal part of the magnetization. Note that the elastic-interfacial s-d exchange interaction $\propto \sigma_z S_z$ is not explicitly taken into account in Eq. (11) since it is already included in the definition of the interfacial spin-dependent DOS [see Eq. (7)].

The efficiency of the electron-magnon interaction Q can be estimated from more complete models (free electron, tight-binding theory, etc.). Considering electron-magnon within a free-electron model (see, e.g., Ref. 28), the efficiency can be identified with the square of the ratio between the interfacial s-d exchange energy J_{sd} and the electron-kinetic energy $p^2/2m$. For a usual ferromagnet ($J_{sd} \approx 1$ eV and $E_F \approx 3$ eV), this yields $Q = (J_{sd}/E_F)^2 \approx 10\%$ consistently with previous experimental results.[29,30] Note however that this ratio strongly depends on the details of the band structure and on the quality of the interfaces.

B. Direct tunneling transfer matrix

The determination of the direct-tunneling matrix is the key point of this section. Our goal is to determine analytically, from exact models (tight binding, free electron, etc.), the spin-dependent form of this matrix.

As we mentioned in the introduction, the transfer Hamiltonian formalism has been used by Slonczewski[8] and Levy and Fert[31] to determine the spin torque in MTJs. The authors assumed that $\hat{T}^d_{\mathbf{kp}}$ is spin-independent, i.e., $\hat{T}^d_{\mathbf{kp}} = (\hat{T}^d_{\mathbf{pk}})^+ = T^d_{\mathbf{kp}}\hat{I}$, where $\hat{I}$ is the 2×2 unitary matrix. In this case, the electron spin is not affected by the tunneling process and remains in the $(\mathbf{S}_l^L, \mathbf{S}_l^R)$ plane defined by the magnetizations. Consequently, the spin torque extracted from this model has *only in-plane component and no out-of-plane component*. This result contrasts with free-electron or tight-binding models that confirm the presence of *both* in-plane and out-of-plane components in the spin torque,[10–13] indicating that the assumption of a diagonal transfer matrix is *not valid* in magnetic tunnel junctions with noncollinear magnetizations.

Actually, explicit calculations based on the free-electron model have shown that the electron spin is *rotated* out of the $(\mathbf{S}_l^L, \mathbf{S}_l^R)$ plane during the tunneling process, due to spin-dependent reflections and transmissions at the Ferromagnet/Insulator interfaces.[11,34] The angle of rotation can be quite significant, leading to a sizable out-of-plane component of the spin current[10–13] even at zero bias, known as interlayer exchange coupling[34] (noted b_0 here). This spin rotation is also present in metallic spin valves but is usually averaged out after integration over the Fermi surface, yielding to a vanishing out-of-plane torque.[35]

To include the above physics in the transfer Hamiltonian formalism, we must abandon the assumption of a diagonal-transfer matrix. Instead, one evaluates the spin-dependence of the transfer matrix by using an exact solution of the free-electron model. The general expression for the transfer matrix is

$$\hat{T}^d_{\mathbf{kp}} = \begin{pmatrix} \langle \Psi_L^{\uparrow} | \Psi_R^{\uparrow} \rangle & \langle \Psi_L^{\downarrow} | \Psi_R^{\uparrow} \rangle \\ \langle \Psi_L^{\uparrow} | \Psi_R^{\downarrow} \rangle & \langle \Psi_L^{\downarrow} | \Psi_R^{\downarrow} \rangle \end{pmatrix}, \tag{14}$$

where $|\Psi_i^{\sigma}\rangle$ are the spin-dependent electron wave functions at the interface $i(i=L,R)$, σ is the spin projection in the right quantization axis. Using the free-electron wave functions defined in Ref. 11, up to the first order in the barrier thickness, the transfer matrix takes the form

$$\hat{T}^d_{\mathbf{kp}} = T^d \begin{pmatrix} e^{i\phi_L}\left(e^{i\phi_R}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) & -e^{i\phi_L}\frac{\sin\theta}{2}(e^{i\phi_R}-1) \\ -\frac{\sin\theta}{2}(e^{i\phi_R}-1) & \left(e^{i\phi_R}\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right) \end{pmatrix}. \tag{15}$$

The angles ϕ_i account for the rotation of the electron spin at the interface i within the barrier. We straightforwardly see that $\hat{T}^d \to T^d\hat{I}$ for an infinitely thick or high-barrier ($\phi_i \to 0$). In the following, we assume that $\phi_i \ll 1$, so that $e^{i\phi_i} \approx 1 + i\phi_i$.

III. TORQUES AND CURRENTS

The influence of magnons on the tunnel transport is clearly illustrated by the "zero-bias anomaly," i.e., the observation of a sharp-resistance drop at low temperature (<77 K) as a function of the applied bias voltage.[24,25] Associated with characteristic peaks around 20 mV in the inelastic electron tunneling spectra[36] (IETS), this strongly suggests that interfacial magnons significantly contribute to the spin transport in MTJs. At low temperature, the magnons emission opens spin-flip channels for hot minority electrons that increase with the bias voltage. Consequently, the conductivity in the antiparallel (AP) state increases much faster than the conductivity in the parallel (P) state, leading to a drop in TMR. We then expect that these spin-flip channels strongly affect the spin-torque bias dependence as well.

In this section we evaluate the current-density J_e and spin-currents $\mathbf{J}_s$ from the spinor current given by Eq. (10). The spin-density continuity equation in a magnetic layer gives[4]

$$\mathbf{T} = -J_{sd}\int \mathbf{m}(x)\mathbf{S}dx = \int \{-\nabla . \mathbf{J}_s(x) + \Gamma[\mathbf{m}(x)]\}dx, \tag{16}$$

where $\mathbf{m}$ is the spin density and $\Gamma(\mathbf{m})$ accounts for spin-flip processes in the magnetic layer. When no spin relaxation (due to bulk magnons, for example[20]) is present in the volume of the layer $[\Gamma(\mathbf{m})=0]$, the torque is directly the interfacial spin-current: $\mathbf{T}=\mathbf{J}_s$. Otherwise, one should directly calculate the spin density in the free layer and integrate it over the layer thickness.[20] Then, in the case of interfacial electron-magnon scattering, the spin-transfer torque is related to the transverse spin current given by the spinor form, Eq. (10): $T_\parallel=\Re[\mathrm{Tr}(\sigma_x\hat{J})]$ and $T_\perp=\Re[\mathrm{Tr}(\sigma_y\hat{J})]$, where Tr is the trace taken on the spin states.

We express the transport quantities in term of the DOS polarization $P=\Delta\rho/\bar{\rho}$ and of the elastic averaged conductance $G_0=2\pi\frac{e^2}{\hbar}|T^d_{\mathbf{kp}}|^2\rho_{\mathbf{k}}\rho_{\mathbf{p}}$. We give the explicit expression of the torques on the right electrode and at positive bias. Symmetry considerations will easily lead to the expressions of the torques on the left electrode and at negative bias.

A. Elastic contribution

The elastic contributions to the current-density J_e, in-plane ($T_\parallel$) and out-of-plane ($T_\perp$) components of the spin torque are

$$J_e^0=\int dE\frac{G_0}{e}(1+P^LP^R\cos\theta)(f_L-f_R), \quad (17)$$

$$T_\parallel^0=\int dE\frac{G_0}{e}\sin\theta P^L(f_L-f_R), \quad (18)$$

$$T_\perp^0=\int dE\frac{G_0}{e}\sin\theta(P_R\phi_Lf_L+P_L\phi_Rf_R). \quad (19)$$

These above expressions are consistent with the previous studies about elastic spin-transfer torques.[8,10–13] The spin torque possesses two components, acting in the plane and out of the plane of the magnetization, respectively, the former vanishing at zero bias. To evaluate the bias dependence of the elastic torques we assume that the DOS are almost constant on the range of the applied bias voltage (so that $\int dE\rho(f_L-f_R)\approx\rho eV$). The exact form of the out-of-plane torque is less straightforward, involving the integration over all the energy range. Nevertheless, symmetry considerations yield that in a symmetric MTJ ($P_R\phi_L$ and $P_L\phi_R$ are equivalent), only a quadratic bias dependence exists, whereas in an asymmetric MTJ, a linear term appears. We will not discuss further the form of the elastic torque since they have already been widely studied in both symmetric and asymmetric junctions.[8,10–13] Therefore, the general expressions of the elastic-transport quantities are

$$J_e^0=G_0V(1+P^LP^R\cos\theta), \quad (20)$$

$$T_\parallel^0=G_0VP^L\sin\theta, \quad (21)$$

$$T_\perp^0=(b_0+b_2V^2)\sin\theta, \quad (22)$$

where b_0 and b_2 can be determined by performing the energy integration of Eq. (19). Since the general expression of these coefficients is tedious, we do not give the exact form of b_0 and b_2 in this paper. Note that the conservative interlayer exchange coupling b_0 (Ref. 34) is generally quenched by the other magnetic fields involved in the system (anisotropy field, orange-peel coupling, dipolar field, etc.). Therefore it is only of little interest for the present study and will be disregarded in the remainder of this article.

B. Inelastic contribution

As stated in Sec. II, the spinor form of the current density is obtained after performing matrix products involving the DOS matrices and the tunneling matrices defined in Eq. (11). After some algebra we obtain the expression of the electrical current, in-plane and out-of-plane torques, as a function of terms proportional to $\langle S_-^iS_+^i\rangle$ and $\langle S_+^iS_-^i\rangle$. The former accounts for magnons absorption at the ith interface, whereas the latter accounts for magnons emission. The general form of the electrical current flowing through the junction is

$$\begin{aligned}J_e^{sw}(E,\mathbf{q})=\frac{G_0}{e}\frac{SQ}{N}\{&[(1-P^R)(1+P^L\cos\theta)\langle S_-^RS_+^R\rangle+(1+P^R)(1-P^L\cos\theta)\langle S_+^RS_-^R\rangle+(1+P^L)(1-P^R\cos\theta)\langle S_-^LS_+^L\rangle\\&+(1-P^L)(1+P^R\cos\theta)\langle S_+^LS_-^L\rangle]f_L(1-f_R)-[(1+P^R)(1-P^L\cos\theta)\langle S_-^RS_+^R\rangle+(1-P^R)(1+P^L\cos\theta)\langle S_+^RS_-^R\rangle\\&+(1-P^L)(1+P^R\cos\theta)\langle S_-^LS_+^L\rangle+(1+P^L)(1-P^R\cos\theta)\langle S_+^LS_-^L\rangle]f_R(1-f_L)\}.\end{aligned} \quad (23)$$

This form extends the results derived in Refs. 24 and 31. For the remainder of this paper, we will only focus on the IP and OP torques. The general form of the spin transfer torque in the presence of interfacial magnons is then

$$\begin{aligned}T_\parallel^{sw}(E,\mathbf{q})=\frac{G_0}{e}\sin\theta\frac{SQ}{N}\{&[P^L(1-P^R)\langle S_-^RS_+^R\rangle+P^L(1+P^R)\langle S_+^RS_-^R\rangle+(1+P^L)(1-\cos\theta P^R)\langle S_-^LS_+^L\rangle\\&-(1-P^L)(1+\cos\theta P^R)\langle S_+^LS_-^L\rangle]f_L(1-f_R)-[(1-P^L)\langle S_-^LS_+^L\rangle-(1+P^L)\langle S_+^LS_-^L\rangle]f_R(1-f_L)\},\end{aligned} \quad (24)$$

$$\begin{aligned}T_\perp^{sw}(E,\mathbf{q})=\frac{G_0}{e}\sin\theta\frac{SQ}{N}\{&\{\phi_L[(1-P^R)\langle S_-^RS_+^R\rangle-(1+P^R)\langle S_+^RS_-^R\rangle]-\phi_R[(1-\cos\theta P^R)\langle S_-^LS_+^L\rangle-(1+\cos\theta P^R)\langle S_+^LS_-^L\rangle]\}f_L(1-f_R)\\&+\{\phi_L[(1-\cos\theta P^L)\langle S_-^RS_+^R\rangle-(1+\cos\theta P^L)\langle S_+^RS_-^R\rangle]-\phi_R[(1-P^L)\langle S_-^LS_+^L\rangle-(1+P^L)\langle S_+^LS_-^L\rangle]\}f_R(1-f_L)\}.\end{aligned} \quad (25)$$

Equations (23)–(25) give the electrical current and spin torques for an electron with energy E interacting with a magnon with wave-vector $\mathbf{q}$. The integration over the planar component of the electron wave-vector $\mathbf{k}$ is already assumed in the definition of the density of states. In the following, we will consider only incoherent electron-magnon interaction, so that the integrations over E and $\mathbf{q}$ are independent. These integrations will be described in more details below.

Interestingly, the IP and OP torques possess contributions from both interfaces, L and R, competing each other. For example, magnon absorption (terms $\propto\langle S_-^i S_+^i\rangle$) increase the IP torque, whereas magnon emission (terms $\propto\langle S_+^i S_-^i\rangle$) decreases it. Furthermore, contrary to the OP torque, the IP torque is asymmetric as a function of the bias voltage: only leftward electrons interacting with the left interface contribute to the IP torque.

C. Integration considerations

Before going one step further, a few words must be said about the treatment of the integration of the components given by Eqs. (23)–(25). These components have to be integrated over the electron energy and magnon wave vector in order to obtain the bias dependence: $\hat{J}=\int\int\hat{j}\frac{d^2\mathbf{q}}{\Omega}dE$ where Ω is the volume of the first Brillouin zone. In the following, we assume that the DOS are roughly constant over the range of integration (of the order of eV), so that one only has to integrate $\langle S_-^i S_+^i\rangle$ and $\langle S_+^i S_-^i\rangle$. Considering electrons emitted by the ith electrode, we can rewrite these quantities as

$$\langle S_-^i S_+^i\rangle f_j(1-f_{\bar{j}}) \to n_{\mathbf{q}}^i f_j(E)[1-f_{\bar{j}}(E+\hbar\omega_{\mathbf{q}})], \quad (26)$$

$$\langle S_+^i S_-^i\rangle f_j(1-f_{\bar{j}}) \to (1+n_{\mathbf{q}}^i) f_j(E)[1-f_{\bar{j}}(E-\hbar\omega_{\mathbf{q}})], \quad (27)$$

where $f_j(E)$ is the Fermi distribution of the jth electrode: $f_L(E)=f(E)$ and $f_R(E)=f(E+\text{eV})$. We also consider that, even at room temperature, the Fermi energy is much larger than k_BT so that $f_i(E)$ is approximated by a step-function $\Theta(E)$. After integrating over the electron energy E, we obtain

$$\int dE\langle S_-^i S_+^i\rangle f_L(1-f_R) = n_{\mathbf{q}}^i(\text{eV}+\hbar\omega_{\mathbf{q}}),$$

$$\int dE\langle S_+^i S_-^i\rangle f_L(1-f_R) = (1+n_{\mathbf{q}}^i)(\text{eV}-\hbar\omega_{\mathbf{q}})\Theta(\text{eV}-\hbar\omega_{\mathbf{q}}),$$

$$\int dE\langle S_-^i S_+^i\rangle f_R(1-f_L) = n_{\mathbf{q}}^i(\hbar\omega_{\mathbf{q}}-\text{eV})\Theta(\hbar\omega_{\mathbf{q}}-\text{eV}),$$

$$\int dE\langle S_+^i S_-^i\rangle f_R(1-f_L) = 0.$$

D. Torques at zero temperature

At zero temperature, $n_{\mathbf{q}}^i=0$. Following Ref. 24, the magnon energy in the ith electrode is written $\hbar\omega_{\mathbf{q}}=E_m^i q^2/\Omega$, and $E_m^i=3k_BT_c^i/(S^i+1)$. E_m^i corresponds to the maximum-magnon energy, T_c^i is the Curie temperature. In this case, the only magnons present in the system are emitted by spin flip (no thermal magnons), with energy $\hbar\omega_{\mathbf{q}}<\max(\text{eV},E_m^i)$. Then

$$\int_0^{\text{eV}}\frac{d^2\mathbf{q}}{\Omega}dE\langle S_+^i S_-^i\rangle f_L(1-f_R) = \frac{e^2V^2}{2E_m^i}, \quad \text{eV}\le E_m^i, \quad (28)$$

$$\int_0^{E_m^i}\frac{d^2\mathbf{q}}{\Omega}dE\langle S_+^i S_-^i\rangle f_L(1-f_R) = \frac{1}{2}\left(\text{eV}-\frac{E_m^i}{2}\right), \quad \text{eV}\ge E_m^i. \quad (29)$$

All the other integrals involved in Eqs. (23)–(25) are zero. As a consequence, at $T=0$ K, the electron-magnon interaction adds a *quadratic* component to the torques at low-bias voltage ($\text{eV}<E_m^i$) and a linear-bias dependence at large voltage ($\text{eV}>E_m^i$). Because Eqs. (23) and (24) change their sign for negative-bias voltage, the quadratic part changes its sign with the bias, consistently with the results obtained on zero-bias anomaly.[24] Then, in a symmetric MTJ, the spin transfer torque is

$$T_{\parallel}^{sw} = G_0\frac{\text{eV}|V|}{2E_m}\frac{SQ}{N}\sin\theta[P(1+P)-(1-P)(1+\cos\theta P)], \quad (30)$$

$$T_{\perp}^{sw} = -G_0\frac{\text{eV}^2}{2E_m}\frac{SQ}{N}\sin\theta(1-\cos\theta)\phi P, \quad (31)$$

where $e|V|/E_m^i$ has to be replaced by $(1-E_m^i/2e|V|)$ when $\text{eV}\ge E_m^i$. The IP torque is similar to the one derived by Levy and Fert.[31] Note also that the electron-magnon interaction modifies the angular dependence of the torque. Assuming $Q=10\%$, $N=1$, $S=3/2$, $E_m=0.12$ eV, $P=0.5$, and $\phi=0.5$ we find that the ratio of the magnon contribution of the IP (OP) torque to the elastic IP torque is on the order of 4% (3%) when $\text{eV}=E_m$.

E. Torques at finite temperature

The case of finite temperature is more complex since it involves both magnons emission and absorption. In this case, the integration for large-bias voltage ($\text{eV}\gg E_m$) can be approximated by

$$\int\frac{d^2\mathbf{q}}{\Omega}dE\langle S_-^i S_+^i\rangle f_L(1-f_R) \approx \frac{\text{eV}}{2}\left(1-\frac{E_m}{2\ \text{eV}}\right) + \frac{k_BT}{2E_m}\text{eV}\ln\frac{k_BT}{E_c},$$

$$\int\frac{d^2\mathbf{q}}{\Omega}dE\langle S_+^i S_-^i\rangle f_L(1-f_R) \approx \frac{\text{eV}+k_BT}{2E_m}k_BT\ln\frac{k_BT}{E_c},$$

where E_c is a cutoff energy due to the magnetic anisotropy or to the finite coherence length of the magnons.[29] At low-bias voltage ($\text{eV}\ll E_m,k_BT$), we find

$$\int \frac{d^2\mathbf{q}}{\Omega} dE \langle S_-^i S_+^i \rangle f_L (1-f_R)$$
$$\approx \frac{(\mathrm{eV}-E_c)^2}{4E_m} - \frac{\mathrm{eV}k_BT}{2E_m}\ln\frac{|\mathrm{eV}|}{E_c} - \frac{k_BT}{2E_m}(\mathrm{eV}-E_c).$$

These approximation have been numerically checked and remains within 5% of the exact value even at large-bias voltage. Both magnon emission and absorption give rise to a linear dependence at large bias. In the case of a symmetric MTJ, for low-bias voltage we find (disregarding the terms independent of the bias voltage)

$$T_\parallel^{sw} \approx G_0 \frac{V}{2E_m}\frac{SQ}{N} k_BT \sin\theta \left\{ 2P\left(\ln\frac{k_BT|\mathrm{eV}|}{E_c^2} - 1\right) - [1-P^2(1+\cos\theta)]\left(\ln\frac{|\mathrm{eV}|}{k_BT} - 1\right) \right\}, \tag{32}$$

$$T_\perp^{sw} \approx - G_0 \frac{|V|}{2E_m}\frac{SQ}{N} k_BT\phi P \sin\theta(1-\cos\theta)\left(\ln\frac{k_BT|\mathrm{eV}|}{E_c^2} - 1\right). \tag{33}$$

In contrast, for high-bias voltage we have (disregarding the terms independent of the bias voltage and those in 1/eV)

$$T_\parallel^{sw} \approx - G_0 \frac{V}{2E_m}\frac{SQ}{N} \sin\theta \left\{ [1-2P-P^2(1+\cos\theta)]E_m - 4Pk_BT \ln\frac{k_BT}{E_c} \right\}, \tag{34}$$

$$T_\perp^{sw} \approx - \phi P G_0 \frac{|V|}{2E_m}\frac{SQ}{N} \sin\theta(1-\cos\theta)\left(E_m + 2k_BT\ln\frac{k_BT}{E_c}\right). \tag{35}$$

Interestingly, the electron-magnon interaction modifies the angular dependence of the spin-transfer torque. These expressions show that the interfacial electron-magnon interaction at finite temperature gives rise to a linear contribution to the spin-transfer torque. The quadratic contribution should be detectable for intermediate bias voltages $k_BT < \mathrm{eV} < E_m$.

IV. COMPARISON WITH EXPERIMENTS

A. Influence of the interfacial magnons

The results obtained above may qualitatively explain the recent results of Petit *et al.*[19] and Li *et al.*[20] In their experiment, Petit *et al.*[19] proposed an estimation of the linear contribution of the OP torque in AlO_x-based MTJ, at room temperature for $V \leq 300$ mV. During their analysis, all the quadratic contributions where regarded as due to the Joule effect and removed. In this case, the authors found that the linear component of the OP torque is about 20% of the IP torque.

Since the experiment is performed at room temperature it is difficult to deduce the electron-magnon efficiency from their data. However, from the discussion given in Sec. II, since the TMR is of the order of 20%, the corresponding effective polarization is about $P=40\%$ and the electron-magnon interaction efficiency should be around $Q \approx 15\%$ (this is also consistent with Ref. 6). Assuming that the junction is at $T=350$ K (Ref. 19), the IP torque is of the form $T_\parallel = \alpha G_0 VP \sin\theta$ [see Eq. (34)] where $\alpha \approx 1.5$. The spin-rotation angle can be estimated from the free-electron model and is of the order of $\phi \approx 0.3$. In this case, the ratio between the OP torque and the IP torque is $\Delta T_\perp^{sw}/T_\parallel \approx 7\%$. This value is consistent with the one measured by Petit *et al.*[19] The discrepancy may be due to uncertainty in the estimation of the critical current in Ref. 19 and to an underestimation of the spin rotation angle ϕ.

In contrast, Li *et al.*[20] studied the stability phase diagram of MgO-based MTJ applying large-current pulses. The junction consists of MgO-based MTJ, with TMR=146%, and a corresponding effective polarization of $P=77\%$. Assuming a linear IP torque, the authors find that the OP torque is antisymmetric in bias voltage, of the form $b_j \propto J|V|$. The authors argued that this form may arise from electron-magnon scattering present in the volume of the free layer.

Electron-magnon interaction at the interfaces does not generate such an antisymmetric quadratic bias dependence but rather a linear bias dependence $\propto |V|$. The ratio $\Delta T_\perp^{sw}/T_\parallel$ has been measured to be of the order of 100% at $V=1$ V.[20] Assuming that the electron-magnon interaction efficiency is about $Q \approx P^2 = 50\%$, we find that $\alpha \approx 3$. Assuming a spin-rotation angle of $\phi = 0.5$, we find that the ratio between the OP torque and the IP torque is $\Delta T_\perp^{sw}/T_\parallel \approx 20\%$.

These two estimations are approximative since we do not exactly know the magnitude of the electron-magnon interaction efficiency Q. The evaluation of the angle ϕ and the effective polarization P, which are crucial to estimate the effect of electron-magnon interaction on the OP torque, is convenient in the case of amorphous AlO_x-based MTJs (large-barrier height) but questionable in the case of crystalline transport occurring in MgO-based MTJs (low-barrier height). For example, assuming $\phi \approx 2$, one obtain $\Delta T_\perp^{sw}/T_\parallel \approx 80\%$, in agreement with Li *et al.*[20] measurements. In order to account for such large-angle ϕ, the present model should be extended to lower-barrier heights, using the general form of Eq. (14).

In spin-diode experiments,[15,16] the authors found that the out-of-plane torque follows the elastic quadratic bias dependence. Actually, as discussed above, at low-bias voltage, the linear contribution coming from the electron-magnon interaction is of the order of 5%–10% of the in-plane torque. This linear contribution may not be easily detectable using "spin-diode" experiments.[37]

Finally, the spin-transfer torque is a superposition between elastic and inelastic tunneling, and the resulting bias dependence should reflect the presence of both tunneling processes.

B. Interplay with other scattering sources

It is important to mention that the magnon bandwidth is usually on the order of 0.1 eV.[36] Thus, at higher bias voltages, multiple magnons scattering and other effects that we did not consider in the model, like the energy dependence of the interfacial densities of states, become important and should be taken into account for a comprehensive understanding and modeling of the spin transport in tunnel junctions.

In the previous section, we showed that the electron-magnon interaction at the interfaces of a magnetic tunnel junction can modify the bias dependence of the spin torque, and more specifically, induce an asymmetry in the torque as a function of the bias polarity. Note that other sources of such an asymmetry have been proposed. Within the framework of the transfer Hamiltonian formalism, Slonczewski and Sun[9] discussed the influence of an asymmetric concentration of defects or dopants at the interfaces of the barrier, as well as the presence of inelastic spin-conserving mechanisms within the barrier or within the electrodes. The author showed that these elastic and inelastic interactions may also lead to an asymmetry in the torque with the bias polarity. Wilczynski *et al.*[12] studied the case of ferromagnetic electrodes with different *s-d* exchange coupling. Similarly, they found that this asymmetry in the junction structure implies an asymmetry in the bias dependence of the spin torque with the voltage polarity.

At finite temperature, in addition to the magnon absorption at the interfaces of the barrier, we also expect an absorption of the magnons present in the bulk of the ferromagnet. This contribution has two major consequences on the spin transport. First, the spin torque is no more equal to the absorption of the interfacial transverse spin current, since a spin relaxation exists in the volume of the ferromagnet $[\Gamma(\mathbf{m})\neq 0]$. One should then calculate the resulting spin density and integrate it over the layer thickness. Second, the temperature dependence of this contribution will be different from the case of interfacial electron-magnon interaction, since their densities of states are different $(\int n_{\mathbf{q}}d^2\mathbf{q}\rightarrow\int n_{\mathbf{q}}d^3\mathbf{q})$. The absorption mechanism of bulk magnons will be proportional to $T^{3/2}$.

The electron-phonon interaction should also influence the spin-dependent transport.[27,38] Since this process is spin-independent, both majority and minority spin are scattered and relax their energy: this is equivalent to effectively decrease the bias voltage ($eV-\hbar\omega_{\mathbf{p}}$, $\hbar\omega_{\mathbf{p}}$ being the phonon energy) and therefore affects the electron-magnon scattering. Interestingly, in the presence of spin-orbit scattering the electron-phonon interaction becomes spin dependent and can contribute to spin relaxation, similarly to the electron-magnon interaction.[39] However, this is a second-order contribution (combination of spin-orbit and electron-phonon interaction) and the relaxation rate associated with this interaction ($\tau_{s-\text{ph}}\approx 100$ ps) is several orders of magnitude smaller than the electron-magnon interaction[22] ($\tau_{e-m}\approx 1$ fs).

Temperature-dependent measurements of the amplitude of both components of the spin-transfer torque would be of great interest in order to determine the relative contribution of elastic and inelastic spin torque at low and high bias, as well as to distinguish magnons contributions from impurities, disorder or structural asymmetries. An effective temperature of the order of 400–600 K, attributed to a self-heating in the junction, is often assumed to interpret the experimental data[40] and should significantly enhance the influence of interfacial magnons. Furthermore, as shown by Eqs. (34) and (35), at large-bias voltage, the IP torque increases with the temperature, whereas the OP torque decreases with it. These temperature-dependent measurements would bring new insights to estimate the actual influence of such effective temperature compared to the inelastic OP torque in the magnetization dynamics.

V. CONCLUSION

Based on the transfer-matrix approach, we analyzed the tunneling transport in a magnetic-tunnel junction in the presence of interfacial electron-magnon scattering. We first found that the previous assumed diagonal tunneling matrix must be modified in the noncollinear magnetization structure. Due to rotation of the reflected spin at the tunneling barrier, the spin direction in the barrier possesses a perpendicular component that must be described by a 2×2 spinor-transfer matrix. We have determined this matrix within a free-electron model.

We then showed that at zero temperature, the contribution of magnon scattering is essentially an antisymmetric quadratic component to the spin torque at low bias and a linear component at large bias. At finite temperature, this scattering adds both a linear and a quadratic component at low bias and a linear component a high-bias voltage. These components are antisymmetric as a function of the bias voltage for the in-plane torque and symmetric for the out-of-plane torque.

This inelastic contribution to the bias dependence of the spin-transfer torque can interpret the results obtained by Petit *et al.*[19] and by Li *et al.*[20] at large-bias voltage. The actual bias dependence of the spin-transfer torque in a magnetic-tunnel junction is then a superposition between elastic and temperature-dependent and independent inelastic tunnelings (impurities, disorder, interfacial, and bulk magnons, phonons). Finally, we suggest that temperature studies could provide interesting elements to better understand the role of electron-magnon scattering and the ratio between elastic and inelastic contributions.

ACKNOWLEDGMENTS

The authors acknowledge fruitful discussions with J. Z. Sun at IBM T. J. Watson Research Center and C. Wang at Cornell University. This work was supported by NSF (Contract No. DMR-0704182) and DOE (Contract No. DE-FG02-06ER46307).

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