# Magnetoconductivity of quantum dots with Rashba interaction

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We address the magnetoconductivity of a quantum dot with Rashba spin-orbit interaction within linear-response theory. As a consequence of the generalized Kohn's theorem, the magnetoconductivity of the dot is zero when the spin-orbit coupling is neglected. The inclusion of the spin-orbit interaction violates the mentioned theorem and gives rise to a nonzero magnetoconductivity. We derive a simple expression for this quantity valid up to the second order in the Rashba parameter. In the limit of vanishing lateral confinement, i.e., for a quantum well, a similar calculation yields the quantum Hall-effect result.

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## I. INTRODUCTION

The quantization of the transverse conductivity in a quantum well, i.e., the two-dimensional electron gas (2DEG) formed at the interface of a semiconductor heterostructure, namely, the quantum Hall effect, is one of the most remarkable discoveries of the last three decades.[1–5] The Hall conductivity in a 2DEG can be derived within the linear-response theory calculating the transverse current response of this system to a time-dependent in-plane electric field when it is simultaneously submitted to a perpendicular magnetic field and can in principle be extended to other confined electron systems such as quantum dots or quantum wires.

During the last few years, the effects of the spin-orbit (SO) coupling in semiconductor nanostructures have also been the object of an intensive study due to their potential applications in, e.g., spintronics and in quantum computation (see, e.g., Ref. 6 and references therein). In this sense, the Rashba SO contribution has received special attention since it was proven that its intensity can be externally tuned via the application of gate voltages.[7] For the particular case of quantum dots, this interaction has been considered in the description of the ground-state structure and the infrared response.[8]

In this work, we follow the approach indicated above to calculate the transverse magnetoconductivity of a parabolically confined GaAs quantum dot in the presence of the Rashba SO coupling. We show that when the Rashba term is neglected the magnetoconductivity of the dot is zero owing to the generalized Kohn's theorem. On the contrary, the violation of the latter when the SO interaction is taken into account yields a nonzero contribution to the magnetoconductivity that is given by a simple expression valid up to the second order in the Rashba parameter and in which the key quantity is the orbital angular momentum of the electrons in the ground-state configuration. For this system, the magnetoconductivity arises from the effect of the SO interaction on the cyclotron resonance, and it might be measured from far-infrared photoabsorption experiments as in quantum wells.[9]

This paper is organized as follows. In Sec. II we describe the method used to calculate the magnetoconductivity. The obtained results are presented in Sec. III, and some conclusions are drawn in Sec. IV. The calculation of the magnetoconductivity for a quantum well is presented in the Appendix.

## II. THEORETICAL APPROACH

We consider a two-dimensional quantum dot containing $N$ electrons with charge $-e$ and effective mass $m$ confined by a parabolic potential with frequency $\omega_0$. The system is submitted to a uniform magnetic field $B$ perpendicularly applied to the plane where the electrons move—the $x$-$y$ plane—that arises from the vector potential $\mathbf{A}=B(-y,x,0)/2$. Within linear-response theory, if $\mathcal{H}$ is the Hamiltonian of the system and an external time-dependent electric field $\mathcal{E}$ is considered to be applied along the $x$ direction, the transverse magnetoconductivity $\sigma_{yx}$ of the dot is given by[10,11]

$$\sigma_{yx}(\omega)=\frac{\langle j_y\rangle}{E}=\frac{i}{\omega}\sum_n\left(\frac{\langle 0|j_y|n\rangle\langle n|j_x|0\rangle}{\omega-\omega_{n0}}-\frac{\langle 0|j_x|n\rangle\langle n|j_y|0\rangle}{\omega+\omega_{n0}}\right), \tag{1}$$

where $|0\rangle$ and $|n\rangle$ are, respectively, the ground and excited states of $\mathcal{H}$ with corresponding excitation energies $\omega_{n0}$; $\omega$ is the frequency of the external electric field, and $j_q$ is the current in the system along the $q$ direction, with $q=x,y$ related to the velocity operator $v_q^k$ as

$$j_q=-e\sum_{k=1}^{N}v_q^k,\quad v_q^k=-i[q^k,\mathcal{H}]. \tag{2}$$

We have calculated the excited states $|n\rangle$ and energies $\omega_{n0}$ entering Eq. (1) with and without taking into account the spin-orbit coupling.

Without SO interaction, the electronic Hamiltonian (in $\hbar=1$ units) of the dot reads as

$$\mathcal{H}=\sum_{k=1}^{N}\left[\frac{\mathbf{P}_k^2}{2m}+\frac{1}{2}m\omega_0^2r_k^2+g^*\mu_BBs_k^z\right]+\sum_{k<j}^{N}V(|\mathbf{r}_k-\mathbf{r}_j|), \tag{3}$$

where $\mathbf{r}_k$, $\mathbf{P}_k=\mathbf{p}_k+\frac{e}{c}\mathbf{A}(\mathbf{r}_k)$ and $s_k^z$ are, respectively, the position vector, canonical momentum, and spin $z$ component of the $k$th electron; $g^*$ is the effective gyromagnetic factor and $\mu_B$ is the Bohr magneton. It can be checked that $\mathcal{H}$ satisfies the relation[10]

$$[\mathcal{H}, m\omega_0^2 Q_\pm - i\omega_\pm P_\pm] = \omega_\pm (m\omega_0^2 Q_\pm - i\omega_\pm P_\pm), \tag{4}$$

where $\omega_\pm = \sqrt{\omega_0^2 + \omega_c^2/4} \pm \omega_c/2$, $\omega_c = eB/mc$ is the cyclotron frequency, and where we have used the general notation for the operators $F_\pm \equiv F_x \pm iF_y$.

The result obtained from Eq. (4) is known as the generalized Kohn's theorem and it tells us that the states

$$\frac{1}{\sqrt{2m\Omega N}}\left(\frac{m\omega_0^2}{\omega_\pm} Q_\pm - iP_\pm\right)|0\rangle, \tag{5}$$

with $\Omega = \sqrt{4\omega_0^2 + \omega_c^2}$, are exact and normalized $N$-electron eigenstates of $\mathcal{H}$ with respective energies $E_0 + \omega_\pm$. These states are known as the bulk and edge magnetoplasmon resonances of the quantum dot and, according to Eq. (4), are the only states that can be excited in photoabsorption reactions with long-wavelength (far-infrared) photons. Such excitations have been observed in several experiments,[12] and it must be stressed that their frequencies are not affected by the electron-electron (e-e) interaction since $[V(|\mathbf{r}_i - \mathbf{r}_j|), m\omega_0^2 Q_\pm - i\omega_\pm P_\pm] = 0$.

When the spin-orbit interaction is included, the generalized Kohn's theorem is violated. Indeed, the inclusion of the Bychkov-Rashba[13] SO Hamiltonian

$$\mathcal{H}_R = \lambda_R \sum_{i=1}^{N} [P_y \sigma_x - P_x \sigma_y]_i, \tag{6}$$

in Eq. (4) yields

$$[\mathcal{H} + \mathcal{H}_R, m\omega_0^2 Q_\pm - i\omega_\pm P_\pm] = \omega_\pm (m\omega_0^2 Q_\pm - i\omega_\pm P_\pm) \pm \lambda_R m\left(\omega_0^2 + \frac{1}{2}\omega_c\omega_\pm\right) S_\pm, \tag{7}$$

where $S_\pm = \Sigma_{k=1}^{N} \sigma_\pm^k$. This expression shows that in the presence of SO terms, the bulk and edge magnetoplasmons given by Eq. (5) are mixed with the spin-flip excitations induced by the operators $S_\pm$ when acting on the ground state of the system. We take into account here this mixing when evaluating the transverse magnetoconductivity given by Eq. (1) by solving only the one-body part of the Hamiltonian and neglecting higher-order corrections to $\sigma_{yx}$ due to the two-body interaction. To simplify the expressions, in the following we shall use effective atomic units ($\hbar = m = 1$), except when giving the numerical values of the GaAs constants.

We therefore define the *single-particle* (sp) operators,

$$a^\pm = \frac{1}{\sqrt{2\Omega}}\left[P_\pm \pm \frac{i}{2}\Omega(1-\gamma) Q_\pm\right],$$

$$b^\pm = \frac{1}{\sqrt{2\Omega}}\left[P_\mp \pm \frac{i}{2}\Omega(1+\gamma) Q_\mp\right],$$

with $[a^-, a^+] = [b^-, b^+] = 1$, and $\gamma \equiv \omega_c/\Omega$. In terms of these operators, the sp Hamiltonian $h_0$ of the quantum dot including the Rashba interaction reads as

$$h_0/\Omega = \frac{1}{2} + \frac{1+\gamma}{2} a^+ a^- + \frac{1-\gamma}{2} b^+ b^- - \frac{1}{2}\frac{\omega_L}{\Omega}\sigma_z - \frac{1}{4} i\tilde{\lambda}_R [(1+\gamma) \times (a^+\sigma_- - a^-\sigma_+) + (1-\gamma)(b^-\sigma_- - b^+\sigma_+)], \tag{8}$$

where $\omega_L = |g^* \mu_B B|$ is the Larmor frequency, and $\tilde{\lambda}_R \equiv \lambda_R \sqrt{2/\Omega}$.

We have solved the Schrödinger equation $\frac{h_0}{\Omega}|\phi_j\rangle = \varepsilon_j |\phi_j\rangle$ where, in the presence of the Rashba coupling, $|\phi_j\rangle$ is a general sp state represented by a two-component spinor $|\phi_j\rangle \equiv \binom{\phi_1^j}{\phi_2^j}$. The components $\phi_1^j$ and $\phi_2^j$ are expanded into oscillator states $|n,m\rangle$ as $\phi_1^j = \Sigma_{n,m=0}^{\infty} a_{n,m}^j |n,m\rangle$ and $\phi_2^j = \Sigma_{n,m=0}^{\infty} b_{n,m}^j |n,m\rangle$, on which the operators $a^+$ and $a^-$, and $b^+$ and $b^-$ act in the usual way, namely, $a^+|n,m\rangle = \sqrt{n+1}|n+1,m\rangle$, $a^-|n,m\rangle = \sqrt{n}|n-1,m\rangle$, and $a^-|0,m\rangle = 0$; $b^+|n,m\rangle = \sqrt{m+1}|n,m+1\rangle$, $b^-|n,m\rangle = \sqrt{m}|n,m-1\rangle$, and $b^-|n,0\rangle = 0$. We also recall that the angular-momentum operator is given by $\hat{L} = a^+a^- - b^+b^-$ with eigenvalues $\hat{L}|n,m\rangle = (n-m)|n,m\rangle \equiv \ell_{n,m}|n,m\rangle$. Substituting $|\phi_j\rangle$ into the Schrödinger equation, one ends up with the following infinite system of equations for the coefficients $a_{n,m}^j$ and $b_{n,m}^j$ and energies $\varepsilon_j$ needed to compute the magnetoconductivity via Eq. (1):

$$\left[\frac{1+\gamma}{2} n + \frac{1-\gamma}{2} m + \alpha - \varepsilon_j\right] b_{n,m}^j - \frac{i}{2}\tilde{\lambda}_R [(1+\gamma)\sqrt{n}\, a_{n-1,m}^j + (1-\gamma)\sqrt{m+1}\, a_{n,m+1}^j] = 0,$$

$$\left[\frac{1+\gamma}{2} n + \frac{1-\gamma}{2} m + \beta - \varepsilon_j\right] a_{n,m}^j + \frac{i}{2}\tilde{\lambda}_R [(1+\gamma)\sqrt{n+1}\, b_{n+1,m}^j + (1-\gamma)\sqrt{m}\, b_{n,m-1}^j] = 0, \tag{9}$$

with $n$, $m \geq 0$, $a_{-1,m}^j = a_{n,-1}^j = b_{-1,m}^j = b_{n,-1}^j = 0$, and $\alpha \equiv (1 + \omega_L/\Omega)/2$, $\beta \equiv (1 - \omega_L/\Omega)/2$. We present in Sec. III an approximate but very accurate analytical solution of Eq. (9) that is valid up to the second order in $\lambda_R^2$.

## III. RESULTS

When the Rashba interaction is not included ($\lambda_R = 0$) from the generalized Kohn's theorem, we know that only the states given by Eq. (5) can enter Eq. (1). This yields for the real part of $\sigma_{yx}$ per unit of surface,

$$\frac{\mathrm{Re}[\sigma_{yx}(\omega)]}{L^2} = -\frac{e^2\rho}{2\Omega}\mathcal{P}\left(\frac{\omega_+^2}{\omega^2 - \omega_+^2} - \frac{\omega_-^2}{\omega^2 - \omega_-^2}\right), \tag{10}$$

where $\rho$ is the electron areal density and $\mathcal{P}$ means the principal value. From this expression, one can draw the following conclusions. (i) In the direct-current (DC) limit, i.e., when $\omega=0$, the transverse magnetoconductivity of the quantum dot vanishes since the contributions in Eq. (10) coming from the two magnetoplasmons with energies $\omega_{\pm}$ cancel each other. (ii) In the limit of vanishing lateral confinement $\omega_0=0$—i.e., when the system is a quantum well—one gets $\mathrm{Re}[\sigma_{yx}(\omega)]/L^2=-e^2\rho\omega_c\mathcal{P}(\frac{1}{\omega^2-\omega_c^2})$. Thus, if $\nu$ Landau levels are filled in the system with electron density $\rho=\nu eB/ch$, in the DC regime one obtains the well-known expression of the quantum Hall effect, namely, $\mathrm{Re}[\sigma_{yx}]/L^2=ec\rho/B=\nu e^2/h$.

Let us now calculate the magnetoconductivity of the quantum dot including the Rashba SO coupling following the method indicated in Sec. II. For the GaAs, one has $\lambda_R\sim 10^{-11}$ eV m ($\sim$0.1 effective a.u.) and $m=0.067m_e$, $m_e$ being the electron mass. This yields $\lambda_R^2 m/\hbar^2 \lesssim 100$ $\mu$eV, whereas the confinement energy $\hbar\omega_0$ is often on the order of several meV. Therefore, for this material, even when $B=0$ ($\Omega=\omega_0$), one can safely take the limit $\lambda_R^2/\Omega \ll 1$ in Eq. (9). This allows one to obtain an approximate solution for $a^j_{n,m}$, $b^j_{n,m}$, and $\varepsilon_j$ by neglecting in these equations all the terms except those which couple, through the SO interaction, each level $|n,m\rangle$ to $|n\pm 1,m\rangle$ or $|n,m\pm 1\rangle$. Since the coupling to all the other levels is of order $(\lambda_R^2/\Omega)^2$ or higher, our solution is valid up to order $\lambda_R^2/\Omega$.

Defining $\gamma_{\pm}\equiv(1\pm\gamma)/2$ and labeling the sp energies $\varepsilon_j$ and spinors $|\phi_j\rangle$, respectively, as $\varepsilon^{\pi}_{n,m}$ and $|\phi^{\pi}_{n,m}\rangle$ with $\pi=u,d$, one then obtains[10]

$$\varepsilon^d_{n,m}=\gamma_+ n+\gamma_- m+\alpha+2\lambda_R^2\left[\frac{\gamma_+^2 n}{\gamma_+\Omega+\omega_L}-\frac{\gamma_-^2(m+1)}{\gamma_-\Omega-\omega_L}\right], \tag{11}$$

which corresponds to the spinor

$$|\phi^d_{n,m}\rangle=\begin{pmatrix} a^{\varepsilon^d_{n,m}}_{n-1,m}|n-1,m\rangle+a^{\varepsilon^d_{n,m}}_{n,m+1}|n,m+1\rangle \\ b^{\varepsilon^d_{n,m}}_{n,m}|n,m\rangle \end{pmatrix},$$

where the coefficients are given by

$$a^{\varepsilon^d_{n,m}}_{n-1,m}=i\tilde{\lambda}_R\frac{\gamma_+\Omega\sqrt{n}}{\gamma_+\Omega+\omega_L},$$

$$a^{\varepsilon^d_{n,m}}_{n,m+1}=-i\tilde{\lambda}_R\frac{\gamma_-\Omega\sqrt{m+1}}{\gamma_-\Omega-\omega_L},$$

$$b^{\varepsilon^d_{n,m}}_{n,m}=1-\frac{1}{2}\tilde{\lambda}_R^2\Omega^2\left[\frac{\gamma_+^2 n}{(\gamma_+\Omega+\omega_L)^2}+\frac{\gamma_-^2(m+1)}{(\gamma_-\Omega-\omega_L)^2}\right],$$

and

$$\varepsilon^u_{n,m}=\gamma_+ n+\gamma_- m+\beta-2\lambda_R^2\left[\frac{\gamma_+^2(n+1)}{\gamma_+\Omega+\omega_L}-\frac{\gamma_-^2 m}{\gamma_-\Omega-\omega_L}\right], \tag{12}$$

corresponding to the spinor

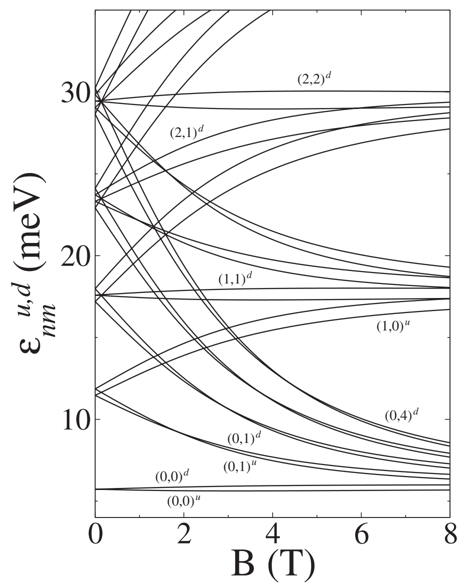


FIG. 1. Single-particle energies (meV) as a function of $B$ (T) for a quantum dot with $\hbar\omega_0=3.5$ meV and $\lambda_R^2 m/\hbar^2=0.12$ meV. For the sake of clarity, only a few states $|\phi^{\pi}_{n,m}\rangle$ are indicated.

$$|\phi^u_{n,m}\rangle=\begin{pmatrix} a^{\varepsilon^u_{n,m}}_{n,m}|n,m\rangle \\ b^{\varepsilon^u_{n,m}}_{n+1,m}|n+1,m\rangle+b^{\varepsilon^u_{n,m}}_{n,m-1}|n,m-1\rangle \end{pmatrix},$$

where

$$b^{\varepsilon^u_{n,m}}_{n+1,m}=i\tilde{\lambda}_R\frac{\gamma_+\Omega\sqrt{n+1}}{\gamma_+\Omega+\omega_L},$$

$$b^{\varepsilon^u_{n,m}}_{n,m-1}=-i\tilde{\lambda}_R\frac{\gamma_-\Omega\sqrt{m}}{\gamma_-\Omega-\omega_L},$$

$$a^{\varepsilon^u_{n,m}}_{n,m}=1-\frac{1}{2}\tilde{\lambda}_R^2\Omega^2\left[\frac{\gamma_+^2(n+1)}{(\gamma_+\Omega+\omega_L)^2}+\frac{\gamma_-^2 m}{(\gamma_-\Omega-\omega_L)^2}\right].$$

Here, the labels $u$ and $d$ refer to what are often called quasi-spin-up and quasi-spin-down states since in the $\lambda_R=0$ limit $|\phi^u_{n,m}\rangle$ and $|\phi^d_{n,m}\rangle$ become, respectively, $|n,m\rangle\binom{1}{0}$ and $|n,m\rangle\binom{0}{1}$.

We plot in Fig. 1 the single-particle energies given by Eqs. (11) and (12) as a function of $B$ up to principal quantum number $M\equiv n+m=4$ for a GaAs quantum dot with $\hbar\omega_0=3.5$ meV and $\lambda_R^2 m/\hbar^2=0.12$ meV. Each of the sets of lines that tend to converge at $B=0$ corresponds to one value of $M$, from 0 to 4 in increasing energy. The upward, downward, and nearly flat tendencies of the sp states as the magnetic field increases depend on whether the corresponding angular momentum $\ell_{n,m}$ is positive, negative, or null (when $n=m$), respectively. For high values of $B$, the lowest-energy state of each pair is always the quasi-spin-up one, and one can see that in the quantum-well limit, i.e., when $\Omega\sim\omega_c$, the states with the same $n$ (and $m=0,1,2,\ldots$) show a tendency to collapse giving rise to the well-known Landau-band structure. Also, the lifting of the degeneracy of the sp levels by the Rashba interaction can be observed by comparing their energies to the ones obtained when neither the SO coupling nor the Zeeman spin splitting is considered (see, e.g., Fig. 3.2 of Ref. 14).

The accuracy of this analytical solution has been assessed by comparing it with an exact numerical one obtained by direct diagonalization of the Hamiltonian $h_0$. A similar agreement has been found for the quantum-well case when both Rashba and Dresselhaus SO interactions are simultaneously taken into account.[15] One should keep in mind, however, that the e-e interaction, and in particular the exchange-correlation contribution, might alter the ordering of the occupied sp levels. This effect could be especially important at high values of $B$ because of its competition with the Zeeman term.

We have next calculated the DC magnetoconductivity of the dot when $\lambda_R \neq 0$ from the relation

$$\sigma_{yx}(0) = -2ie^2 \sum_n \langle 0|\sum_{k=1}^{N} y_k|n\rangle\langle n|\sum_{j=1}^{N} x_j|0\rangle, \qquad (13)$$

which is obtained rewriting Eq. (1) as $\sigma_{yx}(\omega) = 2i\sum_n \frac{\langle 0|j_y|n\rangle\langle n|j_x|0\rangle}{\omega^2-\omega_{n0}^2}$, taking the limit $\omega=0$ and using Eq. (2), where now $v_q^k = -i[q^k, h_0]$. Equation (13) can be rewritten in terms of the sp states as

$$\sigma_{yx}(0) = -2ie^2 \sum_{\substack{n,m,\pi \\ n',m',\pi'}} \langle \phi^{\pi}_{n,m}|y|\phi^{\pi'}_{n',m'}\rangle\langle \phi^{\pi'}_{n',m'}|x|\phi^{\pi}_{n,m}\rangle, \qquad (14)$$

where $|\phi^{\pi}_{n,m}\rangle$ and $|\phi^{\pi'}_{n',m'}\rangle$ are, respectively, occupied and empty sp states. Expressing the operators $x$ and $y$ in terms of the creation and annihilation operators defined in Eq. (8), the real part of the magnetoconductivity reads as

$$\mathrm{Re}[\sigma_{yx}(0)] = \frac{e^2}{\Omega} \sum_{\substack{n,m,\pi \\ n',m',\pi'}} \langle \phi^{\pi}_{n,m}|(a^+ + a^-) - (b^+ + b^-)|\phi^{\pi'}_{n',m'}\rangle \times \langle \phi^{\pi'}_{n',m'}|(a^+ - a^-) - (b^+ - b^-)|\phi^{\pi}_{n,m}\rangle. \qquad (15)$$

After a straightforward but very tedious calculation, one finally gets

$$\mathrm{Re}[\sigma_{yx}(0)] = -4e^2\lambda_R^2 \frac{\gamma_+\gamma_-}{(\gamma_+\Omega + \omega_L)(\gamma_-\Omega - \omega_L)} \sum_{n,m} (n-m). \qquad (16)$$

This equation is the main result of this work. In it, the sum runs only over the occupied sp states that can be connected with an empty one such that $\Delta n = n' - n = 1$ or $\Delta m = m' - m = 1$. Thus, from Fig. 1, if one takes, e.g., $N=4$ and $B=1$ T, the states contributing to $\sigma_{yx}(0)$ are the $|\phi^{u,d}_{0,0}\rangle$ and $|\phi^{u,d}_{0,1}\rangle$ ones, which can be connected to $|\phi^{u,d}_{1,0}\rangle$ and $|\phi^{u,d}_{0,2}\rangle$, respectively, yielding $\Sigma_{n,m}(n-m) = -2$.

We have also used this approach in the Appendix to obtain the magnetoconductivity of a quantum well. In this case, the calculation is much easier to perform and shows that up to order $\lambda_R^2$, the Rashba SO interaction does not contribute to the quantum Hall effect. This is in agreement with the findings of Ref. 16.

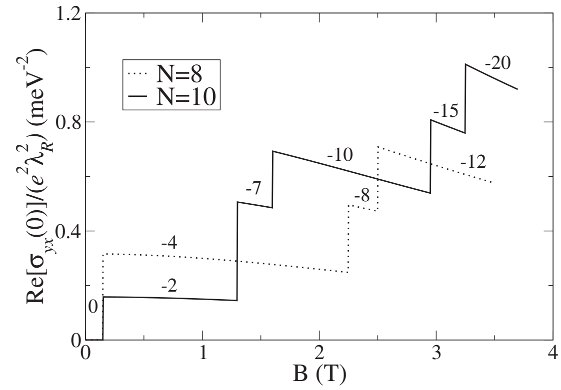


FIG. 2. Magnetoconductivity for the quantum dot of Fig. 1 with $N=8$ and 10 as a function of the magnetic field. The numbers indicate the corresponding value of the quantity $\Sigma_{n,m}(n-m)$.

Figure 2 shows the magnetoconductivity of dots hosting $N=8$ and 10 electrons. One can see that it displays jumps at the magnetic fields for which the value of the sum $\Sigma_{n,m}\ell_{n,m}$ changes due to the crossings[17] of the sp levels (see Fig. 1). Notice that the mentioned possible effect of the e-e interaction on the single-particle levels could also change the values of the magnetoconductivity. At low magnetic fields, however, the energy spectrum is more realistically described by the one-body Hamiltonian and so are the obtained values for $\sigma_{yx}(0)$. One might also expect a slight smearing of the plateaus as an effect of the experimental temperature, similar, e.g., to the thermal smearing of the conductance plateaus of a quantum wire.[18] Finally, we want to stress that contrarily to other observables for which the SO contribution is just a small correction, often proportional to $\lambda_R^2$, Eq. (16) represents a purely spin-orbit effect. Therefore, its experimental observation might provide a direct measurement of the SO strength, which is usually difficult to determine since it depends not only on the electric fields inside the nanostructure but also on the boundary conditions at the interfaces.[19] Although the DC magnetoconductivity is an intrinsic property of the system, its experimental determination would necessarily require a weak coupling of the dot to the environment. Such weak coupling could be achieved by means of external gate contacts, similarly as done in quantum Hall measurements.

## IV. CONCLUSIONS

Within the linear-response theory, we have calculated the transverse magnetoconductivity of a quantum dot with parabolic confinement in the presence of the Rashba spin-orbit coupling. When the intensity of the Rashba SO interaction is set to zero, there is no magnetoconductivity due to Kohn's theorem; whereas when the Rashba coupling is taken into account the theorem is violated and the magnetoconductivity of the dot turns out to be proportional to the square of the Rashba parameter. For a quantum well, the method we have used yields the known result of the quantum Hall effect with no correction to the second order in the Rashba intensity parameter.


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## APPENDIX

In this appendix, we carry out the calculation of the magnetoconductivity of a quantum well following the analytical approach described in Sec. II. The starting point is Eq. (14). In this case, we work in the Landau gauge, namely, $\mathbf{A} = B(0,x,0)$ and, in analogy with the quantum dot case, we label the sp states as $|\phi_n^{\pi}\rangle$, which are expanded in terms of one-dimensional(1D)-oscillator states $|n\rangle$. We suppose that all the electrons are in the quasi-spin-up state, although the same result is found assuming that they fill the quasi-spin-down state. Therefore, the quantities to be evaluated are $\langle\phi_n^u|q|\phi_{n'}^{\pi'}\rangle$, with $\pi'=u,d$. We first recall the commutation relations $[q,P_q]=i$, which for, e.g., $q=x$, yield

$$\langle n|[x,P_x]|n'\rangle = \sum_l (X_{n,l}P_x^{l,n'} - P_x^{n,l}X_{l,n'}) = i\delta_{n,n'}, \quad \text{(A1)}$$

where $X_{n,l} \equiv \langle n|x|l\rangle$ and $P_x^{n,l} \equiv \langle n|P_x|l\rangle$. Writing the $P_\pm$ operators defined in Sec. II as function of the creation/annihilation operators[15] ($P_\pm = \sqrt{2\omega_c}a^\pm$), one has

$$P_x^{n,l} = \langle n|\frac{1}{2}(P_+ + P_-)|l\rangle = \frac{1}{2}[\sqrt{2\omega_c(n+1)}\delta_{n,l+1} + \sqrt{2\omega_c l}\delta_{n,l-1}], \quad \text{(A2)}$$

which, using the property $X_{n,l}=X_{l,n}^*$, yields

$$\sqrt{2\omega_c(n+1)}(X_{n,n+1} - X_{n,n+1}^*) + \sqrt{2\omega_c n}(X_{n,n-1} - X_{n,n-1}^*) = 2i. \quad \text{(A3)}$$

From Eq. (A3), and since the $P_x^{n,l}$ are real, it is easy to check that the non-null matrix elements are such that $X_{n-1,n} = -X_{n,n-1} = i\sqrt{n/(2\omega_c)}$. Analogously, for $q=y$, one finds $Y_{n-1,n} = Y_{n,n-1} = -\sqrt{n/(2\omega_c)}$.

If only the Rashba SO coupling is considered, the quasi-spin-up and quasi-spin-down sp states of the quantum well can be exactly written as[15]

$$|\phi_n^d\rangle = \begin{pmatrix} a_{n-1}^d|n-1\rangle \\ b_n^d|n\rangle \end{pmatrix}, \quad |\phi_n^u\rangle = \begin{pmatrix} a_n^u|n\rangle \\ b_{n+1}^u|n+1\rangle \end{pmatrix}, \quad \text{(A4)}$$

where the coefficients are given by

$$a_{n-1}^d = i\tilde{\lambda}_R\sqrt{n}\Lambda, \quad a_n^u = 1 - \frac{1}{2}\tilde{\lambda}_R^2(n+1)\Lambda^2,$$

$$b_n^d = 1 - \frac{1}{2}\tilde{\lambda}_R^2 n\Lambda^2, \quad b_{n+1}^u = i\tilde{\lambda}_R\sqrt{n+1}\Lambda, \quad \text{(A5)}$$

with $\Lambda \equiv \omega_c/(\omega_c+\omega_L)$.

The contribution to $\sigma_{yx}$ coming from the state $|\phi_{n'}^u\rangle$ is

$$\sum_{n'} \langle\phi_n^u|y|\phi_{n'}^u\rangle\langle\phi_{n'}^u|x|\phi_n^u\rangle = \sum_{n'} \{|a_n^u|^2|a_{n'}^u|^2 Y_{n,n'}X_{n',n} + a_n^{u*}a_{n'}^u b_{n'+1}^{u*}b_{n+1}^u Y_{n,n'}X_{n'+1,n+1} + b_{n+1}^{u*}b_{n'+1}^u a_{n'}^{u*}a_n^u Y_{n+1,n'+1}X_{n',n} + |b_{n+1}^u|^2|b_{n'+1}^u|^2 Y_{n+1,n'+1}X_{n'+1,n+1}\}. \quad \text{(A6)}$$

From the relations (A5) and the expressions for the matrix elements $X_{n,l}$ and $Y_{n,l}$ (which limit the sum to the cases $n'=n\pm1$), to order $\lambda_R^2$, one has

$$\sum_{n'} \langle\phi_n^u|y|\phi_{n'}^u\rangle\langle\phi_{n'}^u|x|\phi_n^u\rangle = \frac{i}{2\omega_c}(1+\tilde{\lambda}_R^2\Omega^2). \quad \text{(A7)}$$

Analogously, one can find that the contribution coming from the state $|\phi_{n'}^d\rangle$ is

$$\sum_{n'} \langle\phi_n^u|y|\phi_{n'}^d\rangle\langle\phi_{n'}^d|x|\phi_n^u\rangle = \sum_{n'} \{|a_n^u|^2|a_{n'-1}^d|^2 Y_{n,n'-1}X_{n'-1,n} + a_n^{u*}b_{n+1}^u a_{n'-1}^d b_{n'}^{d*} Y_{n,n'-1}X_{n',n+1} + b_{n+1}^{u*}a_n^u b_{n'}^d a_{n'-1}^{d*} Y_{n+1,n'}X_{n'-1,n} + |b_{n+1}^u|^2|b_{n'}^d|^2 Y_{n+1,n'}X_{n',n+1}\}. \quad \text{(A8)}$$

Since in this case only $n'=n$ and $n'=n+2$ contribute, it reduces to

$$\sum_{n'} \langle\phi_n^u|y|\phi_{n'}^d\rangle\langle\phi_{n'}^d|x|\phi_n^u\rangle = -\frac{i}{2\omega_c}\tilde{\lambda}_R^2\Omega^2. \quad \text{(A9)}$$

Recalling the relation between the density $\rho=N/L^2$ and the filling factor $\nu$, namely, $\rho=\nu eB/ch$, one finally obtains

$$\frac{\sigma_{yx}(0)}{L^2} = -\frac{2ie^2}{L^2}\sum_{n',\pi'} \langle\phi_n^u|y|\phi_{n'}^{\pi'}\rangle\langle\phi_{n'}^{\pi'}|x|\phi_n^u\rangle = \frac{e^2N}{L^2\omega_c} = \nu\frac{e^2}{h}, \quad \text{(A10)}$$

which is the usual expression of the quantum Hall effect for the 2DEG.

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