

Domain walls in a tetragonal chiral p -wave superconductor

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Domain walls in a tetragonal chiral p -wave superconductors with broken time-reversal symmetry are analyzed in the framework of the Ginzburg-Landau theory. The energy and the jump of the magnetic induction on the wall were determined for different types of walls as functions of the parameters of the Ginzburg-Landau theory and orientation of the domain wall with respect to the crystallographic axes. We discuss possible implications of the analysis for Sr_2RuO_4 where no stray magnetic fields from domain walls were detected experimentally.

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I. INTRODUCTION

There were a number experimental evidences of the broken time-reversal symmetry (TRS) in the unconventional superconductor Sr_2RuO_4 .¹⁻³ It was suggested that this phenomenon was connected with the p -wave Cooper pairing with the wave function in the momentum space proportional to $p_x + ip_y$ (chiral p -wave superconductivity).^{4,5} In chiral p -wave superconductors the spontaneous magnetic flux must be present near domain walls (DWs) or sample boundaries. However, Kirtley *et al.*⁶ have not detected any stray magnetic fields, which the spontaneous magnetic flux should produce above the sample surface. This put in question the scenario of the chiral p -wave pairing and stimulated theoretical investigations of the problem. In particular, the relation between the stray fields and the magnetic flux, which appears near the DW in the bulk, was derived.^{7,8} Another challenge for the theory was to find the spontaneous magnetic flux itself. It is determined by the product of the London penetration depth and the jump of the magnetic induction on the domain wall. Although it is known that the latter is on the order of the first critical magnetic field H_{c1} and was already analyzed in the past for particular types of the DWs in chiral superconductors,⁹⁻¹¹ the reliable theory of magnetic jump at the interface boundary in unconventional superconductor, in particular the dependence of the flux on orientations of the domain wall, is still lacking.

There are two possible explanations why the experiment could not detect the stray fields above the sample surface. The first one is that there is a domain structure with a period so small that stray fields decay very fast in space at a distance small compared to the distance of the experimental probe from the sample surface. There were some experimental evidences of domain structure in Sr_2RuO_4 ,^{12,13} although Xia *et al.*³ did not reveal any domain structure at studying the Kerr effect. The theory predicts that in superconductors with broken TRS usual ferromagnetic domains, which depend on the size and the shape of the sample (extrinsic domains), cannot appear,¹⁴ but another type of domains, which decrease the bulk magnetostatic energy at the expense of destroying the Meissner state, becomes possible.^{15,16} The size of these domains is roughly on the order of the London penetration depth λ and does not depend on either shape or

size of the sample (intrinsic domains; see discussion in Ref. 17). The analysis of intrinsic domain structure in superconductors with broken TRS was recently extended on the case of finite external magnetic fields, and the equilibrium magnetization curves in the state with intrinsic domains (cryptoferromagnetic state) were found theoretically.¹⁸ The intrinsic domain structure may lead to serious suppression of stray fields above the sample, although definite quantitative conclusions are not yet possible because of the absence of essential information on Sr_2RuO_4 (magnetic crystal anisotropy, e.g.). Therefore, the other possible explanation of the negative result of Kirtley *et al.*⁶ must be also considered: a very small spontaneous magnetic flux penetrating along DWs in Sr_2RuO_4 . This problem is investigated in the present work.

The magnetic flux near DWs originates from the intrinsic orbital moment of Cooper pairs (orbital ferromagnetism). As was demonstrated in Ref. 19, in the case of orbital ferromagnetism, one should not use the Landau-Lifshitz theory²⁰ since in this case one cannot define local spontaneous magnetization. Therefore one should rely on magnetization currents generated by the orbital moment of Cooper pairs, which cannot be reduced to the curl of any vector moment. The total magnetization current along the DW leads to the jump of the magnetic induction on the DW, which eventually determines the magnetic flux connected with the DW. In the present work we extend previous theoretical investigations of the magnetic flux around DWs in tetragonal chiral p -wave superconductors analyzing conditions for appearance of different types of DW structure depending on the strength of crystal in-plane anisotropy and DW orientation with respect to crystallographic axes. The analysis was done using the Ginzburg-Landau (GL) theory, and the magnetic induction near DWs depends on the parameters of the GL theory. For the parameters obtained in the weak-coupling limit the magnetic induction near DWs is scaled by the first critical magnetic field H_{c1} differing from the latter by a numerical factor. But for other values of the GL parameters the magnetic induction near DWs can be much smaller and even become negative with respect to the intrinsic magnetic moment of Cooper pairs. This stresses again that the orbital moment of the Cooper pair in the chiral p -wave state does not lead to any definite local magnetization.

The practical outcome of the presented analysis is that one can explain very weak stray fields from DWs if the GL

parameters differ from those calculated in the weak-coupling limit. The final conclusion on the reason for the absence of detectable stray fields around Sr_2RuO_4 samples is possible only after these parameters are found from the experiment or from the strong-coupling theory.

II. MODEL

The unconventional superconductor Sr_2RuO_4 belongs to the tetragonal crystallographic symmetry group D_{4h} . Considering the p -wave state for this material they usually believe that strong crystal anisotropy keeps both spin and orbital momentum of the Cooper pair parallel to the c axis. Then the p -wave state corresponds to the two-dimensional representation $\Gamma_5^- = \{k_x \hat{z}, k_y \hat{z}\}$, and the order parameter $(\eta_x k_x + \eta_y k_y) \hat{z}$ is fully described by a complex two-component vector $\eta = (\eta_x, \eta_y)$.^{4,5} Then the GL free-energy density is

$$\begin{aligned} \mathcal{F} = & P_1 |\eta|^2 + \beta_1 |\eta|^4 + \beta_2 (\eta_x^* \eta_y - \eta_x \eta_y^*)^2 + \beta_3 (|\eta_x|^2 |\eta_y|^2 \\ & + K_1 |\mathbf{D}_{x,y} \cdot \eta|^2 + K_2 (|D_x \eta_y|^2 + |D_y \eta_x|^2) \\ & + K_3 [(D_x \eta_x)^* (D_y \eta_y) + (D_x \eta_x) (D_y \eta_y)^*] \\ & + K_4 [(D_x \eta_y)^* (D_y \eta_x) + (D_x \eta_y) (D_y \eta_x)^*] \\ & + K_5 (|D_z \eta_x|^2 + |D_z \eta_y|^2). \end{aligned} \quad (1)$$

Here $D_i = \partial_i - i(2e/c\hbar)A_i$ is a covariant derivative and $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic induction. In the case of full axial symmetry in the plane (cylindrical Fermi surface) $\beta_3 = 0$ and $K_1 = K_2 + K_3 + K_4$. In the Bardeen-Cooper-Schrieffer (BCS) theory (weak-coupling limit) $K_1/3 = K_2 = K_3 = K_4$.²¹

The complex components of the two-component order parameter η can be represented in a form

$$\begin{aligned} \eta_x &= \eta_0 \cos \Theta \exp(i\phi), \\ \eta_y &= \eta_0 \sin \Theta \exp[i(\phi + \zeta)], \end{aligned} \quad (2)$$

where angles (phases) Θ , ϕ , and ζ are coordinate-dependent functions in general, and $\eta_0 = |\eta|$. The focus of the present study is the chiral state with the twofold-degenerate ground state $\eta = 2^{-1/2} \eta_0 \exp(i\phi)(1, \pm i)$, which corresponds to $\Theta = \pi/4$ and $\zeta = \pm \pi/2$ and is realized if $\beta_2 > 0$ and $\beta_1 > \beta_2 - \beta_3/4 > 0$. The order-parameter amplitude

$$\eta_0^2 = \frac{2|P_1|}{4(\beta_1 - \beta_2) + \beta_3} \quad (3)$$

is determined by minimization of the GL free energy. The chiral state has a nonzero orbital moment $\mathbf{I} = i\eta \times \eta^* / \eta_0^2 = l_z \hat{z}$ with the only component

$$l_z = i(\eta_x \eta_y^* - \eta_y \eta_x^*) / \eta_0^2 = \sin 2\Theta \sin \zeta. \quad (4)$$

Other components of the orbital moment are suppressed by strong crystal anisotropy. Although they are necessary for orbital magnons,¹⁹ they can be ignored in our analysis of DWs.

Substituting the general expression for the two-component order parameter [Eq. (2)] into the free-energy density \mathcal{F} [Eq. (1)], one can derive the expression for the superconducting current as

$$\mathbf{j} = \frac{e}{m} \partial_{\mathbf{v}} \mathcal{F} = \mathbf{j}^{(tr)} + \mathbf{j}^{(m)}, \quad (5)$$

which consists of two contributions. The transport current $\mathbf{j}^{(tr)} = e\hat{n}\mathbf{v}$ is determined by the gauge-invariant superfluid velocity

$$\mathbf{v} = \frac{\hbar}{2m} \nabla \phi - \frac{e}{mc} \mathbf{A}, \quad (6)$$

where \hat{n} is the superfluid electron-density matrix with the components

$$\begin{aligned} n_{xx} &= \frac{8m}{\hbar^2} \eta_0^2 (K_1 \cos^2 \Theta + K_2 \sin^2 \Theta), \\ n_{yy} &= \frac{8m}{\hbar^2} \eta_0^2 (K_1 \sin^2 \Theta + K_2 \cos^2 \Theta), \\ n_{xy} = n_{yx} &= \frac{4m}{\hbar^2} \eta_0^2 (K_3 + K_4) \sin 2\Theta \cos \zeta, \\ n_{zz} &= \frac{8m}{\hbar^2} \eta_0^2 K_5. \end{aligned} \quad (7)$$

The second contribution to the current is the magnetization current with the components

$$\begin{aligned} j_x^{(m)} &= \frac{2e}{\hbar} \eta_0^2 [2K_2 \sin^2 \Theta \partial_x \zeta + 2(K_3 \cos^2 \Theta \\ & + K_4 \sin^2 \Theta) \sin \zeta \partial_y \Theta + K_3 \sin 2\Theta \cos \zeta \partial_y \zeta], \\ j_y^{(m)} &= \frac{2e}{\hbar} \eta_0^2 [2K_1 \sin^2 \Theta \partial_y \zeta + 2(K_3 \sin^2 \Theta \\ & + K_4 \cos^2 \Theta) \sin \zeta \partial_x \Theta + K_4 \sin 2\Theta \cos \zeta \partial_x \zeta], \\ j_z^{(m)} &= \frac{4e}{\hbar} \eta_0^2 K_5 \sin^2 \Theta \partial_z \zeta. \end{aligned} \quad (8)$$

In the following we shall neglect the terms $\propto K_5$ considering only DWs parallel to the axis z when parameters do not vary along this axis. We also assume that the London penetration depth λ is much larger than the thickness of the DW, which is on the order of the coherence length ξ . This allows us to ignore Meissner currents studying currents inside the DW.

Inside domains $\Theta = \pi/4$ and $n_{xx} = n_{yy} = (4m/\hbar^2) \eta_0^2 (K_1 + K_2)$. Then the London penetration depth and the first critical magnetic field are

$$\lambda = \sqrt{\frac{\hbar^2 c^2}{16\pi e^2 \eta_0^2 (K_1 + K_2)}}, \quad H_{c1} = \frac{4\pi e}{\hbar c} \eta_0^2 (K_1 + K_2) \ln \frac{\lambda}{\xi}. \quad (9)$$

The magnetization current cannot be presented as $\mathbf{j}^{(m)} = (1/c) \nabla \times \mathbf{M}$, which is the main assumption of the Landau-Lifshitz theory. Therefore for chiral superconductors the Landau-Lifshitz theory is not valid, and the local magnetization \mathbf{M} cannot be defined.¹⁹ Whereas in the Landau-Lifshitz

theory the jump of the tangential component of the magnetic induction on the DW is given by the universal relation $\delta B = 8\pi M$ independently of the DW structure, in chiral superconductors δB does depend on the DW type and on the DW orientation relative to crystallographic axes. This will be demonstrated in Sec. III.

III. DOMAIN WALLS

The two degenerate ground states correspond to two possible directions of the orbital moment $l_z = \pm 1$ parallel or antiparallel to the c axis (the axis z). This leads to the existence of domains with $l_z = \pm 1$ separated by domain walls. Two types of DWs are known. The DW of type I is characterized by a gradual change in the phase $\zeta = \zeta(\mathbf{r})$ at constant $\Theta = \pi/4$, i.e., the absolute values of the components $|\eta_x| = |\eta_y|$ remain constant inside the DW.⁹ We address this type of the DW as ζ wall. In the DW of type II the relative phase $\zeta = -\pi/2$ remains fixed, and the transition from $l_z = -1$ to $l_z = +1$ is realized via variation of the angle Θ , so that in the center of the DW one of the components η_x or η_y vanishes. This type of DW was considered by Sigrist *et al.*¹⁰ assuming that the component of η , which does not vanish, remains constant. In the present work we assume that inside the DW the order parameter η rotates in space at fixed order-parameter modulus $|\eta| = \eta_0$. This assumption is justified if $\beta_1 \gg \beta_2, \beta_3$. Further we address this type of the DW as Θ wall. However, the difference between the two types of DW depends on a choice of the coordinate system: rotating the in-plane coordinate frame xy , in which the components of η are defined, through the angle $\pi/4$ (45°), the ζ wall becomes a Θ wall and vice versa.

A. Variation of relative phase: ζ wall

Let us consider a DW, which is characterized by continuous variation of the relative phase $\zeta = \zeta(\mathbf{r})$ at fixed angle $\Theta = \pi/4$. The transformation of the $l_z = -1$ domain to the $l_z = +1$ domain is possible via counterclockwise rotation of ζ from $-\pi/2$ to $\pi/2$ or clockwise rotation of ζ from $-\pi/2$ to $-3\pi/2$. The structure of this DW, its surface energy, and the jump of magnetic induction depend on its orientation with respect to chosen crystallographic axes (xy). Assuming the arbitrary orientation of the DW plane in the xy plane we transform the gradient terms in Eq. (1) to the new coordinate system ($x'y'$) connected to the DW: the axis x' is normal and the axis y' is parallel to the DW, i.e.,

$$\begin{aligned}\partial_x &= \cos \psi \partial_{x'} - \sin \psi \partial_{y'}, \\ \partial_y &= \sin \psi \partial_{x'} + \cos \psi \partial_{y'}.\end{aligned}\quad (10)$$

The transformation is a counterclockwise rotation of the original coordinate axes xy around the z axis through an angle ψ about the origin. Note that only the gradients were rotated, with the order parameter η being untouched and characterized by the components $\eta_{x,y}$ in the original coordinate system xy . After the transformation the free-energy density with respect to the energy of the uniform chiral phase is

$$\begin{aligned}\Delta \mathcal{F} &= \frac{\beta_2 \eta_0^4}{2} (1 + \cos 2\zeta) + \frac{K_1 \eta_0^2}{2} \{(\cos \psi D_{x'} \phi - \sin \psi D_{y'} \phi)^2 \\ &+ [\sin \psi (D_{x'} \phi + \partial_{x'} \zeta) + \cos \psi (D_{y'} \phi + \partial_{y'} \zeta)]^2\} \\ &+ \frac{K_2 \eta_0^2}{2} \{(\sin \psi D_{x'} \phi + \cos \psi D_{y'} \phi)^2 + [\cos \psi (D_{x'} \phi \\ &+ \partial_{x'} \zeta) - \sin \psi (D_{y'} \phi + \partial_{y'} \zeta)]^2\} + K_3 \eta_0^2 \cos \zeta (\cos \psi D_{x'} \phi \\ &- \sin \psi D_{y'} \phi) [\sin \psi (D_{x'} \phi + \partial_{x'} \zeta) + \cos \psi (D_{y'} \phi \\ &+ \partial_{y'} \zeta)] + K_4 \eta_0^2 \cos \zeta (\sin \psi D_{x'} \phi + \cos \psi D_{y'} \phi) \\ &\times [\cos \psi (D_{x'} \phi + \partial_{x'} \zeta) - \sin \psi (D_{y'} \phi + \partial_{y'} \zeta)].\end{aligned}\quad (11)$$

The relevant components of the superfluid electron-density matrix are

$$\begin{aligned}n_{x'x'} &= \frac{4m}{\hbar^2} \eta_0^2 [K_1 + K_2 + (K_3 + K_4) \sin 2\psi \cos \zeta], \\ n_{x'y'} = n_{y'x'} &= \frac{4m}{\hbar^2} \eta_0^2 (K_3 + K_4) \cos 2\psi \cos \zeta.\end{aligned}\quad (12)$$

The magnetization currents are defined by

$$\begin{aligned}j_{x'}^{(m)} &= \frac{e}{\hbar} \eta_0^2 [K_1 + K_2 - (K_1 - K_2) \cos 2\psi \\ &+ (K_3 + K_4) \sin 2\psi \cos \zeta] \partial_{x'} \zeta, \\ j_{y'}^{(m)} &= \frac{e}{\hbar} \eta_0^2 \{[-K_3 + K_4 + (K_3 + K_4) \cos 2\psi] \cos \zeta \\ &+ (K_1 - K_2) \sin 2\psi\} \partial_{x'} \zeta.\end{aligned}\quad (13)$$

The current normal to the DW plane must vanish: $j_{x'} = e n_{x'x'} v_{x'} + j_{x'}^{(m)} = 0$. This gives the following expression for the x' component of the velocity:

$$\begin{aligned}v_{x'} &= \frac{\hbar}{2m} D_{x'} \phi \\ &= -\frac{\hbar}{4m} \left[1 - \frac{(K_1 - K_2) \cos 2\psi}{K_1 + K_2 + (K_3 + K_4) \sin 2\psi \cos \zeta} \right] \partial_{x'} \zeta.\end{aligned}\quad (14)$$

After its exclusion the total current $j_{y'}$ parallel to the DW is

$$\begin{aligned}j_{y'} &= \frac{e}{\hbar} \eta_0^2 \left[-(K_3 - K_4) \cos \zeta \right. \\ &\left. + (K_1 - K_2) \frac{(K_1 + K_2) \sin 2\psi + (K_3 + K_4) \cos \zeta}{K_1 + K_2 + (K_3 + K_4) \sin 2\psi \cos \zeta} \right] \partial_{x'} \zeta.\end{aligned}\quad (15)$$

Using Eq. (15) and the Maxwell equation one can calculate the jump of the magnetic induction on the ζ wall as follows:

$$\begin{aligned} \delta B_{\xi}(\psi) &= -(4\pi/c) \int dx' j_y, \\ &= \frac{8\pi e}{\hbar c} \eta_0^2 \left\{ (K_3 - K_4) \mp \frac{K_1 - K_2}{\sin 2\psi} \right. \\ &\quad \times \left. \left[\frac{\pi}{2} - \frac{2 \cos^2 2\psi}{(1 - q^2)^{1/2}} \arctan \sqrt{\frac{1 \mp q}{1 \pm q}} \right] \right\}, \end{aligned} \quad (16)$$

where

$$q = \frac{K_3 + K_4}{K_1 + K_2} \sin 2\psi. \quad (17)$$

The upper and the lower signs correspond to ζ rotations from $-\pi/2$ to $\pi/2$ or to $-3\pi/2$, respectively.

After the exclusion of the transport velocity the free-energy density becomes

$$\Delta \mathcal{F} = \frac{\beta_2 \eta_0^4}{2} (1 - \cos 2\gamma) + (K_1 + K_2) \eta_0^2 f(\gamma) \frac{\partial_{x'} \gamma^2}{8}, \quad (18)$$

where

$$f(\gamma) = 1 \mp q \sin \gamma - \left(\frac{K_1 - K_2}{K_1 + K_2} \right)^2 \frac{\cos^2 2\psi}{1 \pm q \sin \gamma}. \quad (19)$$

Here we used the angle γ equal to $\gamma = \zeta + \pi/2$ or $\gamma = -\zeta - \pi/2$ for ζ rotations from $-\pi/2$ to $\pi/2$ or to $-3\pi/2$, respectively. So across the DW γ varies from 0 to π .

The structure of the ζ wall is determined by the Euler-Lagrange equation obtained by variation of the free energy with density (18) with respect to γ ,

$$f(\gamma) \partial_{x'}^2 \gamma + \frac{1}{2} f(\gamma)' (\partial_{x'} \gamma)^2 - \frac{\sin 2\gamma}{\Delta^2} = 0, \quad (20)$$

where the scale

$$\Delta = \frac{1}{2\eta_0} \left(\frac{K_1 + K_2}{\beta_2} \right)^{1/2} \quad (21)$$

determines the thickness of the DW. The first integration of Eq. (20) yields

$$(\partial_{x'} \gamma)^2 = \frac{1 - \cos 2\gamma}{\Delta^2 f(\gamma)}. \quad (22)$$

After this one can find the surface energy of the DW,

$$\sigma_{\xi}(\psi) = \eta_0^3 \sqrt{\frac{\beta_2 (K_1 + K_2)}{2}} \int_0^{\pi} d\gamma \sin \gamma \sqrt{f(\gamma)}. \quad (23)$$

Figure 1 shows the plots of the magnetic-induction jump δB_{ξ} and the surface energy as functions of the angle ψ between the DW and one of the crystallographic axes for the weak-coupling case when $K_2 = K_3 = K_4 = K_1/3$. The plots for the clockwise rotation (ζ varies within DW from $-\pi/2$ to $\pi/2$, curve 1) are obtained from the counterclockwise rotation (ζ varies within DW from $-\pi/2$ to $-3\pi/2$, curve 2) by reflection $\psi \rightarrow -\psi$. The angular dependences have two extrema at $\psi = \pm \pi/4$. The spatial distribution of the magnetization currents along the DW for different orientations of the DW is shown in Fig. 2.

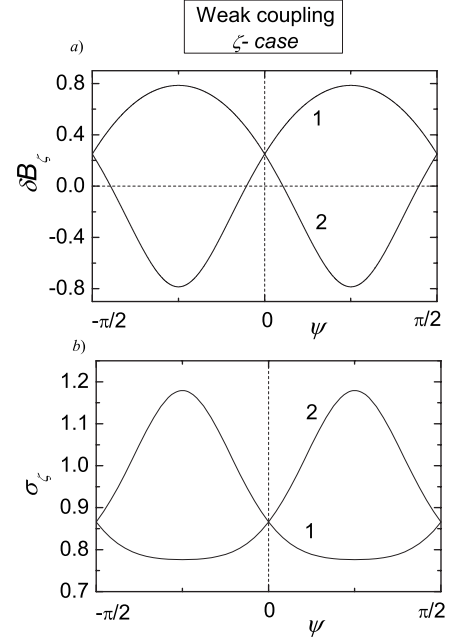


FIG. 1. Shown are the angular dependence (a) of the jump of reduced magnetic induction $\delta B_{\xi} \times 3\hbar c / 32\pi e \eta_0^2 K_1$ and (b) the reduced surface energy $\sigma_{\xi} \times 3/4 \eta_0^3 (2\beta_2 K_1)^{1/2}$ of the DW for two configurations when ζ increases from $-\pi/2$ to $\pi/2$ (curve 1), or decreases from $-\pi/2$ to $-3\pi/2$ (curve 2) with increase in x' .

Let us consider some important particular cases. The case $\psi=0$ corresponds to the DW parallel to one from chosen crystallographic axes. The surface energy and the jump of magnetic induction in this case are

$$\sigma_{\xi}(0) = 2\eta_0^3 \sqrt{\frac{2\beta_2 K_1 K_2}{K_1 + K_2}},$$

$$\delta B_{\xi}(0) = \frac{16\pi e}{\hbar c} \eta_0^2 \frac{K_2 K_3 - K_1 K_4}{K_1 + K_2}. \quad (24)$$

This agrees with the results of Volovik and Gor'kov⁹ who assumed that $K_3 = K_4$. In the case $\psi=0$ there is degeneracy

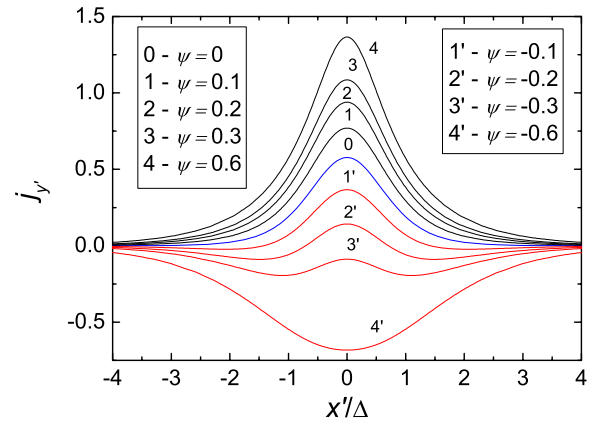


FIG. 2. (Color online) The coordinate dependence of the reduced superconducting current $j_y \times 3^{1/2} \hbar / 2^{3/2} e \eta_0^3 (K_1 \beta_2)^{1/2}$ at different orientations of the DW ($|\zeta| \leq \pi/2$).

between DWs corresponding to two senses of rotation of the phase ζ : the surface energies and the field jumps for two cases coincide.

Another particular case is the angle $\psi = \pi/4$, where the plots in Fig. 1 have extrema. In this case according to Eq. (16) the jump of magnetic induction is

$$\delta B_{\zeta}\left(\frac{\pi}{4}\right) = \frac{8\pi e}{c} \eta_0^2 \left[K_3 - K_4 \mp \frac{\pi}{2}(K_1 - K_2) \right] \quad (25)$$

and is different for both configurations. The DW surface energy can be obtained from Eqs. (19) and (23) as follows:

$$\begin{aligned} \sigma_{\zeta}\left(\frac{\pi}{4}\right) &= \eta_0^3 \sqrt{\frac{\beta_2(K_1 + K_2)}{2}} \int_0^{\pi} d\gamma \sin \gamma \sqrt{1 \mp q \sin \gamma} \\ &= \eta_0^3 \sqrt{2\beta_2(K_1 + K_2)} \left[{}_3F_2\left(-\frac{1}{4}, \frac{1}{4}, 1; \frac{1}{2}, \frac{3}{2}; q^2\right) \right. \\ &\quad \left. \mp \frac{\pi q}{8} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; 2; q^2\right) \right], \end{aligned} \quad (26)$$

where

$$q = \frac{K_3 + K_4}{K_1 + K_2} \quad (27)$$

and ${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function.²²

B. Rotation of order parameter: Θ wall

Let us consider now the DWs with rotation of the order parameter η in the configuration space. Inside this domain wall the phase Θ rotates from $-\pi/4$ to $+\pi/4$ (counterclockwise rotation) or from $-\pi/4$ to $-3\pi/4$ (clockwise rotation). We address this type of the DW as Θ wall. The whole analysis, which was performed above for the ζ wall, can be repeated for the Θ wall. This yields the expressions obtained from those for the ζ wall by the following substitution: $\zeta \rightarrow 2\Theta$, $\psi \rightarrow \psi + \pi/4$, $K_1 - K_2 \rightleftharpoons K_3 + K_4$, and $\beta_2 \rightarrow \beta_2 - 4\beta_3$. In particular, the final expressions for the magnetic-induction jump and the DW surface energy are

$$\begin{aligned} \delta B_{\Theta}(\psi) &= -(4\pi/c) \int dx' j_y, \\ &= \frac{8\pi e}{c} \eta_0^2 \left\{ (K_3 - K_4) \mp \frac{K_3 + K_4}{\cos 2\psi} \left[\frac{\pi}{2} \right. \right. \\ &\quad \left. \left. - \sin^2 2\psi \frac{2}{(1 - q^2)^{1/2}} \arctan \sqrt{\frac{1 \mp q}{1 \pm q}} \right] \right\}, \end{aligned} \quad (28)$$

where

$$q = \frac{K_1 - K_2}{K_1 + K_2} \cos 2\psi \quad (29)$$

and

$$\sigma_{\Theta}(\psi) = \eta_0^3 [2\beta_2(K_1 + K_2)]^{1/2} \int_0^{\pi} d\gamma \sin \gamma \sqrt{f(\gamma)}, \quad (30)$$

where

$$f(\gamma) = 1 \mp \frac{K_1 - K_2}{K_1 + K_2} \cos 2\psi \sin \gamma - \left(\frac{K_3 + K_4}{K_1 + K_2} \right)^2 \frac{\sin^2 2\psi}{1 \pm q \sin \gamma}. \quad (31)$$

Here $\gamma = 2\Theta + \pi/2$ or $\gamma = -2\Theta - \pi/2$ for Θ rotations from $-\pi/4$ to $\pi/4$ or to $-3\pi/4$, respectively. The DW surface energy has extrema at $\psi = 0$ where the magnetic-induction jump is

$$\delta B_{\Theta}(0) = \frac{8\pi e}{c} \eta_0^2 \left[K_3 - K_4 \mp \frac{\pi}{2}(K_3 + K_4) \right]. \quad (32)$$

IV. DISCUSSION

Let us discuss consequences of the obtained analytical and numerical results. We start from the case of full axial symmetry in the xy plane when $K_1 = K_2 + K_3 + K_4$ and $\beta_3 = 0$. In this case the ψ dependencies for the ζ and the Θ walls are identical except for the shift of ψ by $\pi/4$. Naturally the choice of crystallographic axes does not matter in this case, only their orientation with respect to the DW being important. The minimum of the surface energy for the ζ wall corresponds [see Fig. 1(b)] to the angle $\psi = \pm \pi/4$ (45°) between the DW and the axes for η . However, choosing for η the coordinate system connected with the DW, the DW becomes a Θ wall.

Let us consider now the tetragonal symmetry with $\beta_3 \neq 0$. Without any loss of generality one may assume that $\beta_3 < 0$. Indeed, if $\beta_3 > 0$ with $\pi/4$ rotation of the crystal axes one obtains the fourth-order terms with $\beta_2 - \beta_3/4$ instead of β_2 and $-\beta_3$ instead of β_3 . Thus after this rotation the new β_3 is negative. At our choice of crystallographic axes the ζ wall has a smaller surface energy with the minimum at $\psi = \pm \pi/4$. Thus the stable DW is a ζ wall with respect to the order-parameter axes, but is a Θ wall, if one uses the order parameter η defined in the coordinates related to the stable DW. The stable configuration of the DW corresponds to the maximum jump of the magnetic induction given by Eq. (25) with the lower sign before the second term. On the other hand, the jump is minimal and is given by Eq. (24) if the DW is parallel to one of the two crystallographic axes. It is interesting to compare these extremal values with the first critical magnetic field given by Eq. (9),

$$\frac{\delta B_{\max}}{H_{c1}} = \frac{\pi(K_1 - K_2)}{(K_1 + K_2) \ln(\lambda/\xi)}, \quad \frac{\delta B_{\min}}{H_{c1}} = \frac{4(K_1 K_4 - K_2 K_3)}{(K_1 + K_2)^2 \ln(\lambda/\xi)}. \quad (33)$$

For the values $\lambda = 190$ nm and $\xi = 66$ nm (Ref. 6) and assuming the weak-coupling relations $K_1/3 = K_2 = K_3 = K_4$, these ratios are 1.48 and 0.48, respectively.

Since now there is no freedom for the choice of the axes for the order parameter η , there is a force which tends to orient the DW at 45° to the crystallographic axes. This force may compete with other forces on the DW, e.g., those from the sample shape. This would result in various configurations of DW not necessary those dictated by the bulk tetragonal anisotropy.

Whereas in the Landau-Lifshitz theory of ferromagnetism the magnetic-induction jump is given by the universal value $8\pi M$, where M is the local spontaneous magnetization, in our case this jump depends on the structure and the orientation of the DW. Moreover, if one tried to introduce formally the effective magnetization via the relation $\tilde{M} = \delta B / 8\pi$ the latter has no straightforward connection with the orbital moment \mathbf{l} of the Cooper pair and even can have a negative sign with respect to the intrinsic magnetic moment of the Cooper pair.

Returning back to the question why they could not detect stray fields generated by the magnetic-induction jumps outside Sr_2RuO_4 samples,⁶ our analysis demonstrates that at the present stage the theory cannot make definite predictions on the strength of these fields without reliable information on the parameters in the GL theory. In principle, one could suggest that the DWs in the experiments were not in the ground state, or the GL parameters are essentially different from their values for a cylindrical Fermi surface in the weak-coupling-limit. But further experimental and theoretical works are needed to check these suggestions.

V. CONCLUSIONS

We investigated properties of DWs in a tetragonal chiral p -wave superconductor. Various cases of the DW structure and orientation with respect to in-plane crystallographic axes were analyzed. The magnetic-induction jump on the DW changes from case to case, in contrast to the Landau-Lifshitz theory of ferromagnetism, where this jump has the universal value proportional to the local spontaneous magnetization. This conclusion stresses again that the latter is not defined for orbital ferromagnetism. The magnetic-induction jump is responsible for stray magnetic fields, which have not yet detected outside Sr_2RuO_4 samples.⁶ A quantitative evaluation of stray fields can be done with a more detailed information (from the microscopic theory or from the experiment) of the parameters of the GL theory.

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