

Theory of topological spin current in noncentrosymmetric superconductors

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We study the spin transport properties of the noncentrosymmetric superconductor with time-reversal symmetry where spin-triplet ($p_x \pm ip_y$)-wave and spin-singlet s -wave pair potentials can mix with each other. We show that when the amplitude of the ($p_x \pm ip_y$)-wave pair potential is larger than that of s -wave one, the superconducting state belongs to the topologically nontrivial class analogous to the quantum spin Hall system, and the resulting helical edge modes as Andreev bound states are topologically protected. We find that the incident angle dependent spin polarized current flows through the interface due to the presence of the helical edge modes. With a weak magnetic field, also the angle-integrated current is strongly spin polarized.

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The topological properties of the electronic states have been attracting intensive interest in condensed-matter physics. Especially, it was highlighted by the discovery of the quantum Hall system (QHS) showing the accurate quantization of the Hall conductance σ_H which is related to the topological integer.^{1,2}

Recently, the concept of the QHS has been generalized to the time-reversal (T) symmetric system, i.e., the quantum spin Hall system (QSHS).³⁻⁵ QSHS could be regarded as the two copies of QHS for up and down spins with the opposite chiralities. In the generic case, however, the mixture of up and down spins occurs due to the spin-orbit interaction, which necessitates the topological number to characterize QSHS.^{3,5} In QSHS, there exist the helical edge modes, i.e., the time-reversal pair of right- and left-going one-dimensional modes, which has been experimentally demonstrated for the quantum well of the HgTe system by the measurement of the charge conductance.⁶

In the field of superconductivity, the chiral p -wave superconductors such as Sr₂RuO₄ (Ref. 7) can be considered as an analog of the QHS, and remarkable phenomena such as one-dimensional Majorana fermion (real fermion) modes at the edge^{8,9} and the non-Abelian statistics of the vortex^{9,10} have been proposed there. Besides these issues, the noncentrosymmetric (NCS) superconductors such as CePt₃Si are a central topic.^{11,12} Also the two-dimensional NCS superconductors are expected at the interfaces and/or surfaces due to the strong potential gradient. An interesting example is the superconductivity at the LaAlO₃/SrTiO₃ interface.¹³ In NCS superconductors, the spin-orbit interaction comes into play. Especially, Frigeri *et al.*¹² showed that the ($p_x \pm ip_y$)-pairing state has the highest T_c within the triplet channel in CePt₃Si. However, the singlet (s -wave) and triplet (p -wave) pairings are mixed, and several associated properties such as the large upper critical field beyond the Pauli limit have been focused on.¹² On the other hand, the pure ($p_x \pm ip_y$)-pairing state has been studied from the viewpoint of the superconducting analog of QSHS.¹⁴ Therefore, it is an important and urgent issue to study the spin transport properties of the NCS superconductors from the topological viewpoint.

In this paper, we study the spin transport properties of the NCS superconductor¹¹ with T symmetry, where

($p_x \pm ip_y$)-wave and spin-singlet s -wave pair potentials can mix with each other. We show that when the amplitude of the ($p_x \pm ip_y$)-wave pair potential is larger than that of s -wave one, the superconducting state belongs to the topologically nontrivial class analogous to the quantum spin Hall system and the resulting helical edge modes as Andreev bound states (ABSs) are topologically protected. We study Andreev¹⁵ reflection at low energy, which is determined mostly by the helical edge modes, and find the incident angle dependent spin polarized current flowing through the interface. When the magnetic field is applied, even the angle-integrated current is spin polarized.

We start with the Hamiltonian of the NCS superconductor

$$\check{H}_S = \begin{pmatrix} \hat{H}(\mathbf{k}) & \hat{\Delta}(\mathbf{k}) \\ -\hat{\Delta}^*(-\mathbf{k}) & -\hat{H}^*(-\mathbf{k}) \end{pmatrix},$$

with $\hat{H}(\mathbf{k}) = \xi_{\mathbf{k}} + \mathbf{V}(\mathbf{k}) \cdot \hat{\boldsymbol{\sigma}}$, $\mathbf{V}(\mathbf{k}) = \lambda(\hat{x}k_y - \hat{y}k_x)$, and $\xi_{\mathbf{k}} = \hbar^2 k^2 / (2m) - \mu$. Here, μ , m , $\hat{\boldsymbol{\sigma}}$, and λ denote the chemical potential, effective mass, Pauli matrices, and coupling constant of Rashba spin-orbit interaction, respectively.¹² The pair potential $\hat{\Delta}(\mathbf{k})$ is given by

$$\hat{\Delta}(\mathbf{k}) = [\mathbf{d}(\mathbf{k}) \cdot \hat{\boldsymbol{\sigma}}] i \hat{\sigma}_y + i \psi(\mathbf{k}) \hat{\sigma}_y. \quad (1)$$

We choose a ($p_x \pm ip_y$)-wave pair for the spin-triplet component with $\mathbf{d}(\mathbf{k}) = \Delta_p(\hat{x}k_y - \hat{y}k_x)/|\mathbf{k}|$ (Ref. 12) and an s -wave one with $\psi(\mathbf{k}) = \Delta_s$, with $\Delta_p \geq 0$ and $\Delta_s \geq 0$. The superconducting gaps $\Delta_1 = \Delta_p + \Delta_s$ and $\Delta_2 = |\Delta_p - \Delta_s|$ open for the two spin-split band, respectively, in the homogeneous state.¹⁶

However, as seen below, surface states are crucially influenced by the relative magnitude between Δ_p and Δ_s . Let us consider a wave function including ABS localized at the surface. Consider a two-dimensional semi-infinite superconductor on $x > 0$ where the surface is located at $x = 0$. The corresponding wave function is given by¹⁷

$$\Psi_S(x) = \exp(ik_y y) [c_1 \psi_1 \exp(iq_{1x}^+ x) + c_2 \psi_2 \exp(-iq_{1x}^- x) + c_3 \psi_3 \exp(iq_{2x}^+ x) + c_4 \psi_4 \exp(-iq_{2x}^- x)],$$

$$q_{1(2)x}^{\pm} = k_{1(2)x}^{\pm} \pm \frac{k_{1(2)}}{k_{1(2)x}^{\pm}} \sqrt{\frac{E^2 - \Delta_{1(2)}^2}{\lambda^2 + 2\hbar^2\mu/m}}, \quad (2)$$

with $k_{1(2)x}^+ = k_{1(2)x}^- = k_{1(2)x}$ for $|k_y| \leq k_{1(2)}$ and $k_{1(2)x}^+ = -k_{1(2)x}^- = k_{1(2)x}$ for $|k_y| > k_{1(2)}$. Here, k_1 and k_2 are Fermi momenta of the small and large magnitudes of the Fermi surface given by $-m\lambda/\hbar^2 + \sqrt{(m\lambda/\hbar^2)^2 + 2m\mu/\hbar^2}$ and $m\lambda/\hbar^2 + \sqrt{(m\lambda/\hbar^2)^2 + 2m\mu/\hbar^2}$, respectively. $k_{1(2)x}$ denotes the x component of the Fermi momentum $k_{1(2)}$, with $k_{1(2)x} = \sqrt{k_{1(2)}^2 - k_y^2}$. The wave functions are given by ${}^T\psi_1 = (u_1, -i\alpha_1^- u_1, i\alpha_1^- v_1, v_1)$, ${}^T\psi_2 = (v_1, -i\tilde{\alpha}_1^- v_1, i\tilde{\alpha}_1^- u_1, u_1)$, ${}^T\psi_3 = (u_2, i\alpha_2^- u_2, i\gamma\alpha_2^- v_2, -\gamma v_2)$, and ${}^T\psi_4 = (v_2, i\tilde{\alpha}_2^- v_2, i\gamma\tilde{\alpha}_2^- u_2, -\gamma u_2)$, with $\gamma = \text{sgn}(\Delta_p - \Delta_s)$. In the above, $u_{1(2)}$ and $v_{1(2)}$ are given as $\sqrt{\frac{1}{2}(1 + \frac{\sqrt{E^2 - \Delta_{1(2)}^2}}{E})}$ and $\sqrt{\frac{1}{2}(1 - \frac{\sqrt{E^2 - \Delta_{1(2)}^2}}{E})}$. Here we have presented $\alpha_1 = (k_{1x}^+ - ik_y)/k_1$, $\alpha_2 = (k_{2x}^+ - ik_y)/k_2$, $\tilde{\alpha}_1 = (-k_{1x}^- - ik_y)/k_1$, and $\tilde{\alpha}_2 = (-k_{2x}^- - ik_y)/k_2$. E is the quasiparticle energy measured from the Fermi energy.

By postulating $\Psi_S(x)=0$ at $x=0$, we can determine the ABS. The bound-state condition can be expressed by

$$\sqrt{(\Delta_1^2 - E^2)(\Delta_2^2 - E^2)} = \frac{1 - \zeta}{1 + \zeta} (E^2 + \gamma\Delta_1\Delta_2), \quad (3)$$

$$\zeta = \begin{cases} \frac{\sin^2 \left[\frac{1}{2}(\phi_1 + \phi_2) \right]}{\cos^2 \left[\frac{1}{2}(\phi_1 - \phi_2) \right]} & |\phi_2| \leq \phi_C, \\ 1 & \phi_C < |\phi_2| \leq \pi/2, \end{cases} \quad (4)$$

with $\zeta \leq 1$, $\cos \phi_1 = k_{1x}/k_1$, and $\cos \phi_2 = k_{2x}/k_2$. The critical angle ϕ_C is defined as $\sin^{-1}(k_1/k_2)$. For $\lambda=0$, Eq. (3) reproduces the previous result.¹⁶ As seen from Eq. (3), the ABS including zero energy state is only possible for $|\phi_2| \leq \phi_C$ and $\gamma=1$, i.e., $\Delta_p > \Delta_s$. The present ABS is just the edge state, where the localized quasiparticle can move along the edge. The energy level of the edge state depends crucially on the direction of the motion of the quasiparticle. The inner gap edge modes are absent for large magnitude of k_y , i.e., ϕ_2 . The parameter regime where the edge modes survive is reduced with the increase of the magnitude of λ . However, as far as we concentrate on the perpendicular injection, the edge modes survive as the midgap ABS (Refs. 18 and 19) irrespective of the strength of λ . If we focus on the low energy limit, the ABS can be written as

$$E = \pm \Delta_p \left(1 - \frac{\Delta_s^2}{\Delta_p^2} \right) \frac{k_1 + k_2}{2k_1k_2} k_y, \quad (5)$$

with $\Delta_s < \Delta_p$ for any λ with small magnitude of k_y . For $\Delta_s \geq \Delta_p$, the present ABS vanishes since the value of the right side of Eq. (3) becomes negative due to the negative sign of γ for $|E| < \Delta_1$ and $|E| < \Delta_2$. It should be remarked that the present ABS does not break the time-reversal symmetry, since the edge current carried by each Kramers doublet flows in the opposite direction. Thus they can be regarded as heli-

cal edge modes, where two modes are connected to each other by time-reversal operation.

Now we give an argument why the superconducting state with $\Delta_p > \Delta_s$ has the ABS from the viewpoint of the Z_2 (topological) class.³ We commence with the pure $(p_x \pm ip_y)$ -wave state without the spin-orbit interaction λ . The spin Chern number⁵ for the Bogoliubov–de Gennes (BdG) Hamiltonian is 2. Turning on λ adiabatically, which leaves the T symmetry intact and keeps the gap open, one can arrive at the BdG Hamiltonian of interest. Upon this adiabatic change of λ , the number of the helical edge mode pairs does not change. Then we increase the magnitude of Δ_s from zero. As far as $\Delta_p > \Delta_s$ is satisfied, the number of helical edge modes does not change, since it is a topological number. However, if Δ_s exceeds Δ_p , the helical edge mode disappears. In this regime, the topological nature of the superconducting state belongs to the pure s -wave state without λ . It is remarkable that, just at $\Delta_s = \Delta_p$, one of the energy gaps of the quasiparticle in the bulk closes, where a quantum phase transition occurs.

Now we turn to the spin transport property governed by the ABS in the NCS superconductors.²⁰ First, we point out that the spin Hall effect, i.e., the appearance of the spin Hall voltage perpendicular to the superconducting current, is suppressed by the compressive nature of the superconducting state by the factor of $(k_F\lambda_m)^{-2}$ (k_F is the Fermi momentum and λ_m is the penetration depth).²¹ Instead, we will show below that the spin transport through the junction between the ballistic normal metal at $x < 0$ and NCS superconductor, i.e., (N/SC) junction, can be enhanced by the Doppler effect at the Andreev reflection. The Hamiltonian \check{H}_N of N is given by putting $\hat{\Delta}(\mathbf{k})=0$ and $\lambda=0$ of \check{H}_S . We assume an insulating barrier at $x=0$ expressed by a delta-function potential $U\delta(x)$. The wave function for spin σ in the normal metal $\Psi_N(x)$ is given by

$$\Psi_N(x) = \exp(ik_{Fy}y) \left[\left(\psi_{i\sigma} + \sum_{\rho=\uparrow,\downarrow} a_{\sigma,\rho} \psi_{a\rho} \right) \exp(ik_{Fx}x) + \sum_{\rho=\uparrow,\downarrow} b_{\sigma,\rho} \psi_{b\rho} \exp(-ik_{Fx}x) \right], \quad (6)$$

with ${}^T\psi_{i\uparrow} = {}^T\psi_{b\uparrow} = (1, 0, 0, 0)$, ${}^T\psi_{i\downarrow} = {}^T\psi_{b\downarrow} = (0, 1, 0, 0)$, ${}^T\psi_{a\uparrow} = (0, 0, 1, 0)$, and ${}^T\psi_{a\downarrow} = (0, 0, 0, 1)$. The corresponding $\Psi_S(x)$ is given by Eq. (2). The coefficients $a_{\sigma,\rho}$ and $b_{\sigma,\rho}$ are determined by postulating the boundary condition $\Psi_N(0) = \Psi_S(0)$ and $\hbar\check{v}_{Sx}\Psi_S(0) - \hbar\check{v}_{Nx}\Psi_N(0) = -2iU\check{\tau}_3\Psi_S(0)$, with $\hbar\check{v}_{S(N)x} = \partial\check{H}_{S(N)}/\partial k_x$ and diagonal matrix $\check{\tau}_3$ given by $\check{\tau}_3 = \text{diag}(1, 1, -1, -1)$. The quantities of interest are the angle-resolved spin conductance $f_S(\phi)$ and charge conductance $f_C(\phi)$ defined by²²

$$f_S(\phi) = [(|a_{\uparrow,\uparrow}|^2 - |a_{\uparrow,\downarrow}|^2 - |b_{\uparrow,\uparrow}|^2 + |b_{\uparrow,\downarrow}|^2) - (|a_{\downarrow,\downarrow}|^2 - |a_{\downarrow,\uparrow}|^2 - |b_{\downarrow,\downarrow}|^2 + |b_{\downarrow,\uparrow}|^2)] \frac{\cos \phi}{2},$$

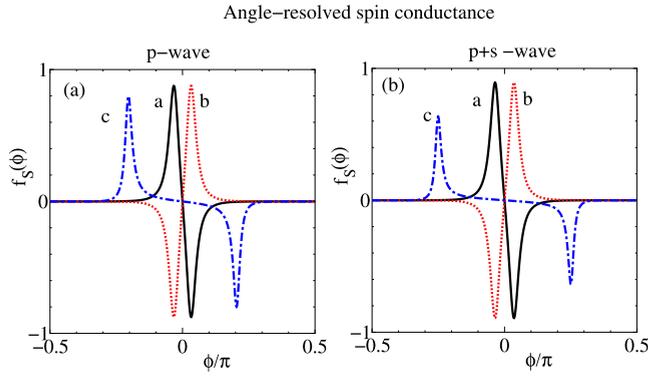


FIG. 1. (Color online) Angle-resolved spin conductance for $Z = 5$. a: $eV = 0.1\Delta_p$, b: $eV = -0.1\Delta_p$, and c: $eV = 0.6\Delta_p$, with $\lambda k_F = 0.1\mu$. (a) Pure $(p_x \pm ip_y)$ -wave case with $\Delta_s = 0$. (b) $\Delta_s = 0.3\Delta_p$.

$$f_C(\phi) = \left[2 + \sum_{\sigma,\rho} (|a_{\sigma,\rho}|^2 - |b_{\sigma,\rho}|^2) \right] \frac{\cos \phi}{2}, \quad (7)$$

where ϕ denotes the injection angle measured from the normal to the interface. First we consider the pure $(p_x \pm ip_y)$ -wave state. In Fig. 1, the angle-resolved spin conductance is plotted as a function of injection angle ϕ and bias voltage V with $E = eV$. Note here that the k_y is related to ϕ as $k_y = k_F \sin \phi$. It is remarkable that spin conductance has a nonzero value although the NCS superconductor does not break time-reversal symmetry. $f_S(\phi)$ has a peak when the angle ϕ or k_y gives the energy E in the energy dispersion of the ABS. With this condition, the spin-dependent Andreev reflection occurs to result in the spin current. Besides this property, we can show that $f_S(\phi) = -f_S(-\phi)$ is satisfied. By changing the sign of eV , $f_S(\phi)$ changes sign as seen in Fig. 1(a). Next, we look at the case where the s -wave component coexists. We can calculate spin conductance similar to the pure $(p_x \pm ip_y)$ -wave case. For $\Delta_s < \Delta_p$, where helical edge modes exist, $f_S(\phi)$ has a sharp peak and $f_S(\phi) = -f_S(-\phi)$ is satisfied [see Fig. 1(b)]. These features are similar to those of the pure $(p_x \pm ip_y)$ -wave case. On the other hand, for $\Delta_s > \Delta_p$, where the helical edge modes are absent, sharp peaks of $f_S(\phi)$ as shown in Fig. 1 are absent.

We have checked that there is a negligible quantitative change by taking the $\lambda = 0$ limit compared to Fig. 1, e.g., less than 0.5% change of the peak height. In this limit, for a pure $(p_x \pm ip_y)$ -wave state, $f_S(\phi)$ is given simply as follows:

$$\frac{-8\sigma_N^2(1 - \sigma_N)\sin 2\phi \sin 2\varphi \cos \phi}{[4(\sin^2 \phi - \sin^2 \varphi) + \sigma_N[\exp(-2i\varphi)(\sigma_N - 2) + 2 \cos 2\phi]]^2}$$

for $|E| < \Delta_p$ and $f_S(\phi) = 0$ for $|E| > \Delta_p$, with $\sin \varphi = E/\Delta_p$. Transparency of the interface σ_N is given by $4 \cos^2 \phi / (4 \cos^2 \phi + Z^2)$ with a dimensionless constant $Z = 2mU/\hbar^2 k_F$. The magnitude of $f_S(\phi)$ is largely enhanced at $E = \pm \Delta_p \sin \phi$ corresponding to the energy dispersion of the ABS. The origin of nonzero $f_S(\phi)$ even without λ is due to the spin-dependent ABS. We have checked that even if we take into account the spatial dependence of the $(p_x \pm ip_y)$ -wave pair potential explicitly, the resulting $f_S(\phi)$ does not qualitatively change.²⁰

Summarizing these features, we can conclude that the presence of the helical edge modes in the NCS superconductor is the origin of the large angle-resolved spin current through N/NCS superconductor junctions. However, the magnitude of the angle averaged normalized spin conductance becomes zero since $f_S(\phi) = -f_S(-\phi)$ is satisfied.

Magnetic field offers an opportunity to observe the spin current in a much more accessible way, where T symmetry is broken by the shielding current at the interface. Here we consider the angle averaged normalized spin conductance σ_S and charge conductance σ_C as a function of magnetic field which are given by^{19,22}

$$\sigma_S = \frac{\int_{-\pi/2}^{\pi/2} f_S(\phi) d\phi}{\int_{-\pi/2}^{\pi/2} f_{NC}(\phi) d\phi}, \quad \sigma_C = \frac{\int_{-\pi/2}^{\pi/2} f_C(\phi) d\phi}{\int_{-\pi/2}^{\pi/2} f_{NC}(\phi) d\phi}, \quad (8)$$

where $f_{NC}(\phi)$ denotes the angle-resolved charge conductance in the normal state with $\Delta_p = \Delta_s = 0$. Now we consider the magnetic field H applied perpendicular to the two-dimensional plane, which induces a shielding current along the N/NCS superconductor interface. When the penetration depth of the NCS superconductor is much longer than the coherence length, the vector potential can be approximated as $\mathbf{A}(\mathbf{r}) = (0, A_y(x), 0)$ with $A_y(x) = -\lambda_m H \exp(-x/\lambda_m)$, with the penetration depth λ_m . Here we consider the situation where the quantization of the Landau level can be neglected. Then the quasiclassical approximation becomes available. The applied magnetic field shifts the quasiparticle energy E in the wave function of $\Psi_S(x)$ to $E - H\Delta_p \sin \phi/H_0$ with $H_0 = \hbar/(2e\pi^2\xi\lambda_m)$ and $\xi = \hbar^2 k_F/(\pi m \Delta_p)$.²³ For typical values of $\xi \sim 10$ nm, $\lambda_m \sim 100$ nm, the magnitude of H_0 is of the order of 0.2 T. Here the order of the energy of the Doppler shift is given by $H\Delta_p/H_0$. Since the Zeeman energy is given by $\mu_B H$, the order of the energy of the Doppler shift is $k_F \lambda_m$ times larger than that of Zeeman energy. Thus, we can neglect the Zeeman effect in the present analysis. This is in sharp contrast to a QSHS where the Zeeman energy is the main effect of H , which opens the gap in the helical edge modes and modulates the transport properties.⁶ The enhanced spin current due to the Doppler shift is specific to a superconducting state not realized in the QSHS.

As shown in Fig. 1, to discuss the topological nature of the helical edge modes, it is sufficient to consider the pure $(p_x \pm ip_y)$ -wave state. In the following, we choose the $(p_x \pm ip_y)$ -wave case. In Fig. 2, the spin conductance σ_S and charge conductance σ_C normalized by the charge conductance in the normal state are plotted. It should be noted that σ_S becomes nonzero in the presence of the magnetic field H (see curves b, c, and d), since $f_S(\phi)$ is no longer an odd function of ϕ due to the imbalance of the helical edge modes. For the $\lambda = 0$ limit, the corresponding helical edge modes are given by $E = \Delta_p(1 + H/H_0)\sin \phi$ and $E = -\Delta_p(1 - H/H_0)\sin \phi$. As seen from the curves b and c, the sign of σ_S is reversed by changing the direction of the applied magnetic field. On the other hand, the resulting charge conductance has different features. For $H = 0$, the resulting line

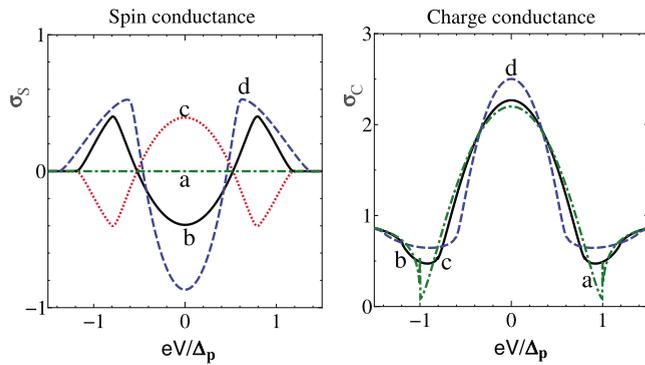


FIG. 2. (Color online) Angle averaged spin conductance σ_S (left panel) and charge conductance σ_C (right panel) as a function of eV with bias voltage V with $\lambda k_F = 0.1\mu$. a: $H=0$, b: $H=0.2H_0$, c: $H=-0.2H_0$, and d: $H=0.4H_0$. Curves *b* and *c* of the right panel are identical.

shape of σ_C is the same as that of a chiral p -wave superconductor (see curve *a* of the right panel).^{16,17,24} As seen from curves *b* and *c* of right panel, σ_C does not change with the change of the direction of the magnetic field H .

Finally, we show in Fig. 3 the zero-voltage σ_S and σ_C . σ_S is nearly linearly proportional to H . Note that with a small magnetic field $H \cong 0.4H_0 \sim 100$ Oe, σ_S is already of the order of 1. Meanwhile, σ_C is almost independent of H .

In conclusion, we have studied the spin transport property of the NCS superconductor from the viewpoint of topology and ABS. We have found the incident angle dependent spin

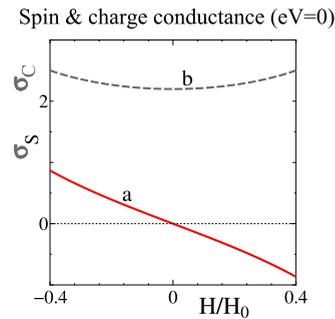


FIG. 3. (Color online) Angle averaged spin and charge conductance for $eV=0$ as a function of H with $\lambda k_F = 0.1\mu$. a: Spin conductance and b: charge conductance.

polarized current flowing through the interface. When the weak magnetic field is applied, even the angle-integrated current is largely spin polarized. As the analogy to the QSHS, the ABS in the NCS superconductor corresponds to the helical edge modes in the QSHS. The Andreev reflection via helical edge modes produces the enhanced spin current specific to NCS superconductor.

Note added. Recently, the topological aspect of NCS superconductor is discussed by Sato and Fujimoto.²⁵

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¹ See, e.g., *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, Berlin, 1987), and references therein.

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