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Electron spin resonance in heavy-fermion systems

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The electron-spin-resonance (ESR) linewidth for localized moments is studied within the framework of the Kondo impurity and Kondo lattice models. An ESR signal for an impurity can only be observed if the Kondo temperature is sufficiently small. For the Kondo lattice, short-range ferromagnetic correlations between the localized spins are necessary to obtain an observable signal. These results are very similar to those derived by Abrahams and Wölfle [Phys. Rev. B **78**, 104423 (2008)] for the Anderson impurity and Anderson lattice models. It is concluded that from ESR data alone it is not possible to distinguish if the resonance is due to localized spins or conducting heavy electron spins.

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I. INTRODUCTION

Until recently it was commonly believed that an electron-spin-resonance (ESR) signal could not be observed in heavy-fermion compounds due to the broad linewidth, a consequence of the large relaxation of the spins in the electron gas. This picture was derived from the single-ion Kondo effect, where the relaxation rate is proportional to the Kondo temperature T_K. This common belief was recently proven wrong. An ESR signal was first observed in YbCuAl, an intermediate-valence compound with moderately heavy carriers.[1] Then an ESR signal was found in single crystals of the quantum critical system $YbRh_2Si_2$,[2] which was attributed to the Yb^{3+} ions despite their rather large Kondo temperature. Since then several other compounds showing ESR resonance were discovered, e.g., $YbIr_2Si_2$,[3] CeRuPO, YbRh,[4] and $YbCo_2Zn_{20}$.[5] The resonance is then not special to the Yb ions since a Ce compound also displays similar properties. It has been concluded that in all of the above compounds there are ferromagnetic correlations among the rare-earth spins.[4] The resonance in $YbRh_2Si_2$ has been confirmed by other groups[6,7] and on a different batch of samples,[7] as well as followed up to 360 GHz.[8]

The observed resonances have a Dysonian line shape,[9] as expected from the skin depth and spin diffusion in a metallic environment. For $YbRh_2Si_2$ the estimated intensity of the signal corresponds to nearly all Yb ions resonating within skin depth for the microwave electromagnetic field.[2] An accurate determination of the fraction of ions resonating is usually difficult because of uncertainties in the penetration depth of the microwave. The analysis of the data was performed within the known framework of ESR of magnetic impurities in metals,[10] i.e., assuming that the resonating electrons are localized. This analysis appears to be consistent since the g factor anisotropy follows that expected for Yb^{3+} ions in a tetragonal crystalline electric field.

The ESR of magnetic ions in a metallic environment corresponds to a Dysonian resonance, and also the resonance of conduction electrons has a similar line shape.[9] In the case of heavy fermions, the g shift is dominated by the one of the f electrons and is going to have the crystalline field anisotropies of the rare-earth sites. Based solely on ESR it is then difficult, if not impossible, to decide if the resonances arise from localized moments or the carriers in a heavy electron band.

In a recent paper Abrahams and Wölfle[11] studied the linewidth of the ESR signal for a heavy-fermion compound within the framework of the Anderson lattice. They obtained that the heavy mass in conjunction with ferromagnetic fluctuations can lead to observable narrow resonances. In this paper we show that similar conclusions hold if such an analysis is carried out for the related Kondo lattice, i.e., for localized spins. Our analysis is not a new calculation but is mostly based on known results for an impurity and is now adapted to the Kondo lattice case.

II. MODEL

The model is the well-known Kondo lattice, consisting of the kinetic energy of the conduction electrons and a spin $S=1/2$ at every lattice site interacting via spin exchange J with the conduction states, i.e., $H=H_0+H_{\rm sd}$ with

$$H_0=\sum_{\mathbf{k}\sigma}\epsilon_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}-B\sum_j S^z_j,$$

$$H_{\rm sd}=\frac{J}{N}\sum_{\mathbf{k}\mathbf{k}'\sigma\sigma' j} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_j}c^\dagger_{\mathbf{k}\sigma}\mathbf{S}_j\cdot\mathbf{s}_{\sigma\sigma'}c_{\mathbf{k}'\sigma'}. \quad (1)$$

Here j labels the spin sites, $\mathbf{R}_j$ denotes the position of the site j, $\mathbf{S}_j$ are the spin-1/2 operators for the localized spin at site j, and $\mathbf{s}_{\sigma\sigma'}$ denotes 1/2 times the Pauli matrices for the conduction states. B is the Zeeman splitting, i.e., $B=hg_f\mu_B$. Similarly for the conduction electrons we have $\epsilon_{\mathbf{k}\sigma}=\epsilon_{\mathbf{k}}-\sigma B_c/2$, where $B_c=hg_c\mu_B$. It is convenient to work with the Hartree-Fock factorization of $H_{\rm sd}$, i.e., we replace

$$B\rightarrow B'=B-J\rho_F g_c\mu_B h/2. \quad (2)$$

This corresponds to the Knight shift of the magnetic resonance.

III. ESR OF AN IMPURITY

For the case of an impurity we suppress the sum over j in Eq. (1) and choose the impurity site as our origin. The ESR

response is determined by the transversal dynamical spin susceptibility defined by the correlation function,

$$\chi^T(z) = -(g_f\mu_B)^2\frac{1}{2}\langle\langle S^+;S^-\rangle\rangle_z, \tag{3}$$

where $S^{\pm}=S^x\pm iS^y$ and the factor 1/2 is introduced for normalization (the S^x and S^y correlation functions are the same).

Following Götze and Wölfle[12] we write

$$\chi^T(z) = \frac{N^T(z)-(g_f\mu_B)^2\langle S^z\rangle}{z-B'+N^T(z)/\chi_0^T}, \tag{4}$$

which defines the function $N^T(z)$. $\langle S^z\rangle$ is the polarization of the impurity and χ_0^T is the static transversal susceptibility. Here $N^T(z)$ is a function that is analytical in the complex upper- and lower-frequency half-planes and falls off as $1/z$ for large z.

The relaxation function $N^T(z)$ has been calculated to second order in the exchange J in Ref. 12,

$$N^T(z) = (g_f\mu_B)^2\frac{\pi}{4}(J\rho_F)^2\left[i+\frac{2}{\pi}\langle S^z\rangle\phi(z)\right],$$

$$\phi(z) = \ln(D/2\pi T) - \frac{z-B'}{z}\psi\left(1-i\frac{z-B'}{2\pi T}\right) - \frac{B'}{z}\psi\left(1+i\frac{B'}{2\pi T}\right), \tag{5}$$

where ψ is the digamma function, T is the temperature, and D is the band cutoff. If $g_f=g_c$, i.e., the g factors of the f and conduction electrons are equal, the total magnetization of the Kondo model is conserved (spin conservation). The Kondo model then does not provide a mechanism for spin relaxation and the ESR linewidth should be zero (bottleneck situation). However, the spin-lattice relaxation of the conduction electrons breaks this bottleneck, introducing a finite linewidth. The situation of the coupled resonance of f and conduction electrons has been studied in Refs. 11 and 13. If the g factors differ significantly as for Yb^{3+} in tetragonal symmetry, this coupling is not very relevant.

A. Korringa relaxation

For $z=\omega+i0$ we may expand the function $N^T(z)$ for small $(\omega-B')$ as $N^T(z)=\Delta'+i\Delta''+(\gamma'+i\gamma'')(\omega-B')+\cdots$. The dynamical susceptibility has then a Lorentzian shape with a relaxation rate[12]

$$1/T_{\rm rel} = \Delta''/(\chi_0^T+\gamma'), \tag{6}$$

where

$$\Delta'' = (g_f\mu_B)^2\frac{\pi}{4}(J\rho_F)^2\left[i+\frac{2}{\pi}\langle S^z\rangle{\rm Im}\ \psi\left(1+i\frac{B'}{2\pi T}\right)\right],$$

$$\gamma' = (g_f\mu_B)^2\frac{\pi}{4}(J\rho_F)^2\frac{2}{\pi}\frac{\langle S^z\rangle}{B'}{\rm Re}\left[\psi\left(1+i\frac{B'}{2\pi T}\right)-\psi(1)\right], \tag{7}$$

and Im and Re denote imaginary and real parts, respectively. Here γ' contains the retardation effects due to the frequency dependence of $N^T(z)$. To obtain the relaxation rate to second order in J, we only need the free ion $\langle S^z\rangle=(1/2)\tanh(B'/2T)$ and $\chi_0^T=\langle S^z\rangle/B'$.[12]

In the limit $B'\to 0$ we obtain

$$\frac{1}{T_{\rm rel}} = \pi(J\rho_F)^2T, \tag{8}$$

which is the well-established Korringa relaxation rate. This relaxation is proportional to T. If, on the other hand, $B'\gg T$ the result is

$$\frac{1}{T_{\rm rel}} = \frac{\pi}{4}(J\rho_F)^2\frac{B'}{1+\frac{1}{2}(J\rho_F)^2\ln(B'/2\pi T)}, \tag{9}$$

so that the linewidth increases almost linearly with the external field. Hence, at low T the residual linewidth in X-band ESR should be less than the corresponding one for Q band.

In general, the relaxation rate in a metal can be separated into two terms: one corresponding to relaxation by spin flips $(1/T_1)$ and the other to phase coherence $1/T_2$, i.e., $1/T_{\rm rel}=1/T_1+1/T_2$, with

$$\frac{1}{T_1} = \frac{\pi}{4}(J\rho_F)^2\frac{B'}{\tanh(B'/2T)} \times \frac{1-\frac{2}{\pi}\tanh(B'/2T){\rm Im}\ \psi\left(1+i\frac{B'}{2\pi T}\right)}{1+\frac{1}{2}(J\rho_F)^2\ {\rm Re}\left[\psi\left(1+i\frac{B'}{2\pi T}\right)-\psi(1)\right]}, \tag{10}$$

$$\frac{1}{T_2} = \frac{\pi}{4}(J\rho_F)^2\frac{B'}{\tanh(B'/2T)} \times \frac{1}{1+\frac{1}{2}(J\rho_F)^2\ {\rm Re}\left[\psi\left(1+i\frac{B'}{2\pi T}\right)-\psi(1)\right]}. \tag{11}$$

In zero field $T_1=T_2$, while as a function of field the spin flips are gradually suppressed, so that $1/T_1$ tends to zero.[12]

B. Kondo effect

For simplicity we will now neglect γ', i.e., the retardation effect, which has played only a secondary role in the Korringa rate discussed above. Perturbatively in J the Kondo effect introduces logarithmic divergencies as a function of T and B', which eventually give rise to the screening of the spin. The Kondo effect affects both the relaxation function $N^T(z)$ and the static susceptibility χ_0^T.

Let us first consider the case $T\gg T_K$, where the corrections to second order in $J\rho_F$ are logarithmic. They are taken into account by dressing the interaction vertex. The renormalized vertex introduces an enhancement and a weak T dependence (on a logarithmic scale) to $N^T(z)$. Also χ_0^T acquires logarithmic corrections. Hence, the result resembles the Korringa relaxation rate; it is linear in T with logarithmic corrections

and a strongly enhanced effective exchange coupling. These results can be found in Ref. 14 with application to ^{171}Yb and ^{174}Yb impurities in Au. A slightly different approach is presented in Ref. 15.

If $T, B' \ll T_K$, on the other hand, the impurity spin and the conduction electrons form a spin singlet as a consequence of the Kondo effect. According to the Fermi-liquid theory, the relaxation function is then determined by the unitarity bound $[N^T(z=0)=i2(g_f\mu_B)^2/\pi]$ and is independent of the exchange coupling J.[16] As a function of ω and T, the imaginary part of N^T decreases as $(\omega/T_K)^2$ and $(T/T_K)^2$. The static susceptibility for the spin singlet is a constant (there are Van Vleck-type transitions into the excited spin triplet) of the order of $1/T_K$. The relaxation rate is then of the order of T_K instead of being proportional to the temperature [see Eq. (6)], so that only if T_K is less than 100 mK it would be possible to observe an X-band ESR. A calculation of the longitudinal dynamical spin susceptibility was carried out in Ref. 17. In this limit we expect the longitudinal and transversal dynamical susceptibilities to be essentially the same.

In summary, for a Kondo impurity an ESR resonance can only be observed if T_K is very small. Otherwise the resonance width is going to be too broad to be seen. One possible way to overcome this is to measure it in rather high magnetic fields with a correspondingly larger frequency of the electromagnetic field. The above considerations lead to the commonly accepted statement that ESR of a Kondo ion cannot be observed.

IV. KONDO LATTICE

A. Spin conservation

The Kondo lattice, defined by Hamiltonian (1), conserves the total spin. This is also the case for the Anderson lattice model.

Rare-earth ions (Ce or Yb) have a large spin-orbit coupling. In a lattice with tetragonal symmetry the crystalline electric field splits the J multiplet into $(J+\frac{1}{2})$ Kramers doublets. In general only the one with lowest energy needs to be considered. This doublet is usually represented by a spin 1/2 and this naturally leads to the Kondo or Anderson lattice model. However, the Kramers doublet has orbital content and is not a true spin since the orbital momentum depends on the chosen origin of coordinates. Hence, the "spin" is only locally, but not globally, conserved. This, however, does neither affect the single impurity problem nor the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between rare-earth sites (where only the interaction amplitude is modified), which are SU(2) invariant.

The orbital content of the Kramers doublets induces a strong spin-orbit coupling into the conduction states, favoring a fast spin-lattice relaxation. The spin information, contained, e.g., in an ESR spin flip, can then not travel very far via the conduction states. Hence, the Kondo and Anderson lattice models have to be taken with some caution when spin conservation is important.

B. Is ESR in a heavy-fermion compound a local or an extended probe?

The experimental results for $YbRh_2Si_2$ have all been interpreted as if the resonance is due to localized f electrons.[2–4,7] In other words, as for ESR on an impurity, if the microwave induces a spin flip at one site, the response of the system is measured at the same site. The response function in that case is the local susceptibility. On the other hand, Abrahams and Wölfle[11] described the ESR of the metallic system with the $\mathbf{q}\to 0$ limit of the global dynamical susceptibility, which appears to be the natural approach for extended conducting states.

The global dynamical susceptibility is the Fourier transform over the site pairs, $S(\mathbf{q},\omega)$. The longitudinal dynamical susceptibility, for instance, is measured by inelastic neutron scattering. Assuming that the magnetic field is small compared to T_K, the $\mathbf{q}\to 0$ limit corresponds to the low-energy window for forward scattering in an inelastic neutron spectrum (only accessible as an extrapolation).

In principle, one could experimentally distinguish between these two situations by partially substituting the rare-earth ions by their nonmagnetic analog. The dilution increases the average distance between the magnetic ions. The local picture would be consistent with a resonance that does not significantly change with concentration. Unfortunately, alloying in a metal inhomogeneously broadens the resonance, so that this procedure is not practicable.

This difficulty can be bypassed for the cubic low-carrier density system $Eu_xCa_{1-x}B_6$, which crystallizes for all x and ESR can be observed for all x at all T (measured up to room temperature).[18] For $x>0.15$ the resonance is a Dysonian (metallic), while for low x it is Lorentzian with fully resolved fine structure (insulator). In the metallic regime there are not enough carriers to generate a Korringa relaxation rate. For $x>0.15$ there are no dramatic changes in the resonance as a function of x, i.e., as the average distance between the Eu ions is reduced. This experiment can be taken as an indication that ESR for local moments is a local probe.

Furthermore, inhomogeneous broadening of ESR lines (e.g., La- or Lu-diluted $YbRh_2Si_2$) is a concept that only makes sense if the resonance is at least relatively local. For $Eu_xCa_{1-x}B_6$ with $x=0.07$ and $x=0.10$, the superpositions of two resonances, one corresponding to insulating and the other to metallic patches of the sample, are observed. Such phase separation is also representative of local measurements.

To reconcile the differences at least partially we have to invoke the spin-lattice relaxation of the conduction electrons and the orbital content of the spin. This limits the range over which the spin can travel. If this relaxation is strong we are close to the local picture.

C. Dynamical susceptibility

We first consider ESR as a local probe, i.e., as if it measures the response at one site to a perturbation at the same site. Hence, the correlation function to be considered is the same one as for the impurity, but with Hamiltonian (1). The local transversal dynamical susceptibility can be cast into the same form as Eq. (4). It depends on *local* thermodynamic quantities, such as χ_0^T and $\langle S^z\rangle$, and the local relaxation function. The relaxation function can again be calculated perturbatively in the exchange J. To second order in $J\rho_F$ we obtain

the same result as for an impurity [Eq. (5)]. Kondo logarithms and intersite correlations start to play a role to the order higher than second in J. Kondo logarithms were already present in the Kondo impurity case. New are the intersite correlations which eventually are responsible for the coherence in the Kondo lattice and the formation of heavy electron states at low T, i.e., low-energy excitations corresponding to quasiparticles with large effective mass. This is the consequence of the strong-coupling fixed point for the Kondo lattice. The relaxation kernel $N^T(\omega)$ will approach an imaginary constant value $i\Delta''$ as $\omega \to 0$, which corresponds to the unitarity bound. Unfortunately, a Shiba relation[16] determining Δ'' is not known for the Kondo lattice, but it is expected that $\Delta''=\alpha(g_f\mu_B)^2$, where α is of the order of unity. The exact value of $N^T(0)$ is not relevant here. If the field is small, the low ω and T dependences of $N^{T''}$ are Fermi-liquid type, i.e., $(\omega/T_K)^2$ and $(T/T_K)^2$.

The polarization $\langle S_i^z \rangle$ is proportional to the transversal susceptibility χ_0^T and the proportionality constant is B'. Neglecting γ', i.e., the retardation effects, $1/T_{\rm rel} \approx \Delta''/\chi_0^T$, the relaxation rate has primarily the temperature dependence of χ_0^T. This is similar to the impurity case, except that χ_0^T now contains intersite correlations.

If we consider ESR as a nonlocal probe, the correlation function is $\chi^T(z)=-(g_f\mu_B)^2\frac{1}{2N}\Sigma_{ij}\langle\langle S_i^+;S_j^-\rangle\rangle_z$. This means that we also consider the possibility that a spin flip induced at the site j is observed at the site i. We again cast the response function into the general form of Eq. (4). The contribution to $N^{T''}(\omega)$ arising from the pair of sites i and j is

$$(g_f\mu_B)^2\frac{\pi}{4\omega}\frac{J^2}{N^2}\sum_{\mathbf{k},\mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')(\mathbf{R}_i-\mathbf{R}_j)}\delta(\omega-\epsilon_{\mathbf{k}'}+\epsilon_{\mathbf{k}}) \times [f(\epsilon_{\mathbf{k}})-f(\epsilon_{\mathbf{k}'})](\langle S_i^z S_j^z\rangle+\langle S_i^+ S_j^-\rangle), \quad (12)$$

where we neglected the magnetic field (below we will consider the limit $B' \ll T_K$). Converting the sums over the wave vectors into integrals we obtain for $\omega \to 0$ and $T \to 0$,

$$(g_f\mu_B)^2\frac{\pi}{4}(J\rho_F)^2\frac{\sin^2(k_F R_{ij})}{(k_F R_{ij})^2}(\langle S_i^z S_j^z\rangle+\langle S_i^+ S_j^-\rangle). \quad (13)$$

It is now important to introduce relaxation mechanisms for the conduction states, e.g., the spin-lattice relaxation. This gives rise to a spin mean-free path l and expression (13) is to be multiplied by the exponential $\exp(-R_{ij}/l)$. Expression (13) for a fixed $\mathbf{R}_i$ has to be summed over all $\mathbf{R}_j$. The outcome depends on the expectation values, i.e., the correlation between the spins. For ferromagnetic coupling the expectation values are roughly independent of $\mathbf{R}_{ij}$ and the contribution is a constant that strongly depends on the mean-free path. In this case the exponential $\exp(-R_{ij}/l)$ is of fundamental importance. For antiferromagnetic correlations the integrand oscillates and the intersite contribution to $N^T(0)$ is small. The main contribution is then the $i=j$ term. The quantity $N^{T''}(\omega)$ has to be positive definite because it represents a dissipation.

In the strong-coupling limit the relaxation function for $\omega \to 0$ and $T \to 0$ is expected to approach a constant imaginary value. This value is given by the $\pi/2$ phase shift of the electrons at the Fermi surface, but since the Shiba relation[16] for the lattice is not known, it remains undetermined. This is similar to the local $N^{T''}(\omega)$. In both cases the expected result is $(g_f\mu_B)^2$ times a constant of the order of unity. As a function of ω and T the function decreases as expected for a Fermi liquid.

The relaxation rate is given by $N^{T''}(0)/\chi_0$. Below we discuss the effect from the static susceptibility. We have assumed that $B' \ll T_K$.

D. Magnetic correlation effects

Usually, in a heavy-fermion compound, the rare-earth spins are antiferromagnetically correlated, even if the system does not undergo a phase transition to long-range order. The correlations have short-range character and the susceptibility follows a Curie-Weiss law with antiferromagnetic Weiss temperature θ, $\chi_0=C/(T+\theta)$, where θ is of the order of T_K. Hence, the linewidth roughly follows a Korringa law, with a residual $T=0$ linewidth proportional to θ. This is similar to the case of a Kondo impurity. The resonance can only be observed if θ is very small, i.e., of the order of 100 mK or less for X-band microwaves. This would require an extremely narrow heavy-fermion band or fermions with an effective mass of $10^5 m_e$, where m_e is the free-electron mass. The T dependence in this case would be linear in T, i.e., a Korringa law.

However, if the rare-earth spins are ferromagnetically correlated the static susceptibility is given by $\chi_0=C/(T-T_C)$ for $T>T_C$, where $T_C>0$ is the Curie temperature. As $T \to T_C$ the susceptibility diverges and hence, according to expression (6), the ESR linewidth becomes very narrow. In general we would have to replace it by $\chi_0 \sim |t|^{-\gamma}$, where $t=(T-T_C)/T_C$ is the reduced temperature and γ is the corresponding critical exponent, which is larger than 1. Hence, the narrowing of the line proceeds even faster as $t \to 0$. This result has to be regarded with caution because we have neglected the relaxation through collective excitations, i.e., magnons.

If the ferromagnetic correlations are not strong enough to produce long-range order at any temperature, i.e., their nature is short ranged, then the susceptibility is proportional to $T^{-\gamma}$ and again the relaxation rate is strongly reduced at low T. In this case relaxation through magnons does not play a relevant role because low-energy excitations have a wavelength larger than the range of the correlations. Shorter wavelength magnons cannot be excited because their energy is larger than that of the thermal bath. Hence, there is the possibility that the electron spin resonance can be observed.

For $YbRh_2Si_2$ the ferromagnetic correlations are predominantly in the ab plane. The linewidth should then depend on whether the field is in the plane or along the c axis. For a field in the ab plane the short-range ordered domains tend to align. This should give rise to a large static susceptibility along the field direction, as well as a large χ_0^T. In the case of a field oriented along the c axis, the spins are tilted out of the plane, which should give rise to smaller susceptibilities. The resonance linewidth is then expected to be narrower for the field in the ab plane, in agreement with the experimental observations.[7,19] Since the in-plane value of the g factor is

much smaller than the one along the c axis, there is also a magnetic field broadening of the resonance [see Eq. (9)], in addition to the above-mentioned broadening effect.

V. DISCUSSION

(1) Magnetic impurities are often introduced in compounds that by themselves do not show an ESR signal. These are usually Kramers rare-earth ions, such as Nd^{3+}, Eu^{2+}, Gd^{3+}, Er^{3+}, and Dy^{3+}. Except for Eu^{2+} and Gd^{3+}, which are $S=7/2$ S-state ions, the crystalline electric field yields a doublet (in rare cases a quadruplet) ground state. The spin exchange with the metallic host is such that there is no Kondo effect. The relaxation rate in this case is Korringa type, i.e., at low T it has a component that is proportional to T.

(2) On the other hand, Ce^{3+} and Yb^{3+} impurities are Kondo ions and consequently an ESR signal cannot be observed unless the Kondo temperature is very small. One example is Yb impurities in Au.[14] The Kondo effect involves a spin-singlet formation at low T, so that the linewidth is proportional to T_K.

(3) For the Kondo lattice, the short-range correlations among the localized spins play a fundamental role. If the spins are antiferromagnetically correlated, the linewidth is of the order of the Curie-Weiss temperature of the susceptibility. θ is usually too large for ESR to be observed. On the other hand, if the spins are ferromagnetically correlated the linewidth is strongly suppressed and there is the possibility of an ESR signal. In this case the ferromagnetic short-range correlations prevent the spin flip from being passed on to other sites. To some degree this situation can be considered a narrowing of the signal due to bottleneck.[7]

(4) The above conclusions for Kondo impurities and the Kondo lattice (i.e., involving localized spins) are similar to those derived for the Anderson impurity and Anderson lattice by Abrahams and Wölfle[11] (i.e., for heavy electrons that are delocalized).

(5) The ESR line shape in a metallic environment is Dysonian. This is the case for resonating localized spins,[10] as well as for conduction states.[9] For the case of a heavy-fermion compound the line shape can then not distinguish between localized spins or conduction states with heavy mass.

(6) Similarly from the g values and their anisotropy, we cannot distinguish between localized and conducting states resonating. For the localized states the g tensor is determined by the crystalline electric field scheme. Heavy fermions arise from the hybridization of the f electrons with the conduction states. Close to the Fermi level they are dominated by the f character and, hence, the g tensor is predominantly given by the crystalline field scheme of the f states.

(7) Introducing impurities into the compound, e.g., substituting Yb ions by Lu ions in $YbRh_2Si_2$, broadens the resonance.[7] This is expected within the heavy-fermion picture because the impurities break the coherence of the heavy-fermion states and enhance the spin-lattice relaxation. Similarly, the relaxation with impurities is faster for the Kondo lattice. The localized spins relax into the conduction-electron bath and the conduction states due to the scattering off the impurities relax faster via spin-lattice relaxation. In addition the line may become inhomogeneously broadened.

(8) Intuitively, ferromagnetic correlations, within either picture, should increase the effective g value significantly similar to the exchange interaction in the Knight shift. In Ref. 11 it is argued that the g shift is almost completely off set by vertex corrections. This is the consequence of the assumed spin-rotational invariance and the spin conservation.[20] Similarly, for the Kondo lattice with ferromagnetic short-range correlations, we may assume that the Knight shift is affected by spins that are further away from the resonating ion than the range of the ferromagnetic correlation. Then patches with different polarizations could compensate each other, leading to a g shift that is small in magnitude. The RKKY interaction is predominantly responsible for the ferromagnetic correlations, and also in this case the interaction is isotropic and conserves the total localized pseudospin [global SU(2) invariance].

An enhancement of the g value has not been observed for $YbRh_2Si_2$. In Ref. 10 the g factor first decreases with field before it increases slightly.

(9) At low T the bulk susceptibility of the $YbRh_2Si_2$ samples appears to have an antiferromagnetic (rather than ferromagnetic) Weiss temperature.[21] However, in the temperature and field ranges, where ESR is measured, the susceptibility appears to have the characteristics of ferromagnetic correlations.[4]

(10) The spin fluctuations of the heavy electrons in $YbRh_2Si_2$ have also been probed locally on the ^{29}Si site with nuclear magnetic resonance.[22] At low T and fields above 1 T, the Knight shift follows the temperature dependence of the bulk susceptibility but shows anomalous T dependencies at low fields. Also strong deviations from the Korringa relation are observed and at low T the nuclear T_1-relaxation time is anomalously short. These measurements were carried out along the Fermi-liquid to non-Fermi-liquid crossover boundary, while the ESR data stem from a different region of the phase diagram.

VI. CONCLUSIONS

In summary, using only ESR results it is not possible to distinguish between localized states or itinerant heavy fermions resonating. The ESR in a Kondo or heavy-fermion lattice is primarily related to short-range ferromagnetic correlations. A resonance has been observed in several systems, which all appear to have ferromagnetic correlations. The ESR in $YbRh_2Si_2$ is then not related to the quantum critical point, which is induced by antiferromagnetic spin fluctuations.

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