## **Giant frictional drag in strongly interacting bilayers near filling factor one**

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We study the frictional drag in high mobility, strongly interacting GaAs bilayer hole systems in the vicinity of the filling factor  $\nu=1$  quantum Hall state (QHS) at the same fillings where the bilayer resistivity displays a re-entrant insulating phase. Our measurements reveal a very large longitudinal drag resistivity  $(\rho_{xx}^D)$  in this *xx* regime, exceeding 15 k $\Omega/\square$  at filling factor  $\nu$ =1.15.  $\rho_{xx}^D$  shows a weak temperature dependence and appears to saturate at a finite, large value at the lowest temperatures. Our observations are consistent with theoretical models positing a phase separation, e.g., puddles of  $\nu = 1$  QHS embedded in a different state, when the system makes a transition from the coherent  $\nu = 1$  QHS to the weakly coupled  $\nu = 2$  QHS.

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Closely spaced bilayer carrier systems have been the test ground for a multitude of novel electronic states with no counterpart in the single-layer case. The most important are quantum Hall states (QHSs) possessing interlayer coherence<sup>1</sup> at total filling factor  $\nu = 1/2$  $\nu = 1/2$  (layer filling  $\nu_{\text{layer}} = 1/4$ ) (Refs. 2 and [3](#page-3-2)) and  $\nu=1$  ( $\nu_{\text{layer}}=1/2$ ).<sup>[4](#page-3-3)[,5](#page-3-4)</sup> These QHSs are stabilized when the interaction between carriers in the same layer is comparable to that of carriers residing in opposite layers. The  $\nu$ =1 QHS has been shown to exhibit enhanced interlayer tunneling $6$  reminiscent of a Josephson junction, as well as a peculiar charge-neutral superfluid in counterflow transport[.7](#page-3-6)[,8](#page-3-7) In a simple picture the  $\nu = 1$  QHS can be regarded as a condensate of excitons,<sup>9</sup> where carriers and vacancies pair up in the opposite half-filled layers and form excitons, which condense at lowest temperatures.

An equally interesting ground state also explored in conjunction with the emergence of high quality, interacting bi-layer systems is the Wigner crystal (WC).<sup>[10](#page-3-9)</sup> Experimentally, transport measurements in electron and hole bilayers show a suppression of QHSs beyond a given filling factor, namely,  $\nu = 1/2$  in interacting electron bilayers<sup>11</sup> and  $\nu = 1$  in interacting hole bilayers. $12$  This is similar to the suppression of fully developed QHSs in single layers beyond  $\nu = 1/5$  for electrons and  $\nu = 1/3$  for dilute holes which has been interpreted as a signature of the WC being stabilized at sufficiently low fillings.<sup>10</sup> Furthermore, the quenching of QHSs at sufficiently low fillings is accompanied by the presence of a re-entrant insulating phase (RIP) around the lowest filling QHS, suggesting an onset of the WC state. To further understand this RIP, here we study the frictional drag in interacting GaAs hole bilayers in the vicinity of the phase-coherent  $\nu = 1$  QHS, in the same filling factor range where the bilayer resistivity exhibits a RIP. Our results show an anomalous record large longitudinal drag resistivity  $(\rho_{xx}^D)$  on the flanks of  $\nu = 1$ , larger than 15 k $\Omega/\square$ . Equally anomalous is the relatively weak temperature (*T*) dependence of  $\rho_{xx}^D$ ; it follows a power law  $\rho_{xx}^D \sim T^{\alpha}$ , with  $\alpha < 1$ , and saturates at a finite value at the lowest *T*.

Our sample is a Si-modulation-doped GaAs double-layer hole system grown on GaAs (311)A substrate. It consists of two GaAs quantum wells which have a width of 150 Å each and are separated by a 75-Å-wide AlAs barrier. The sample is patterned in a Hall bar aligned along the  $[01\bar{1}]$  crystal direction[.13](#page-3-12) Diffused InZn Ohmic contacts are placed at the end of each lead. We use front and back gates $14$  to selectively deplete one of the layers near each contact. As grown, the areal densities were  $p_T = 2.6 \times 10^{10}$  cm<sup>-2</sup> and  $p_B = 3.2$  $\times 10^{10}$  cm<sup>-2</sup> for the top and bottom layers, respectively. The mobility along  $\lceil 011 \rceil$  at these densities is approximately  $200\,000\,$  cm<sup>2</sup>/V s. Metallic top and bottom gates are added on the active area to control the densities in the layers. The measurements are performed down to a temperature of *T*  $= 30$  mK and using standard low-current  $(0.5-1n)$ , lowfrequency lock-in techniques.

Two types of measurement configurations are used in our study. In one (bilayer) configuration, current is passed through both top and bottom layers and the Ohmic contacts connect both layers simultaneously. The voltage drops along and across the Hall bar, divided by the total bilayer current, represent the longitudinal  $(\rho_{xx}^B)$  and Hall  $(\rho_{xy}^B)$  bilayer resistivities. In a second (drag) configuration, current is passed in one (drive) layer only by using the selective depletion technique around the Ohmic contacts such that they connect to a single layer only.<sup>14</sup> The voltage drops measured in the opposite (drag) layer, divided by the drive current, represent the longitudinal  $(\rho_{xx}^D)$  and Hall  $(\rho_{xy}^D)$  drag resistivities. The drag resistivity provides a measure of the electron-electronscattering rate between the carriers in the drive layer and those in the drag layer. For the data presented here we adopt the following sign convention: the longitudinal (Hall) drag resistivity is defined as positive when the voltage drop along (across) the drag layer is *opposite* to the voltage drop along (across) the drive layer. We performed the usual consistency checks associated with drag measurements[.15](#page-3-14) Owing to the proximity of the two layers in our sample, there is a small but finite interlayer leakage current. This leakage translates into an uncertainty in frictional drag measurements, which does not exceed  $\pm 6\%$  in our study.

In Fig. [1](#page-1-0) we show  $\rho_{xx}^B$  and  $\rho_{xx}^D$  vs the applied perpendicular magnetic field  $(B)$ , both measured at  $T=30$  mK. The total bilayer density is  $p_{\text{tot}} = 5.5 \times 10^{10} \text{ cm}^{-2}$ , equally distributed between the two layers (balanced). We observe a fully developed QHS at  $\nu = 1$ , stabilized here solely by interlayer coherence[.16](#page-3-15) In a simple picture the emergence of a QHS at

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FIG. 1. (Color online) Bilayer and longitudinal drag resistivities  $(\rho_{xx}^B \text{ and } \rho_{xx}^D)$  measured at  $T=30 \text{ mK}$  for a balanced bilayer with  $p_{\text{tot}}$ = 5.5 × 10<sup>10</sup> cm<sup>-2</sup>. Both traces are plotted on the same scale. Inset: *T* dependence of  $\rho_{xx}^B$  data.

 $\nu = 1$  can be understood by considering the pairing of carriers and vacancies in the opposite layers. At a total filling factor of 1 each layer has the lowest Landau level half full, i.e., has an equal number of carriers and vacancies. Owing to the close proximity of the two layers and the ensuing interlayer interaction, it is energetically favorable to form carriervacancy pairs in the opposite layers, which condense at the lowest temperature. A spectacular signature of this phenomenon is the emergence of a neutral superfluid, experimentally observed when equal and opposite currents are passed in the two layers[.7,](#page-3-6)[8](#page-3-7) The ratio between the interaction energy of carriers in the same layer and in different layers is commonly quantified by  $d/l_B$ , where *d* is the interlayer distance and  $l_B$  $=\sqrt{\hbar/eB}$  is the magnetic length at  $\nu=1$ . For the case exam-ined in Fig. [1](#page-1-0)  $d/l_B = 1.33$ .

Figure [1](#page-1-0) inset shows  $\rho_{xx}^B$  measured at different temperatures for the same layer densities as in the main panel. These data show that as *T* is reduced a RIP develops on the flanks of the  $\nu$ =[1](#page-1-0) QHS.<sup>17</sup> Most interestingly, Fig. 1 data show a very large longitudinal drag on the left flank of  $\nu = 1$ , in the same filling factor range where  $\rho_{xx}^B$  exhibits a RIP.<sup>18</sup> In contrast to typical drag measurements where the drag resistivity is 1–3 orders of magnitude smaller than the single-layer resistance,<sup>[1](#page-1-0)5</sup> Fig. 1 data reveal that  $\rho_{xx}^D$  and  $\rho_{xx}^B$  are of the *same* order of magnitude, which testifies to the strong interlayer coupling at these filling factors. Clearly frictional drag constitutes a substantial component of the longitudinal resistivity here, in contrast to frictional drag at  $B=0$  T where drag is a very small perturbation.

In Fig. [2](#page-1-1) we show  $\rho_{xx}^D$  (top panel) and  $\rho_{xy}^D$  (bottom panel) vs *B*, measured at different *T* ranging from 30 to 630 mK and at the same layer densities as the data of Fig. [1.](#page-1-0) At the lowest *T* the data of Fig. [2](#page-1-1) show a nearly vanishing  $\rho_{xx}^D$  at  $\nu=1$  and  $\rho_{xy}^D$  quantized at  $h/e^2 = 25.88 \text{ k}\Omega$ .<sup>[19](#page-3-18)</sup> The *T* dependence of the Hall drag measured at and around  $\nu = 1$  is relatively weak: as *T* is increased  $\rho_{xy}^D$  remains close to the quantized value for *T* 

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FIG. 2. (Color online)  $\rho_{xx}^D$  (top panel) and  $\rho_{xy}^D$  (bottom panel) vs *B* measured at different temperatures. The total bilayer density is  $p_{\text{tot}} = 5.5 \times 10^{10} \text{ cm}^{-2}$ . Note the very large  $\rho_{xx}^D$  on the left flank of  $\nu$ = 1, concomitant with the observed RIP.

as high as 500 mK. The weak *T* dependence of  $\rho_{xy}^D$  at  $\nu=1$  is consistent with previous results in GaAs hole bilayers, which show a vanishing counterflow Hall resistivity for *T*  $\leq$  500 mK (Ref. [8](#page-3-7)), and indicates a strong pairing of the carriers and vacancies in opposite layers. Figure [2](#page-1-1) data (top panel) substantiate our observation of an anomalously large drag in the vicinity of  $\nu = 1$  QHS. As *T* is increased  $\rho_{xx}^D$  increases, reaching a record of 17 k $\Omega$ /  $\square$  at *T*=630 mK at *v* = 1.10. Equally noteworthy is that the onset of the anomalously large  $\rho_{xx}^D$  *coincides* with the onset of the nonzero  $\rho_{xy}^D$ , indicating that the particle-vacancy pairing which stabilizes the  $\nu = 1$  QHS is also responsible for the observed anomalously large  $\rho_{xx}^D$ .

In Fig. [3](#page-2-0) we show the *T* dependence of the anomalously large longitudinal drag observed near  $\nu = 1$ , from 30 to 630 mK. An increase in interlayer current prevents an accurate measurement of the frictional drag near  $\nu = 1$  above *T* = 700 mK. The  $\rho_{xx}^D$  vs *T* data were measured in two different cooldowns at  $\nu = 1.10$  and 1.15, namely, fillings where the

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FIG. 3. *T* dependence of  $\rho_{xx}^D$  measured at  $\nu = 1.10$  and  $\nu = 1.15$  in two different cooldowns. The different *T* dependences in separate cooldowns suggest that sample disorder affects the measured  $\rho_{xx}^D$ .

RIP reaches the maximum resistance. Note that  $\rho_{xx}^D$  maximum shifts slightly from  $\nu = 1.15$  at the lowest *T* to  $\nu = 1.10$ at the highest  $T$ , as apparent from Fig. [2](#page-1-1) (top panel) data. Several features of Fig. [3](#page-2-0) data are noteworthy. First,  $\rho_{xx}^D$  exhibit a weak, slightly sublinear *T* dependence in the range of  $T = 100 - 500$  mK, which contrasts the more common  $\rho_{xx}^D$  $\propto T^2$  characteristic of the Coulomb drag in two-dimensional (2D) electron systems<sup>15</sup> or  $\rho_{xx}^D \propto T^{4/3}$  observed in drag measurements between composite fermions.<sup>20</sup> Second,  $\rho_{xx}^D$  appears to saturate at a constant, *finite* value below *T*  $= 100 \text{ mK}.^{21}$  Third, the large  $\rho_{xx}^D$  near  $\nu = 1$  shows a cooldown dependence, suggesting that sample disorder affects the measured  $\rho_{xx}^D$ .

Before discussing our observation of enhanced frictional drag in the vicinity of  $\nu = 1$  within existing theoretical models, we summarize the salient features of the experimental data. First, the longitudinal drag is greatly enhanced in the vicinity of the bilayer  $\nu = 1$  QHS, exceeds 15 k $\Omega / \square$ , and becomes comparable to the single-layer longitudinal resistivity. Second, the observed giant longitudinal drag emerges concomitantly with the large Hall drag near  $\nu = 1$ , indicating that particle-vacancy pairing is present. Third, the giant longitudinal drag has a weak, sublinear *T* dependence and appears to saturate at a finite and large value ( $\approx$ 5 k $\Omega$ / $\Box$ ) at the lowest *T*. Finally, the frictional drag exhibits a cooldown dependence, suggesting that disorder affects the measured  $\rho_{xx}^D$  value. These features contrast with the frictional drag between two 2D carrier systems, which typically has a small  $(1-100 \ \Omega)$  magnitude and a  $\rho_{xx}^D \propto T^2$  temperature dependence.<sup>15</sup> Although our measurements are performed in the quantum Hall regime where an agreement with Fermiliquid theor[y22](#page-3-20)[,23](#page-3-21) should not be expected, these highlighted differences are nonetheless stark.

Our data can qualitatively be explained by theoretical models which invoke the coexistence of two phases as the system makes a transition, driven by filling factor in our case, from the  $\nu = 1$  QHS to the weakly coupled  $\nu = 2$  QHS.

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FIG. 4. (Color online)  $\rho_{xx}^D$  vs  $\rho_{xy}^D$  in units of  $h/e^2$  for different temperatures. The end points represent the weakly ( $\nu \geq 2$  and  $\rho_{xy}^D$  $= 0$ ) and strongly ( $\nu = 1$  and  $\rho_{xy}^D = -h/e^2$ ) coupled bilayer regimes. The dashed line represents the semicircle relation  $(\rho_{xy}^D + 1/2)^2$  $+(\rho_{xx}^D)^2 = 1/4$  expected according to Ref. [24.](#page-3-22)

The  $\nu = 2$  QHS consists of a pair of  $\nu<sub>layer</sub> = 1$  QHSs, one in each of the two layers. Stern and Halperin $^{24}$  examined theoretically the transition between the strongly coupled  $\nu = 1$ QHS and two weakly coupled layers, each at  $v_{\text{layer}}=1/2$ . By postulating that in the transition regime the system is composed of puddles of  $\nu = 1$  QHS phase and assuming the conductivity tensors in the two regimes, namely, a strongly coupled  $\nu = 1$  QHS on one hand and two weakly coupled layers at  $\nu = 1/2$  each on the other hand, they derived an expression for the longitudinal and Hall drags as a function of the fraction of the  $\nu=1$  QHS across the transition. Their model predicts a large longitudinal drag, as high as *h*/2*e*<sup>2</sup> in the transition regime, concomitantly with a nonzero Hall drag. Their results can analytically be approximated by a simple semicircle relation for the drag resistivity tensor,

$$
(\rho_{xy}^D + 1/2)^2 + (\rho_{xx}^D)^2 = 1/4, \tag{1}
$$

<span id="page-2-2"></span>with the resistivity expressed in units of *h*/*e*<sup>2</sup> . Kellogg *et al.*[25](#page-3-23) experimentally probed this transition by varying the total bilayer density, which in turn changes  $d/l_B$ . They observed an enhanced longitudinal drag in the transition region, in qualitative agreement with the theoretical model. $^{24}$ 

In order to quantitatively compare our experimental re-sults with the model of Ref. [24,](#page-3-22) in Fig. [4](#page-2-1) we show  $\rho_{xx}^D$  vs  $\rho_{xy}^D$ at different  $T$  along with the semicircle law of Eq.  $(1)$  $(1)$  $(1)$ . The end points of the semicircle, namely,  $\rho_{xx}^D = \rho_{xy}^D = 0$  and  $\rho_{xx}^D = 0$ ,  $\rho_{xy}^D$ =−1, represent the weakly and strongly coupled bilayer regimes at  $\nu \geq 2$  and  $\nu = 1$ , respectively. As the system makes the transition from weakly to strongly coupled,  $\rho_{xx}^D$  and  $\rho_{xy}^D$ depart from zero simultaneously, with  $\rho_{xx}^D$  reaching a temperature-dependent maximum.  $\rho_{xx}^D$  is close to  $h/2e^2$ 

= [1](#page-2-2)2.9 k $\Omega / \Box$  predicted by Eq. (1). At intermediate temperatures  $T \approx 300 \text{ mK}$  the  $\rho_{xx}^D$  vs  $\rho_{xy}^D$  data are in very good quantitative agreement with the semicircle law of Eq.  $(1)$  $(1)$  $(1)$  but depart from it at the lowest *T*. The stronger *T* dependence of  $\rho_{xx}^D$  compared to that of  $\rho_{xy}^D$  partly explains the departure from the semicircle rule at the lowest *T*. We note however that Eq.  $(1)$  $(1)$  $(1)$  is expected to hold quantitatively if the drag resistivity is large compared to the symmetric (parallel flow) bilayer resitivity at all fillings and also neglects the bilayer and drag resistivities in the weakly coupled regime. In light of these approximations, the agreement with the simple semicircle law is satisfactory.

A separate model, also invoking the coexistence of two phases, that may explain the giant frictional drag data was proposed by Spivak and Kivelson[.26](#page-3-24) The model of Ref. [26](#page-3-24) considers the frictional drag between a passive layer and a low-density 2D system where the ground state consists of bubbles of WC embedded in Fermi liquid. Each WC bubble in the active layer casts an image in the passive layer, which  $(2009)$ 

can be pictured as a hard wall potential being dragged in the passive layer. This in turn results in a significant scattering for the electrons in the passive layer and hence an anomalously large frictional drag. We speculate that one plausible scenario for the large drag in the vicinity of  $\nu = 1$  is a "microemulsion" of the  $\nu = 1$  QHS coexisting with a WC state.

In summary we report the observation of giant frictional drag in the vicinity of the strongly coupled bilayer  $\nu = 1$ QHS. The giant longitudinal drag emerges concomitantly with a nonzero Hall drag, indicating the particle-vacancy pairing in this regime. Our observations are consistent with theoretical models $24,26$  $24,26$  which invoke the coexistence of two distinct phases as the system makes a transition from the  $\nu$  $= 1$  bilayer OHS, e.g., puddles of  $\nu = 1$  OHS embedded in a weakly coupled bulk state or in a Wigner crystal state.

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