Paraconductivity in layered cuprates behaves as if due to pairing of nearly free quasiparticles

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We revisit the Aslamazov-Larkin theory of paraconductivity in two dimensions to identify its universal and its specific model-dependent features. We show that both the numerical prefactor and the temperature dependence of the experimental paraconductivity in underdoped $La_{2-x}Sr_xCuO_4$ provide evidence that the paired carriers behave *as if* they were nearly free fermionic quasiparticles. This conclusion is strengthened by the analysis of paraconductivity data in the presence of a finite magnetic field. In this case we show that there exists a temperature range above T_c where the superconducting fluctuations are purely Gaussian and their destruction is determined by a rather low magnetic field scale $H_{c2}^G(T=0)$.

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I. INTRODUCTION

Layered superconducting (SC) cuprates are characterized by a pseudogap state in the underdoped region of the phase diagram, below a temperature T^* which at low doping is much larger than the SC critical temperature T_c and merges with it near optimal doping, where T_c is maximum. A possible explanation relies on the formation of incoherent SC Cooper pairs below T^*, the modulus $|\Delta|$ of the SC order parameter acting as the pseudogap detected by various thermodynamical and transport measurements. Superconductivity is prevented by fluctuations of the phase of the order parameter and develops only below T_c, where phase coherence is eventually established and the preformed pairs condense. The observation of a sizable Nernst effect[1] and a strong diamagnetic response[2,3] above T_c have been interpreted in this sense.[4] If this were the case, however, the most anisotropic cuprates [e.g., $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO)] should display an exponential temperature dependence in the enhancement of conductivity due to SC fluctuations at temperatures $T>T_c$ (the so-called *paraconductivity*) associated with vortical fluctuations, typical of a Kosterlitz-Thouless transition in two dimensions.[5] Instead, it is well documented[6–17] that paraconductivity in all the families of cuprates is fully accounted for by the standard Aslamazov-Larkin (AL) theory[18,19] based on Gaussian SC fluctuations, with the real and imaginary parts of the SC order parameter Δ fluctuating around zero. While $YBa_2Cu_3O_{7-x}$ is less anisotropic and displays the AL behavior characteristic of three-dimensional systems, all other compounds, which have a more anisotropic structure, display the standard AL behavior for two-dimensional (2D) systems. In particular recent experiments in underdoped $La_{2-x}Sr_xCuO_4$ (LSCO) recovered the normal state under strong magnetic field, thereby allowing for a reliable determination of paraconductivity,[17] which can be fitted without contribution of vortical phase fluctuations over the broad temperature range relevant for the pseudogap. This result challenges the phase-fluctuation scenario raising the following issue: How stringent is the above conclusion based on the AL expression for paraconductivity? Within a general phenomenological Ginzburg-Landau (GL) approach, we show that the AL functional form in two dimensions $[\propto(T-T_c)^{-1}]$ is fairly general because it ultimately stems from two general principles, namely, gauge invariance and the hydrodynamic form of the pair collective modes. On the contrary, the numerical prefactor is specific of the fermionic state and therefore provides valuable information on the microscopic state of the system. Specifically, guided by experiments in cuprates, we analyze the case in which the prefactor has the precise AL value. In this case one is led to assume that the BCS theory is a natural starting point, and we show that this specific value of the prefactor either directly stems from fermions with very narrow spectral weight (i.e., nearly free fermionic quasiparticles) or it stems from strong cancellations occurring in the Cooper channel between self-energy and vertex corrections. These cancellations ultimately give rise to an effective behavior mimicking that of nearly free weakly paired fermions. This puts a strong constraint on strong-coupling theories aiming at describing paraconductivity in cuprates. This result is one of the two central points of this work and, together with experiments in Ref. 17, which dictate the specific value and temperature dependence of the paraconductivity prefactor, clearly indicates that in underdoped LSCO fluctuations are not only Gaussian but also arise from pairing of carriers behaving as if they were weakly coupled nearly free quasiparticles.

To challenge this quite surprising result, we present here paraconductivity data in magnetic fields. We find that there exists a temperature range slightly above T_c, where paraconductivity is still fully compatible with weakly paired quasiparticles. Here we also introduce the concept of critical field at zero temperature, $H_{c2}^G(0)$, related to the Gaussian fluctuations only. This is the magnetic-field scale ruling the destruction of the Gaussian fluctuations in the regime (if any) where they dominate paraconductivity. Its value is remarkably lower than the one usually reported in the literature, strengthening our conclusion that paraconductivity is related to superconductivity due to seemingly nearly free quasiparticles in the sense discussed above. This is the second remarkable point of this work. This evidence of weakly coupled quasiparticles are surprising because their presence could hardly be guessed from the quite anomalous form of the normal-state resistivity and is at odds with the broad spectral lines usually observed in photoemission experiments in cuprates.[20] Our aim is not to solve this apparent contradiction but rather to draw attention to this feature. To extract all information from the data, we preliminarily revisit the theoretical derivation of the Gaussian theory, putting precise bounds to the meaning and generality of the 2D AL expression.

II. GAUGE INVARIANT HYDRODYNAMIC DESCRIPTION OF PARACONDUCTIVITY

A superconductor can be described within a generic model of fermions coupled by a λ-wave pairing interaction (most frequently *s* or *d* wave has been considered for singlet superconductors). As customary, by integrating out the fermions one derives an effective action for the pair field $\Delta(\mathbf{r},\tau)$ (here $\mathbf{r}$ is the coordinate vector and τ is the imaginary time within the finite-temperature formulation). The quadratic (Gaussian) part of the resulting action is

$$\mathcal{S}_G = \int_0^\beta d\tau \int d^D\mathbf{q}\Delta^*(\mathbf{q},\tau)[a + C\mathbf{q}^2 + \gamma\partial_\tau]\Delta(\mathbf{q},\tau), \quad (1)$$

where D is the space dimensionality, $\Delta(\mathbf{q},\tau)$ is the Fourier transform of $\Delta(\mathbf{r},\tau)$ with respect to $\mathbf{r}$, and $\mathbf{q}$ is the corresponding wave vector. Whereas the explicit expressions of the coefficients a, C, and γ depend on the details of the microscopic model, e.g., the pairing symmetry and the fermionic density of states (DOS), Eq. (1) holds generically whenever a hydrodynamic description for the pair field is adequate and is indeed phenomenologically adopted in the time-dependent Ginzburg-Landau approach.[19] In this paper we consider Gaussian fluctuations above T_c and keep only the quadratic action [Eq. (1)], discarding higher-order terms. In the GL approach, one may conventionally take $\gamma=\gamma_{\mathrm{GL}}=1$, which amounts to rescale Δ so that its equation of motion is the Schrödinger equation. Thus, in the Gaussian approximation, physical quantities only depend on two parameters, the *mass* $a_{\mathrm{GL}} \equiv a/\gamma$ and the *stiffness* $C_{\mathrm{GL}} \equiv C/\gamma$.

The pair field, with a charge $2e$, is coupled to a spatially uniform electromagnetic field $\mathbf{A}(\tau)$ taking $\mathbf{q}\rightarrow\mathbf{q}-2e\mathbf{A}(\tau)$, as dictated by gauge invariance. The AL contribution to the current-current response, and hence to paraconductivity, is associated with the current density $4eC\mathbf{q}\Delta^*(\mathbf{q},\tau)\Delta(\mathbf{q},\tau)$, and the prefactor in the current vertex can be identified with the stiffness. Under the assumption of a *gauge-invariant hydrodynamic description* for the SC pair fluctuations, the above arguments hold irrespective of the Fermi-liquid or non-Fermi-liquid character of the normal state. Of course, any microscopic derivation of Eq. (1) must obey gauge invariance. In the case of strongly interacting fermions such a derivation is overwhelmingly difficult and beyond the scope of this paper. On the other hand, in Sec. III we provide an example of the current-stiffness relation in the case of weakly coupled fermions.

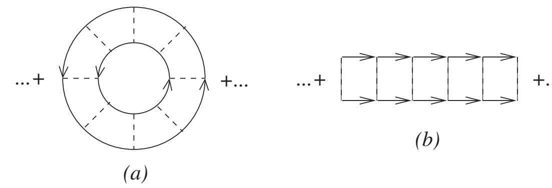

FIG. 1. Typical diagram for the Baym-Kadanoff functional (a) and $\mathcal{T}$-matrix propagator of Gaussian fluctuations (b) adopted in this paper. Dashed and solid lines represent, respectively, the pairing interaction and the fermion propagator (see text).

III. WEAK-COUPLING MICROSCOPIC DERIVATION

Our treatment closely follows the gauge-invariant approach in Ref. 21. We start from a Baym-Kadanoff functional (i.e., the microscopic equivalent of the GL functional) and obtain the paraconductivity by insertion of current vertices. For weakly coupled fermions one can adopt the Baym-Kadanoff functional shown in Fig. 1(a).

For definiteness we assume a separable potential $V(\mathbf{k},\mathbf{k}')=Vw_\lambda(\mathbf{k})w_\lambda(\mathbf{k}')$ of strength V, promoting λ-wave pairing [in cuprates, e.g., d wave, with $w_d=\cos(k_x)-\cos(k_y)$]. A weak-coupling $\mathcal{T}$-matrix approximation yields the pair propagator of Fig. 1(b), i.e., the inverse of the coefficient of action (1),

$$\mathcal{K}_\lambda(\mathbf{q},\omega_\ell) = \frac{1}{V^{-1}-\Pi_\lambda(\mathbf{q},\omega_\ell)} \approx \frac{1}{a_\lambda + C_\lambda\mathbf{q}^2 + \gamma_\lambda|\omega_\ell|}, \quad (2)$$

with the τ variable Fourier transformed into the Matsubara frequency ω_ℓ. The λ-wave particle-particle bubble is

$$\Pi_\lambda(\mathbf{q},\omega_\ell) \equiv T\sum_{\mathbf{k},\varepsilon_n} w_\lambda^2(\mathbf{k})\mathcal{G}(\mathbf{k}+\mathbf{q},\varepsilon_n+\omega_\ell)\mathcal{G}(-\mathbf{k},-\varepsilon_n), \quad (3)$$

where $\mathcal{G}(\mathbf{k},\varepsilon_n)\equiv(i\varepsilon_n-\xi_{\mathbf{k}})^{-1}$ is the fermion propagator and $\xi_{\mathbf{k}}$ is the fermion dispersion. An expansion of $\Pi_\lambda(\mathbf{q},\omega_\ell)$ at small $\mathbf{q}$ and ω_ℓ yields, respectively, C_λ and γ_λ. The mass $a_\lambda \equiv V^{-1}-\Pi_\lambda(0,0)$ linearly vanishes at $T=T_c$.

The insertion of two current vertices in the diagrams of Fig. 1(a) yields the current-current correlation functions[21] shown in Fig. 2. The diagrams of Figs. 2(c1) and 2(c2) give the AL contributions, once the ladder resummation of Fig. 1(b) is adopted for the pair propagator. These contributions are different from the others as they vanish if the fermionic loops with one current vertex are evaluated for zero frequency and momentum of the pair propagators due to the vector character of the current vertex. The first nonzero contribution to each loop is $\tilde{C}\mathbf{q}$,[18] where $\tilde{C}$ is a constant prefactor. Gauge invariance imposes a definite relation between $\tilde{C}$ and the stiffness C. This relation is enforced by a Ward iden-

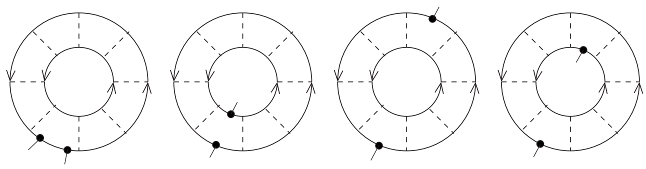

FIG. 2. Diagrams of the current-current response functions generated from the Baym-Kadanoff functional of Fig. 1(a): (a) DOS correction, (b) Maki-Thompson vertex correction, and [(c1) and (c2)] AL contributions. The full circle with a thin line represents a current-vertex insertion (see text).

tity, which can be derived from the Baym-Kadanoff functional, and to first order in the momentum difference $\mathbf{s}$ reads

$$\mathcal{K}_\lambda^{-1}(\mathbf{q}+\mathbf{s},\omega_\ell)-\mathcal{K}_\lambda^{-1}(\mathbf{q},\omega_\ell) = T\sum_{\mathbf{k},\varepsilon_n} w_\lambda^2(\mathbf{k})\mathcal{G}(\mathbf{k},\varepsilon_n)\mathcal{G}(\mathbf{k},\varepsilon_n)\mathcal{G}(-\mathbf{k}+\mathbf{q},-\varepsilon_n)\mathbf{v}_\mathbf{k}\cdot\mathbf{s}, \tag{4}$$

where $\mathbf{v}_\mathbf{k}\equiv\partial_\mathbf{k}\xi_\mathbf{k}$ is the fermion velocity, acting as a current vertex in the fermion loops. The direct calculation in the weak-coupling limit yields indeed $2C=\widetilde{C}$, where the factor of 2 stems from the $2e$ charge of the pair field.

IV. WHAT CAN BE INFERRED FROM OBSERVATION OF AL PARACONDUCTIVITY

The identification of the coefficient $\widetilde{C}$ of the AL current vertex with the stiffness C is the reason why the AL paraconductivity in two dimensions assumes an expression which is independent of C. Indeed, the AL current-current response reads[18,19]

$$\delta\chi_{\mathrm{AL}}(\Omega_n)=4e^2T\sum_{\omega_\ell}\int d^D\mathbf{q}\frac{1}{a_{\mathrm{GL},\lambda}+C_{\mathrm{GL},\lambda}\mathbf{q}^2+|\omega_\ell|} \times\frac{1}{a_{\mathrm{GL},\lambda}+C_{\mathrm{GL},\lambda}\mathbf{q}^2+|\omega_\ell+\Omega_n|}C_{\mathrm{GL},\lambda}^2\mathbf{q}^2, \tag{5}$$

where the dependence on γ_λ was eliminated in the GL spirit, introducing the two independent parameters $a_{\mathrm{GL},\lambda}\equiv a_\lambda/\gamma_\lambda$ and $C_{\mathrm{GL},\lambda}\equiv C_\lambda/\gamma_\lambda$, as discussed above. In the classical limit the sum over ω_ℓ is dominated by the term $\omega_\ell=0$. After the analytic continuation $i\Omega_n\to\omega+i0^+$, the AL paraconductivity is found as $[\mathrm{Im}\,\delta\chi_{\mathrm{AL}}(\omega)/\omega]_{\omega\to0}$. In two dimensions the change in variables $C_{\mathrm{GL},\lambda}\mathbf{q}^2\to x$ makes $C_{\mathrm{GL},\lambda}$ disappear, yielding the well-known result,[18]

$$\delta\sigma_{\mathrm{AL}}(\varepsilon)=\frac{e^2}{2\pi\hbar d}\frac{T_c}{a_{\mathrm{GL},\lambda}}\equiv\frac{e^2}{16\hbar d\varepsilon}, \tag{6}$$

where d is the interlayer distance translating the 2D result into the paraconductivity of a layered system, and $\varepsilon\equiv\pi a_{\mathrm{GL},\lambda}/(8T_c)$ is the dimensionless mass. We stress once more that Eq. (6) stems from the assumption of a gauge-invariant hydrodynamical description for the Gaussian pair fluctuations, which in two dimensions imposes the independence from $C_{\mathrm{GL},\lambda}$, and is thus generic for 2D Gaussian fluctuations.[22,23]

Since we aim to extract as much physical content as possible from the fitting of experimental data with Eq. (6), we now detail the specific value of the coefficients in the various physical situations. All information on the microscopic physical properties is contained in ε. As soon as the fermion DOS changes with temperature (e.g., with the opening of a pseudogap) one may wonder how this is reflected in the temperature dependence of a_{GL} for the various pairing regimes. In a BCS model of weakly coupled fermions, the explicit calculation of the particle-particle bubble Π_λ can be carried out, yielding

$$\gamma_\lambda=-\sum_\mathbf{k} w_\lambda^2(\mathbf{k})\int dz\,A(\mathbf{k},z)A(\mathbf{k},-z)\partial_z f(z), \tag{7}$$

$$a_\lambda=V^{-1}-\sum_\mathbf{k} w_\lambda^2(\mathbf{k})\int dy\,dz\,A(\mathbf{k},y)A(\mathbf{k},z)\mathcal{R}(y,z), \tag{8}$$

where $f(z)$ is the Fermi function, $\mathcal{R}(y,z)=[1-f(y)-f(z)]/(y+z)$, and $A(\mathbf{k},z)$ is the fermion spectral function. If the latter is narrower than $\partial_z f(z)$, it can be replaced by $\delta(\xi_\mathbf{k}-z)$. In this case, a symmetry-dependent weighted DOS $\mathcal{N}_\lambda\equiv\Sigma_\mathbf{k}w_\lambda^2(\mathbf{k})\delta(\xi_k)$ appears, generalizing the standard s-wave expressions of the γ and a coefficients.[19] This factor enters both in γ_λ and a_λ and disappears in $a_{\mathrm{GL}}\propto a_\lambda/\gamma_\lambda$ leaving the paraconductivity unaffected by the T dependence of the DOS. In this case the numerical prefactor relating a_{GL} to $(T-T_c)$ is given by the standard result $\varepsilon=\log(T/T_c)$. On the other hand, we explicitly checked that taking a large (but still T independent) broadening of the fermion spectral function strongly affects the numerical value of the prefactor in front of the logarithm in ε. For instance, taking the broadening about five times T_c leads to a prefactor of order of 0.1 (i.e., 90% smaller than the BCS value). Furthermore, if an additional temperature dependence is considered in the broadening of $A(\mathbf{k},z)$, even the logarithmic dependence of ε is spoiled. This shows that, starting from the above expressions of γ and a, the logarithmic dependence and its unity prefactor in ε are a rather fragile consequence of narrow fermion spectral functions entering the Cooper channel and are no longer trivially valid if the spectral density is broad. By this we mean that if a BCS-like paraconductivity [i.e., Eq. (6) with $\varepsilon=\log(T/T_c)$ or Eq. (9) in the presence of a magnetic field, see below] is observed the most natural attitude is to *assume* the BCS theory as a good starting point taking Eqs. (7) and (8) as valid. If broad spectral functions are concomitantly observed (such as, e.g., in photoemission experiments in cuprates) suitable self-energy corrections should be introduced to dress and broaden the A's. In this case our numerical check allows us to claim that the simple replacement of broad spectral functions associated with large single-particle self-energy corrections is inconsistent with the observation of BCS-like paraconductivity. Large vertex corrections in the Cooper channel dressing $w_\lambda(\mathbf{k})$ should also be introduced to compensate for the large single-particle self-energies thereby leading to an effective theory where the fermions in the Cooper channel behave as if they were weakly coupled nearly free quasiparticles.

We also find it instructive to comment on the well-known Eliashberg theory of strongly coupled superconductors. In this case the paraconductivity has been calculated in three dimensions[24] finding that the Ginzburg-Landau functional form is robust and the standard AL three-dimensional result $\delta\sigma^{3D} \propto \sqrt{T_c/(T-T_c)}$ is recovered. However, in this theory, where the Migdal theorem allows one to disregard vertex corrections, the value of $\varepsilon = b(T-T_c)/T_c$ is found, with $b \neq 1$. Therefore, this standard Eliashberg case is in agreement with our observation that broadening the fermions with self-energies without vertex corrections immediately results in a different coefficient with respect to the BCS-like case where $\varepsilon = \log(T/T_c) \sim (T-T_c)/T_c$. Again this standard example demonstrates how fragile the coefficient of $T-T_c$ is, and therefore, if $b=1$ is experimentally found, a related strong-coupling theory with dressed fermionic lines and particle-particle vertices should fulfill the strong cancellation constraint. We now apply these theoretical arguments to the data obtained in underdoped LSCO.

V. EVIDENCE OF NEARLY FREE QUASIPARTICLE PAIRING

The resistance of several LSCO samples at different dopings has been recently measured as a function of T with and without strong magnetic fields H.[17] The complete destruction of the SC state at $H=47$ T uncovers a highly unusual normal state with a resistivity well reproduced, over an extended temperature range below 200 K, by the superposition of a linear and a logarithmic term $\rho_N(T) \equiv \rho(T, H=47\text{ T}) = AT - B\ln(T/T_0)$, which naturally introduces a temperature scale at which a minimum in the resistivity occurs in underdoped cuprates under strong magnetic fields.[17,25] For a sample with $x=0.09$ and $T_c=19.0$ K our fit gives $A=7.54$ $\mu\Omega$ cm/K, $B=490$ $\mu\Omega$ cm, and $T_0=80.3$ K. We propose no explanation or hypothesis for this unusual normal state and rather focus on the SC state appearing when H is reduced. Following Ref. 17, we define the paraconductivity as $\delta\sigma(T) \equiv \rho^{-1}(T, H=0) - \rho_N^{-1}(T)$ and report the results in Fig. 3 (black dots) as a function of $\varepsilon \equiv \ln(T/T_c)$, in comparison with the 2D AL result in the BCS limit (solid line). Despite the unusual ρ_N, $\delta\sigma(T)$ is very well described by the standard AL expression with the pure BCS coefficients and without fitting parameters. Most importantly, we find that not only the temperature dependence is clearly linear in ε^{-1} but even the numerical prefactor is that of the weak-coupling theory for nearly free fermions, within error bars of less than 5%. Since the paraconductivity diverges at T_c, uncertainties in the determination of ρ_N are rather immaterial for $T \approx T_c$ and our finding is quite robust. The contribution of Gaussian fluctuations to paraconductivity spreads over a broad temperature range, $T - T_c \sim T_c$, similarly to that found in underdoped BSCCO,[6] where, however, the need to guess the reference normal state made the analysis much less stringent.

Rewriting $\varepsilon = (\xi_0/\xi)^2$ and assuming $\xi_0 \sim 20$ Å, we can estimate the coherence length ξ of the Gaussian fluctuations. Even for $\varepsilon \approx 0.01$, i.e., $T \approx 1.01 T_c$, we find $\xi \sim 10\xi_0$

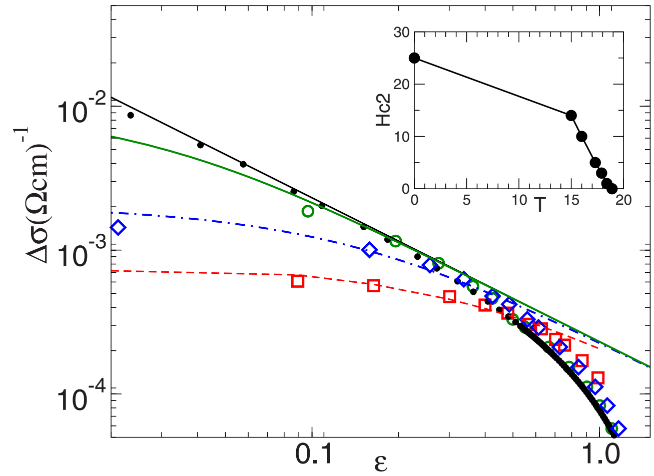

FIG. 3. (Color online) Comparison between the theoretical Gaussian paraconductivity $\delta\sigma(T,H)$, Eq. (9) (lines), and the experimental data (symbols) taking an interlayer distance $d=6.6$ Å [data at $H=0$ T (black dots) should be compared with the theoretical result, Eq. (6) (straight solid line)]. $H=1$ T (solid line and circles), 5 T (dotted-dashed line and diamonds), and 14 T (dashed line and squares). Inset: Gaussian critical temperatures vs H and estimated $H_{c2}^G(T=0)$ (see text). The line is a guide for the eyes.

~ 200 Å, which is much smaller than the value estimated for Kosterlitz-Thouless vortical phase fluctuations in magnetometry experiments in BSCCO.[2] This discrepancy can hardly be due to the different materials because paraconductivity experiments in BSCCO (Ref. 6) give values of ξ consistent with those obtained here for LSCO.

We now focus on data showing the gradual suppression of Gaussian fluctuations for small to moderate H. Since dissipating vortices, introduced by the magnetic field, largely contribute to the resistivity, the Gaussian paraconductivity is difficult to extract. Nevertheless we tested the 2D AL theory at finite H using the expression reported in Sec. 8.1 of Ref. 19,

$$\delta\sigma(T,H) = \frac{e^2}{8\hbar d}\left(\frac{\varepsilon_H}{h^2}\right)\left[\Psi\left(\frac{1}{2}+\frac{\varepsilon_H}{2h}\right) - \Psi\left(\frac{\varepsilon_H}{2h}\right) - \frac{h}{\varepsilon_H}\right], \tag{9}$$

where $h \equiv H/H_{c2}^G(T=0)$, $\varepsilon_H \equiv \log T/T_c^G(H)$, and Ψ is the digamma function. This expression is derived for Gaussian fluctuations and does not include the effects of dissipating vortices. Therefore, to pinpoint the contribution of Gaussian fluctuations to the magnetoparaconductivity, we should apply this formula in a temperature regime (if any), where these effects are not dominating. We thus attempt a fit of the paraconductivity data using $H_{c2}^G(T=0)$ and $T_c^G(H)$ as fitting parameters. According to Eq. (9), these quantities can be interpreted as the critical field at zero temperature and the critical temperature, respectively, in the absence of dissipating vortices, i.e., in a system where only Gaussian fluctuations are present around the transition line. Of course, in the real system vortices are present due to the magnetic field (and, possibly, also pre-existing, as phase fluctuations), which ultimately determine the true T_c and critical field. Nevertheless, if our fitting procedure is successful, this indicates that before entering the vortex-dominated regime, there is a regime where the fluctuations are essentially Gaussian and are ruled by the related scales $H_{c2}^G(T=0)$ and $T_c^G(H)$. In Fig. 3 we report our results. The choice of $H_{c2}^G(T=0)$ and of $T_c^G(H)$ is made to optimize the agreement with the data. For $H=1,5,14$ T we find $T_c^G=18.4,17.3,15.0$ K, respectively (see the inset of Fig. 3), which are substantially larger than the experimental $T_c(H)$, determined by vortex dissipation. Therefore our analysis reliably indicates that 2D Gaussian fluctuations persist under substantial magnetic fields in a temperature range above the crossover to the dissipating vortices' regime. Similarly we find a $H_{c2}^G(T=0)=25$ T much lower than the values at which superconductivity is actually destroyed and usually reported for LSCO at $x=0.09$.[26] This indicates that a regime exists where the dominant fluctuations are Gaussian and vortex fluctuations only become relevant at higher fields and/or lower temperatures. Our fitting procedure would instead completely fail if only preformed pairs with vortical excitations were present well above the critical temperature.

VI. CONCLUSIONS

In this work we started from the preliminary remark that AL paraconductivity is ubiquitously observed in cuprates. This leads us to re-examine the theoretical grounds of AL theory in order to fully ascertain the physical implication of this phenomenological remark. We showed that under general conditions (i.e., gauge invariance and hydrodynamics) 2D paraconductivity is independent of the fluctuation stiffness and depends on a single parameter, the dimensionless mass ε, which contains all information on the specific character of the paired fermions. Therefore the robustness of the AL functional form in two dimensions stems from general physical principles, but the specific numerical prefactors may shed light on the nature of the paired fermions. In particular we showed that paraconductivity of the AL functional form with the precise and specific AL prefactors strongly constrains the theoretical frameworks: within a (strong-coupling) BCS-like approach the presence of broad single-particle spectra is inconsistent with the BCS value $\varepsilon=\log(T/T_c)$ in Eq. (6), unless substantial cancellations due to particle-particle vertex corrections occur.

As far as the experimental part of our work is concerned, we concentrated on LSCO only because the data in strong magnetic field allowed for the direct determination of the reference normal state, but our analysis applies to all families of cuprates. Thus we investigated the experimental paraconductivity in underdoped LSCO showing that it is fully accounted for by Gaussian fluctuations, both in the absence and in the presence of a magnetic field. We also introduce the concept of a critical field related to weakly bound pairs only, which rules the suppression of paraconductivity due to Gaussian fluctuations under magnetic field. Of course in real systems this magnetic-field scale is usually masked by the presence of dissipating vortices, which shift the critical field to higher values. Nevertheless, we remarkably find that there is a temperature and magnetic-field range where the suppression of paraconductivity in LSCO is described by Eq. (9), which only takes into account Gaussian fluctuations.

The supporting theoretical analysis allows us to conclude that (i) within the experimental errors the observed paraconductivity can be fitted without contributions due to vortical phase fluctuations, which seem instead to be present in other experimental quantities.[1–3] Moreover, (ii) the specific value of the numerical prefactor and the temperature dependence of the experimental dimensionless mass indicate that, despite the very anomalous normal state uncovered by the magnetic field, paraconductivity is quantitatively fitted by Gaussian fluctuations of weakly coupled nearly free quasiparticles [i.e., Eq. (9) with $\varepsilon_H \equiv \log T/T_c^G(H)$]. One possibility, which cannot be excluded *a priori*, is that some (so far unknown) strong-coupling theory matches the T and H dependencies of the weak-coupling system within a completely different non-BCS scheme. A more natural possibility is that the BCS theory is a good starting point, but single-particle self-energies are present to account for the broad single-particle spectra and the anomalous transport in the normal state. Our analysis then forces the introduction of substantial particle-particle vertex corrections to implement the needed cancellations yielding the resulting weak-coupling paraconductivity behavior. One last possibility is that the broad fermionic spectra displayed by cuprates largely arise from excitations not involved in the Cooper pairing. Only a small and narrow part of excitations embedded in the broad spectra would instead be involved in superconducting pairing. In this case, the weak-coupling theory would be straightforwardly applicable to these excitations. Such a situation would also directly account for some photoemission observations[27] and would agree with the recent observation of a (small) Fermi surface of nearly free electrons in underdoped $YBa_2Cu_3O_{6.5}$.[28] This indication of a possible pairing of weakly coupled quasiparticles, whose presence can hardly be guessed from other physical properties of the cuprates, is perhaps the most surprising and intriguing result of our analysis.

One might speculate that the weakly bound pairs probed by paraconductivity coexist with more tightly bound pairs related to the vortical phenomenology. This coexistence, already implicit in previous analyses of a two-gap model[29] and in fermion-boson models,[30] could also be consistent with recent observations of different gap scales.[31]

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