# Cooperativity among defect sites in $AO_{2+x}$ and $A_4O_9$ (A=U,Np,Pu): Density functional calculations

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Actinide dioxides derived from the  $AO_2$  fluorite lattice are of high technological relevance due to their application in nuclear reactor fuels. In this paper we use density functional theory calculations to study the oxidation of uranium, neptunium and plutonium dioxides,  $AO_2$  (A=U, Np, or Pu), in  $O_2$  and  $O_2/H_2O$  environments. We pay particular attention to the formation of oxygen clusters (cooperativity) in  $AO_{2+x}$  and how this phenomenon governs oxidation thermodynamics and the development of ordered  $A_4O_9$  compounds. The so-called split di-interstitial, composed of two nearest-neighbor octahedral oxygen interstitials that dislocate one regular fluorite lattice oxygen ion to form a cluster of triangular geometry, is predicted to be the fundamental building block of the most stable cluster configurations. We also identify how the formation of oxygen defect clusters and the degree of oxidation in  $AO_{2+x}$  are both governed by the ability of the O 2p orbitals of the interstitial-like (+x) ions to hybridize with regular fluorite lattice ions.

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# I. INTRODUCTION

The technological importance of actinide dioxides,  $AO_2$  (A = U, Np or Pu), largely originates from their application in nuclear reactor fuels. Today many reactors use  $UO_2$  as the primary fuel component. While low-burn up spent fuels contain large amounts of radioactive materials, in order to reduce the amount of waste requiring long-term storage, there are on-going efforts to develop fast reactors capable of utilizing a larger fraction of the fuel for energy production. This approach calls for complex multicomponent fuels that, in addition to the  $UO_2$  matrix, contain significant amounts of transuranic elements (Np, Pu, etc.). Oxidation of  $AO_2$  compounds emerges as a central theme in fuel fabrication, reactor operation, long-term storage forms for both spent fuels and surplus weapons materials, and environmental actinide migration.

The reduction-oxidation properties of AO2 demonstrate significant variability along the actinide series, e.g., while  $UO_2$  easily reacts with oxygen,<sup>1–3</sup> PuO<sub>2</sub> and later actinide oxides are less prone to oxidation.<sup>1,4,5</sup> PuO<sub>2</sub> seems to require special conditions in order to accommodate excess oxygen.<sup>4,6–8</sup> These complexities are all related to the unique behavior of the A 5f electrons and the unusually high degree of covalent mixing or hybridization in chemical bonds between oxygen and actinides. Upon oxidation UO<sub>2+x</sub> retains the parent fluorite structure up to the terminal  $U_4O_9$  compound, which exhibits an ordered arrangement of interstitiallike oxygen ions within the fluorite lattice.<sup>9,10</sup> PuO<sub>2</sub> was long thought to be the highest stable oxide of plutonium,<sup>1,5</sup> however Haschke and co-workers<sup>6,7</sup> found that PuO<sub>2</sub> reacts with moist air to form  $PuO_{2+x}$  with  $x \le 0.27$ . Subsequent EXAFS studies by Conradson and co-workers<sup>11,12</sup> demonstrated that PuO<sub>2+x</sub> compounds are more accurately represented as  $PuO_{2+x-2y}(OH)y \cdot z(H_2O)$  due to the affinity to  $H_2O$  and/or OH<sup>-</sup> species. No ordered Pu<sub>4</sub>O<sub>9</sub> compound has been identified.<sup>7</sup> NpO<sub>2+x</sub> compounds that are derived from the NpO<sub>2</sub> fluorite lattice have not been reported in the literature and the experimental phase diagram points to a two-phase-field that involves NpO<sub>2</sub> and Np<sub>2</sub>O<sub>5</sub> below  $\approx$ 700 K.<sup>13,14</sup> Even though the NpO<sub>2+x</sub> fluorite phase is metastable with respect to NpO<sub>2</sub>+Np<sub>2</sub>O<sub>5</sub> (Refs. 13 and 14) or NpO<sub>2</sub>+O<sub>2</sub>,<sup>4,13,14</sup> its properties are important for multicomponent (U,Np,Pu,...)O<sub>2+x</sub> materials since the hyperstoichiometric fluorite phase should be stabilized in uranium containing solid solutions, (U,Np)O<sub>2+x</sub>.

Theoretical studies of actinide dioxides predict oxidation of  $UO_2$  to be exothermic<sup>15–20</sup> while oxidation of  $PuO_2$  is predicted to be endothermic.<sup>16,21,22</sup> These studies are complicated by the strongly correlated nature of A 5f electrons, which cause conventional exchange-correlation functionals, e.g., the local-density approximation (LDA), to describe actinide dioxides as ferromagnetic metals instead of antiferromagnetic Mott insulators. There are, however, promising approaches to address this issue, such as the LDA+Umethod,<sup>18-20,23-27</sup> hybrid density functionals,<sup>21,28-30</sup> the selfinteraction corrected methodology (SIC) (Ref. 22) and dynamical mean field theory (DMFT).<sup>31</sup> In addition to the delicate electronic properties of  $AO_{2+x}$ , the interstitial-like excess oxygen ions exhibit complex structural properties that include cluster formation (cooperativity) and strong interaction with the host fluorite lattice.<sup>9–12,32–36</sup> With exception for the recent publication by Geng et al.,<sup>37</sup> existing theoretical studies predominantly rely on simplistic structure models of  $AO_{2+x}$  and/or  $A_4O_9$  that essentially ignore clustering effects.15-22

The present paper is motivated by the persisting incomplete experimental and theoretical understanding of actinide dioxides, in particular for issues regarding oxygen clustering phenomena and their relation to the thermodynamic and kinetic properties of  $AO_{2+x}$ . We investigate the oxidation of  $UO_2$ , NpO<sub>2</sub>, and PuO<sub>2</sub> by applying density functional theory

TABLE I. Calculated lattice parameters  $(a_0)$  and bulk moduli  $(B_0)$  of AO<sub>2</sub> (A=U, Np or Pu) compounds. Experimental reference values are shown within parenthesis.

	UO <sub>2</sub>	NpO <sub>2</sub>	PuO <sub>2</sub>	
$ \frac{a_0(\text{\AA})}{B_0(\text{GPa})} $	5.448(5.47 <sup>a</sup> )	5.398(5.43 <sup>a</sup> )	5.354(5.394 <sup>b</sup> )	
	218 (207 <sup>c,d</sup> )	228 (200 <sup>c,d</sup> )	226 (178 <sup>d</sup> )	

<sup>a</sup>Reference 44.

<sup>b</sup>Reference 6. <sup>c</sup>Reference 45.

<sup>d</sup>Reference 46.

(DFT) calculations to study the structure, thermodynamic stability, and electronic characteristics of interstitial oxygen clusters in  $AO_{2+x}$  and how these clusters may order in the terminal  $A_4O_9$  compounds. The role of  $H_2O$  and hydrolysis products (OH<sup>-</sup>) in the oxidation process is also addressed.

Our paper is organized as follows. Section II describes details of the theoretical methodology to be used. Section III A provides a brief overview of existing structure models for the  $UO_{2+x}$  system and explains the fundamental aspects of their electronic structure. Section III B presents new concepts and results for the geometry and stability of defect clusters in  $AO_{2+x}$ . The corresponding ordering patterns in the terminal  $A_4O_9$  compounds are explored in Sec. III C. We also analyze the particular features of the A 5f electronic structure that drive clustering (cooperativity) in  $AO_{2+x}/A_4O_9$ compounds and formulate a common framework for  $UO_{2+r}$ ,  $NpO_{2+x}$  and  $PuO_{2+x}$ .

#### **II. METHODOLOGY**

The DFT calculations were performed with the Vienna Ab Initio Simulation Package (VASP) (Refs. 38-40) using the projector augmented wave (PAW) method.41,42 The rotationally invariant LDA+U functional due to Lichtenstein et al.<sup>43</sup> was employed to describe the exchange and correlation effects. This method applies the local-density approximation (LDA) together with an Hubbard U term that improves handling of the intraband Coulomb repulsion among the 5f electrons. Unlike conventional LDA or generalized gradient approximation (GGA), the LDA+U functional captures the Mott insulating properties of the strongly correlated U 5f, Np 5f, and Pu 5f electrons in AO<sub>2</sub> adequately. The LDA + U approach contains two a priori unknown parameters; the spherically averaged screened Coulomb energy, U, and the exchange energy, J. For UO<sub>2</sub>, U and J parameters of 4.5 and 0.51 eV, respectively, were originally derived by Dudarev et al.<sup>23</sup> and have since been widely applied.<sup>18-20,37</sup> Here we adopt these parameters. We applied U=4.0 eV and J =0.7 eV for the Pu 5f orbitals, which is within the parameter range specified in the literature.<sup>24,25</sup> The U and J parameters for the Np 5f orbitals were set to 4.25 eV and 0.6 eV, which represent an average between the U 5f and Pu 5f values. This choice accounts rather accurately for the experimental lattice parameter and bulk modulus of NpO2 (see Table I). The occupied Np 5f orbitals are predicted to form a narrow band right at the top of the valence O 2p band, which is similar to the band structure obtained from parameter-free hybrid functionals,<sup>28</sup> though the latter predicts a tiny gap between the O 2p band and the Np 5f peak. Spin polarization was applied for all  $AO_{2+x}$  compounds and, unless otherwise stated, antiferromagnetic spin alignment was used. This configuration was obtained as the ground-state for  $UO_2$  and PuO<sub>2</sub>, while NpO<sub>2</sub> preferred ferromagnetic ordering by 0.004 eV. Spin-orbit coupling was not considered, since, even though spin-orbit coupling will change the details of the magnetic ground state by splitting the f orbitals, this effect is believed to have limited influence on the thermodynamic and structural properties of current interest.<sup>30</sup> The LDA+U lattice constant and bulk modulus predictions for the AO<sub>2</sub> compounds are summarized in Table I, together with experimental estimates. The calculated lattice parameters agree well with experiments and, even though there are some discrepancies for the bulk moduli, the calculated values are considered to be reasonable, particularly in view of known uncertainties for the experimental PuO<sub>2</sub> bulk modulus.<sup>24</sup> A  $2 \times 2$  $\times 2$  supercell expansion of the cubic fluorite unit cell was used to study defect properties. The volume of the supercell was fixed at the calculated volume of UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub>, respectively. All defect calculations used a  $2 \times 2 \times 2$ Monkhorst-Pack k-point mesh<sup>47</sup> with a Gaussian smearing of 0.05 eV. The plane-wave cut off was set to 400 eV. All internal structural parameters were relaxed until the total energy and the Hellmann-Feynman forces on each ion were <0.02 eV/Å. As discussed in Sec. III C, a number of ordered  $A_4O_9$  compounds have been studied. The unit cells of these structures are spanned by two different sets of vectors; [1 1 0],[0 1 1],[1 0 1] and [2 2 0],[0 0 1],[0.5 -0.5 0], which are defined with respect to the conventional cubic fluorite unit cell. For the first cell we applied a  $2 \times 2 \times 2$  Monkhorst-Pack k-point mesh and for the second cell we applied a 2  $\times 4 \times 6$  mesh. Volume relaxation was taken into account for both cells. In order to assess the oxidation thermodynamics in O<sub>2</sub> and H<sub>2</sub>O chemical environments (gaseous form is assumed throughout this work), reference energies for these molecules (chemical potentials) must be established. These energies were calculated by placing an isolated molecule inside a large supercell and relaxing the geometry. Since DFT within the LDA formulation is known to overestimate the binding energy of many molecules, we have applied correction terms of 2.4 eV (Ref. 48 and 49) and 1.5 eV (Ref. 48) for O<sub>2</sub> and H<sub>2</sub>O molecules, respectively. The oxidation energy of  $AO_{2+x}$  is defined as the energy associated with the  $\frac{1}{x}AO_2 + \frac{1}{2}O_2 \rightarrow \frac{1}{x}AO_{2+x}$  reaction or, when appropriate, the corresponding reaction that includes H<sub>2</sub>O. Negative oxidation energies imply spontaneous oxidation and positive numbers are equivalent to instability of the hyperstoichiometric compounds.

 $AO_{2+x}$  compounds contain a distribution of  $A^{5+}$  ions as charge-compensating defects and there are two  $A^{5+}$  ions for each excess oxygen ion. The total energy of  $AO_{2+x}$  compounds depends, not only on the distribution of excess O<sup>2-</sup> ions, but also on the distribution of  $A^{5+}$  ions, as identified by orbital projected spin densities, with respect to the position of the excess ions. In addition to the ground-state structures, it is possible to obtain metastable  $A^{5+}$  distributions from the DFT calculations. All numbers that are reported here refer to



FIG. 1. (Color online) Idealized schematics of the structure of the undistorted  $(I_2^O)$ , lower case letters) and split  $(I_2^X)$ , upper case letters) di-interstitial structures. The cubes represent the simplecubic oxygen sublattice; for clarity, uranium ions are not shown. The  $I_2^O$  structure has two oxygen interstitials (*a* and *b*) in octahedralinterstitial sites in the lattice. The  $I_2^X$  structure constitutes an equiaxed or equilateral triangle that is situated on top of an empty fluorite lattice oxygen site.

the lowest energy solution that we are able to attain for each structure.

#### **III. RESULTS AND DISCUSSION**

# A. Existing interstitial clustering models and analysis of their electronic structure

### 1. Uranium dioxide

The AO<sub>2</sub> fluorite lattice has large empty octahedral sites that appear to be the natural location for excess oxygen ions. Willis and co-workers<sup>34,35</sup> found that in  $UO_{2+r}$  such interstitials tend to aggregate into clusters composed of multiple interstitial ions interacting with regular oxygen ions to form complex defect clusters.<sup>33–36</sup> They initially proposed the socalled 2:2:2 cluster, which is effectively composed of two oxygen vacancies and four interstitials (see for example Fig. 3 in Ref. 33). Bevan *et al.*<sup>9</sup> revised the  $UO_{2+x}$  structure model and suggested that the excess oxygen ions occupy the cubo-octahedral holes of the fluorite lattice. In this model each defect cluster contains 12 ions, of which four are ascribed as excess oxygen ions and the remaining eight ions are displaced from their fluorite lattice positions (see for example Fig. 2 in Ref. 9). The best agreement with experiments is achieved when the octahedral-interstitial hole enclosed by the cubo-octahedron cluster is also occupied.<sup>10</sup> EXAFS measurements by Conradson et al.32 have further illustrated the complexity of  $UO_{2+x}$  by suggesting a U-O coordination that includes 1.74 Å bonds.

Ichinomiya *et al.*<sup>50</sup> and Govers *et al.*<sup>51</sup> used empirical potentials to investigate the stability of di-interstitial clusters in  $UO_{2+x}$  and, in contrast to experiments, they found the 2:2:2 cluster to be metastable, or even unstable. In its place they predicted the stable di-interstitial oxygen cluster to be composed of three oxygen ions arranged in an equiaxed or equilateral triangle that is situated in a {111} plane above an empty regular oxygen site (see Fig. 1). Andersson *et al.*<sup>52</sup>



FIG. 2. (Color online) Density of states (DOS) and projected density of states (PDOS) for  $U_4O_9$  constructed from an ordered arrangement of split di-interstitials,  $U_4O_9(1-10)$ , and from octahedral interstitials,  $U_4O_9(Oct.)$ . Notice that the U 5f-O 2p gap remains for the octahedral-interstitial structure, while for the split di-interstitial structure the O 2p states overlapping with U 5f peak in the octahedral structure moves down to overlap with the main O 2p band. The highest occupied state is at 0 eV.

confirmed these conclusions using DFT calculations and they labeled this defect as split di-interstitial,  $I_2^X$ . Here the "*I*" denotes an interstitial type of defect, the subscript 2 denotes the number of interstitials that are involved and the superscript "*X*" denotes their particular geometric configuration. This notation is used throughout this paper to designate different types of excess oxygen defects. The split di-interstitial is created from two nearest-neighbor interstitial ions (*a* and *b* in Fig. 1) that move toward a common regular oxygen site, dislocating it (*c* in Fig. 1), thus creating the split di-interstitial and the vacant site that serves as its center of mass (*A*, *B*, and *C* in Fig. 1).

Adding oxygen to UO<sub>2</sub> results in charge transfer from the 5f orbitals of  $U^{4+}$  ions to the 2p orbitals of the excess oxygen ions. The high stability of split di-interstitals in  $UO_{2+r}$ can be explained by the fact that this structure allows the orbitals occupied by the transferred electrons to hybridize with the uranium orbitals and thus imitate the uraniumoxygen interaction in the ideal fluorite lattice, in particular the covalent contribution to the uranium-oxygen bonding. This process enables the excess O 2p orbitals to move down from the U 5f states at the Fermi level to the main O 2pband, thus decreasing the band energy and, unless the corresponding increase in electrostatic energy among ions dominates, also the total energy (illustrated for  $U_4O_9$  in Fig. 2). As a consequence, the O 2p-U 5f gap found in UO<sub>2</sub> diminishes or even disappears. Oxygen ions in octahedral-interstitial sites are unable to interact with the uranium ions as efficiently as split di-interstitials or regular fluorite oxygen ions and, consequently, the excess O 2p orbitals for octahedral interstitials cannot merge with the main O 2p band. For this reason they do not experience the same gain in band energy



FIG. 3. (Color online) The density of states for UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub>. The NpO<sub>2</sub> and PuO<sub>2</sub> DOS have been shifted from the zero line. The position of the A 5f bands is shown in dashed lines, including both localized states (shaded in gray) and states hybridized with the O 2p band. The arrows indicate the separation of the localized A 5f states and the O 2p bands. The highest occupied state is at 0 eV.

as split di-interstitials. However, due to the larger separation of excess  $O^{2-}$  ions and of charge compensating  $U^{5+}/U^{6+}$ ions, octahedral interstitials are more favorable from the perspective of Coulomb repulsion among ions. Qualitatively, this implies that the defect stability in  $UO_{2+x}$  is a competition between hybridization (decrease of the band energy) and Coulomb repulsion among ions. The former favors the split di-interstital type of structures while the latter favors structures based on isolated octahedral interstitals. Figure 2 illustrates the band structures of two  $U_4O_9$  compounds; one based on octahedral interstitials and another one composed of ordered split di-interstials. We pay particular attention to the highlighted difference in the band-structure position of the excess O 2p orbitals, and thus hybridization, between these two structures.

Let us take a closer look at the underlying mechanisms for the hybridization features discussed in the previous paragraph. The projected DOS for UO<sub>2</sub> (partially illustrated in Fig. 3) reveals that there is a sizable U 5f/U 6d contribution in the main O 2p band.<sup>28</sup> At the same time, an electron count shows that all the U 5f electrons that are expected from the formal valence of the  $U^{4+}$  ions (two) are located in the U 5f peak above the edge of the main O 2p band. This implies that the U 5f/U 6d contribution in the O 2p band originates from overlap or hybridization of nominally filled O 2p orbitals with nominally empty U 5f/U 6d orbitals. This UO<sub>2</sub> hybridization is not available for the octahedral-interstitial oxygen ions in U<sub>4</sub>O<sub>9</sub>(Oct.) since their interaction with the uranium ions is shielded by the cubic cage of regular fluorite oxygen ions. Instead the nearest-neighbor uranium interaction for the octahedral oxygen interstitials is such that their O 2p orbitals almost overlap with the U 5f states in the band gap, as indicated by Fig. 2. The split di-interstitial  $U_4O_9$ structures, on the other hand, allow the O 2p orbitals of both the regular fluorite oxygen ions and the excess ions to overlap or hybridize with the nominally empty U 5f/U 6d orbitals, which, as mentioned above, facilitates for the excess O 2p orbitals to move down from the U 5f states at the Fermi level to the main O 2p band (Fig. 2) and thus decrease the band energy. We conclude that the overlap of excess O 2p orbitals with the main O 2p band is in fact driven by the hybridization of excess O 2p orbitals with nominally empty U 5f/U 6d orbitals made possible by the particular structural distortion for split di-interstitials. Figure 2 also reveals that the calculations predict the band gap (valence to conduction band) to decrease by  $\approx 0.45$  eV for structures built from split di-interstitials  $[U_4O_9(1-10)]$  in this particular case], as compared to octahedral interstitials, and by  $\approx 0.70$  eV compared to UO<sub>2</sub>. This is an indicator of the changes in O 2p-U 5f/U 6d hybridization that takes place for split di-interstitial structures.

#### 2. Neptunium and plutonium dioxides

Band-structure calculations reveal that the 5f electrons in UO<sub>2</sub> are situated  $\approx 0.6$  eV above the O 2*p* valence-band edge, while the corresponding Np 5f states are right at the top of the O 2p band. For PuO<sub>2</sub> the 5f states overlap with the O 2p band and together they form a more or less continuous band. Due to the shift in position of the occupied 5f orbitals, the gain in band energy coming from charge transfer and hybridization between neptunium ions and the excess oxygen ions should be smaller than for  $UO_{2+x}$ . Another way of stating this would be that the difference between the delocalization energy of Np 5f electrons (Np<sup>4+</sup> $\rightarrow$ Np<sup>5+</sup>) and the energy of these electrons occupying the (hybridized) O 2p band decreases compared to UO2. This effect is even stronger for  $PuO_{2+x}$  and, in fact, the  $Pu^{4+}$  to excess oxygen charge transfer may result in increasing band energy. Figure 3 highlights the change in band structure between UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub>. We note that the shift in the separation of the O 2p and A 5fbands is a qualitative measure of the ease of oxidation; large positive separation (long arrow pointed to the left in Fig. 3) implies negative oxidation energy and as the A 5f states move down below the main O 2p band-edge oxidation becomes more difficult (shorter "left" arrow and arrow pointed to the right in Fig. 3). Qualitatively, upon oxidation the A 5felectrons move to the top of the O 2p band and, consequently, the relative A 5f/O 2p position is closely related to the oxidation thermodynamics, via the band energy. We remark that for  $UO_{2+x}$  this picture corresponds to maximum gain in band energy and, as we have seen in Fig. 2, the real band structure for  $UO_{2+x}$  is somewhat more complicated but, at least qualitatively, the concept presented above is still valid. An immediate consequence of this band-structure effect is that the excess oxygen ions in  $PuO_{2+x}$  exhibits a different valence state than for  $UO_{2+x}$ . Analyses of spin densities reveal that excess oxygen ions in  $UO_{2+x}$  act as  $O^{2-}$ , while the corresponding state for  $PuO_{2+x}$  is  $O^{(2-\delta)-}$ , where  $0 < \delta < 0.5$  and the exact  $\delta$  value is structure dependent.  $\delta$ values close to 0 correspond to split di-interstitial structures and the higher values to structures based on regular octahedral interstitials, i.e., the higher degree of hybridization for split di-interstitials facilitates higher valence but compared to

TABLE II. The oxidation energy for various defects in  $AO_{2+x}$ , differently ordered  $A_4O_9$  compounds and hydrogen-containing  $AO_{2+x}/A_4O_9$  compounds. The  $UO_{2+x}$  data are partially reproduced from Ref. 52. The type of defect and defect ordering is indicated within parenthesis, e.g.,  $A_{32}O_{65}(I_1)$  for one single interstitial in the  $2 \times 2 \times 2$  supercell. See text for labeling. The values within parenthesis for  $Pu_4O_9(Oct.)$  and  $Np_4O_9(Oct.)$  correspond to an octahedral-interstitial compound that contains  $O_2$  species (see text). All energies are in eV and measured per excess oxygen ion. For the small unit cells, the values within brackets represent the change in lattice parameter (in Å) compared to  $AO_2$  (volume relaxation was not performed for the cases where this number is missing). N/A designates that this particular compound was not studied.

	$A_{32}O_{65}(I_1)$	$A_{32}O_{66}(I_2^0)$	$A_{32}O_{66}(I_2^S)$	$A_{32}O_{66}(I_2^X)$	$A_{32}O_{68}(I_4^X)$	$A_{32}O_{68}(I_4^S)$
UO <sub>2</sub>	-2.53	-2.21	-2.38	-2.34	-2.71	-2.32
NpO <sub>2</sub>	-0.32	-1.29	-0.16	-0.34	-0.71	-0.18
PuO <sub>2</sub>	0.44	0.50	0.13	0.45	-0.12	0.04
	$A_{32}O_{68}(I_4^C)$	$A_{32}O_{69}(I_5^C)$	$A_4O_9(\text{Oct.})$	$A_4O_9(111_0)$	$A_4O_9(111)$	$A_4O_9(1-10)$
UO <sub>2</sub>	-2.25	-2.50	-1.86[-0.05 Å]	-2.05[-0.03 Å]	-2.11[-0.01 Å]	-2.25[-0.02 Å]
NpO <sub>2</sub>	N/A	N/A	0.39(-0.37)[0.01 Å]	-0.18[-0.01 Å]	-0.22[-0.01 Å]	-0.56[-0.01 Å]
PuO <sub>2</sub>	N/A	N/A	0.34(-0.13)[0.03 Å]	0.00[0.02 Å]	0.14[0.01 Å]	0.46[-0.01 Å]
	$A_4O_9(bcc)$	$A_4O_9(bccr)$	$A_4O_9(Cubo)$	AO <sub>2</sub> (H <sub>2</sub> O) <sub>1/32</sub>	AO <sub>2</sub> (OH) <sub>1/32</sub>	$A_4O_8(OH)_2$
UO <sub>2</sub>	-2.63	-2.62	-2.13	N/A	-2.11	-1.81[0.03 Å]
NpO <sub>2</sub>	-0.86	-0.83	N/A	N/A	-0.55	-0.69[0.03 Å]
PuO <sub>2</sub>	-0.14	-0.09	N/A	0.47	0.09	-0.60[0.03 Å]

 $UO_{2+x}$  this process is still incomplete. The chargecompensating plutonium ions also deviate from the pentavalent state of U<sup>5+</sup> ions and exhibit partially reduced valency. All together this indicates that complete charge transfer between localized Pu 5f and excess O 2p orbitals is not favorable. Here, charge transfer refers to delocalizing one electron from the 5f orbitals and allowing it to participate in A-O bonding, which, as discussed above, includes both direct charge transfer and hybridization. The valence states in  $NpO_{2+r}$  suggest that it should be classified in the same category as  $PuO_{2+x}$ , but with smaller  $\delta$  values, which is consistent with the occupied Np 5f states being situated at the top of the O 2p band and thus in between the position of the occupied U 5f and Pu 5f orbitals. The fact that charge transfer is incomplete in  $PuO_{2+x}$  and  $NpO_{2+x}$  creates unfilled states in the O 2p valence band, which, from a chemical point of view, is unfavorable and, as discussed in Secs. III C and III D, this has important consequences for the oxidation thermodynamics. Following the trends for the position of the 5f electrons along the actinide series we expect that actinide dioxides to the left of UO<sub>2</sub> (PaO<sub>2</sub>) should oxidize easily, while actinide oxides to the right of PuO<sub>2</sub> should be very hard to oxidize.

The band-structure concepts presented above will be used as basis for the forthcoming discussion of  $AO_{2+x}/A_4O_9$  structures and their thermodynamic stability.

# B. New geometries and stabilities of oxygen clusters in $AO_{2+x}$

### 1. Uranium dioxide

Table II summarizes the oxidation energies (stabilities) of di-interstitial defect clusters in  $UO_{2+x}$ ,  $NpO_{2+x}$ , and  $PuO_{2+x}$ .

For now we focus on mono- and di-interstitials,  $I_1$  and  $I_2$ , in  $UO_{2+x}$  and ignore other data in Table II. Andersson *et al.*<sup>52</sup> showed that in the low-concentration regime the split diinterstitials are essentially degenerate with two isolated interstitials  $(I_2^{S})$ , the latter being somewhat lower in energy. These two defects constitute the most stable configurations of two excess oxygen ions within the  $2 \times 2 \times 2$  supercell and most notably undistorted, but still relaxed, di-interstitials  $(I_2^0)$ , i.e., two nearest-neighbor interstitials without the accompanying dislocation of any regular oxygen ions, are predicted to be higher in energy than split di-interstitials. The oxidation energy of a single isolated oxygen interstitial in the  $2 \times 2 \times 2$ supercell  $(I_1)$  is still lower than for either  $I_2^X$  or  $I_2^S$ , which means that in the dilute limit the binding energy for any di-interstitial cluster is negative and they should thus dissolve. This energy balance is an effect of the aforementioned competition between Coulomb repulsion among ions and hybridization.

The cubo-octahedron cluster  $(I_4^C)$  proposed for UO<sub>2+x</sub> (Refs. 9 and 10) contains four excess oxygen ions and thus represents a larger and more extended defect than the diinterstitial type of clusters. In order to evaluate its stability we have studied a single cubo-octahedral cluster embedded in the 2×2×2 supercell and, similarly, we have also studied the cubo-octahedron structure with the central octahedral hole occupied by another excess oxygen ion  $(I_5^C)$ , giving a cluster composed of five excess oxygen ions. For comparison, we have also treated clusters composed of two split di-interstitials, hereby denoted split quadinterstitial  $(I_4^X)$ , as well as four octahedral interstitials homogeneously distributed within the 2×2×2 supercell  $(I_4^S)$ . The geometry of the most stable split quadinterstitial configuration is such that the two split di-interstitials are aligned along the [0.5 0 0] lattice



FIG. 4. (Color online) Idealized schematics of the split quadinterstitial  $(I_4^X)$ . The cubes represent the simple-cubic oxygen sublattice; for clarity, uranium ions are not shown. The dashed lines indicate the particular cube diagonals, along which the interstitial and regular oxygen ions that belong to the defect cluster are displaced. The two split di-interstitials that make up the quadinterstitial are highlighted by A and B, respectively. The parallellogram formed by the adjoining base lines of the two split di-interstitials is outlined.

vector and rotated 180° with respect to each other (see Fig. 4). The stabilities (oxidation energies) of these structures are collected in Table II (labeled as  $I_4$  and  $I_5$ ). The split quadinterstitial cluster is the most stable configuration, followed by the cubo-octahedron cluster with occupied octahedral hole  $(I_5^{C})$  and both of them are more stable than the structure based on regular octahedral interstitials. The simple cubooctahedron  $(I_4^C)$  is significantly less stable than the  $I_5^C$  structure, which agrees with the findings by Geng et al.<sup>37</sup> Moreover, we confirm the observation that the central octahedral oxygen ion in the  $I_5^C$  cluster is rather significantly displaced from its ideal position.<sup>37</sup> The addition of an excess oxygen ion to the center of the cubo-octahedron cluster creates a structure that in many aspects resembles the split quadinterstitial structure, in particular after the displacement of the central ion from the high-symmetry position. The key electronic structure difference between the split quad- and octahedral-interstitial structures is that the former enables efficient O 2p hybridization for the excess ions and thus lowers the band energy, in the same way as for the split diinterstitial structures, while the latter pins the excess O 2pstates at the Fermi level and, as a result, does not allow the same lowering of the band energy. The cubo-octahedron structure shares some of the electronic structure features with the split quadinterstitials, but, as indicated by their higher oxidation energy, these features are not optimized to the same extent for cubo-octahedrons as for split quadinterstitials. We recall that, compared to split di-interstitials, octahedral-interstitial structures benefit from lower Coulomb repulsion among ions. However, unlike the split diinterstitial/octahedral-interstitial energy balance in the lowconcentration range  $(I_2^X \text{ vs } I_2^S)$ , for the intermediate oxygen concentration range the gain in band energy due to hybridization outweighs the increase in Coulomb repulsion to clearly favor split quadinterstitials over seperated octahedral interstitials  $(I_4^X \text{ vs } I_4^S)$ . The oxidation energy of split quadinterstitials is lower than for split di-interstitials, which means that the split quadinterstitial is a bound state of two split di-interstitials (0.70 eV per split quadinterstitial cluster). The high stability for the split quadinterstitial follows from a combination of further increased hybridization and more efficient handling of the repulsive ionic Coulomb contribution from excess  $O^{2-}$  ions and charge-compensating  $U^{5+}$  ions. Moreover, the binding energy of split quadinterstitials with respect to isolated interstitials in the dilute limit is positive by as much as 0.72 eV per split quadinterstitial.

### 2. Neptunium and plutonium dioxides

Comparing the results for  $NpO_{2+x}$  and  $PuO_{2+x}$  with  $UO_{2+x}$ , the main conclusion is that the oxidation energies for  $NpO_{2+x}$  and  $PuO_{2+x}$  are much less negative and, as a result, NpO<sub>2</sub> and PuO<sub>2</sub> should be more difficult to oxidize.  $PuO_{2+x}$ is even predicted to have positive oxidation energy, which implies that  $PuO_2$  should not react with  $O_2$ , unless another lower energy reaction path is available (discussed in Secs. III C and III D). The observed UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub> sequence of increasing oxidation energies correlates with the change in the relative position of the A 5f orbitals and the O 2p band between UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub>. The associated decreased gain in O 2p band energy compared to the 5f delocalization energy explains the high oxidation energy for NpO<sub>2+x</sub> and PuO<sub>2+x</sub>. A notable feature for NpO<sub>2+x</sub> is the very low oxidation energy for  $I_2^0$  compared to other defect clusters, contrasting the properties of  $UO_{2+x}$  and  $PuO_{2+x}$ . This suggest that optimal bonding in NpO<sub>2+x</sub> occurs for the  $I_2^0$ geometry, though at present we are unable to explain this particular feature. The stability of split di-interstitials and regular octahedral interstitials is switched between  $NpO_{2+r}$ and  $UO_{2+x}$ . As discussed in Sec. III C 2 the oxidation energies of  $PuO_{2+x}$  are closely related to the ability to avoid unfilled O 2p states at the top of the valence band, which, in turn, is related to the degree of hybridization between O 2p/A 5f orbitals. Increased hybridization removes the unfilled states by occupying empty O 2p states and separating the corresponding Pu 5f states into either the band gap or to the bottom of the conduction band. Similar, but less pronounced, conclusions hold true for  $NpO_{2+r}$ .

Table II also contains the stability of split quadinterstitial clusters and four separated octahedral interstitials in NpO<sub>2+x</sub> and PuO<sub>2+x</sub>. The PuO<sub>2+x</sub> results confirm the trends for UO<sub>2+x</sub>, in the sense that the split quadinterstitial is predicted to be the most stable state and also bound with respect to  $I_1$  and  $I_2$  structures. The high stability of  $I_2^0$  for NpO<sub>2+x</sub> shifts the relative stabilities, as compared to UO<sub>2+x</sub> and PuO<sub>2+x</sub>, and  $I_4^X$  is the second most stable defect structure. Compared to UO<sub>2+x</sub>, the oxidation energy of split quadinterstitials remains high for both NpO<sub>2+x</sub> and PuO<sub>2+x</sub>.

## C. Ordering of oxygen clusters in $A_4O_9$

# 1. Uranium dioxide

Below 500–1400 K  $UO_{2+x}$  phase separates into  $UO_2$  ( $UO_{2+x}$ ) and ordered  $U_4O_9$  ( $U_4O_{9-y}$ ).<sup>9,53</sup> The temperature at which the two-phase field transforms into a solid solution is



FIG. 5. (Color online) Viewed along the {111} lattice direction, the fluorite lattice is composed of the -U-O-O-U-O-O-U-O-O-U-O-O-sequence of uranium and oxygen planes. This figure demonstrates ordering patterns of split di-interstitials for various  $U_4O_9$  compounds. (a) The hexagonal pattern of oxygen ions in a {111} plane of the UO<sub>2</sub> fluorite lattice. (b), (c), and (d) illustrate the ordering of split di-interstitials (green/light) in {111} planes for (b)  $U_4O_9(111)$ , (c)  $U_4O_9(1-10)$ , and (d)  $U_4O_9(bcc)$ . In the "-U-O-O-U-O-O-U-O-O-U-O-O-" label above figures (a), (b), (c), and (d), the red/dark "O" character symbolizes a regular oxygen plane and the green/light O character symbolizes an oxygen plane that contains excess oxygen ions or defects. Note that this notation does not specify the actual crustallographic periodicity of {111} planes. Red/dark spheres denote regular oxygen ions and green/light spheres represent oxygen ions that are part of the split di-interstitial clusters.

a strong function of composition.<sup>9,53</sup>  $U_4O_9$  is envisioned to be an ordered arrangement of the oxygen defects or clusters in  $UO_{2+r}$ . Early neutron-diffraction experiments indicated that  $U_4O_9$  should be described within a  $4 \times 4 \times 4$  expansion of the conventional fluorite unit cell and Allen et al.<sup>54</sup> proposed an ordered arrangement of 2:2:2 clusters along  $\begin{bmatrix} 1 & -1 \end{bmatrix}$ 0] lattice vectors. This model was later rejected due to incompatibilities with intensities of neutron-diffraction data.9,20 The most recent  $U_4O_9$  ( $U_4O_{9-v}$ ) structure model is an intricate arrangement of the cubo-octahedron type of clusters within the  $4 \times 4 \times 4$  unit cell.<sup>10</sup> Proceeding from the groundstate configurations of the split di- and quadinterstitials, we now investigate how these may order at the  $U_4O_9$  composition. We first describe the different structure motifs under study, then we discuss their thermodynamic stability, and finally we compare predicted local structure properties with experimental EXAFS measurements.

Two different ordering patterns of split di-interstitials have been considered; split di-interstitials ordered in every fourth {111} plane  $[U_4O_9(111)]$  and split di-interstitials ordered along [1 -1 0] lattice vectors  $[U_4O_9(1-10)]$ .  $U_4O_9(111)$  is a natural extension of the fact that split diinterstitials occupy {111} planes and  $U_4O_9(1-10)$  is inspired by the  $U_4O_9$  structure proposed by Allen *et al.*,<sup>54</sup> but instead of the 2:2:2 cluster, the  $U_4O_9(1-10)$  structure uses the split di-interstitial as the expansion unit.

Viewed along the [1 1 1] lattice direction the fluorite lattice is composed of hexagonal U and O layers arranged in a -O-O-U-O-O-U-O-O- sequence; see Fig. 5(a). Figure 5(b) shows the geometry of one of the {111} planes in  $U_4O_9(111)$ that contains split di-interstitials. The split di-interstitial oxygen ions are slightly displaced out of the {111} plane, as compared to the regular fluorite oxygen ions in the same {111} plane. The charge-compensating  $U^{5+}$  ions in  $U_4O_9(111)$  fully occupy the next-nearest-neighbor uranium plane with respect to the split di-interstitials. Sickafus *et al.*<sup>55</sup> discuss how fluorite derived structures such as pyrochlores and various sequioxides can be rationalized in terms of {111} stacking motifs and the  $U_4O_9(111)$  structure obviously belongs to this class, though this particular stacking pattern has not been previously identified. Below we outline how all the split di- and quadinterstitial  $U_4O_9$  structures proposed here can be portrayed in terms of {111} stacking motifs based on split di-interstitials.

The starting point for the  $U_4O_9(1-10)$  structure is a chain of split di-interstitials separated by the [0.5 - 0.5 0] lattice vector. Upon geometry optimization this structure relaxes significantly and the [0.5 - 0.5 0] ordering of triangular split di-interstitial motifs transforms into a new stacking pattern, still along [0.5 –0.5 0], but composed of four interstitial-like oxygen ions arranged in a parallelogram. This parallelogram is created from the split di-interstitial by displacing one additional regular oxygen ion from its lattice site, which may also be interpreted as the two excess oxygen ions dislocating two regular oxygen ions, instead of one for the split diinterstitial. This parallelogram is directed along the  $[0.5 \ 0 \ 0]$ lattice direction and then stacked with a periodicity of [0.5]-0.5 0], which is prescribed by our choice of unit cell. We note that the adjoining split di-interstitials in the split quadinterstitial cluster also creates a parallelogram (see Fig. 4), which is equivalent to the parallelogram in the  $U_4O_0(1-10)$ structure. Thus,  $U_4O_0(1-10)$  can be viewed as an interconnected ordering of split quadinterstitials along the [1 - 1 0]lattice vector (note that the periodicity is [0.5 - 0.5 0]). The  $U^{5+}$  ions in the  $U_4O_9(1-10)$  structure occupy the four nearest-neighbor positions with respect to the defect oxygen parallelogram. As shown in Fig. 5(c), the  $U_4O_9(1-10)$  structure can also be visualized within a {111} stacking model. The cluster ions form chains in the {111} planes and, since the excess ions/defects occupy every {111} plane instead of every fourth plane as for  $U_4O_9(111)$ , the concentration of excess ions or defects in each  $U_4O_9(1-10)$  {111} plane is 1/4 of the corresponding  $U_4O_9(111)$  {111} concentration. Relaxation results in small out of plane shifts within each  $U_4O_9(1-10)$  {111} plane.

The split quadinterstitial clusters may also be arranged as separate units within the fluorite lattice and, in principle, arbitrary superstructures of any composition can be created by varying the stacking and orientation of these units. We have investigated two of the most probable configurations within the  $2 \times 2 \times 2$  supercell; in the first case the split quadinterstitials are ordered along the  $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$  vector  $\begin{bmatrix} U_4 O_0(bcc) \end{bmatrix}$ and in the second case, one of these split quadinterstitial units is rotated 90° with respect to the other cluster  $[U_4O_9(bccr)]$ . Notice that the split quadinterstitials in  $U_4O_9(bcc)$  and  $U_4O_9(bccr)$  form a bcc lattice, hence the bcc and bccr notation. Even though the true ground-state structure of U<sub>4</sub>O<sub>9</sub> may exhibit a longer stacking sequence of split quadinterstitials than allowed within the  $2 \times 2 \times 2$  cell, we believe that  $U_4O_0(bcc)$  and  $U_4O_0(bccr)$  serve as appropriate modeling structures, capable of capturing the relevant physics of the  $U_4O_9$  superstructures. Due to the specific geometry of split quadinterstitials, both  $U_4O_9(bcc)$  and  $U_4O_9(bccr)$  in fact involve stacking of {111} planes that contain an ordered arrangement of split di-interstitials. In these two cases the split di-interstitials occupy every oxygen plane and Fig. 5(d)illustrates the {111} in-plane geometry for  $U_4O_9(bcc)$ . The {111} planes in  $U_4O_9(bccr)$  exhibit the same type of in-plane pattern of split di-interstitials as  $U_4O_9(bcc)$ , but the stacking sequence differs between the two structures. The split quadinterstitials are made up of two split di-interstitials overlapping between nearest-neighbor {111} planes. There are rather significant out-of-plane distortions for the split di-interstitials in both  $U_4O_9(bcc)$  and  $U_4O_9(bccr)$ .

Following the recipe for  $U_4O_9(bcc)$ , we have also built  $U_4O_9$  structure models by inserting two cubo-octahedral clusters separated by the [1 1 1] vector in the  $2 \times 2 \times 2$  cell, labeled  $U_4O_9(Cubo)$ . These structures represent a simplified version of the  $4 \times 4 \times 4$  cubo-octahedron structure reported by Bevan *et al.*<sup>9</sup> As reference, we have calculated the stability of  $U_4O_9$  constructed from regular octahedral interstitials  $[U_4O_9(Oct.)]$  and undistorted di-interstitials  $[U_4O_9(111_0)]$ .

Table II shows that  $U_4O_9(bcc)$  is the most stable  $U_4O_9$ compound, closely followed by  $U_4O_9(bccr)$ .  $U_4O_9(1-10)$ and  $U_4O_9(111)$ , as well as any structures based on octahedral interstitials, are considerably higher in energy. With reference to Fig. 5 and Table II, we emphasize the stabilizing contribution coming from the covalent bonding (hybridization) in split di-interstitial type of defects and, additionally, the fact that distributing these defects among several planes [compare for example  $U_4O_9(bcc)$  and  $U_4O_9(111)$ ] and maximizing the intraplanar separation of split di-interstitials [compare for example  $U_4O_9(bcc)$  and  $U_4O_9(1-10)$ ] lowers the contribution from Coulomb repulsion among ions. The stability curve in Fig. 6 demonstrates that the low oxidation energy of single split quadinterstitials  $(I_4^X)$  precludes the existence of a pure UO2-U4O9 two-phase field, instead the calculated stability curve suggests the existence of a phase field involving UO<sub>2</sub> and a nonstoichiometric  $U_4O_{9-\nu}$  phase. Our calculations indicate that the rationale for the preference of



FIG. 6. (Color online) The stability (*E*) of  $UO_{2+x}$ , NpO<sub>2+x</sub>, and PuO<sub>2+x</sub>, measured relative to  $AO_2$  and  $O_2$  (oxidation energy). The numbers represent the most stable configuration for each composition. We have also included hydrogen-containing compounds and the cubo-octahedron based  $UO_{2+x}$  compounds. In this plot, phase separation would follow if the oxidation energies exhibit negative curvature. Due to the high stability of  $I_4^X$ , our calculations do not predict a pure  $UO_2$ -U<sub>4</sub>O<sub>9</sub> phase equilibrium, but instead they suggest the existence of a  $UO_2$ +U<sub>4</sub>O<sub>9-y</sub> phase field.

nonstoichiometric  $U_4O_{9-y}$  over stoichiometric  $U_4O_9$  is that oxidation all the way up to  $U_4O_9$  causes frustration among excess oxygen ions and particularly U<sup>5+</sup> ions, leading to optimal energy balance for the nonstoichiometric phase. In agreement with this reasoning, the experimental phase diagram contains a  $UO_{2+x}$ - $U_4O_{9-y}$  phase field rather than  $UO_{2+x}$ - $U_4O_{9}$ .<sup>9,53</sup> The  $UO_{2+x}$ - $U_4O_{9-y}$  two-phase field only emerges when the split quadinterstitial type of clusters is taken into account, i.e.,  $I_4^X$ ,  $U_4O_9(bcc)$  or  $U_4O_9(bccr)$ , and this emphasizes the strong influence that clustering or cooperativity has on the  $UO_{2+x}$  phase diagram. The  $U_4O_9$  structures based on cubo-octahedron clusters within the  $2 \times 2$  $\times 2$  cell are not as stable as  $U_4O_9(bcc)/U_4O_9(bccr)$ . This contradicts the established experimental U<sub>4</sub>O<sub>9</sub> structure models,9,10 which favors an ordering pattern based on cubooctahedrons, as an alternative we propose that the lowtemperature U<sub>4</sub>O<sub>9</sub> compound constitutes an ordered array of split quad-/di-interstitial clusters. We have not been able to investigate  $U_4O_{9-\nu}$  structures based on cubo-octahedron  $I_5^C$ clusters, but the more negative oxidation energy of  $I_4^X$  compared to  $I_5^C$  discussed in Sec. III B emphasizes the importance of the former defect as the fundamental component of  $U_4O_{9-\nu}$  structures.

For  $U_4O_9(111)$ ,  $U_4O_9(bcc)$ , and  $U_4O_9(bccr)$ , the {111} inplane patterns of split di-interstitials are similar to the ordering of triangular oxygen clusters that was obtained by Campbell and Ellis<sup>56</sup> from LEED measurements of the surface structure in oxidized  $UO_2$ ,<sup>57</sup> thus supporting the pivotal role of the split di- or quadinterstitial as the fundamental building block in  $UO_{2+x}$ . Some discrepancies still exist between the structure patterns in Fig. 5 and Refs. 56 and 57, which could be due to thermal disorder, kinetic trapping, or more intricate ordering of split di- or quadinterstitials in the experiments.

The experimental oxidation energy for  $U_4O_9$  is  $\approx -1.8$  eV,<sup>58</sup> which is notably higher (less negative) than the calculated value of -2.63 eV for U<sub>4</sub>O<sub>9</sub>(bcc) or -2.71 eV for the split quadinterstitial. This discrepancy may partially be due to uncertainties in the choice of the Hubbard U parameter and/or the exchange-correlation potential. Test calculations using GGA+U with U=3.5 and J=0.35 eV give an oxidation energy of -2.02 eV for U<sub>4</sub>O<sub>0</sub>(bcc), which agrees better with experiments. The  $GGA + U UO_2$  lattice parameter is overestimated by 0.06 Å and the bulk modulus is predicted to be 194 GPa, in good agreement with experiments (see Table I). The relative stabilities of  $U_4O_9$  compounds within GGA+U are  $U_4O_9(Oct.)$  (-0.79 eV), (-1.47 eV), $U_4 O_9 (1-10)$ (-1.63 eV),  $U_4O_9(111)$  $U_4O_9(bccr)$  (-1.94 eV), and  $U_4O_9(bcc)$  (-2.02 eV), which is the same stability sequence as obtained for LDA+U and even the stability differences are rather similar  $[U_4O_9(Oct.)]$ deviates slightly from this conclusion]. Also the balance between  $U_4O_9(bcc)$  and the  $I_4^X$  cluster, the latter with an oxidation energy of -2.13 eV, is reproduced. From this we conclude that even though the current LDA+U functional overestimates the oxidation energy of  $UO_{2+r}$  by 0.8 eV, this shift seems to be rather constant and, most importantly, this implies that the relative defect stabilities, and thus also the proposed oxygen ordering patterns, should be reliable.

In UO<sub>2</sub> there is a single U-O distance of 2.36 Å (for the calculated lattice constant), but oxidation of UO<sub>2</sub> creates a distribution of U-O bond lengths. For  $U_4O_9(Oct.)$ , the calculations predict the nearest-neighbor U-O bond length to be 2.20 Å and this is largely achieved by displacing the fluorite oxygen ions while the uranium sublattice remains essentially intact. For the split di-interstitial  $U_4O_9$  compounds there is obviously a significant displacement of the oxygen sublattice, but, unlike the  $U_4O_9(Oct.)$  compound, there is also an accompanying distortion of the uranium sublattice. Figure 7 illustrates these properties in terms of the uranium radial distribution function for  $U_4O_9(Oct.)$  and  $U_4O_9(bccr)$ . Extended x-ray-absorption fine-structure (EXAFS) measurements by Conradson et al.<sup>32</sup> have provided new information about the local structure in  $UO_{2+x}$  compounds. Most notably they predict very short oxo type of U-O bonds (1.74 Å).<sup>32</sup> Figure 7 compares the radial distribution function, g(r), from the  $U_4O_9$  structure models that are used in the present work with g(r) derived from EXAFS measurements.<sup>32</sup> Qualitatively, the U<sub>4</sub>O<sub>9</sub>(bccr) structure model reproduces the broad distribution of U-O bonds between 2.15 and 2.75 Å. The short U-O (2.1-2.2 Å) distances obtained from the structure models proposed here uniquely originate from U<sup>5+</sup>-O<sup>2-</sup> distances. The oxo bonds at 1.7-1.8 Å in the EXAFS data are not present in any of the calculated distribution functions, which suggests that the oxo type of bonds should originate from domains, interfaces or surfaces where the regular fluorite structure breaks down.

#### 2. Neptunium and plutonium dioxides

The ordering patterns established above for  $U_4O_9$  have also been applied to the neptunium and plutonium based oxides. The NpO<sub>2+x</sub> and PuO<sub>2+x</sub> stabilities are collected in Table II. Np<sub>4</sub>O<sub>9</sub> exhibits the same sequence of ordered com-



FIG. 7. (Color online) The uranium partial radial distribution function, g(r), calculated from U<sub>4</sub>O<sub>9</sub>(bccr) and U<sub>4</sub>O<sub>9</sub>(Oct.) and compared to g(r) from EXAFS measurements on two different UO<sub>2+x</sub> samples (Ref. 32). The U-O and U-U labels (dotted vertical lines) indicate the bond distances in UO<sub>2</sub>.

pounds as U<sub>4</sub>O<sub>9</sub>, though overall the oxidation energy increases significantly. The Np<sub>4</sub>O<sub>9</sub>(Oct.) structure is even unstable, though, its stability can be increased considerably (0.76 eV) by allowing half of the octahedral interstitials to form molecular O<sub>2</sub> species with regular fluorite oxygen ions, a process which is further discussed for Pu<sub>4</sub>O<sub>9</sub>. In conjunction with  $U_4O_9$ ,  $Np_4O_9(bcc)$  is the most stable  $Np_4O_9$  compound followed by Np<sub>4</sub>O<sub>9</sub>(bccr). As pointed out for NpO<sub>2+x</sub>, the high oxidation energy for  $Np_4O_9$  compared to  $U_4O_9$  is a consequence of the smaller gain in O 2p band energy relative the A 5f delocalization energy, which in turn follows from the decreased separation of A 5f and O 2p orbitals in NpO<sub>2</sub> compared to UO<sub>2</sub>. In order to verify that the decreased stability of Np<sub>4</sub>O<sub>9</sub> is principally a band-structure effect and not due to the smaller volume of  $NpO_2$  compared to  $UO_2$ , we calculated the NpO<sub>2</sub> oxidation thermodynamics at the UO<sub>2</sub> lattice constant without finding any significant changes. Unlike  $UO_{2+x}$  and  $PuO_{2+x}$ , the overall most negative  $NpO_{2+x}$ oxidation energy occurs for the  $I_2^0$  cluster. The NpO<sub>2+x</sub> and Np<sub>4</sub>O<sub>9</sub> oxidation energies are negative, however, in this context we recall that according to the experimental phase diagram  $NpO_{2+x}$  is unstable or possibly metastable with respect to  $NpO_2 + Np_2O_5$  (Refs. 13 and 14) or  $NpO_2 + O_2$ .<sup>4,13,14</sup> According to Ref. 14 the Np<sub>2</sub>O<sub>5</sub> oxidation energy is -0.15 eV, which, in relation to our calculated data, implies that Np<sub>4</sub>O<sub>9</sub> could exist as a stable compound. However, from  $U_4O_9$  we also know that the LDA + U calculations tend to overestimate the stability of oxidation products. If this overestimation is of the same order of magnitude for  $Np_4O_9$  as for  $U_4O_9$  (0.8 eV), the stability of  $Np_4O_9$  is slightly higher than  $Np_2O_5$ . In both cases the oxidation energy is close to 0 eV, implying that disassociation into NpO2+O2 should begin even at moderate temperatures and for small changes in the oxygen chemical potential. These conclusions qualitatively agree with the experimental analyses by Neck at al.<sup>4</sup> and they are also consistent with the difficulty of preparing NpO<sub>2+x</sub> by direct oxidation of NpO<sub>2</sub>,<sup>13</sup> however the low oxidation energy for  $I_2^0$ clusters indicates that partial oxidation of NpO<sub>2</sub> may nevertheless be possible. Moreover, since the oxidation energies of Np<sub>4</sub>O<sub>9</sub> and Np<sub>2</sub>O<sub>5</sub> are not very different, the fluorite based NpO<sub>2+x</sub> phase can probably be stabilized by the addition of uranium.

Table II demonstrates that the properties of the  $Pu_4O_9$ compounds are quite different from the corresponding uranium and neptunium compounds.  $Pu_4O_9(1-10)$  is the structure of highest energy (this structure is low in energy for  $U_4O_9$  and  $Np_4O_9$ ) and  $Pu_4O_9(111_0)$  is one of the most stable structures (this structure is high in energy for  $U_4O_9$  and  $Np_4O_9$ ).  $Pu_4O_9$ (bcc) is the most stable compound. There is also an octahedral-interstitial structure that is almost as stable as  $Pu_4O_9(bcc)$ , but this  $Pu_4O_9$  structure includes an  $O_2$ molecular species that is created from an octahedralinterstitial oxygen ion and one of its nearest-neighbor regular fluorite oxygen ions. The O-O bond distance for this molecular species is 1.43 Å. In our calculations, the  $O_2$  species constitute half of the interstitials and the other half acts as regular octahedral interstitials. There is minimal charge transfer from plutonium ions to the  $O_2$  species, which makes it an uncharged molecule rather than an  $O_2^-$  or  $O_2^{2-}$  ion. The formation of O<sub>2</sub> species lowers the energy by 0.46 eV compared to regular octahedral interstitials. Even for the most stable structures the oxidation energy for  $Pu_4O_9$  is just below zero and PuO<sub>2</sub> should consequently be rather difficult to oxidize, especially if we assume a similar overestimation of the stability of oxidation products as was found for  $UO_{2+r}$  $[\approx 0.8 \text{ eV per excess oxygen ion}]$ . We have already concluded that the instability of  $PuO_{2+x}$  follows from the fact that in  $PuO_2$  the 5f orbitals completely overlap with the O 2p band, implying that the Pu 5f to excess O 2p charge transfer does not result in any significant gain in band energy, even if maximum hybridization is assumed. Consequently, charge transfer must be more difficult and this also explains why the excess oxygen ions are not able to attract two full electrons from the plutonium ions, or more precisely attain the O<sup>2-</sup> valence state (see Sec. III A). Analogous arguments partially hold for NpO<sub>2+x</sub>, but since the Np 5f states are situated at the top of the O 2p band charge transfer is still easier than for  $PuO_{2+x}$ . The incomplete charge transfer in  $PuO_{2+x}$  (and  $NpO_{2+x}$ ) creates unfilled states in the O 2p valence band (see Fig. 8), which from a chemical point of view, is unfavorable. The formation of O<sub>2</sub> species within the lattice in part removes the unfilled O 2p states and separates the remaining states from the valence band, thus explaining the increased stability for this compound. The most stable  $Pu_4O_9(bcc)$ compound exhibits a high degree of hybridization and thus also yields high plutonium to excess oxygen charge transfer (i.e., valence states close to  $Pu^{5+}$  and  $O^{2-}$ ). Increased hybridization yields largely filled O 2p states for Pu<sub>4</sub>O<sub>9</sub>(bcc) and, correspondingly, the now empty Pu 5f states either move into the band gap or attaches to the lower part of the conduction band. Our calculations show that the relative stabilities of  $PuO_{2+x}$  structures are correlated with the ability to increase O 2p hybridization and thus avoid unfilled O 2pstates at the top of the valence band, which introduces a rather high sensitivity to structural details, in particular for



FIG. 8. (Color online) Density of states and projected density of states for  $Pu_4O_9(Oct.)$  and  $Pu_4O_8(OH)_2$ . For the  $Pu_4O_9(Oct.)$  structure we observe the unfilled O 2p states at the top of the valence band. By introducing distortions to the octahedral  $Pu_4O_9$  structure, the unfilled O 2p states may become occupied due to increased hybridization and instead the emptied Pu 5f states are transferred into either the band gap or the bottom conduction band, which is a process that lowers the energy. The unoccupied O 2p states at the top of the valence band disappear for  $Pu_4O_8(OH)_2$  and instead the OH<sup>-</sup> band appears below the main O 2p band. The highest occupied state is at 0 eV.

defect configurations that exhibit high oxidation energy. This conclusion is similar to the observation that the energy of  $Pu_4O_9(Oct.)$  decreases by displacing the octahedral interstitial from its high-symmetry position made by Prodan *et al.*<sup>21</sup>

### **D.** Hydrogen affinity of $AO_{2+x}$ and $A_4O_9$

The PuO<sub>2</sub> oxidation thermodynamics presented in Secs. III B 2 and III C 2 partially contradict the findings by Haschke and co-workers,<sup>6,7</sup> who claimed to be able to oxidize  $PuO_{2+x}$  up to x=0.27. On the other hand our results support failures to produce bulk  $PuO_{2+x}$  from  $PuO_2$  by using pure oxygen as oxidation agent.<sup>59</sup> The fact that Haschke et al. used moist air as oxidation agent and the fact that Conradson et al.<sup>12</sup> found  $PuO_{2+x}$  to readily incorporate H<sub>2</sub>O or its radiolysis products (e.g., OH<sup>-</sup>) in the lattice point to the critical role of these species in the oxidation process. Based on analogies to  $UO_{2+x}$ , Neck *et al.*<sup>4</sup> further emphasized the importance of  $H_2O$  by estimating anhydrous  $PuO_{2+x}$  to be unstable, which disagrees with the oxidation mechanism proposed by Haschke and co-workers.<sup>6,7</sup> Their estimated  $Pu_4O_9$ oxidation energy of 0.13 eV (Ref. 4) is higher than the present calculations (-0.14 eV), though from U<sub>4</sub>O<sub>9</sub> we know that the LDA+U calculations have a tendency to overestimate oxidation energies. Assuming the Pu<sub>4</sub>O<sub>9</sub> error to be similar to  $U_4O_9$ , e.g., 0.8 eV, we predict even more positive oxidation energies than Neck et al.,<sup>4</sup> thus emphasizing the endothermic character of this reaction. On the other hand Neck *et al.* predicted hydrous  $Pu_4O_9$  ( $Pu_4O_9$  associated with  $H_2O$ ) to have slightly negative oxidation energy (-0.12 eV).<sup>4</sup>

These observations motivate us to study oxidation thermodynamics based on absorption of  $OH^-$  or  $H_2O$  in the  $AO_2$ lattice. This was achieved by inserting hydrogen atoms into the  $2 \times 2 \times 2$  supercell that contains single oxygen interstitial. The hydrogen atoms prefer to bind to the oxygen ion in the octahedral-interstitial position and align themselves in the [1 1 1] directions, which means that they are situated in between two oxygen ions but the distance to the octahedral oxygen ion is shorter than to the regular fluorite ion. In the case of H<sub>2</sub>O the two hydrogen atoms form an approximately 180° angle with the interstitial oxygen ion. Using similar procedures as for the  $2 \times 2 \times 2$  cell we have also studied  $A_4O_8(OH)_2$  within the same cell as was used for  $A_4O_9(111)$ . The oxidation energies are summarized in Table II and they are also illustrated in Fig. 6. For  $PuO_{2+x}$  the presence of OH<sup>-</sup> species lowers the oxidation energy significantly, in particular for  $Pu_4O_8(OH)_2$ , so that oxidation of  $PuO_2$  becomes a more exothermic reaction. Measured against oxidation based on OH<sup>-</sup> species, absorption of H<sub>2</sub>O species is less likely to occur; as an example the oxidation energy of  $PuO_2(H_2O)_{1/32}$ is 0.47 eV.

Our results suggest that oxidation of PuO<sub>2</sub> requires the presence of H<sub>2</sub>O in order to take place at any significant rate. This would explain why Gouder et al.<sup>59</sup> failed to oxidize  $PuO_2$  using pure oxygen as oxidation agent, while Haschke et al. observed oxidation in moist air. The lattice constant of  $Pu_4O_8(OH)_2$  exhibits a small expansion, which agrees with the experimental observation by Haschke et al.<sup>6</sup> In this context we observe that  $U_4O_9(1-10)$  and  $Np_4O_9(1-10)$  both contract by 0.01–0.02 Å. Experimentally,  $U_4O_9$  contracts by 0.03 Å, measured as an average for the  $4 \times 4 \times 4$  unit cell.<sup>9</sup> The reason for  $Pu_4O_8(OH)_2$  being the most stable oxidation product of PuO<sub>2</sub> lies in the change of the band structure induced by hydrogen. Figure 8 illustrates how the orbitals of the OH<sup>-</sup> species are situated about 0.50 eV below the main O 2p band, which implies that the electron transferred from plutonium to the OH<sup>-</sup> species moves down from the hybridized O 2p-Pu 5f band to the OH band and this results in a gain in the band energy, much like the process for split di-interstitials in  $UO_{2+x}$ . Also, the electron donated by hydrogen means that only one plutonium electron needs to be transferred to achieve filled O 2p orbitals. The negative oxidation energy of  $Pu_4O_8(OH)_2$  is consistent with the oxidation energy of -0.12 eV predicted by Neck *et al.*<sup>9</sup> for hydrous  $PuO_2$ . Our experience from  $U_4O_9$  tell us that we slightly overestimate the stability of oxidation products and, consequently, we expect the oxidation energy of  $Pu_4O_8(OH)_2$  to increase (become less negative), however even if the upper limit of the error for  $U_4O_9$  is applied as correction (0.8 eV) the oxidation energy of Pu<sub>4</sub>O<sub>8</sub>(OH)<sub>2</sub> is at least close to zero and may thus be thermodynamically allowed. In any case,  $Pu_4O_8(OH)_2$  is more stable than the corresponding hydrogen-free compounds.

Even though the  $UO_2(OH)_{1/32}$  and  $U_4O_8(OH)_2$  compounds have negative oxidation energies, both are less stable

than the corresponding pure oxides and thus they release any absorbed hydrogen, which agrees with established oxidation thermodynamics of  $UO_2$ .<sup>3</sup>  $U_4O_8(OH)_2$  exhibits a similar OH<sup>-</sup> band below the main O 2*p* band as  $Pu_4O_8(OH)_2$ . The fact that  $UO_{2+x}$  does not attract hydrogen means that the contribution from decreased band energy due to O 2*p* hybridization is more important. Notice that  $U_4O_8(OH)_2$  is indeed more stable than the  $U_4O_9(Oct.)$  structure, for which hybridization effects are small compared to the most stable  $U_4O_9$  compounds. The Np<sub>4</sub>O<sub>8</sub>(OH)<sub>2</sub> compound is somewhat higher in energy than Np<sub>4</sub>O<sub>9</sub>(bcc).

#### **IV. CONCLUSIONS**

The excess oxygen ions in hyperstoichiometric actinide dioxides,  $AO_{2+x}$  (A=U, Np, or Pu), form clusters of interstitial oxygen ions and, using density functional theory calculations, we have established the so-called split di-interstitial, illustrated in Fig. 1, as the fundamental building block of these defect clusters. The most stable configuration constitutes two split di-interstitials that are rotated 180° with respect to each other, as illustrated in Fig. 4. Our calculations demonstrate that the stability of these clusters is an effect of the increased hybridization between actinide ions and excess oxygen ions. We also predict a new ground-state structure for U<sub>4</sub>O<sub>9</sub> that is based on a superstructure of split quadinterstitials, which, alternatively, can be visualized as a stacking of {111} planes that contain an ordered arrangement of split di-interstitials.

The structural insights acquired from the present density functional theory calculations allow us to accurately assess the oxidation thermodynamics of UO<sub>2</sub>, NpO<sub>2</sub>, and PuO<sub>2</sub> within fluorite derived structures. UO<sub>2</sub> exhibits strongly negative oxidation energy and reacts readily with oxygen, while NpO<sub>2</sub> is significantly harder to oxidize and PuO<sub>2</sub> is predicted to have positive, or just slightly negative, oxidation energy. The degree of  $AO_2$  oxidation is a function of the position of the A 5f electrons relative the O 2p band. If these states are situated above the O 2p band, oxidation easily occurs (UO<sub>2</sub>), while the overlap of A 5f and O 2p states in PuO<sub>2</sub> suppresses oxidation. The presence of H<sub>2</sub>O and hydrolysis products is able to offset this effect and turn oxidation of PuO<sub>2</sub> into an exothermic process.

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