RAPID COMMUNICATIONS

PHYSICAL REVIEW B **79**, 020506(R) (2009)

Flux-line pinning by point defects in anisotropic biaxial type-II superconductors

G. P. Mikitik[1,2] and E. H. Brandt[1]

[1]*Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany*

[2]*B. Verkin Institute for Low Temperature Physics and Engineering, Kharkov 61103, Ukraine*

(Received 28 October 2008; published 15 January 2009)

The pinning force for an arbitrarily oriented individual vortex in a biaxial type-II superconductor is calculated from the collective pinning theory. It is shown that in anisotropic superconductors the critical force at which the vortex starts to move can be smaller than the pinning force since the vortex will move at an angle to the acting force. Explicit expressions for this angle and for the critical force and critical current density are given.

DOI: 10.1103/PhysRevB.79.020506 PACS number(s): 74.25.Qt, 74.25.Sv

In a recent paper[1] magnetic force microscopy was employed to image and manipulate individual vortices in a single-crystal $YBa_2Cu_3O_{6.991}$. Several unexpected results were obtained in that paper. In particular, the authors found enhanced anisotropy of vortex pinning in this crystal. Besides this, they discovered a dramatic enhancement of the response of a vortex to pulling by a magnetic tip when the tip is wiggled transversely. Importantly, such experiments allow one to measure directly the interaction of an individual vortex with the local disorder potential of the superconductor.

Within the collective pinning theory, pinning of an isolated vortex in a *uniaxial* anisotropic superconductor was theoretically investigated in Ref. 2 (see also review).[3] Using the ideas of collective pinning theory,[3] Auslaender *et al.*[1] estimated the pinning force in a *biaxial* superconductor for a flux line aligned with the c axis. However, in their experiment[1] the vortex was generally tilted at large angles $\theta \sim 1$ to the c axis. So, to provide a basis for the analysis of the experimental data obtained in Ref. 1 or in similar experimental investigations, in this Rapid Communication we study pinning of a tilted vortex in a biaxial superconductor. We first obtain the line tension for such a vortex and then calculate the pinning force. For clarity in this derivation we follow the ideas in Ref. 2 rather than the scaling approach.[3,4] We also discover an effect that is absent in isotropic crystals: in an anisotropic superconductor the critical force for a given direction can be *smaller* than the maximum pinning force for the same direction. The critical force is defined as the force at which the vortex starts to move, and it is proportional to the critical current density. This effect is due to the deviation of the direction of vortex motion from the direction of the acting force.

Consider a biaxial superconductor and let the x, y, and z axes of the coordinate system coincide with the anisotropy axes a, b, and c of this superconductor, respectively. The line tension ε_l of a vortex can be derived from its line energy[5] $e_l(\theta,\varphi)$, where the angles θ and φ define the direction of the vortex, i.e., we describe this vortex direction by the unit vector $(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ (see Fig. 1). Within anisotropic London theory, the line energy of the vortex in a biaxial superconductor is[6,7]

$$e_l(\theta,\varphi) = \varepsilon_0\sqrt{\varepsilon^2\eta(\varphi)\sin^2\theta + \cos^2\theta}, \tag{1}$$

where $\varepsilon \equiv \lambda_{ab}/\lambda_c$ is the parameter of the anisotropy and $\varepsilon_0 = (\Phi_0/\lambda_{ab})^2\ln(\lambda_{ab}/\xi_{ab})/(4\pi\mu_0)$; Φ_0 is the flux quantum, $\lambda_{ab} = \sqrt{\lambda_a\lambda_b}$, $\xi_{ab} = \sqrt{\xi_a\xi_b}$, ξ_a and ξ_b are the coherence lengths, λ_c, λ_a, and λ_b are the London penetration depths, and we have used the notation

$$\eta(\varphi) = \zeta\cos^2\varphi + \zeta^{-1}\sin^2\varphi, \tag{2}$$

in which $\zeta = \lambda_a/\lambda_b$ is the parameter of the anisotropy in the a-b plane. In particular, for $YBa_2Cu_3O_{6.995}$ one has $\zeta \approx 1.3$ and $\varepsilon \approx 1/7$,[8] i.e., $\varepsilon^2 \ll 1$.

When the vortex described by the angles θ and φ is symmetrically deformed over a distance $2L$ by an amplitude $L\delta\alpha$ ($L\delta\alpha$ is the height of an isosceles triangle with basis $2L$ and two small angles α) in the direction defined by the angle ψ (Fig. 1), the energy increase in one segment with the length L is

$$\delta e_l = \frac{\partial e_l}{\partial L}\delta L + \frac{\partial e_l}{\partial\theta}\delta\theta + \frac{\partial e_l}{\partial\varphi}\delta\varphi + \frac{1}{2}\frac{\partial^2 e_l}{\partial\theta^2}\delta\theta^2 + \frac{1}{2}\frac{\partial^2 e_l}{\partial\varphi^2}\delta\varphi^2 + \frac{\partial^2 e_l}{\partial\theta\,\partial\varphi}\delta\theta\delta\varphi. \tag{3}$$

Geometrical considerations yield $\delta L \approx L(\delta\alpha)^2/2$, and

$$\delta\theta \approx \cos\psi\delta\alpha + \frac{\cot\theta}{2}\sin^2\psi(\delta\alpha)^2,$$

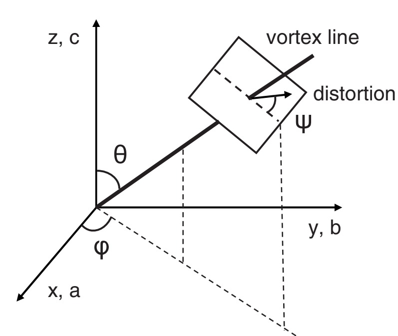

FIG. 1. Definition of the angles θ, φ, and ψ. The angles θ and φ specify the direction of the vortex shown as bold solid line. The angle ψ in the plane perpendicular to the vortex defines the direction of a small vortex deformation; ψ is measured from the line that is the intersection of this plane with the plane containing the vortex and the z axis.

$$\delta\varphi \approx \frac{\sin\psi}{\sin\theta}\delta\alpha - \cot\theta\frac{\sin\psi\cos\psi}{\sin\theta}(\delta\alpha)^2. \qquad (4)$$

Since the total deformation involves two segments of length L with opposite $\delta\alpha$, the contributions to δe_l that are linear in $\delta\alpha$ are canceled in the second and third terms of Eq. (3). That is why we also take into account the terms proportional to $(\delta\alpha)^2$ in formulas (4). These terms are due to the fact that the vortex deformation at $\psi=\pi/2$ changes not only the angle φ but also the angle θ a little. Similarly, the deformation in θ slightly influences $\delta\varphi$.

The line tension ε_l of the vortex in the direction specified by the angle ψ is defined by the relation

$$\frac{1}{2}\varepsilon_l(\theta,\varphi,\psi)L(\delta\alpha)^2 = \delta e_l. \qquad (5)$$

Substitution of formulas (3) and (4) into this relation gives

$$\varepsilon_l(\theta,\varphi,\psi) = e_l + \cot\theta\left(\frac{\partial e_l}{\partial\theta}\sin^2\psi - 2\frac{\partial e_l}{\partial\varphi}\frac{\sin\psi\cos\psi}{\sin\theta}\right) + \frac{\partial^2 e_l}{\partial\theta^2}\cos^2\psi + \frac{\partial^2 e_l}{\partial\varphi^2}\frac{\sin^2\psi}{\sin^2\theta} + 2\frac{\partial^2 e_l}{\partial\theta\,\partial\varphi}\frac{\sin\psi\cos\psi}{\sin\theta}. \qquad (6)$$

At the specific case $\psi=0$ this expression reduces to Eq. (18) of Ref. 9. Using formulas (1) and (6), one can directly calculate $\varepsilon_l(\theta,\varphi,\psi)$. In the subsequent analysis we shall imply that the vortex is not too close to the a-b plane, i.e., $\varepsilon^2\tan^2\theta \ll 1$. Then, we arrive at

$$\varepsilon_l(\theta,\varphi,\psi) = \frac{\varepsilon_0\varepsilon^2}{\cos\theta}\zeta\left(\frac{\cos\varphi\cos\psi}{\cos\theta} - \sin\varphi\sin\psi\right)^2 + \frac{\varepsilon_0\varepsilon^2}{\cos\theta}\frac{1}{\zeta}\left(\frac{\sin\varphi\cos\psi}{\cos\theta} + \cos\varphi\sin\psi\right)^2. \qquad (7)$$

If $\zeta=1$ and $\psi=0$ or $\pi/2$, formula (7) reproduces the two so-called out-of-plane and in-plane line tensions in a uniaxial superconductor.[2,3] At $\theta=0$ we obtain from expression (7) the line tension,

$$\varepsilon_l(\varphi,\psi) = \varepsilon_0\varepsilon^2\eta(\varphi+\psi), \qquad (8)$$

which is much smaller than the line energy $e_l=\varepsilon_0$ [Eq. (1)].[10] Since at $\theta=0$ the planes of φ and ψ coincide, the angle $\varphi+\psi$ specifies the vortex-distortion direction measured from the x axis. In this case we reproduce the result in Ref. 1. Expression (7) is immediately applicable if the characteristic length of the vortex deformation is essentially larger than the appropriate London penetration depth. In the opposite case, for example, in the problem of vortex pinning, which involves short-wavelength distortion of the vortex line, the logarithmic factor should be omitted in ε_0.[3]

We now use the collective pinning theory[3] to find the pinning force produced by point defects. The pinning energy of a vortex segment of length L_c is of the order of $E_{\rm pin} \sim U_p(nSL_c)^{1/2}$, where U_p is the characteristic pinning energy produced by one of the point defects, n is their density, S is the cross-section area of the vortex core, and the collective pinning length L_c is characterized by the condition that for this segment the averaged amplitude of the vortex distortions due to pinning is of the order of the coherence length ξ. This length L_c is found from the balance of the pinning energy $E_{\rm pin}$ and the elastic energy of the segment, $E_{\rm el} \sim \varepsilon_l(\xi/L_c)^2L_c$. The balance leads to the formula

$$L_c \sim \left(\frac{\varepsilon_l^2\xi^4}{U_p^2 nS}\right)^{1/3}. \qquad (9)$$

The coherence length ξ is anisotropic in a biaxial superconductor and can be obtained from the Ginzburg-Landau functional, yielding

$$\xi^2(\theta,\varphi,\psi) = \xi_{ab}^2\Big[\zeta(\sin\varphi\cos\psi\cos\theta + \cos\varphi\sin\psi)^2 + \frac{1}{\zeta}(\cos\varphi\cos\psi\cos\theta - \sin\varphi\sin\psi)^2 + \varepsilon^2\sin^2\theta\cos^2\psi\Big], \qquad (10)$$

where the last term should be omitted in our approximation $\varepsilon^2\tan^2\theta \ll 1$. Here and below we indicate the direction of ξ using the angles θ and φ for the appropriate vortex and the angle ψ defining this direction in the plane perpendicular to the vortex line. The dimension of the vortex core is determined by $\xi(\psi)$, and Eq. (10) shows that the cross section of this core is an ellipse with area $S=\pi\xi_{ab}^2\cos\theta$. Since ε_l and ξ depend on the angle ψ, one will obtain different L_c for different ψ. But the vortex relaxes to the pinning potential choosing the mode characterized by the smallest collective pinning length. Thus, to calculate L_c, it is necessary to find the minimum of $\varepsilon_l(\psi)\xi^2(\psi)$ over the angle ψ. Both $\varepsilon_l(\psi)$ [Eq. (7)] and $\xi^2(\psi)$ [Eq. (10)] are quadratic forms in the variables $\cos\psi$, $\sin\psi$. Omitting the last term in Eq. (10) and making the substitution $\psi=\tilde{\psi}+\pi/2$ in $\xi^2(\psi)$, one finds that the quadratic forms $\varepsilon_l(\psi)$ and $\xi^2(\tilde{\psi})$ are proportional to each other. Thus, they have the same eigenvectors which are generally not along the directions $\psi=0$ and $\psi=\pi/2$. Along these vectors the quadratic forms reach their maximum and minimum values. Due to the relation $\psi=\tilde{\psi}+\pi/2$, $\varepsilon_l(\psi)$ is maximum when $\xi^2(\psi)$ is minimum and vice versa. The product $\varepsilon_l(\psi)\xi^2(\psi)$ turns out to be minimum along these vectors and equals $\varepsilon_0\varepsilon^2\xi_{ab}^2/\cos\theta$. Thus, we find

$$L_c = \frac{L_c^c}{\cos\theta}, \quad E_{\rm pin} = E_{\rm pin}^c, \qquad (11)$$

where $L_c^c \sim \varepsilon^{4/3}(\varepsilon_0^2\xi_{ab}^2/U_p^2 n\pi)^{1/3}$ is the collective pinning length and $E_{\rm pin}^c \sim \varepsilon^{2/3}U_p(n\pi\xi_{ab}^2L_c^c)^{1/2}$ is the pinning energy for the vortex directed parallel to the c axis in the superconductor without the anisotropy in the a-b plane (i.e., at $\zeta=1$). Note that the obtained L_c and $E_{\rm pin}$ in the biaxial crystal exactly coincide with the appropriate quantities calculated[2,3] for the uniaxial superconductor.

The pinning force is perpendicular to the vortex. Its value (per unit vortex length) in the direction ψ is determined by

$$f_p(\theta,\varphi,\psi) \sim \frac{E_{\text{pin}}}{L_c(\theta)\xi(\theta,\varphi,\psi)} = f_p^c \frac{\xi_{ab}\cos\theta}{\xi(\theta,\varphi,\psi)}, \qquad (12)$$

where $f_p^c \sim E_{\text{pin}}^c/(L_c^c \xi_{ab})$ is the pinning force for the vortex along the c axis in the uniaxial superconductor. Consider f_p in some special cases. At $\theta=0$ Eq. (12) yields

$$f_p(0,\varphi,\psi) = \frac{f_p^c}{\eta(\varphi+\psi+\pi/2)}, \qquad (13)$$

where $\eta(\phi)$ is defined by formula (2). Since at $\theta=0$ the planes of the angles φ and ψ coincide with each other, the sum $\varphi+\psi$ simply specifies the direction of the force in the a-b plane relative to the a axis. Expression (13) slightly corrects the appropriate formula in the supplementary information to Ref. 1.

If $\theta \neq 0$, but $\psi=0$, we obtain from formula (12) that

$$f_p(\theta,\varphi,0) = \frac{f_p^c}{\eta(\varphi+\pi/2)}. \qquad (14)$$

Note that this force does not depend on θ, i.e., on the tilt of the vortex, and in fact, it coincides with the force for the vortex aligned with the c axis [Eq. (13)]. This property of the force is quite similar to that of the uniaxial superconductors.[2,3]

At $\theta \neq 0$ and $\psi=\pi/2$ the pinning force is applied in the a-b plane. In this case it follows from Eq. (12) that

$$f_p(\theta,\varphi,\pi/2) = \frac{f_p^c \cos\theta}{\eta(\varphi)}. \qquad (15)$$

When θ tends to zero, this expression is in accordance with Eq. (13). Finally, in the case of the uniaxial superconductor (i.e., at $\zeta=1$) Eq. (12) reduces to

$$f_p(\theta,\varphi,\psi) = \frac{f_p^c \cos\theta}{(\cos^2\theta\cos^2\psi+\sin^2\psi)^{1/2}}. \qquad (16)$$

This formula agrees with the results in Refs. 2 and 3.

In anisotropic superconductors flux-line pinning has a property that is absent in the isotropic case: under an applied force exerted by the critical current, flux lines can move in a direction that differs from the direction of the force.[11] In the considered case this property can be reformulated as follows: if one applies a force along the direction defined by the angle ψ, the critical value of this force, $f_c(\psi)$, at which the vortex overcomes pinning and begins to move, can be smaller than the maximum pinning force in this direction, $f_p(\psi)$. This occurs because it is favorable for the vortex to start to move in some other direction ψ_1. The mechanism of this effect is explained in Fig. 2. The projection of the force f applied along the direction ψ on some other direction ψ_1 is $f\cos(\psi-\psi_1)$. If this projection reaches $f_p(\psi_1)$, i.e., if

$$f = \frac{f_p(\psi_1)}{\cos(\psi-\psi_1)}, \qquad (17)$$

the vortex can move in this direction ψ_1. The critical force, f_c, is the minimum force f over the angle ψ_1. Differentiation of formula (17) over ψ_1 gives the equation for this angle,

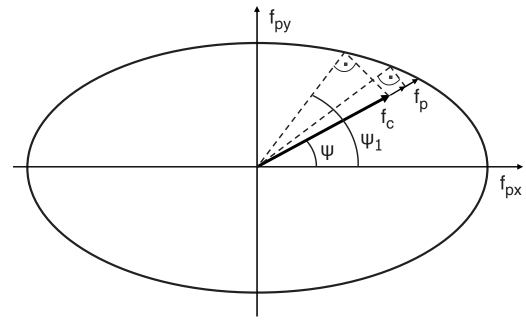

FIG. 2. Schematic plot explaining why in an anisotropic superconductor the critical force f_c at which the vortex starts to move may be smaller than the maximum pinning force f_p. The ellipse shows the angular dependence of the maximum pinning force $f_p(\psi)$. The dashed lines indicate the projections of forces $f<f_p$ acting along ψ on some other directions. All these projections reach the ellipse at angles different from ψ. The thick arrow shows the minimum force of this type, $f_c(\psi)$. At $f=f_c(\psi)$ the vortex starts to move in the direction ψ_1.

$$\tan(\psi-\psi_1) = \frac{f_p'(\psi_1)}{f_p(\psi_1)}, \qquad (18)$$

where the prime means $d/d\psi_1$. On determining ψ_1, one finds $f_c(\psi)$ from formula (17),

$$f_c(\psi) = \sqrt{[f_p(\psi_1)]^2 + [f_p'(\psi_1)]^2}. \qquad (19)$$

At this critical force $f_c(\psi)$ the vortex starts to move in the direction ψ_1 given by Eq. (18). Interestingly, the described difference between f_c and f_p appears even in uniaxial superconductors if $\theta \neq 0$.

The critical force f_c permits one to find the value of the critical current density $j_{c\perp}$ in the plane perpendicular to the vortex,

$$j_{c\perp}(\psi) = \frac{f_c(\psi-\pi/2)}{\Phi_0}, \qquad (20)$$

where Φ_0 is the flux quantum and the angle ψ defines the direction of the current. In experiments it may also be useful to know the critical current density j_c in the a-b plane for a tilted vortex. Let this j_c flow at an arbitrary angle ϕ to the a axis. Using the results in Ref. 11 (Appendix A), we obtain

$$j_c(\phi) = \frac{j_{c\perp}(\psi)}{[1-\cos^2(\phi-\varphi)\sin^2\theta]^{1/2}}, \qquad (21)$$

where ψ is determined from the relation $\tan\psi = \tan(\phi-\varphi)/\cos\theta$, while the vortex direction is still defined by the angles θ and φ. The denominator in formula (21) is due to the fact that the currents in the a-b plane are not perpendicular to the vortex, and this denominator is the sine of the angle between the current and the vortex.

Since according to Eq. (12) the quantity $f_p^2(\psi)$ is described by an ellipse in the plane perpendicular to the vortex, we shall now analyze a simple model for the ψ dependence of f_p that we define by the equation

$$f_p^2(\psi)\left(\frac{\cos^2\psi}{f_{p0}^2}+\frac{\sin^2\psi}{f_{p0}^2\delta}\right)=1, \tag{22}$$

in which f_{p0} and δ are some constants, and for definiteness let be $\delta \le 1$. This model describes an ellipse whose axes are along the directions $\psi=0$ and $\psi=\pi/2$. Formula (22) immediately follows from Eq. (12) if $\varphi=0$. In this case one has $f_{p0}=f_p^c\sqrt{\zeta}$ and $\delta=\cos^2\theta/\zeta^2$. If the angle $\varphi\neq 0$ or $\pi/2$, the only difference is that the axes of the ellipse do not coincide with the directions $\psi=0$ and $\psi=\pi/2$. Formula (22) also describes the pinning force in uniaxial superconductors at any θ and φ. In this case one has $\delta=\cos^2\theta$ and $f_{p0}=f_p^c$. Using Eqs. (18) and (19), in the case of $f_p(\psi)$ given by formula (22) we obtain the dependence $f_c(\psi)$ in the parametric form,

$$\tan\psi=\frac{(2\delta-1+\tan^2\psi_1)\tan\psi_1}{\delta+(2-\delta)\tan^2\psi_1},$$

$$f_c(\psi)=\frac{f_{p0}\sqrt{\delta}}{(\delta\cos^2\psi_1+\sin^2\psi_1)^{1/2}}\left(1+\frac{(1-\delta)^2\tan^2\psi_1}{(\delta+\tan^2\psi_1)^2}\right)^{1/2}. \tag{23}$$

Here the angle ψ_1 plays the role of a curve parameter. If the parameter $\delta\ge 1/2$, the critical force $f_c(\psi)$ does not differ essentially from $f_p(\psi)$ although the angle ψ_1 may noticeably deviate from ψ (Fig. 3). In this case the first formula in Eq. (23) establishes a one-to-one correspondence between ψ and ψ_1. At $\delta<1/2$ the situation changes. In this case spurious branches of $\psi_1(\psi)$ appear. For the physical branch, which provides minimum values of f_c, the angle ψ_1 lies in the interval from ψ_1^{cr} to $\pi/2$ if ψ changes from 0 to $\pi/2$ (Fig. 3). Here $\tan\psi_1^{cr}=\sqrt{1-2\delta}$. Thus, even when the force is applied along the symmetry direction $\psi=0$, the vortex starts to move in one of the directions $\pm\psi_1^{cr}$, and the critical value of this force $f_c(\psi=0)=2f_{p0}\sqrt{\delta(1-\delta)}$ can be noticeably smaller than the pinning force in this direction, $f_p(\psi=0)=f_{p0}$.

In the limit of large anisotropy of the pinning force, $\delta \ll 1$, one obtains $\psi_1^{cr}\approx\pi/4$ and $f_c(\psi=0)\approx 2\sqrt{\delta}f_{p0}$. Since $f_c(\psi=\pi/2)=\sqrt{\delta}f_{c0}$, the anisotropy of the critical force tends to a constant value, $[f_c(\psi=0)/f_c(\psi=\pi/2)]=2$, which is much smaller than the anisotropy of the pinning force, $[f_p(\psi=0)/f_p(\psi=\pi/2)]=\delta^{-1/2}$.

In conclusion, we have derived the anisotropic line tension of an arbitrarily oriented vortex in a biaxial type-II superconductor [Eq. (6)], which reduces to formula (7) in the case $\varepsilon^2\tan^2\theta\ll 1$. From this line tension the pinning force is calculated within collective pinning theory [Eq. (12)]. It is shown that in anisotropic superconductors the critical force at which the vortex starts to move can be smaller than the pinning force in this direction and that, in general, the vortex moves at an angle to the acting force. Formulas (18)–(21) allow us to calculate this critical force and the critical current density in anisotropic superconductors at any given pinning force $f_p(\theta,\varphi,\psi)$.

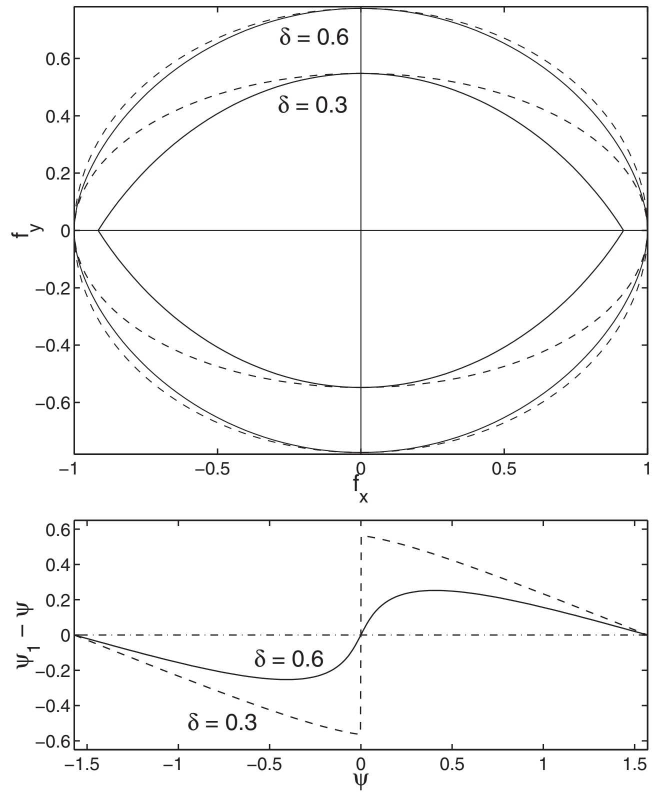

FIG. 3. Top: the ψ dependences of the pinning force f_p [dashed lines, Eq. (22)] and of the critical force f_c [solid lines, Eq. (23)] at δ=0.6 and 0.3. The forces are measured in units of f_{p0}. Here the principal axes of $f_p(\psi)$ are denoted as x and y axes although they generally do not coincide with the x and y axes introduced in Fig. 1. Bottom: dependence of $\psi_1-\psi$ on ψ for δ=0.6 (solid line) and δ=0.3 (dashed line).

This work was supported by the German-Israeli Research Grant (GIF) under Agreement No. G-901-232.7/2005.

[1] O. M. Auslaender *et al.*, Nat. Phys. **5**, 35 (2009).
[2] G. Blatter and V. Geshkenbein, Phys. Rev. B **47**, 2725 (1993).
[3] G. Blatter *et al.*, Rev. Mod. Phys. **66**, 1125 (1994).
[4] G. Blatter, V. B. Geshkenbein, and A. I. Larkin, Phys. Rev. Lett. **68**, 875 (1992).
[5] E. H. Brandt, Phys. Rev. Lett. **69**, 1105 (1992).
[6] A. M. Grishin, Fiz. Nizk. Temp. **9**, 277 (1983) [Sov. J. Low Temp. Phys. **9**, 138 (1983)].
[7] L. I. Burlachkov, Europhys. Lett. **8**, 673 (1989).
[8] T. Pereg-Barnea, P. J. Turner, R. Harris, G. K. Mullins, J. S. Bobowski, M. Raudsepp, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **69**, 184513 (2004).
[9] A. Sudbo and E. H. Brandt, Phys. Rev. Lett. **67**, 3176 (1991).
[10] E. H. Brandt, Rep. Prog. Phys. **58**, 1465 (1995).
[11] G. P. Mikitik and E. H. Brandt, Phys. Rev. B **62**, 6800 (2000).