

Conductance-plateau transitions in quantum Hall wires with spatially correlated random magnetic fields

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Quantum transport properties in quantum Hall wires in the presence of spatially correlated disordered magnetic fields are investigated numerically. It is found that the correlation drastically changes the transport properties associated with the edge state, in contrast to the naive expectation that the correlation simply reduces the effect of disorder. In the presence of correlation, the separation between the successive conductance-plateau transitions becomes larger than the bulk Landau-level separation determined by the mean value of the disordered magnetic fields. The transition energies coincide with the Landau levels in an effective magnetic field stronger than the mean value of the disordered magnetic field. For a long wire, the strength of this effective magnetic field is of the order of the maximum value of the magnetic fields in the system. It is shown that the effective field is determined by a part where the stronger magnetic-field region connects both edges of the wire.

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I. INTRODUCTION

Since the discovery of the quantum Hall effect, the quantum transport property of two-dimensional (2D) systems in strong magnetic fields has been one of the central issues of condensed-matter physics.^{1–6} The energy spectrum of electrons in two dimensions in a strong magnetic field forms equally spaced degenerate energy levels called the Landau levels. In the presence of boundaries, edge states exist that extend along the boundary of the sample.^{7,8} The edge state corresponds to the classical skipping orbit along the boundary and is known to be less influenced by impurities and defects of the system.⁹ Since the edge state is associated with each Landau level, the number of edge states at the Fermi energy is equal to the number of the Landau levels below it. The conductance is therefore quantized to be $n(e^2/h)$ per spin when the Fermi energy lies between the n th and the $(n+1)$ th Landau levels. In the absence of impurity scattering, the transition between quantized values of the conductance occurs when the Fermi energy crosses the Landau level.

The stability of the edge states and the mixing of the edge and the bulk states in the presence of impurities has been studied by many authors.^{10–14} It has been shown that even in the presence of disorder, the edge state is well defined and is extended along the boundary as long as its energy lies away from those of the bulk Landau subbands and the magnetic field is strong enough. On the other hand, when its energy lies in the middle of the bulk Landau subbands, the edge

state mixes with the bulk states by the impurity scattering, which leads to the localization of edge states.^{13,14} It has been shown for long quantum Hall wires with uncorrelated disorder potential that the conductance indeed vanishes when the Fermi energy is close to the centers of the bulk Landau subbands.^{15,16} The zero conductance regime therefore appears between the conductance plateaus $n(e^2/h)$ and $(n+1)(e^2/h)$ in the presence of disorder (Fig. 1). This transition between the quantized conductance and the insulator is called the chiral metal-insulator transition (CMIT).^{15,16}

We focus on the effect of disorder correlation in such systems. For example, the bulk quantum Hall system, the effect of potential correlation has been studied^{17–20} and demonstrated numerically that the potential correlation suppresses the mixing between edge states.¹⁸ It has also been observed that the critical energies of the bulk system are insensitive to the potential correlation as long as the disorder is weak.²⁰ All these results are consistent with the intuitive picture that the potential correlation makes the potential smooth and reduces the scattering with large momentum transfers.

It is therefore expected naively that in the case of the quantum Hall wires, the zero conductance regime observed for the uncorrelated potential is suppressed and the quantized conductance steps are recovered when the potential correlation is introduced. In a previous paper,²¹ we have found indeed the suppression of the CMIT which yields the recovery of the quantized conductance steps. Quite unexpectedly,

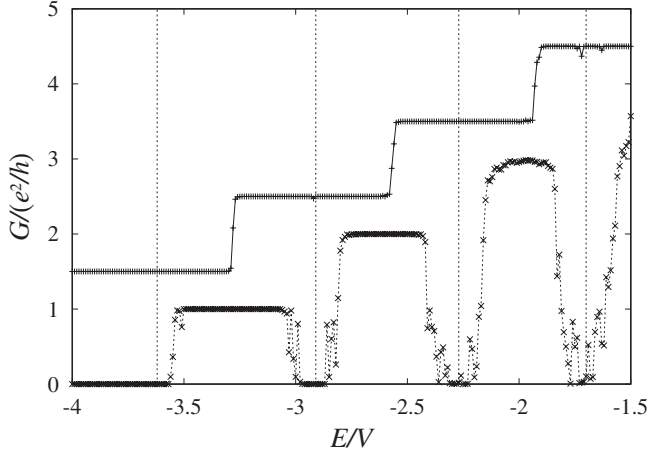


FIG. 1. Conductance for the case of the potential disorder in the square lattice tight-binding model. The system size is 20×1000 and the magnetic flux ϕ per plaquette is assumed to be $\phi/(h/e) = 1/16$. The site energies ε_i obey the Gaussian distribution with variance $\sigma \approx 0.23$ in units of the hopping amplitude. The spatial correlation of site energies is assumed to be $\langle \varepsilon_i \varepsilon_j \rangle = \langle \varepsilon^2 \rangle \exp(-r_{ij}^2/4\eta^2)$, where r_{ij} denotes the distance between the sites i and j . Conductance with potential correlation [$\eta/a=5$ (+)] and that without it [$\eta=0$ (x)] are plotted. For $\eta/a=5$, $G/(e^2/h)+1.5$ is shown instead of $G/(e^2/h)$. Here the lattice constant is denoted by a . The vertical dotted lines represent the positions of the conductance steps in the absence of disorder, namely, the positions of the bulk Landau levels (Ref. 21).

however, it has also been found that the quantized conductance steps shift toward higher energies (Fig. 1). This is remarkable since this means that the potential correlation acts against the conduction of the system contrary to the naive expectation. We have found that the scale of the shift is of the order of the strength of the random potential and is insensitive to the strength of the magnetic field. The shift appears in wires but does not appear in short systems. For understanding the physics behind this shift, we have argued that the correlated potential can act as a potential barrier across the wire, which produces the same amount of shift for each plateau transition. It has also been found,²² by calculating the conductance for various values of energies keeping the disorder potential fixed, that though the size of the shifts depends considerably on the sample, the distance between successive transitions, namely, the plateau width, is insensitive to the sample and is essentially the same as the bulk Landau-level separation. The quantum Hall wire is therefore an interesting system where the potential correlation affects qualitatively its transport property.

In the present paper, we further examine the effect of the disorder correlation in quantum Hall wires by considering a different type of disorder, namely, the disordered magnetic fields. When the disordered component of the magnetic fields is smaller than its uniform component, the system belongs to the quantum Hall universality class except for the band center.²³ It is therefore expected that even in the case of the disordered magnetic field, the conductance-plateau transitions also occur in the same way as the disordered potential case. The disordered magnetic-field system has been studied theoretically in connection to the fractional quantum Hall

effect at filling factor $\nu=1/2$.²⁴ Due to the advances in the fabrication technique, such a system has also been realized experimentally in a semiconductor heterostructure and its magnetotransport has been studied under various conditions.^{25,26} It is therefore important to clarify the effect of correlation in 2D disordered magnetic fields theoretically and experimentally.

We consider a lattice model with spatially correlated disordered magnetic fields and examine numerically how the effect of correlation shows up in the conductance-plateau transitions in such a system. It is then found that the effect appears as the change in the separation of successive plateau transitions, which is qualitatively different from that observed for the correlated disorder potential case. The origin of this change in the separation is also discussed semiclassically.

II. MODELS AND METHODS

We consider a two-dimensional system with disordered magnetic fields described by the Hamiltonian on the square lattice

$$H = \sum_{\langle i,j \rangle} V \exp(i\theta_{i,j}) c_i^\dagger c_j. \quad (1)$$

The phases $\{\theta_{i,j}\}$ are determined so that the summation around the i th plaquette is equal to the magnetic flux $-2\pi\phi_i/\phi_0$ through the plaquette, where $\phi_0 = h/e$ stands for the flux quantum. The flux ϕ_i through the i th plaquette can be expressed as

$$\phi_i = \delta\phi_i + \phi. \quad (2)$$

Here ϕ denotes the uniform component of the magnetic fluxes, which determines the mean value of fluxes. The disordered component around the mean is denoted by $\delta\phi_i$. It is assumed that the disordered component of the flux is distributed with the Gaussian distribution with zero mean as

$$P(\delta\phi) = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} \exp(-\delta\phi^2/2\sigma_\phi^2). \quad (3)$$

The spatial correlation of the disordered components is assumed to be

$$\langle \delta\phi_i \delta\phi_j \rangle = \langle \sigma_\phi^2 \rangle \exp(-|\mathbf{R}_i - \mathbf{R}_j|^2/4\eta_\phi^2). \quad (4)$$

The position vector for the site i is denoted by \mathbf{R}_i . All length scales are measured in units of the lattice constant. The spatially correlated fluxes are constructed from the uncorrelated fluxes $\delta\phi_j^{\text{u.c.}}$'s as^{20,21}

$$\delta\phi_i = \frac{\sum_j \delta\phi_j^{\text{u.c.}} \exp(-|\mathbf{R}_j - \mathbf{R}_i|^2/2\eta_\phi^2)}{\sqrt{\sum_j \exp(-|\mathbf{R}_j - \mathbf{R}_i|^2/\eta_\phi^2)}}. \quad (5)$$

When the uncorrelated fluxes $\delta\phi_j^{\text{u.c.}}$'s obey the Gaussian distribution with variance σ , it is easily verified that $\delta\phi_i$ satisfies the relations in Eqs. (3) and (4). The effective width $w_\phi = \sqrt{12}\sigma_\phi$ is used to specify the disorder strength. In the case of the present lattice model, the flux ϕ_i and $\phi_i + m\phi_0$

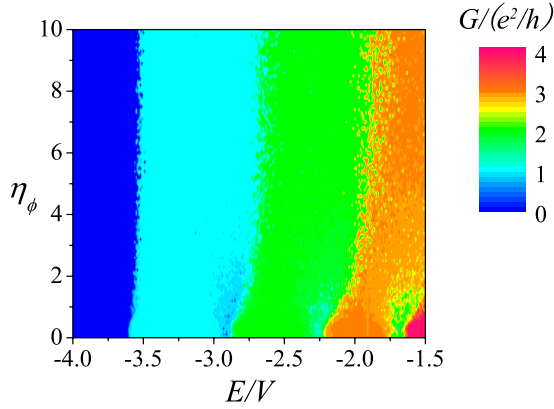


FIG. 2. (Color online) Conductance for $w_\phi/\phi_0=1/32$, $\phi/\phi_0=1/16$, and $L/M=50$ as a function of the correlation η_ϕ and the Fermi energy E/V . The magnetic length for the mean value ϕ is $l \approx 1.6$. Independent sample is assumed for each set of parameters E/v and η_ϕ .

with an integer m are equivalent with each other. To avoid the artifact of this periodicity,²⁷ we confine ourselves to small values of w_ϕ and ϕ and to the Fermi energy away from the band center.

We consider a system with the length L and the width M having two leads attached to both ends of the system. The fixed boundary condition is assumed in the transverse direction. For the realization of the isotropic correlation for the magnetic fields in the sample region $L \times M$, we consider additional regions of the width $5\eta_\phi$ outside of the sample in performing the summation over the uncorrelated fluxes $\delta\phi_i^{u.c.}$. By this procedure, we realize isotropic correlations of disordered fluxes in the sample region.²¹

The two-terminal conductance G is obtained by means of the Landauer formula

$$G = (e^2/h)\text{Tr } T^\dagger T, \quad (6)$$

where T is the transmission matrix, which is evaluated by the transfer-matrix method.²⁸ The width M is set to be 20. The smallest magnetic flux ϕ/ϕ_0 per plaquette is $1/32$, and the corresponding magnetic length $l = \sqrt{\hbar/eB} = \sqrt{\phi_0/(2\pi\phi)}$ is about 2.3, much smaller than the system width M . This leads to the existence of edge states along the boundaries of the present systems.

III. TWO-TERMINAL CONDUCTANCE

We show in Fig. 2 the conductance of a long wire ($L/M=50$) as a function of the Fermi energy E/V and the correlation length η_ϕ of the random magnetic fields. The parameters for the magnetic fields are $\phi/\phi_0=1/16$ and $w_\phi/\phi_0=1/32$. Independent samples are assumed for each set of parameters η_ϕ and E/V throughout this section. It is clearly seen here that the plateau transitions shift toward higher energies as the correlation of random magnetic fields is increased. It should be noted in Fig. 2 that the distance between successive transitions, namely, the plateau width, increases as the correlation of magnetic fields is increased. This means that the shift of the critical energy of the plateau

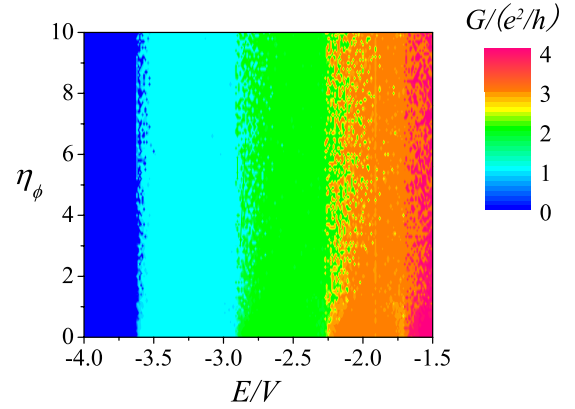


FIG. 3. (Color online) Conductance for $w_\phi/\phi_0=1/32$, $\phi/\phi_0=1/16$, and $L/M=2$ as a function of the correlation η_ϕ and the Fermi energy E/V . The distance between the plateau transitions is insensitive to the strength of the correlation. Independent sample is assumed for each set of parameters E/V and η_ϕ .

transition associated with lower Landau levels seems to be smaller than that associated with higher Landau levels. It is to be recalled that in the case of the correlated disorder potential,²¹ the scale of shifts is common to all plateau transitions and the distance between plateau transitions remains to be the same as the bulk Landau-level separation (Fig. 1). The change in the critical energy in the random-field case is therefore qualitatively different from that observed for the case of the potential correlation. This change in the separations of plateau transitions is absent for a short system (Fig. 3), and hence this change is a specific feature to long wires. Insensitivity of the critical energy to the correlation in a short system is consistent with the fact that the positions of the bulk Landau levels are insensitive to the random magnetic field correlation as long as the disorder is rather weak.²⁹ For a long wire, the present enhancement of the plateau width with increasing correlation is also observed for the case of $\phi/\phi_0=1/32$ and $w_\phi/\phi_0=1/64$ (Fig. 4). This is therefore a general feature of the conductance along long wires in spatially correlated magnetic fields. In both cases, the crossover is likely to take place when the correlation length η_ϕ is of the order of the magnetic length l .

It is also to be mentioned in Fig. 2 that the CMIT for small correlation length η_ϕ does not necessarily occur at every bulk Landau level. In this case, the conductance does not vanish at higher Landau levels even in the absence of correlation (see also Fig. 5). This indicates that in the present case, the mixing of the edge states and the bulk states is not strong enough to reduce the conductance to be zero. Note that in the case of the potential disorder, the conductance vanishes at the centers of the bulk Landau levels when the correlation is weak.^{15,16,21} In Fig. 4, it is also confirmed that the signature of the CMIT survives for larger correlation length η_ϕ than in the case of Fig. 2, because the magnetic length is larger.

In order to clarify the origin of the present increase in the plateau width in the presence of the correlation of the random components of the magnetic fields, we investigate the positions of the critical energy more carefully. We find that in the absence of correlation, the critical energies coincide with

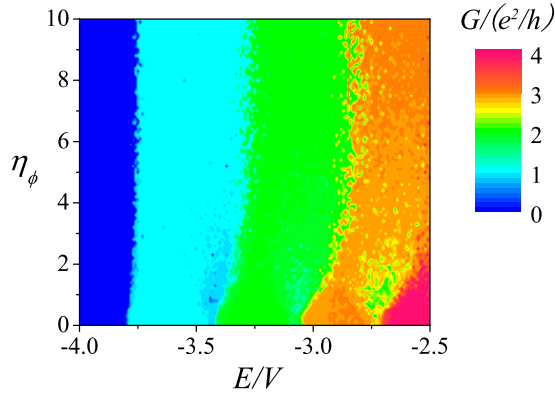


FIG. 4. (Color online) Conductance for $\phi/\phi_0=1/32$, $w_\phi/\phi_0=1/64$, and $L/M=50$ as a function of the correlation η_ϕ and the Fermi energy E/V . The distance between the plateau transitions is increased in the presence of correlation. Note that the plotted energy range is different from Figs. 2 and 3. The magnetic length for ϕ is $l \approx 2.3$. Independent sample is assumed for each set of parameters E/V and η_ϕ .

the centers of the bulk Landau subbands. The plateau width is therefore determined by the mean value ϕ of the disordered magnetic fields. On the other hand, in the presence of correlation, it is likely that the plateau width becomes larger and is determined by a stronger field $\phi+w_\phi/2$, which is effectively the maximum of the distribution of disordered magnetic fields.

In Fig. 5, the conductances for $\eta_\phi=5$ and for $\eta_\phi=0$ are shown. The parameters are assumed to be the same as those in the case of Fig. 2, namely, $L/M=50$, $\phi/\phi_0=1/16$, and $w_\phi/\phi_0=1/32$. Note that the correlation length $\eta_\phi=5$ is larger than the magnetic length for the mean value ϕ of the magnetic field. The vertical lines represent the positions of the critical energies for $w_\phi=0$ and $\phi/\phi_0=1/13$. This value of the magnetic field is effectively the maximum value realized

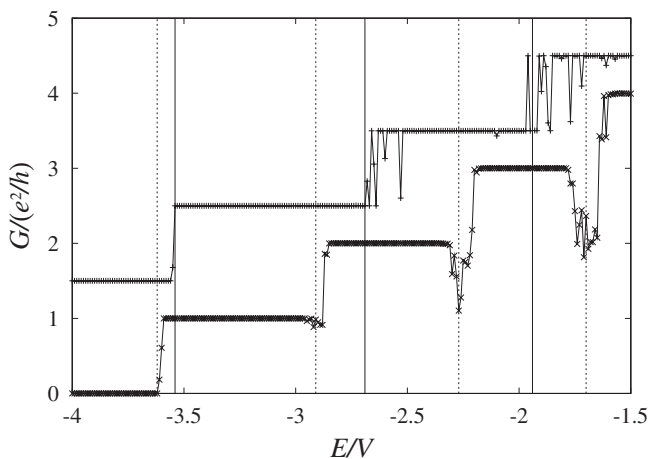


FIG. 5. Conductance for $\eta_\phi=0$ (\times) and $\eta_\phi=5$ ($+$) in the case of $L/M=50$, $\phi/\phi_0=1/16$, and $w_\phi=1/32$ as a function of the Fermi energy E/V . For $\eta_\phi=5$ ($+$), $G/(e^2/h)+1.5$ is plotted instead of $G/(e^2/h)$. The vertical lines and dotted lines represent the positions of the critical energies in the absence of disorder ($w_\phi=0$) for $\phi/\phi_0=1/13$ and $\phi/\phi_0=1/16$, respectively. Independent sample is assumed for each value of E/V .

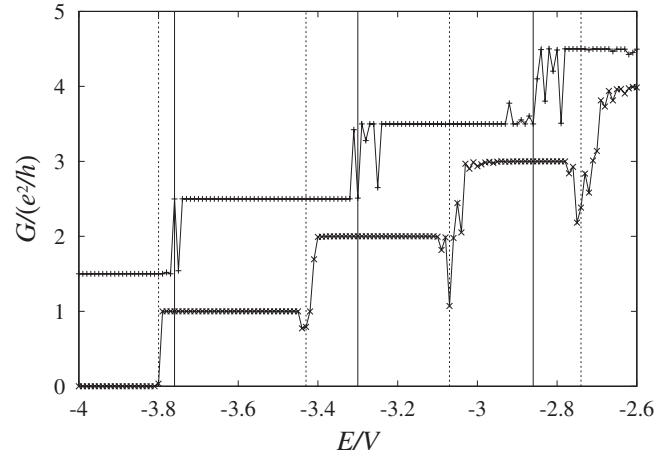


FIG. 6. Conductance for $\eta_\phi=0$ (\times) and $\eta_\phi=5$ ($+$) in the case of $L/M=50$, $\phi/\phi_0=1/32$, and $w_\phi/\phi_0=1/64$ as a function of the Fermi energy E/V . For $\eta_\phi=5$ ($+$), $G/(e^2/h)+1.5$ is plotted instead of $G/(e^2/h)$. The vertical lines and dotted lines represent the positions of the critical energies in the absence of disorder ($w_\phi=0$) for $\phi/\phi_0=1/32+1/128$ and $\phi/\phi_0=1/32$, respectively. Independent sample is assumed for each value of E/V .

in the present system since $1/13 \approx (\phi+w_\phi/2)/\phi_0=1/16+1/64$. The vertical dotted lines are those for $w_\phi=0$ and $\phi/\phi_0=1/16$, equivalent to the positions of the bulk Landau subbands. The results clearly suggest that the critical energy in the presence of correlation ($\eta_\phi=5$) is in good agreement with the Landau-level position for the case of $w_\phi=0$ and $\phi/\phi_0=1/13$. It is to be emphasized that when the correlation is switched off ($\eta_\phi=0$), the critical energies coincide with the Landau-level positions for the mean value of the fields $\phi/\phi_0=1/16$ instead of $\phi/\phi_0=1/13$, even in the case of $w_\phi \neq 0$. These facts mean that the conductance-plateau transitions of a long wire in the presence of correlation is effectively determined by the stronger field $\phi+w_\phi/2$.

We have carried out the same analysis also for a different strength of the magnetic fields $\phi/\phi_0=1/32$ and $w_\phi/\phi_0=1/64$. The results are shown in Fig. 6. Here we find again that the critical energies in the presence of correlation $\eta_\phi=5$ agree with the positions of the Landau levels in the effective magnetic field $\phi/\phi_0=1/32+1/128$. On the other hand, for the uncorrelated case ($\eta_\phi=0$), the critical energies are identical to the Landau-level positions for the mean value $\phi/\phi_0=1/32$. All these results support the conclusion that for the case of the correlated magnetic fields, the critical energies of the conductance-plateau transition in long wires are determined by the effective field $\phi+w_\phi/2$, which is of the order of the maximum value of the disordered magnetic fields in the system. The plateau width therefore becomes larger accordingly.

IV. CONDUCTANCE OF INDIVIDUAL SAMPLES

In Figs. 5 and 6, the fluctuation of the conductance around the critical energies is seen, especially at higher energies. To see the origin of this fluctuation of the conductance, we have calculated the conductance for various energies keeping the

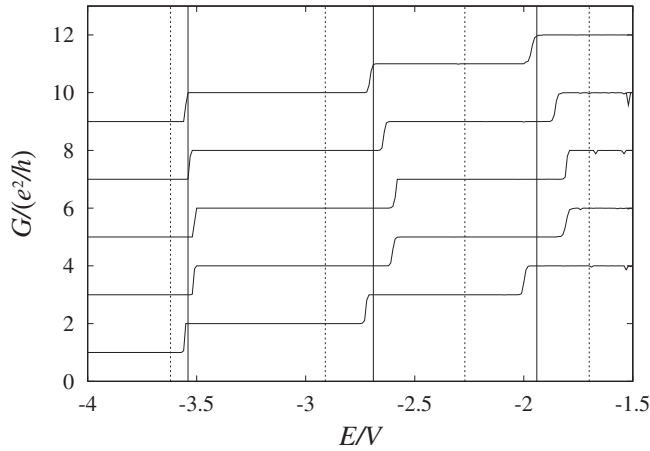


FIG. 7. Conductance for five samples shown with different offsets. The conductance in the region $E/V < -3.8$ is zero for all samples. The parameters are taken to be $\eta_\phi=5$, $L/M=50$, $\phi/\phi_0=1/16$, and $w_\phi/\phi_0=1/32$. The vertical lines and dotted lines represent the positions of the critical energies in the absence of disorder ($w_\phi=0$) for $\phi/\phi_0=1/13$ and $\phi/\phi_0=1/16$, respectively.

magnetic-field configuration fixed. The results for five samples of different random magnetic-field configurations are shown in Fig. 7. Here it is clearly seen that the fluctuation of the conductance seen in Fig. 5 is due to the sample dependence of the critical energies, which originates from the plateau width dependence on the sample. The critical energies therefore depend on the sample particularly at higher steps. This plateau width dependence is naturally understood as a consequence of the sample dependence of the effective field which determines the critical energies of the system. As we have shown in Sec. III, the effective field is of the order of the maximum value of the disordered magnetic field in the system. The critical energy E_c would be determined by the effective field as

$$E_c = (n + 1/2)\hbar\omega_{\text{eff}}, \quad (7)$$

where ω_{eff} denotes the cyclotron frequency corresponding to the effective magnetic field. The fluctuation of the effective field is equivalent to the fluctuation of ω_{eff} , which is independent of the energy. The fluctuation of E_c is therefore larger for larger n . It is to be noted that this plateau width dependence on the sample has not been observed for the case of the correlated potential.²²

Now we discuss what determines the effective field of the sample which governs the conductance-plateau transitions of a long wire in correlated disordered magnetic fields. Let us consider the case of the second sample from the bottom in Fig. 7. We show in Fig. 8 the magnetic-field landscape of the important part of the sample. The length L' of this part is only $L'=30$. Remarkably, this small part effectively determines the transport of the whole system ($L=1000$). We evaluate the conductance of this small part separately and compare it with the conductance of the whole system (Fig. 9). In spite of the fact that the length of this small part is only 3% of the whole system, the conductance steps are almost reproduced. It is therefore clear that this small part is crucial

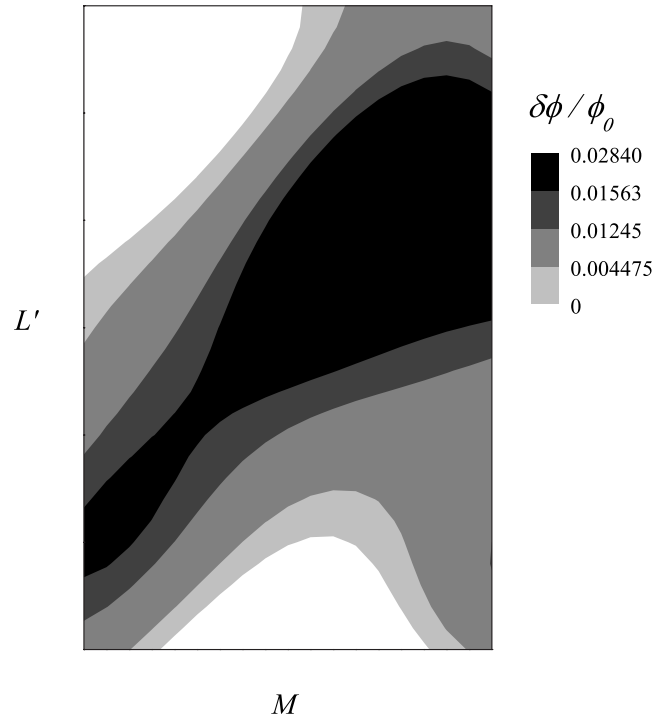


FIG. 8. The magnetic-field landscape of a sample (the second from the bottom of Fig. 7) with $\phi/\phi_0=1/16$, $w_\phi/\phi_0=1/32$, and $\eta_\phi=5$. The part ($L'=30$), which is crucial to the conductance of the whole system, is shown. The horizontal (vertical) direction is across (along) the wire. The region where the magnetic field is stronger than its average is shown by the gray scale. In particular, the magnetic fluxes larger than $\phi+w_\phi/2$ are presented by the black region.

to the conductance of the whole wire and that the effective field, which governs the conductance-plateau transition, is determined by the magnetic-field strength of this part.

It is important to note that in this part, a region where the magnetic field is stronger than the mean lies across the wire (Fig. 8). In such a region, the number of edge states is less

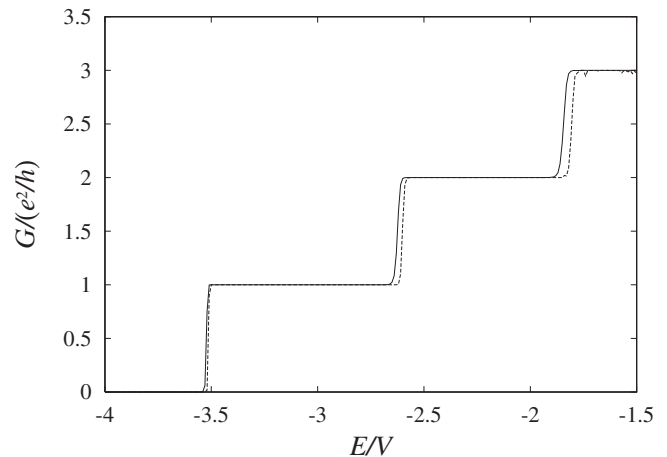


FIG. 9. Conductance of the small part presented in Fig. 8 (solid curve). The conductance of the whole system is also presented by the dashed curve for comparison. The excellent coincidence between the solid and the dashed curves clearly proves that this small part effectively determines the conductance of the whole system.

than the average. It is therefore natural to expect that such bridge structure of a stronger magnetic-field region acts as an effective barrier against the transport along the wire and the conductance is determined by the number of edge states there. If the correlation is absent, the typical size of the region with stronger magnetic field is of the order of the lattice constant and the probability to have such a region across the wire is exponentially small as has been discussed in our previous paper.²¹ For the case of short systems, it is obvious that the probability is also small. In fact, this bridge structure of a stronger magnetic-field region appears only once or twice in the sample with $L/M=50$ in the case of $\eta_\phi=5$. This argument naturally explains why the increase in the plateau width occurs only in a long wire with a large correlation length η_ϕ . The probability to have such a region across the wire is essentially the same as that for the potential barrier discussed in our previous paper.²¹

V. SUMMARY AND CONCLUDING REMARKS

We have investigated numerically the conductance of the quantum Hall wires with correlated magnetic fields and examined how the effect of correlation shows up in the conductance-plateau transition. It has been clearly demonstrated that in the presence of correlation $\eta_\phi > l$ the plateau width becomes larger than that for the uncorrelated case. It has been shown that the larger plateau width in the presence of correlation is determined by the effective field, which is of the order of the maximum value of the magnetic field in the system. In particular, we have shown that the bridge structure of the high magnetic-field region across the wire essentially determines the effective field and therefore governs the conductance-plateau transition of long quantum Hall wires. The change in the critical energy of the plateau transition is qualitatively different from that for the correlated potential case, where the plateau width is unchanged by the potential correlation.²¹ In spite of this apparent difference, the change in the plateau width can also be understood semiclassically by the fact that the number of edge states for a given energy at the bridge structure is smallest and therefore effectively determines the conductance of the whole system.

In the present paper, we have focused on the plateau transitions in the case of the correlated magnetic field, which is more relevant for actual experiments than the uncorrelated case. Before concluding this paper, we make a remark about the CMIT in the case of the uncorrelated magnetic field. In Fig. 10, the conductances in the uncorrelated magnetic field are shown for the cases of $w_\phi/\phi_0=1/16$ and of w_ϕ/ϕ_0

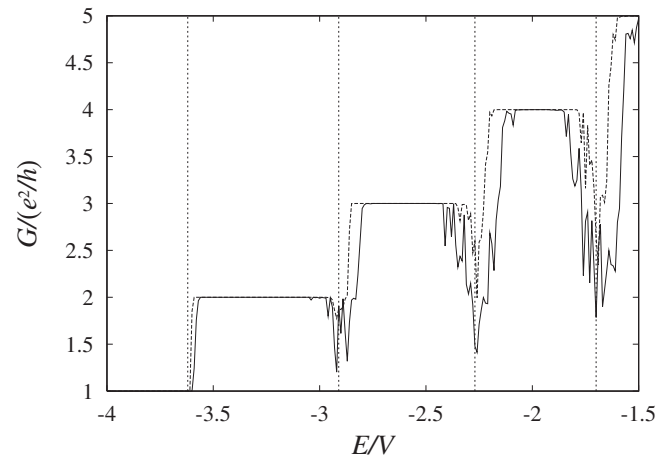


FIG. 10. Conductance of the quantum Hall wires with uncorrelated random magnetic fields $\eta_\phi=0$ as a function of the Fermi energy E/V . The length of the system is $L/M=50$ and the uniform component of the magnetic field is $\phi/\phi_0=1/16$. The solid curves and the dashed curves represent the cases for $w_\phi/\phi_0=1/16$ and for $w_\phi/\phi_0=1/32$, respectively. The vertical dotted lines indicate the bulk Landau-level positions in the absence of disorder ($w_\phi=0$) for $\phi/\phi_0=1/16$. The precursor of the CMIT can be seen around these energies.

$=1/32$. The uniform component ϕ/ϕ_0 is $1/16$ in both cases. For the case of the larger fluctuation $w_\phi/\phi_0=1/16$, the precursor of the CMIT can be seen. It is therefore likely that the fluctuation larger than the mean is necessary to obtain the clear insulating region. On the other hand, the clear conductance plateaus would vanish when the fluctuation is larger than the mean. We have therefore confined ourselves to the case where the fluctuation w_ϕ of the magnetic field is smaller than its mean ϕ to discuss the plateau transitions. It would be worth pointing out that in the previous numerical studies of the transport properties in the two-dimensional disordered magnetic fields,^{30,31} the scattering by the random magnetic fields has been shown to be weak and quantitatively different from that by the random potential.

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