Nonlinear Verdet law in magnetophotonic crystals: Interrelation between Faraday and Borrmann effects

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The magneto-optical Faraday effect is studied in one-dimensional magnetophotonic crystals (MPCs). Mechanisms of a strong enhancement of the Faraday rotation at the edges of the photonic band gap are considered. High difference of refractive indexes of bismuth-substituted yttrium iron garnet (Bi:YIG) and SiO_2 layers provides a strong spatial localization of the optical field in Bi:YIG layers, which leads to manifold Faraday rotation enhancement at the photonic band edges. The Faraday rotation angle in the finite MPCs appears to be a nonlinear function of the total thickness of magnetic material in the stack that can be interpreted as the nonlinear Verdet law. Relation between the enhancement of the Faraday rotation and localization of optical field in magnetic layers is treated as a Borrmann-type effect. This relation shows that the Faraday rotation can be considered as a measure of the density of photonic states trapped within Bi:YIG layers.

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Magneto-optical Faraday rotation in homogeneous magnetic materials scales linearly with the magnetic material thickness in accordance with Verdet law. However, it is not supposed to be the case for magnetic composed multilayered structures with photonic band gap (PBG): magnetophotonic crystals (MPCs). Multiple reflection interference is expected to influence sufficiently on the Faraday rotation in MPCs. The reason for this might be the multipass light propagation in nonreciprocal layers of these composites and modification of dispersion relations for the eigenmodes in multilayered gyrotropic materials which results in the difference of phase velocities for left- and right-circular polarized optical waves.

Apart from modification of dispersion relations, multiple reflection interference significantly modifies the spatial distribution of the optical field in MPCs. The crucial role of the distribution of the electromagnetic field across the crystalline unit cell is known as the Borrmann effect:[1] anomalous x-ray absorption of crystals for Laue diffraction due to interatomic localization of electromagnetic field in the unit cell at certain scattering conditions. The Borrmann effect was interpreted by means of the standing waves that are formed by the interference of the incident and reflected waves. The nodes and antinodes of these standing waves have the periodicity of the lattice planes. For the first type of the wave fields, which undergo a quite small absorption, the nodes lie on the lattice planes. For the second type of the wave fields, which undergo a larger absorption, the antinodes lie on the lattice planes. Rather similar Borrmann-type effects can be observed in photonic crystals[2] for Bragg diffraction: concentration of the field of optical Bloch eigenmodes in the layers with field-dependent properties should manifest itself in the enhancement of the corresponding nonlinear effects. Borrmann-type enhancement of nonlinear optical self-focusing in MPCs was observed in Ref. 3.

The Faraday rotation is independent from the magnitude of the optical field and consequently, at first glance, is not strongly influenced by the spatial distribution of the optical field in the MPC unit cell. Meanwhile, recent experimental studies of the Faraday effect in one-dimensional (1D) magnetophotonic microcavities[4] show that a strong enhancement of the Faraday rotation in these structures with a single magnetic layer sandwiched in between dielectric (nonmagnetic) Bragg reflectors correlates with the concentration of the optical field in the magnetic microcavity layer.

These results open the question of the role of the Borrmann-type mechanism of spatial distribution of optical field across the unit cell in the enhancement of the Faraday effect in multilayered MPCs. Such MPCs consist of a stack of magnetic layers alternated with nonmagnetic layers.[5–9]

In this Brief Report magneto-optical Faraday effect is considered in Bi:YIG-SiO_2 one-dimensional magnetophotonic crystals. The enhancement of the Faraday rotation at the photonic band edges, the nonlinear thickness dependence of this effect, and the role of the Borrmann-type mechanism in both magneto-optical enhancement and the nonlinear thickness dependence are in the focus of these studies.

Two types of experimental 1D MPCs of varied thickness are the stacks of 5.5 and 3.5 pairs of quarter-wavelength-thick ($\lambda_0/4$) layers of amorphous silicon dioxide alternated by either $\lambda_0/4$ or $3\lambda_0/4$ thick layers of microcrystalline bismuth-substituted yttrium iron garnet (Bi:YIG) $Bi_{1.0}Y_{2.0}Fe_5O_x$. The fractional number of pairs relates to the fact that Bi:YIG layers are the bottom and cap layers in both structures. Hereafter λ_0 denotes the central wavelength of the lowermost-energy photonic band gap. The fabrication procedure[7] consists of successive rf sputtering of SiO_2 and Bi:YIG in the Ar^+ atmosphere onto 2.5 mm thick fused-quartz substrate. After the evaporation of each of the successive garnet layers, the MPC is removed from the sputtering machine and annealed in air at 700 °C for 20 min. The hysteresis loop for the polar configuration gives the coercivity of the MPC in polar configuration to be approximately 50 Oe and saturating magnetic field close to 1.5 kOe.

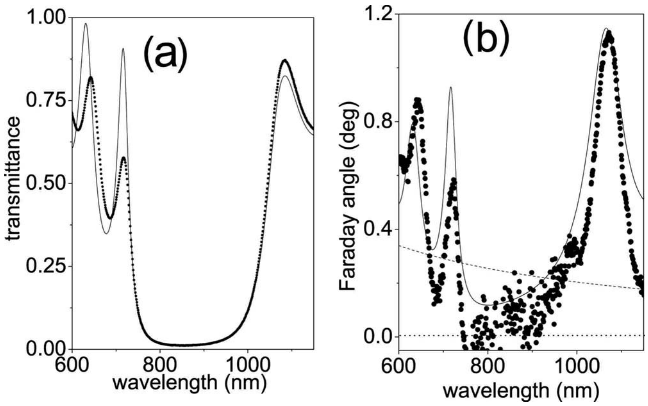

FIG. 1. (a) The experimental spectra (circles) of the transmission and (b) the Faraday rotation angle θ_{pc} of the 5.5 pair MPC measured at the normal incidence and corresponding numerical approximation by the 4×4 matrix method (lines). Dashed line is the spectrum of the Faraday rotation angle θ_0 in homogeneous Bi:YIG slab of the same thickness as the total thickness of Bi:YIG in MPC.

Optical and magneto-optical spectra of the 5.5 pair MPC are shown in Figs. 1(a) and 1(b). The spectral region from 725 to 1025 nm, where the transmission is strongly suppressed, corresponds to the photonic band gap of the MPC. The Faraday rotation angle for the MPC θ_{pc} also decreases in this spectral region. Outside the PBG transmission is increased and shows interference fringes. The Faraday rotation angle also oscillates with the spectrum having the local maxima at 640 and 720 nm correlating well with local transmission maxima. The largest enhancement of θ_{pc} is observed near 1070 nm which coincides with the long-wavelength band edge. The spectrum of the Faraday rotation angle θ_{pc} for a uniform slab of Bi:YIG with the thickness of 520 nm corresponding to the total Bi:YIG thickness in the MPC is calculated using the known spectra of diagonal and off-diagonal components of the permittivity tensor of Bi:YIG with the same Bi concentration.[10] The enhancement of θ_{pc} at the long-wavelength PBG edge attains the factor of 6.5 ± 0.2 in comparison with the homogeneous Bi:YIG slab, while the increases at the short-wavelength band edge appear to be 1.9 ± 0.1.

Comparison of the enhanced Faraday rotation at the long-wavelength band edge for the samples consisting of 5.5 pairs (MPC5) and 3.5 pairs (MPC3) exhibits strong nonlinear dependence on N, the number of pairs in the structure. The Faraday rotation angle normalized on the total thickness of Bi:YIG in the sample is 2.17 and 0.74 grad/μm for MPC5 and MPC3, respectively. This implies that the ratio of θ_{pc} is approximately 2.93 whereas the ratio of the number of pairs is approximately 1.57, which is stronger than quadratic dependence on N.

All calculations for light propagation in layered structures—dispersion relations, optical-field distribution, optical and magneto-optical spectra, etc.—can be performed by the four-by-four matrix technique.[4] The particular applications of the 4×4 matrix formalism for 1D MPCs are discussed in detail in Refs. 11 and 12.

Both experimental spectra in Fig. 1 are fitted simultaneously using the aforementioned 4×4 matrix technique

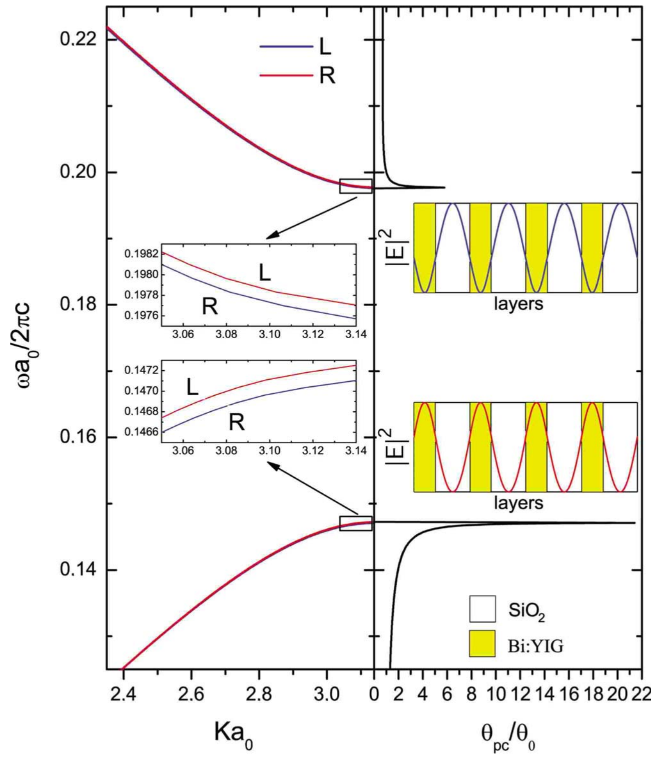

FIG. 2. (Color online) Band structures of Bi:YIG-SiO$_2$ 1D MPC. Red (light gray) lines and blue (dark gray) lines in left-hand panel are dispersion relations for RCP for LCP Bloch modes, respectively. $\mathbf{K}$ is the wave vector of eigenmodes in MPCs; a_0 is the period of the MPC layered structure. Insets in the left-hand panel show magnified behavior of dispersion relations at the edge of the Brillouin zone. Right-hand panel shows corresponding spectra of θ_{pc} normalized on θ_0. Insets in the right-hand panel schematically show spatial distribution of optical field in MPCs at the long- and short-wavelength band edges.

with known spectral dependences of refractive indices and absorption coefficients for Bi:YIG and SiO$_2$. Results of calculations are in good agreement with experimental data.

The Faraday effect in a homogeneous medium originates from the different phase velocities for right- and left-circular polarized (RCP and LCP) waves. The phase difference between eigenmodes results in the polarization rotation at the angle θ_F. Taking into consideration multiple reflection interference this approach can be extended to the Faraday effect in multilayered photonic structures. As this follows from calculations in Refs. 13 and 14 the spectral behavior of θ_{pc} in the vicinity of the band edges can be formerly described by the split of dispersion relations for circular polarized eigenmodes at the edge of the Brillouin zone. Figure 2 shows the dispersion relations $\omega(\mathbf{K}_L)$ and $\omega(\mathbf{K}_R)$ and corresponding difference of wave vectors $\Delta\mathbf{K}=\mathbf{K}_L-\mathbf{K}_R$ for RCP and LCP Bloch eigenmodes for two model MPC structures. The model MPC structures are formed from magnetic, Bi:YIG, and nonmagnetic, SiO$_2$, quarter-wavelength-thick layers. The most difference in dispersion relations for eigenmodes and as a consequence for the Faraday rotation is achieved at the edge of the Brillouin zone which implies that corresponding spectral features of the Faraday effect appear at the photonic band edges. In concordance with our experimental spectrum of θ_{pc} in Fig. 1 a strong Faraday effect is expected at the long-wavelength band edge (Fig. 2).

Meanwhile, in spite of the fact that the enhancement of the Faraday rotation is a clear consequence of the split of the dispersion relations the mechanism of the split itself is not straightforward. One of the mechanisms of the enhanced contribution from the off-diagonal elements of the dielectric constant to the Faraday rotation relates to the spatial distribution of optical fields in a MPC structure. This relation between the Faraday rotation and distribution of optical fields was already mentioned for magnetophotonic microcavities[4] and could be considered as the Borrmann-type effect assisted by multipass light propagation between two Bragg reflectors in the Fabry-Perot etalon. In the case of MPCs, the dispersion curves in Fig. 2 for RCP and LCP waves flatten at the band edge which leads to decreasing the group velocity and consequently to increasing the standing-wave amplitudes. It can be considered as optical-field localization or photonic trap in different layers of a photonic crystal due to the Bloch structure of optical eigenmodes in periodic systems.

Insets in the left panel of Fig. 2 show a schematic view of the spatial distribution of local optical fields in the model MPCs. For $n_m > n_{nm}$ the optical field is concentrated in magnetic layers for the wavelengths corresponding to the long-wavelength PBG edge. Note that for $n_m < n_{nm}$ the optical field would be localized in the nonmagnetic dielectric for the same spectral range.[15] Further detailed calculations show that spectroscopic and thickness-dependent features of the Faraday effect at the photonic band edges in MPCs are related to the optical-field distribution in the unit cell and can be considered in terms of the Borrmann effect and density of photonic states.

Figure 3 shows results of the calculation for both band edges of the fine structure of the spatial distribution of optical field $|E(x)|^2$ and corresponding coordinate dependence of the Faraday rotation angle $\theta_{pc}(x)$ in the high refractive index layer of the model MPC unit cell. In these model calculations the ultrathin magnetic sublayer (MSL) scans across the nonmagnetic high refractive index layer (HIL) of the unit cell of the infinite MPC. The refractive index of the MSL is supposed to be the same as the high refractive index of the HIL. Self-consistent calculations are performed for field distribution $|E(x)|^2$ and the Faraday rotation from the total structure as a function of MSL position x in the unit cell. The insets of Fig. 3(a) show that $\theta_{pc}(x)$ dependence ideally follows $|E(x)|^2$ dependence. It therefore appears that Faraday rotation in this numerical experiment is proportional to the local density of photonic states, space resolved density of states, at the point within the MPC unit cell determined by the MSL position. This relation between the Faraday rotation and optical-field distribution in the MPC unit cell is an analog of the Borrmann effect for photonic crystals.

The Borrmann-type mechanism can be responsible for some specific properties of real MPCs. One of these is the thickness dependence of the Faraday rotation in finite MPCs. In homogeneous magneto-optical materials the Faradayrotation scales linearly with the thickness in accordance with the Verdet law $\theta_F(l)=\theta_V l$, where θ_V is the Verdet constant and l is the thickness of the homogeneous magnetic layer. Thickness dependence of the Faraday rotation and optical field in magnetic layers of the finite Bi:YIG MPC is calculated by the 4×4 matrix method at the long-wavelength band edge. Figure 4 shows very similar nonlinear dependences of θ_{pc} and $\rho=\int_M |E(x)|^2 dx$, the local density of states integrated over the magnetic layers, as a function of the number of periods N. For $N<30$ $\theta_{pc}(N)$ shows nearly cubic power dependence on N. Initial increase in the number of layers in the MPC leads to the better field localization in magnetic layers of the MPC combined with the increase of the total thickness of magnetic material. Such combination results in strong nonlinear thickness dependence of $\theta_{pc}(N)$ as it appears from comparison of experimental values of θ_{pc} for 5.5 and 3.5 pair thick MPCs. Calculated thickness dependence of $\theta_{pc}(N)$ transforms for $N>100$ to the asymptotic linear regime which corresponds to the saturation of optical-field localization in magnetic layers.

Direct relation between the nonlinear Verdet law and Borrmann effect in finite MPCs is elucidated again by the linear dependence of θ_{pc} on ρ shown in the inset of Fig. 4. This linear correlation shows that even for finite structures, the Faraday rotation can be considered as a time average number of photonic states localized within the magnetic subcomponent of the MPC. It is worth noting that a somewhat

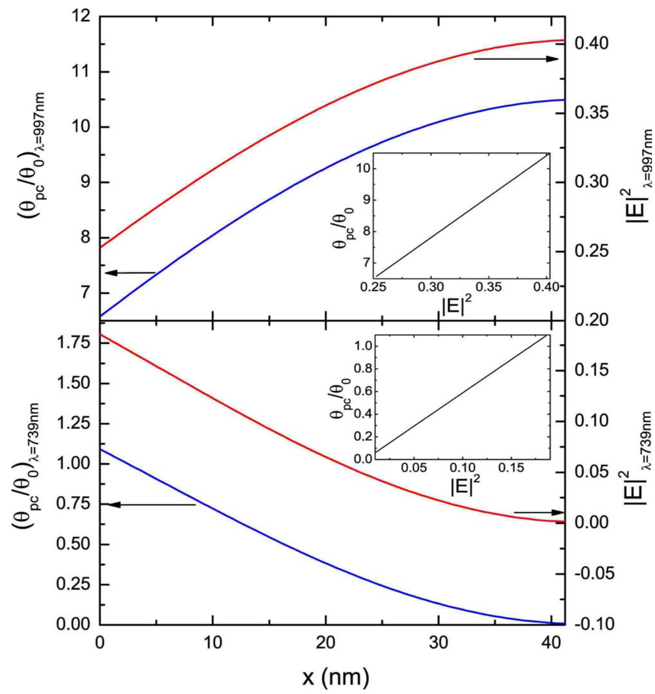

FIG. 3. (Color online) Red (light gray) lines show spatial distribution of the normalized optical field $|E(x)|^2$ and blue (dark gray) lines show coordinate dependence of the Faraday rotation angle $\theta_{pc}(x)$ for the long-wavelength (top panel) and short-wavelength (bottom panel) PBG edges. Calculations are performed for the model infinite MPC when ultrathin magnetic sublayer scanning throw the nonmagnetic high refractive index layer. Insets show corresponding linear dependences of θ_{pc} on $|E|^2$.

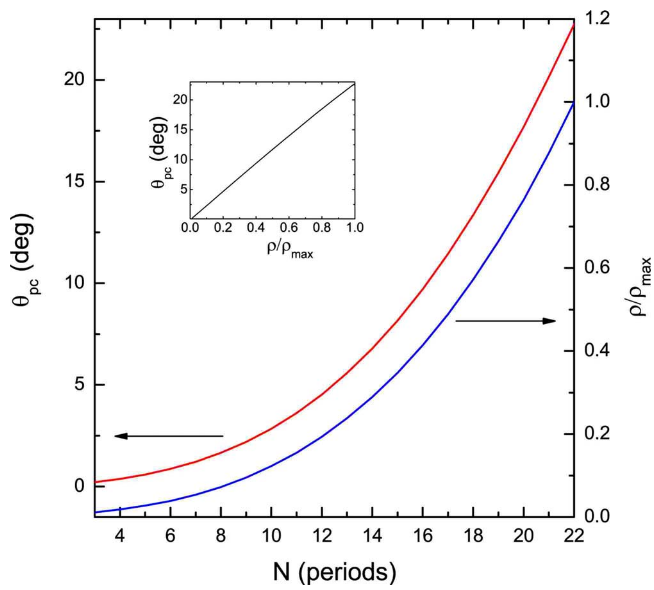

FIG. 4. (Color online) Size dependence of the normalized optical field $|E(N)|^2$ is shown by blue (dark gray) line and the Faraday rotation angle $\theta_{pc}(N)$ is shown by red (light gray) line for the long-wavelength band edge of finite MPCs. Inset shows corresponding linear dependence of θ_{pc} on $|E|^2$.

similar role is prescribed to the Faraday rotation in the case of homogeneous magnetic media. As it was shown recently in Ref. 16 the Faraday rotation can be considered as a measure of time that photons spend within the magnetic material. Figuratively speaking, in our case of MPCs the strong confinement of the electromagnetic field increases the time which photons spend inside magnetic layers giving rise to the increase of both the number of the trapped photons and Faraday rotation.

In conclusion, multiple reflection interference in 1D MPCs results in a perfect correlation of the spatial localization of the optical field in magnetic layers and the enhancement of the Faraday effect. This allows the interpretation of the enhancement mechanism in terms of the Borrmann-type effect. The spatial localization of the optical field in Bi:YIG layers gives rise to the increase of the difference in the phase velocities for right- and left-circularly polarized waves and is shown to be responsible for nonlinear thickness dependence of the Faraday rotation in 1D MPCs. The nonlinear Verdet law in finite MPCs opens up a way for the miniaturization of magneto-optical devices utilizing photonic band-edge effects. Moreover the interrelation between the Faraday rotation and density of photonic states can serve as a tool for various applications in photonics. Density of photonic states can be probed via the Faraday effect for the variety of photonic structures constituting from gyrotropic, magnetic, and nonmagnetic materials.

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