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Thermally activated phase slips in superconducting nanowires

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We reanalyze the problem of thermally activated phase slips (TAPS) which can dominate the behavior of sufficiently thin superconducting wires at temperatures close to T_C. With the aid of an effective action approach we evaluate the TAPS rate which turns out to exceed the rate found by McCumber and Halperin, Phys. Rev. B **1**, 1054 (1970) within the time-dependent Ginzburg-Landau analysis by the factor $\sim(1-T/T_C)^{-1}\gg 1$. Additional differences in the results of these two approaches arise at bias currents close to the Ginzburg-Landau critical current where the TAPS rate becomes bigger. We also derive a simple formula for the voltage noise across the superconducting wire in terms of the TAPS rate. Our results can be verified in modern experiments with superconducting nanowires.

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I. INTRODUCTION

Fluctuations are known to play an important role in superconducting structures with reduced dimensions. In the case of superconducting nanowires,[1] fluctuations may essentially determine the system behavior in a wide temperature interval causing, e.g., nonvanishing wire resistance down to $T=0$.

Over four decades ago it was realized by Little[2] that sufficiently thin superconducting wires may acquire a nonzero resistance below the BCS critical temperature of the bulk material T_C due to nontrivial thermal fluctuations of the order parameter $\Delta=|\Delta|\exp(i\varphi)$. Such fluctuations result in a temporary local destruction of $|\Delta|$ accompanied by the phase slippage in the corresponding points of the wire. According to the Josephson relation $V=\dot{\varphi}/2e$, this process must cause a nonzero voltage drop across the superconducting sample, thus bringing it into a resistive state.

Quantitative theory of these thermally activated phase slips (TAPSs) was worked out by Langer and Ambegaokar,[3] and by McCumber and Halperin (MH).[4] This Langer, Ambegaokar, McCumber, Halperin (LAMH) theory predicts that in a superconducting wire TAPSs are created with the rates $\Gamma_\pm$, which are defined by the activation dependence

$$\Gamma_\pm = B_\pm e^{-\delta F_\pm/T}. \tag{1}$$

Here $\delta F_\pm$ are effective free-energy barriers which the system should overcome in order to create a phase slip corresponding to the overall phase change $\pm 2\pi$. These potential barriers are essentially controlled by the superconducting condensation energy for the volume of the TAPS core where the order parameter $|\Delta|$ gets destroyed by thermal fluctuations. In the absence of any external bias one naturally has $\Gamma_+=\Gamma_-$ and, hence, no net voltage across the sample can occur. Applying an external current I, one lifts the symmetry between "positive" and "negative" TAPSs. As a result, there appears a voltage drop [and, hence, nonzero resistance $R(T)$] proportional to the difference between the two TAPS rates $\Gamma_+-\Gamma_-$.

According to Eq. (1) TAPS remain significant only at temperatures close to T_C while $R(T)$ decreases exponentially as T is lowered well below the critical temperature. This prediction was fully confirmed in experiments[5,6] where the activation behavior of $R(T)$ was detected in small superconducting whiskers with diameters ~ 0.5 μm. Later it was realized that, in thinner wires, not only thermal but also quantum fluctuations of the order parameter (quantum phase slips) become important. It was demonstrated both theoretically[7,8] and experimentally[9–15] that quantum phase slip effects can yield appreciable resistivity of superconducting wires with diameters in the range of ~ 10 nm even well below T_C. For more details we refer the reader to the review.[1]

Turning again to thermal fluctuations near T_C, we note that, while for evaluation of the free-energy barriers for TAPS $\delta F_\pm$ in Eq. (1) it suffices just to solve the standard Ginzburg-Landau (GL) equations, the problem of finding the preexponent $B_\pm$ is in general much more involved, as it requires employing the formalism which properly accounts for *dynamical effects* in superconductors. MH (Ref. 4) treated this problem within the formalism of the so-called time-dependent Ginzburg-Landau (TDGL) equations[16] which was available at that time. Unfortunately, this formalism is known to suffer from serious drawbacks (see, e.g., Refs. 1, 17, and 18 for further discussion) and it is in general hardly applicable below T_C. Thus, although MH calculation[4] of the preexponent $B_\pm$ was correct and sound by itself, their final result needs to be reanalyzed on the basis of a more solid theoretical approach. This task will be accomplished below.

The structure of the paper is as follows. In Sec. II we briefly recapitulate the microscopic effective action formalism[7,8,17] and employ it in order to estimate the fluctuation correction to the order parameter of ultrathin superconducting nanowires. In Sec. III we apply this formalism in order to evaluate the preexponent $B_\pm$ in the expression for the TAPS rates [Eq. (1)]. We will then perform a detailed comparison between our result and that of Ref. 4. In addition, we will present a simple formula which expresses the voltage noise in superconducting nanowires via the TAPS rates [Eq. (1)] evaluated here. Some technical details of our calculation of fluctuation determinants are relegated to the Appendix.

II. EFFECTIVE ACTION AND GAUSSIAN FLUCTUATIONS

Consider a uniform superconducting wire with cross section s and length X. In order to account for superconducting fluctuations in such a wire we will use the effective action approach developed in Refs. 8 and 17. Our starting point is the path-integral representation of the grand partition function,

$$\mathcal{Z} = \int \mathcal{D}\Delta \mathcal{D}V\mathcal{D}\boldsymbol{A} \;\; e^{-S_{\rm eff}}, \tag{2}$$

where $S_{\rm eff}[\Delta, V, \boldsymbol{A}]$ is the imaginary time version of the effective action for a superconducting wire. The fluctuating order-parameter field Δ as well as the scalar and vector potentials V and $\boldsymbol{A}$ depend on coordinate x along the wire (i.e., $-X/2 \le x \le X/2$) and imaginary time τ restricted to the interval $0 \le \tau \le 1/T$. The exact expression for this effective action is obtained by integrating out the electron degrees of freedom and is not easily tractable in a general situation. In order to simplify this general expression for the action $S_{\rm eff}$, one can assume that deviations of the amplitude of the order-parameter field $\Delta(x,\tau)$ from its equilibrium value Δ_0 are relatively small. This assumption allows the expansion of the effective action in powers of $\delta\Delta(x,\tau)=\Delta(x,\tau)-\Delta_0$ and in the electromagnetic fields up to the second-order terms. The next step is to average over disorder. After such averaging the effective action becomes translationally invariant both in space and in time. Performing the Fourier transformation we obtain[8,17]

$$S_{\rm eff} = \frac{s}{2}\int \frac{d\omega dq}{(2\pi)^2}\left\{ \frac{|A|^2}{Ls} + \frac{C|V|^2}{s} + \chi_D \left| qV + \frac{\omega}{c}A\right|^2 + \chi_J \left| V + \frac{i\omega}{2e}\varphi\right|^2 + \frac{\chi_L}{4m^2}\left| iq\varphi + \frac{2e}{c}A\right|^2 + \chi_\Delta |\delta\Delta|^2 \right\}. \tag{3}$$

The functions $\chi_\Delta(\omega,q)$, $\chi_J(\omega,q)$, $\chi_L(\omega,q)$, and $\chi_D(\omega,q)$ are related to the averaged products of the Matsubara Green functions. The corresponding general expressions are established in Refs. 1, 8, and 17. They are rather cumbersome and will not be specified here. In what follows we will use only simplified forms of these functions applicable in certain limits.

As the action $S_{\rm eff}$ [Eq. (3)] is quadratic both in the voltage V and the vector potential A, these variables can be integrated out exactly. After such integration one arrives at the effective action S which only depends on φ and $\delta\Delta$. We get

$$S = \frac{s}{2}\int \frac{d\omega dq}{(2\pi)^2}\{\mathcal{F}(\omega,q)|\varphi|^2 + \chi_\Delta |\delta\Delta|^2\}. \tag{4}$$

Since usually the wire geometric inductance L remains unimportant, in what follows we will disregard this quantity by setting $L=0$. Then we obtain[1,8]

$$\mathcal{F}(\omega,q) = \frac{\left(\frac{\chi_J}{4e^2}\omega^2 + \frac{\chi_L}{4m^2}q^2\right)\left(\frac{C}{s}+\chi_D q^2\right) + \frac{\chi_J\chi_L}{4m^2}q^2}{\frac{C}{s}+\chi_J+\chi_D q^2}. \tag{5}$$

The effective action (4) allows directly evaluating the fluctuation correction to the equilibrium value of the order parameter in superconducting nanowires. Performing Gaussian integration over both φ and $\delta\Delta$, we arrive at the wire free energy

$$F = F_{\rm BCS} - \frac{T}{2}\sum_{\omega,q}\left[\ln\frac{\lambda\mathcal{F}(\omega,q)}{2N_0\Delta_0^2} + \ln\frac{\lambda\chi_\Delta(\omega,q)}{2N_0}\right], \tag{6}$$

where $F_{\rm BCS}$ is the standard BCS free energy and λ is the BCS coupling constant. The order parameter is defined by the saddle-point equation $\partial F/\partial\Delta=0$ and can be written in the form $\Delta=\Delta_0-\delta\Delta_0$, where Δ_0 is the solution of the BCS self-consistency equation $\partial F_{\rm BCS}/\partial\Delta_0=0$ and the fluctuation correction $\delta\Delta_0$ has the form

$$\delta\Delta_0 = -\frac{T}{2}\left(\frac{\partial^2 F_{\rm BCS}}{\partial\Delta_0^2}\right)^{-1}\frac{\partial}{\partial\Delta_0}\sum_{\omega,q}\left[\ln\frac{\lambda\mathcal{F}(\omega,q)}{2N_0\Delta_0^2} + \ln\frac{\lambda\chi_\Delta(\omega,q)}{2N_0}\right]. \tag{7}$$

First let us consider the low-temperature limit $T \ll \Delta_0$. It is useful to note that, at large values of the wave number $|q| \gg \sqrt{\Delta_0/D}$ and/or frequency $|\omega| \gg \Delta_0$, the functions $\mathcal{F}(\omega,q)/\Delta_0^2$ and $\chi_\Delta(\omega,q)$ are weakly affected by superconductivity. Hence, we can restrict the sum in Eq. (7) only to low frequencies $|\omega| \lesssim \Delta_0$ and wave numbers $|q| \lesssim \sqrt{\Delta_0/D}$. It will be convenient for us to introduce dimensionless parameters $y=\omega/\Delta_0$ and $z=q\sqrt{D/\Delta_0}$, and express the kernels as follows:

$$\chi_\Delta = N_0 F_\Delta(y,z), \quad \chi_J = e^2 N_0 F_J(y,z),$$

$$\chi_L = m^2 N_0 D\Delta_0 F_L(y,z), \quad \chi_D = \frac{e^2 N_0 D}{\Delta_0}F_D(y,z), \tag{8}$$

where all the functions F_j are dimensionless. The function $\mathcal{F}(\omega,q)$ acquires the form

$$\mathcal{F}(y,z) = \frac{N_0\Delta_0^2}{4}\left[\frac{(y^2F_J+z^2F_L)(C/s+e^2N_0z^2F_D)}{C/s+e^2N_0F_J+e^2N_0z^2F_D} + \frac{e^2N_0z^2F_JF_L}{C/s+e^2N_0F_J+e^2N_0z^2F_D}\right]. \tag{9}$$

For a wire of length X we obtain $\partial F/\partial\Delta_0=2N_0sX$, and at $T=0$, the correction to Δ_0 reads

$$\delta\Delta_0 \simeq \frac{3}{8}\frac{\Delta_0}{sN_0\sqrt{D\Delta_0}}\int_{-1}^{1}\frac{dydz}{(2\pi)^2}\left[\ln\frac{\mathcal{F}(1,1)}{\mathcal{F}(y,z)} + \ln\frac{F_\Delta(1,1)}{F_\Delta(y,z)}\right]. \tag{10}$$

The integral

$$\int_{-1}^{1}\frac{dydz}{(2\pi)^2}\ln\frac{F_\Delta(y,z)}{F_\Delta(1,1)}$$

is well convergent at small y and z; therefore we can replace it by a constant of order one. The integral

$$\int_{-1}^{1}\frac{dydz}{(2\pi)^2}\ln\frac{\mathcal{F}(y,z)}{\mathcal{F}(1,1)}$$

is only slightly more complicated since $\mathcal{F}(y,z)\to 0$ for $y,z \to 0$. However, since the function $\mathcal{F}(y,z)$ enters only under the logarithm, this integral is convergent as well. Making use of the above expressions for the functions $\mathcal{F}(\omega,q)$ and $\chi_\Delta(\omega,q)$ at $T\to 0$, we obtain

$$\frac{\delta\Delta_0}{\Delta_0} \sim \frac{1}{g_\xi} \sim Gi_{1D}^{3/2}. \qquad (11)$$

Here g_ξ is the dimensionless conductance of the wire segment of length ξ and Gi_{1D} is the Ginzburg number for a superconducting nanowire defined as the value $(T_C-T)/T_C$ at which the fluctuation correction to the wire specific heat becomes equal to the specific-heat jump at the phase-transition point. In the case of quasi-one-dimensional (1D) wires, this number reads[16]

$$Gi_{1D} = \frac{0.15}{(sN_0\sqrt{D\Delta_0})^{2/3}}. \qquad (12)$$

We note that in Eq. (11) fluctuations of both the phase and the absolute value of the order parameter give contributions of the same order. The estimate (11) demonstrates that at low temperatures suppression of the order parameter in superconducting nanowires due to Gaussian fluctuations remains weak as long as $g_\xi \gg 1$ and it becomes important only for extremely thin wires with $Gi_{1D}\sim 1$, in which case the width of the fluctuation region δT is comparable to T_C and the BCS mean-field approach becomes obsolete down to $T=0$.

Turning to higher temperatures we observe that, at T sufficiently close to the critical temperature T_C, it is necessary to retain only the contribution from zero Matsubara frequency. At the same time the terms originating from all nonzero frequencies are small in the parameter $\Delta_0(T)/T_C \ll 1$ and, hence, can be safely omitted. Performing the integration over q, we get

$$\frac{\delta\Delta_0}{\Delta_0(T)} = \frac{T}{\delta F}, \qquad (13)$$

where

$$\Delta_0(T) = \sqrt{\frac{8\pi^2 T(T_C-T)}{7\zeta(3)}}, \quad \zeta(3) \simeq 1.2 \qquad (14)$$

and

$$\delta F = \frac{16\pi^2}{21\zeta(3)} sN_0\sqrt{\pi D}(T_C-T)^{3/2} \qquad (15)$$

turn out to be exactly equal to the magnitude of the effective free-energy barrier for TAPS in the LAMH theory[3,4] in limit of small transport currents (see below). Equations (13) and (15) demonstrate that at temperatures close to T_C Gaussian fluctuations of the superconducting order parameter in thin wires become more significant and effectively wipe out superconductivity at $\delta F \lesssim T_C$, i.e., already in much thicker wires than in the case of low temperatures $T \ll T_C$.

III. THERMALLY ACTIVATED PHASE SLIPS

In what follows let us restrict our attention to superconducting wires in which the condition $\delta F_0 \gg T_C$ is well satisfied and, hence, the effect of Gaussian fluctuations on the order parameter $\Delta_0(T)$ can be safely neglected. This condition requires the wire to be sufficiently thick and/or the temperature should not be too close to T_C, i.e., $(T_C-T)/T_C \gg Gi_{1D}$. At the same time we assume that the temperature is still not far from T_C, i.e., $T_C-T \ll T_C$, in which case the physics is dominated by thermally activated phase slips.[3,4] As we already discussed, sufficiently thin superconducting wires acquire nonzero resistance even below T_C due to TAPS, and this resistance is essentially determined by the TAPS rates [Eq. (1)].

A. Activation exponent

The free-energy barriers $\delta F_\pm$ for TAPS corresponding to overall phase jumps by $\pm 2\pi$ have been evaluated by Langer and Ambegaokar.[3] Here we briefly recapitulate their results. In order to obtain $\delta F_\pm$ entering into Eq. (1), we make use of the standard Ginzburg-Landau free-energy functional for a wire of length X:

$$F[\Delta(x)] = sN_0\int_{-X/2}^{X/2} dx\left(\frac{\pi D}{8T}\left|\frac{\partial\Delta}{\partial x}\right|^2 + \frac{T-T_C}{T_C}|\Delta|^2 + \frac{7\zeta(3)}{16\pi^2T^2}|\Delta|^4\right) - \frac{I}{2e}[\varphi(X/2)-\varphi(-X/2)]. \qquad (16)$$

Here $\varphi(x)$ is the phase of the order parameter $\Delta(x)$ and I is the external current applied to the wire.

The saddle-point paths for this functional are determined by the standard GL equation,

$$-\frac{\pi D}{8T}\frac{\partial^2\Delta}{\partial x^2} + \frac{T-T_C}{T_C}\Delta + \frac{7\zeta(3)}{8\pi^2T^2}|\Delta|^2\Delta = 0. \qquad (17)$$

For any given value of the bias current,

$$I = \frac{\pi e N_0 Ds}{2T}|\Delta|^2 \nabla\varphi, \qquad (18)$$

this equation has a number of solutions. The TAPS free-energy barrier δF_+ is determined by the two of them. The first one, $\Delta_m = |\Delta_m|\exp(i\varphi_m)$, corresponds to a metastable minimum of the free-energy functional. This solution reads

$$|\Delta_m| = \Delta_0(T)\sqrt{\frac{1+2\cos\alpha}{3}}, \quad \varphi_m = \frac{2T}{\pi e N_0 Ds}\frac{Ix}{|\Delta_m|^2}. \qquad (19)$$

Here $\Delta_0(T)$ is the equilibrium superconducting gap defined in Eq. (14) and the parameter,

$$\alpha = \frac{\pi}{3}\theta\left(|I| - \frac{I_C}{\sqrt{2}}\right) + \frac{1}{3}\arctan\frac{2|I|\sqrt{1-(I/I_C)^2}}{I_C(1-2(I/I_C)^2)}, \qquad (20)$$

accounts for the external bias current I. The Ginzburg-Landau critical current I_C is defined by the standard expression:

$$I_C = \frac{16\sqrt{6}\pi^{5/2}}{63\zeta(3)} eN_0\sqrt{D}s(T_C-T)^{3/2}. \qquad (21)$$

The second, saddle-point solution $\Delta_s(x)=|\Delta_s|\exp(i\varphi_s)$ of Eq. (17) has the form

$$\frac{|\Delta_s|}{\Delta_0(T)} = \sqrt{\frac{1+2\cos\alpha}{3} - \frac{2\cos\alpha - 1}{\cosh^2\left[\sqrt{2\cos\alpha - 1}\dfrac{x}{\xi(T)}\right]}},$$

$$\varphi_s = \frac{2TI}{\pi e N_0 D s}\int_0^x dx' \frac{dx'}{|\Delta_s(x')|^2}, \qquad (22)$$

where $\xi(T)=\sqrt{\pi D/4(T_C-T)}$ is the superconducting coherence length in the vicinity of T_C.

The free-energy barrier $\delta F_{+2\pi}$ in Eq. (1) is set by the difference

$$\delta F_+ = F[\Delta_s(x)] - F[\Delta_m(x)] = \delta F\left[\sqrt{2\cos\alpha - 1} - \sqrt{\frac{2}{3}}\frac{I}{I_C}\arctan\left(\frac{\sqrt{3}}{2}\sqrt{\frac{2\cos\alpha - 1}{1-\cos\alpha}}\right)\right], \qquad (23)$$

where δF is defined in Eq. (15). The free-energy barrier δF_- for negative TAPS is determined analogously and is related to δF_+ as follows:

$$\delta F_- = \delta F_+ + \frac{\pi I}{e}. \qquad (24)$$

B. Preexponent

Now let us turn to the preexponent $B_{\pm 2\pi}$ in the expression for the TAPS rate [Eq. (1)]. For simplicity we first analyze the TAPS rate in the zero current limit in which case $\delta F_+ = \delta F_- = \delta F$ and $B_+=B_-=B$. In order to evaluate B one should go beyond the stationary free-energy functional (16) and include time-dependent fluctuations of the order-parameter field $\Delta(x,\tau)$. In Ref. 4 this task was accomplished within the framework of a TDGL-based analysis. Employing TDGL equation it is possible to reformulate the problem in terms of the corresponding Fokker-Planck equation[19] which can be conveniently solved for the problem in question. Since the important time scale within the TDGL approach is the Ginzburg-Landau time

$$\tau_{\rm GL} = \frac{\pi}{8|T_C - T|}, \qquad (25)$$

this time also naturally enters the expression for the preexponent B derived in Ref. 4.

Unfortunately the TDGL approach fails below T_C. For the sake of illustration, let us for a moment ignore both the scalar and the vector potentials. The TDGL action for the wire is then usually written in the form

$$S_{\rm TDGL} = N_0 T s \sum_{\omega_n}\int dx \frac{\pi|\omega_n|}{8T}|\Delta|^2 + \int d\tau F[\Delta(x,\tau)], \qquad (26)$$

where the GL free-energy functional $F[\Delta(x,\tau)]$ is defined in Eq. (16). This form can be obtained from the action (3) by formally expanding the kernel χ_Δ in Matsubara frequencies and wave vectors $\omega_\mu, Dq^2 \ll 4\pi T$. Note, however, that since the validity of the GL expansion is restricted to temperatures $T\sim T_C$, the Matsubara frequencies $|\omega_n|=2\pi|n|T$ cannot be much smaller than $4\pi T$ for any nonzero n. Hence, the expansion $\Psi(1/2+|\omega_n|/4\pi T)-\Psi(1/2)\to\pi|\omega_n|/8T$—which yields TDGL action (26)—is never correct except in the stationary case $\omega_n=0$. Already these simple arguments illustrate the failure of the TDGL action (26) in the Matsubara technique. Further problems with this TDGL approach arise in the presence of the electromagnetic potentials V and A. We refer the reader to the paper[17] for the corresponding discussion.

In view of this problem one should employ a more accurate effective action analysis. Since the microscopic effective action for superconducting wires[1,8,17] cannot be easily reduced to any Fokker-Planck-type of equation, it appears difficult to directly employ the McCumber-Halperin approach[4] in order to evaluate the preexponent B in the expression for the TAPS rate [Eq. (1)]. For this reason, below we will proceed differently and combine our effective action formalism with the well-known general formula for the decay rate of a metastable state expressed via the imaginary part of the free energy. This method is applicable provided the system is not driven far from equilibrium. For the decay rate in the thermal activation regime, one has[20–22]

$$\Gamma = -2\frac{T^*}{T}\,\mathrm{Im}\,F(T), \qquad (27)$$

where T^* is an effective crossover temperature between the activation regime and that of quantum tunneling under the potential barrier. Formally T^* is defined as temperature at which a nontrivial saddle-point solution $\Delta(\tau,x)$ describing quantum phase slip (QPS) first appears upon lowering T. Within the accuracy of our calculation it is sufficient to estimate T^* simply by setting the QPS action $S_{\rm QPS}(T)$ equal to activation exponent, i.e.,

$$S_{\rm QPS}(T^*) \simeq \delta F(T^*)/T^*. \qquad (28)$$

At sufficiently small currents one has[8]

$$S_{\rm QPS}(T) = A s N_0 \sqrt{N_0 \Delta_0(T)}, \qquad (29)$$

where A is a numerical constant of order one.[1] Hence, the condition (28) yields $\Delta_0(T^*)\sim T^*$ or, equivalently, $T^*=aT_C$, where the numerical factor $a<1$ is sufficiently close to unity, i.e., $T^*\sim T_C$. As the whole concept of TAPS is only valid at T close to T_C, one always has $T^*/T\sim 1$. Thus, with the same accuracy one can actually use the expression for the decay rate in the quantum regime[22,23] $\Gamma=-2\,\mathrm{Im}\,F(T)$ (cf. Ref. 1). Here we will retain the parameter T^* for the reasons which will be clear below.

In the limit $\delta F \gg T_C$ it suffices to expand the general expression for the effective action around both solutions (19) and (22) up to quadratic terms in both the phase φ and the amplitude $\delta\Delta$. One can verify that in the limit $\Delta_0(T)\ll T$ the contributions from fluctuating electromagnetic fields can be ignored and we obtain

$$S_{s/m} = F[\Delta_{s/m}] + \delta^2 S_{s/m}, \tag{30}$$

where

$$\delta^2 S_m = \frac{sT}{2}\sum_{\omega_n}\int dxdx'[\delta\Delta(\omega_n,x)\chi_\Delta^{(m)}(|\omega_n|;x-x')\delta\Delta(\omega_n,x') + \varphi(\omega_n,x)k_\varphi^{(m)}(|\omega_n|;x-x')\varphi(\omega_n,x')],$$

$$\delta^2 S_s = \frac{sT}{2}\sum_{\omega_n}\int dxdx'[\delta\Delta(\omega_n,x)\chi_\Delta^{(s)}(|\omega_n|;x,x')\delta\Delta(\omega_n,x') + \varphi(\omega_n,x)k_\varphi^{(s)}(|\omega_n|;x,x')\varphi(\omega_n,x')]. \tag{31}$$

Here $\omega_n = 2\pi Tn$ are Bose Matsubara frequencies. The functions $\chi_\Delta^{(m)}$ and $k_\varphi^{(m)}$ are expressed in terms of the kernels χ_Δ, χ_J, and χ_L as follows:

$$\chi_\Delta^{(m)}(|\omega_n|;x-x') = \int \frac{dq}{2\pi}e^{iq(x-x')}\chi_\Delta(\omega_n,q),$$

$$k_\varphi^{(m)}(|\omega_n|;x-x') = \int \frac{dq}{2\pi}e^{iq(x-x')}\mathcal{F}(\omega_n,q).$$

The functions $\chi_\Delta^{(s)}$ and $k_\varphi^{(s)}$ describe fluctuations around the coordinate dependent saddle point $\Delta_s(x)$, and, therefore, cannot be easily related to χ_Δ, χ_J, and χ_L. Fortunately, the explicit form of $\chi_\Delta^{(s)}$ and $k_\varphi^{(s)}$ is not important for us here.

The preexponent B in Eq. (1) is obtained by integrating over fluctuations $\delta\Delta$ in the expression for the grand partition function. One arrives at a formally diverging expression which signals decay of a metastable state. After a proper analytic continuation, one finds the decay rate in the form (1) with

$$B = 2T^* \,\mathrm{Im}\prod_{\omega_n}\sqrt{\frac{\det \chi_\Delta^{(m)}(\omega_n)\det k_\varphi^{(m)}(\omega_n)}{\det \chi_\Delta^{(s)}(\omega_n)\det k_\varphi^{(s)}(\omega_n)}}. \tag{32}$$

Here it is necessary to take an imaginary part since one of the eigenvalues of the operator $k_\varphi^{(s)}(0)$ is negative.

The key point is to observe that at $T \sim T_C$ all Matsubara frequencies $|\omega_n| = 2\pi T|n|$—except for one with $n=0$—strongly exceed the order parameter, $|\omega_n| \gg \Delta_0(T)$. Hence, for all such values the function $\chi_\Delta(\omega_n,q)$ approaches the asymptotic form,[8,17] which is not sensitive to superconductivity at all at such values of ω_n. In order to illustrate this point, we provide an explicit form of the function $\chi_\Delta(\omega_n,q)$ in the limit $|\omega| \gg 2\pi T_C$:

$$\chi_\Delta(\omega_n,q) \approx 2N_0\left(1 + \frac{2\Delta_0^2}{\omega_n^2}\right)\ln\frac{|\omega_n| + Dq^2}{\pi e^{-\gamma}T_C}, \tag{33}$$

where $\gamma \simeq 0.577$ is the Euler constant. As long as $\Delta_0 \ll T$ the last term in the round brackets can be ignored and we arrive at the expression which does not contain the superconducting order parameter Δ_0 at all. Hence, this expression remains the same even if the order parameter $\Delta(x)$ depends on coordinates, i.e., in this limit we have $\chi_\Delta^{(m)}(\omega_n) = \chi_\Delta^{(s)}(\omega_n)$. Likewise, at $\Delta_0 \ll T$ we obtain $k_\varphi^{(m)}(\omega_n) = k_\varphi^{(s)}(\omega_n)$, implying that $\det \chi_\Delta^{(s)}(\omega_n) \simeq \det \chi_\Delta^{(m)}(\omega_n)$ and $\det k_\varphi^{(s)}(\omega_n) \simeq \det k_\varphi^{(m)}(\omega_n)$. The corresponding determinants in Eq. (32) cancel out and only the contribution from $\omega_n = 0$ remains. It yields

$$B \simeq 2T^* \,\mathrm{Im}\sqrt{\frac{\det \chi_\Delta^{(m)}(0)\det k_\varphi^{(m)}(0)}{\det \chi_\Delta^{(s)}(0)\det k_\varphi^{(s)}(0)}}. \tag{34}$$

The ratio of these determinants can be evaluated at zero current with the aid of the GL free-energy functional (16). This calculation yields (see Appendix):

$$\mathrm{Im}\sqrt{\frac{\det \chi_\Delta^{(m)}(0)\det k_\varphi^{(m)}(0)}{\det \chi_\Delta^{(s)}(0)\det k_\varphi^{(s)}(0)}} = \frac{2\sqrt{6}}{\sqrt{\pi}}\frac{X}{\xi(T)}\sqrt{\frac{\delta F}{T}}, \tag{35}$$

where, as before, δF is defined in Eq. (15).

Combining the above expressions we arrive at the final result for the TAPS rate in the zero-bias limit:

$$\Gamma_\pm \equiv \Gamma = \frac{4\sqrt{6}}{\sqrt{\pi}}T^*\frac{X}{\xi(T)}\sqrt{\frac{\delta F}{T}}\exp\left[-\frac{\delta F}{T}\right]. \tag{36}$$

Turning to the case of nonzero bias one can essentially repeat the whole calculation which now yields two different TAPS rates $\Gamma_\pm$. Of practical importance is the limit of transport currents I sufficiently close to the critical one, i.e., $1 - I/I_C \ll 1$. In this regime Γ_- is negligibly small whereas Γ_+, on the contrary, increases since the free-energy barrier,

$$\delta F_+(I) = \frac{4\times 6^{3/4}}{15}\delta F\left(1 - \frac{I}{I_C}\right)^{5/4}, \tag{37}$$

becomes lower than that at smaller currents. Accordingly, TAPS can be detected easier in this limit.[24]

The preexponent B_+ has essentially the same form as that defined by Eq. (36), one just needs to replace $\delta F \to \delta F_+(I)$ and $T^* \to T^*(I)$. In the limit $T_C - T \ll T_C$ considered here the current dependence of the crossover temperature T^* appears insignificant in most cases and with sufficient accuracy one can set $T^*(I) \simeq T^*$. Indeed, very generally one can express $T^*(I) = T^* f[I/I_C(T^*)]$, where $I_C(T^*)$ is the critical current at temperature T^* and $f(x)$ is some universal function with $f(x \ll 1) \simeq 1$. Having in mind the strong temperature dependence of $I_C(T)$ in the temperature interval $T_C - T \ll T_C$, we find

$$I/I_C(T^*) < I_C(T)/I_C(T^*) \sim (T_C - T)^{3/2}/T_C^{3/2} \ll 1,$$

and, hence, $T^*(I) \approx T^*(0) \equiv T^*$. Thus, in the vicinity of the critical current $I_C(T) - I \ll I_C(T)$ the TAPS rate can be expressed in the form

$$\Gamma_+ \simeq 8.84T^*\frac{X}{\xi(T)}\sqrt{\frac{\delta F}{T}}\left(1 - \frac{I}{I_C}\right)^{5/8}e^{-\delta F_+(I)/T}, \tag{38}$$

where $\delta F_+(I)$ is defined in Eq. (37) and the numerical prefactor is again established from the calculation of the fluctuation determinants which is fully analogous to that presented in the Appendix.

Summarizing all the above results and substituting $T^* = aT_C$, we arrive at the final expression for the TAPS rates

$$\Gamma_{\pm}(I) = \kappa a T_C \frac{X}{\xi(T)} \sqrt{\frac{\delta F_{\pm}(I)}{T}} \exp\left[- \frac{\delta F_{\pm}(I)}{T} \right], \quad (39)$$

where $\kappa(I)$ is a smooth function of I varying from $\kappa(0) \simeq 5.53$ to $\kappa \simeq 8.74$ at $I_C - I \ll I_C$ and, as before, the numerical prefactor a is of order (and slightly smaller than) one. Equation (39) is the central result of this work. This expression is supposed to be valid at $T_C - T \ll T_C$ and at any bias current $I < I_C$ as long as $\delta F_{\pm}(I) \gg T$.

C. Comparison with McCumber-Halperin result

Let us compare our result [Eq. (39)] with the expression for the TAPS rates derived in Ref. 4 from the TDGL-type of analysis. We observe that Eq. (39) does not contain the Ginzburg-Landau time τ_{GL} and exceeds the corresponding expression[4] by the factor $\sim T^* \tau_{GL} \sim (1 - T/T_C)^{-1} \gg 1$. On top of that, in the vicinity of the critical current, the preexponent in the TAPS rate (38) depends on I as $B_+ \propto (1 - I/I_C)^{5/8}$ in contrast to the result[4] $B_{TDGL} \propto (1 - I/I_C)^{15/8}$.

In order to understand the origin of these differences, let us—just for the sake of illustration—for a moment adopt the TDGL action (26) and recalculate the TAPS rate Γ_{TDGL} employing Eq. (27). Since the whole calculation of the fluctuation determinants remains the same (see Appendix) we should only reevaluate the crossover temperature which we now denote as T^*_{TDGL}. To this end we again first set $I \to 0$ and consider fluctuations of the order parameter around the saddle point $\Delta_s(x)$ along the unstable direction [Eq. (A15)] choosing

$$\delta\Delta(\tau, x) = i \frac{\cos(2\pi T^*_{TDGL} \tau)}{\sqrt{2} \cosh(x/\xi)} C, \quad (40)$$

where C is a constant. Substituting this expression into the linearized TDGL equation and formally treating τ_{GL} as an independent parameter, we define the classical-to-quantum crossover temperature T^*_{TDGL} as that at which a nonzero solution ($C \neq 0$) first appears. This definition yields

$$T^*_{TDGL} = 1/4\pi\tau_{GL}. \quad (41)$$

Substituting Eq. (41) into Eq. (36) we arrive at the expression for Γ_{TDGL} just two times bigger than that derived in Ref. 4 in the limit $I \to 0$.

An analogous—although slightly more complicated—analysis can be performed also at nonzero-bias current I. This analysis yields

$$T^*_{TDGL}(I) \sim T^*_{TDGL} \left(1 - \frac{I}{I_C} \right)^{5/4}. \quad (42)$$

Combining Eqs. (41) and (42) with the result [Eq. (38)], we arrive at the preexponent

$$B_{TDGL} \sim \frac{1}{\tau_{GL}} \frac{X}{\xi(T)} \sqrt{\frac{\delta F}{T}} \left(1 - \frac{I}{I_C} \right)^{15/8},$$

which is again in agreement with Ref. 4. Thus, with the aid of the general formula (27) describing thermally activation decay of a metastable state, we confirm that the McCumber-Halperin result[4] for the TAPS rate is essentially correct within the TDGL-type of formalism. Unfortunately, however, the latter formalism is inaccurate by itself. In particular, in the expression for the TAPS rate it does not allow the correctly obtaining of the classical-to-quantum crossover temperature T^*.

D. Temperature-dependent resistance and noise

In order to complete our analysis let us briefly address the relation between the above TAPS rate and physical observables, such as, e.g., wire resistance and voltage noise. Every phase slip event implies changing of the superconducting phase in time in such a way that the total phase difference values along the wire before and after this event differ by $\pm 2\pi$. Since the average voltage is linked to the time derivative of the phase by means of the Josephson relation, $\langle V \rangle = \langle \dot{\varphi}/2e \rangle$, for the net voltage drop across the wire, we obtain

$$V = \frac{\pi}{e} [\Gamma_+(I) - \Gamma_-(I)], \quad (43)$$

where $\Gamma_{\pm}$ are given by Eq. (39). In the absence of any bias current $I \to 0$ both rates are equal $\Gamma_{\pm} = \Gamma$ and the net voltage drop V vanishes. In the presence of small bias current $I \ll I_C$, we obtain

$$\Gamma_{\pm}(I) = \Gamma e^{\pm \pi I/2eT}. \quad (44)$$

Thus, at such values of I and at temperatures slightly below T_C, the $I-V$ curve for quasi-1D superconducting wires takes a relatively simple form:

$$V = \frac{2\pi}{e} \Gamma \sinh \frac{\pi I}{2eT}. \quad (45)$$

The zero-bias resistance $R(T) = (\partial V / \partial I)_{I=0}$ demonstrates exponential dependence on temperature and the wire cross section

$$\frac{e^2 R(T)}{2\pi} = 2\sqrt{6\pi} \frac{a T_C}{T} \frac{X}{\xi(T)} \sqrt{\frac{\delta F}{T}} \exp\left[- \frac{\delta F}{T} \right]. \quad (46)$$

To complete our description of thermal fluctuations in superconducting wires, we point out that in addition to nonzero resistance [Eq. (46)] TAPS also cause the voltage noise below T_C. Treating TAPS as independent events one immediately concludes that they should obey Poissonian statistics. Hence, the voltage noise power,

$$S_V = 2 \int dt \langle \delta V(t) \delta V(0) \rangle,$$

is given by the sum of the contributions of both positive and negative TAPS, i.e.,

$$S_V = \frac{2\pi^2}{e^2} [\Gamma_+(I) + \Gamma_-(I)]. \quad (47)$$

At small currents $I \ll I_C$ this expression reduces to the following simple form:

$$S_V = \frac{4\pi^2}{e^2}\Gamma \cosh\frac{\pi I}{2eT}. \tag{48}$$

Similarly to the wire resistance the voltage noise rapidly decreases as one lowers the temperature away from T_C. Only in the vicinity of the critical temperature this TAPS noise remains appreciable and can be detected in experiments.

In conclusion, employing the microscopic effective action analysis, we have evaluated the rate for thermally activated phase slips in superconducting nanowires. Our main result is summarized in Eq. (39) which remains valid in the temperature interval $Gi_{1D} \ll 1 - T/T_C \ll 1$. Equation (39) turns out to be parametrically bigger as compared to the analogous expression for the TAPS rate derived earlier from the TDGL-type of approach.[4] Although this difference affects only the preexponential factor in Eq. (39), it can nevertheless be significant at temperatures sufficiently close to T_C and it can be detected in experiments with sufficiently thin nanowires. Simultaneous measurements of both TAPS-induced resistance and noise appear to be an efficient way for quantitative experimental analysis of thermally activated phase slips in superconducting nanowires.

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APPENDIX: EVALUATION OF FLUCTUATION DETERMINANTS

Let us set $I=0$ and write the Ginzburg-Landau free energy in the form

$$F = \frac{3\,\delta F}{4}\int_{-X/2\xi}^{X/2\xi} d\eta\left[\frac{u'^2+v'^2}{2} - u^2 - v^2 + \frac{(u^2+v^2)^2}{2}\right], \tag{A1}$$

where we introduced $\eta = x/\xi$, $u = \mathrm{Re}\,\Delta/\Delta_0(T)$, and $v = \mathrm{Im}\,\Delta/\Delta_0(T)$. In terms of these dimensionless variables the metastable solution (19) reads

$$u_m(\eta) = 1, \quad v_m(\eta) = 0, \tag{A2}$$

while the saddle-point solution (22) takes the form

$$u_s(\eta) = \tanh\,\eta, \quad v_s(\eta) = 0. \tag{A3}$$

The second variation in the free energy (A1) around its saddle point with $v(y)=0$ is

$$\delta^2 F = \frac{3\,\delta F}{4}\int_{-X/2\xi}^{X/2\xi} d\eta\left[\frac{\delta u_\eta'^2 + \delta v_\eta'^2}{2} - \delta u^2 - \delta v^2 + u^2(\delta u^2 + \delta v^2) + 2u^2\delta u^2\right]. \tag{A4}$$

Here δu and δv describe fluctuations of, respectively, the absolute value and the phase of the order parameter. Accordingly the operators χ_Δ and k_φ read

$$\chi_\Delta^{(s)}(0) = \frac{3\,\delta F}{4T}\left[-\frac{d^2}{d\eta^2} + 4 - \frac{6}{\cosh^2\,\eta}\right],$$

$$\chi_\Delta^{(m)}(0) = \frac{3\,\delta F}{4T}\left[-\frac{d^2}{d\eta^2} + 4\right],$$

$$k_\varphi^{(s)}(0) = \frac{3\,\delta F}{4T}\left[-\frac{d^2}{d\eta^2} - \frac{2}{\cosh^2\,\eta}\right],$$

$$k_\varphi^{(m)}(0) = \frac{3\,\delta F}{4T}\left[-\frac{d^2}{d\eta^2}\right]. \tag{A5}$$

In order to fix the boundary conditions we note that fluctuations of the absolute value of the order parameter in the bulk leads are negligible. Hence, we set

$$\delta u(-X/2) = \delta v(X/2) = 0. \tag{A6}$$

Likewise, since the current density vanishes in the bulk leads, we can choose

$$\delta v'(-X/2) = \delta v'(X/2) = 0. \tag{A7}$$

Let us evaluate the eigenvalues of, say, the operator $\chi_\Delta^{(s)}(0)$. These eigenvalues $\Lambda_n^{(s)} = (3\,\delta F_0/4T)\lambda_n^{(s)}$ are obtained from the Schrödinger equation,

$$\left[-\frac{d^2}{d\eta^2} - \frac{6}{\cosh^2\,\eta}\right]\delta u = (\lambda - 4)\,\delta u, \tag{A8}$$

with appropriate boundary conditions. The corresponding localized solutions of this equation have the well-known form[25]

$$\lambda_1^{(s)} = 0, \quad \delta u_1^{(s)}(\eta) = \sqrt{\frac{3}{4}}\frac{1}{\cosh^2\,\eta},$$

$$\lambda_2^{(s)} = 3, \quad \delta u_2^{(s)}(\eta) = \sqrt{\frac{3}{2}}\frac{\sinh\,\eta}{\cosh^2\,\eta}. \tag{A9}$$

In order to find the eigenvalues in the continuous spectrum, we introduce transmission, $t(\lambda)$, and reflection, $r(\lambda)$, amplitudes of the potential well,[25]

$$t(\lambda) = \frac{(1 - i\sqrt{\lambda - 4})(2 - i\sqrt{\lambda - 4})}{(1 + i\sqrt{\lambda - 4})(2 + i\sqrt{\lambda - 4})}, \quad r(\lambda) = 0. \tag{A10}$$

Thus, at large negative η the wave function has the form $\delta v(\eta) = C_1 e^{i\sqrt{\lambda-4}\eta} + C_2 t(\lambda) e^{-i\sqrt{\lambda-4}\eta}$, while at large positive η the same wave function is $\delta v(\eta) = C_1 t(\lambda) e^{i\sqrt{\lambda-4}\eta} + C_2 e^{-i\sqrt{\lambda-4}\eta}$. Imposing the boundary conditions (A6) we arrive at the following equation for the eigenvalues $\lambda_n^{(s)}$ $(n=3,4,\ldots)$:

$$F_s(\lambda_n^{(s)}) = 0, \tag{A11}$$

where

$$F_s(\lambda) = \frac{1}{2i}\left(t(\lambda)e^{i\sqrt{\lambda-4}(X/\xi)} - \frac{e^{-i\sqrt{\lambda-4}(X/\xi)}}{t(\lambda)}\right). \tag{A12}$$

In the limit $X/\xi \to \infty$ Eq. (A11) also applies for the discrete eigenvalues $\lambda_1^{(s)}, \lambda_2^{(s)}$ [Eq. (A9)].

The eigenvalues of the operator $\chi_\Delta^{(m)}(0)$ are obtained analogously. They are defined by the equation

$$F_m(\lambda_n^{(m)}) = 0, \quad n = 1,2, \dots , \tag{A13}$$

where $F_m(\lambda)=\sin(\sqrt{\lambda-4}X/\xi)$. Observing that $F_{s,m}(\lambda) \propto \prod_{n=1}^{\infty}(\lambda-\lambda_n^{s,m})$ and extracting the zero eigenvalue of $\chi_\Delta^{(s)}(0)$ in a standard way, after the integration over this zero mode, we obtain

$$\sqrt{\frac{\det \chi_\Delta^m(0)}{\det \chi_\Delta^s(0)}} = \sqrt{\frac{4}{3}\frac{X}{\xi}}\sqrt{\frac{3}{4}\frac{\delta F}{2\pi T}}\lim_{\lambda\to 0}\lim_{X\to\infty}\sqrt{\frac{\lambda F_m(\lambda)}{F_s(\lambda)}} = \frac{2\sqrt{6}}{\sqrt{\pi}}\frac{X}{\xi}\sqrt{\frac{\delta F}{T}}. \tag{A14}$$

The ratio of fluctuation determinants $\det k_\varphi^m(0)/\det k_\varphi^s(0)$ is evaluated analogously. The operator $k_\varphi^{(s)}(0)$ has two localized eigenfunctions $\delta v_1^{(s)}(\eta)$ and $\delta v_2^{(s)}(\eta)$ with the eigenvalues $E_{1,2}^{(s)}=(3\delta F/4T)\epsilon_{1,2}^{(s)}$,

$$\epsilon_1^{(s)} = -1, \quad \delta v_1^{(s)}(\eta) = \frac{1}{\sqrt{2}}\frac{1}{\cosh \eta}, \tag{A15}$$

$$\epsilon_2^{(s)} = 0, \quad \delta v_2^{(s)}(\eta) = \sqrt{\frac{\xi}{X}}\tanh \eta. \tag{A16}$$

The eigenvalue $\epsilon_1^{(s)}$ is negative and, as is usually, it is associated with the unstable direction in the functional space. The ratio of the determinants is expressed as follows:

$$\sqrt{\frac{\det k_\varphi^m(0)}{\det k_\varphi^s(0)}} = \lim_{\lambda\to 0}\sqrt{\frac{G_m(\lambda)}{G_s(\lambda)}}, \tag{A17}$$

where $G_s(\lambda)=\sin(\sqrt{\lambda}X/\xi)$, while $G_s(\lambda)$ reads

$$G_s(\lambda) = \frac{1}{2i}\left(\tilde{t}(\lambda)e^{i\sqrt{\lambda}(X/\xi)} - \frac{e^{-i\sqrt{\lambda}(X/\xi)}}{\tilde{t}(\lambda)}\right), \tag{A18}$$

where $\tilde{t}(\lambda)=(i\sqrt{\lambda}-1)/(1+i\sqrt{\lambda})$. Thus, we get

$$\sqrt{\frac{\det k_\varphi^m(0)}{\det k_\varphi^m(0)}} = i. \tag{A19}$$

Combining Eqs. (A14) and (A19), we arrive at Eq. (35).

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