Magnetic oscillations of resistivity and absorption of radiation in quantum wells with two populated subbands

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The dynamic (ac) conductivity tensor of quantum wells with two populated subbands in the presence of a magnetic field perpendicular to the well layer is calculated theoretically. The microscopic theory is based on the Kubo formalism assuming a detailed consideration of elastic scattering of electrons by the random disorder potential with arbitrary correlation length. The results describe the influence of magnetic field on the linear absorption of low-frequency electromagnetic radiation, and demonstrate the existence of magnetic oscillations that survive at high temperatures and whose maxima correspond to absorption of electromagnetic radiation at combined frequencies, determined by both the magnetic field and the subband separation. Different polarizations of the radiation field with respect to the quantum-well layer are considered. Analytical expressions are derived for the case of sufficiently weak magnetic field when the Landau levels are overlapping. Application of the theory to the static (dc) limit provides a consistent description of the magneto-intersubband oscillations of the resistivity in the systems with two populated subbands.

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I. INTRODUCTION

The discovery of the microwave-induced resistance oscillations[1] whose minima evolve into zero-resistance states[2,3] at sufficiently high intensity of the radiation has stimulated intensive experimental and theoretical activity.[4] These oscillations are related to modification of microwave absorption in the presence of a magnetic field when the density of electron states, owing to the Landau quantization, acquires an oscillating contribution. For this reason, studies of the absorption of electromagnetic radiation by two-dimensional (2D) electrons in a magnetic field perpendicular to the 2D plane are of particular interest. In weak enough magnetic fields when the Landau quantization is not essential, the absorption is described by the classical Drude-Lorentz formula for the dissipative ac conductivity $\sigma_d(\omega)$ and shows a peak corresponding to the cyclotron resonance condition $\omega=\omega_c$, where ω is the frequency of electromagnetic field and ω_c is the cyclotron frequency. In quantizing magnetic fields, $\sigma_d(\omega)$ has another important contribution, which oscillates with the magnetic field as[5]

$$\cos\frac{2\pi\omega}{\omega_c}. \tag{1}$$

The maxima of these oscillations occur under the condition $\omega=k\omega_c$ (k is integer), which corresponds to harmonics of the cyclotron resonance. The existence of these harmonics as a result of electron scattering between different Landau levels was pointed out by Ando.[6] Such harmonics have been observed experimentally in surface space-charge layers on Si.[7] In contrast to the conventional Shubnikov-de Haas oscillations of the dc magnetoresistance, which are exponentially suppressed with the increase in temperature T, the oscillations [Eq. (1)] survive at high temperature.[5] The amplitude of these ac magnetoconductivity oscillations is proportional to the square of the Dingle factor $\exp(-\pi/\omega_c\tau)$, where τ is the quantum lifetime of electrons.

The phenomena of the dc magnetoresistance oscillations under microwave irradiation and of the oscillating ac magnetoconductivity have been studied so far for 2D electron systems realized in quantum wells with a single (ground-state) populated subband. The physics of these phenomena is expected to be more rich and interesting in the case of two-subband occupation (Fig. 1), owing to the intersubband coupling via scattering. This coupling leads to dc magnetoresistance oscillations, which are similar to the ac magnetoconductivity oscillations described above in the sense that these oscillations also survive at high temperatures and are proportional to the square of the Dingle factor. The mechanism responsible for these oscillations relies on periodic modulation of the elastic intersubband scattering by Landau quantization. As the different Landau levels of the two subbands sequentially come in alignment, there appears the oscillating contribution

$$\cos\frac{2\pi\Delta_{12}}{\omega_c}, \tag{2}$$

where Δ_{12} is the subband energy separation (here and below, we use the system of units where Planck's constant $\hbar$ is

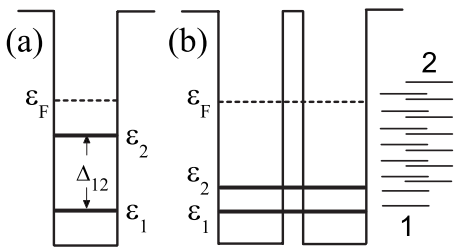

FIG. 1. Schematic of the confinement potential profile and subband structure of the (a) single-quantum-well and (b) double-quantum-well systems. The dashed lines show the position of the chemical potential for the case of two populated subbands. Two staircases of Landau levels originating from the subbands 1 and 2 in a magnetic field are shown on the right.

equal to one). Such oscillations, often called the magneto-intersubband (MIS) oscillations, have been theoretically predicted by Polyanovsky[8] and experimentally observed in single-quantum wells with two populated 2D subbands.[9–12] Microscopic theories of the MIS oscillations have been presented in Refs. 13 and 14. Recent investigations of magnetotransport in high-mobility electron systems with small subband separation, which are realized in double-quantum wells (DQWs), have revealed long-period MIS oscillations with large amplitudes[15] so the interest to this phenomenon is expected to be renewed.

The aim of this paper is to develop a microscopic theory of the linear ac magnetoconductivity in quantum wells with two occupied subbands. The consideration presented below shows that the oscillating properties of the ac magnetoconductivity in these systems are determined by the *interference* of the mechanisms responsible for the oscillations [Eqs. (1) and (2)]. This leads to the magnetic oscillations whose periodicity is described by the functions

$$\cos\frac{2\pi(\Delta_{12}+\omega)}{\omega_c}, \quad \cos\frac{2\pi(\Delta_{12}-\omega)}{\omega_c}. \tag{3}$$

These oscillations, whose maxima correspond to absorption of electromagnetic radiation at combined frequencies $\omega=|\Delta_{12}-k\omega_c|$, can be possibly detected in experiments on transmission or reflection of electromagnetic radiation in quantum wells.[7,16,17]

Whereas the origin of the oscillations [Eq. (3)] is clear from the explanation given above, their description based on a microscopic theory is a complicated problem that requires a careful consideration of electron scattering in the case of many-subband occupation. Analytical consideration of the ac magnetoconductivity can be done for sufficiently weak magnetic fields when the Landau levels are overlapping. Even then, extensive calculations are necessary and the general expressions derived in this paper appear to be complicated. Nevertheless, relatively simple analytical expressions are found under additional reasonable assumptions about scattering and subband occupation.

The problem remains complicated even in the static (dc) limit, $\omega=0$, which has been already studied theoretically[13,14] in application to the MIS oscillations in single-quantum wells with two populated subbands. These previous studies, however, are not complete because they are done under a simplifying approximation of the short-range disorder potential. Moreover, the analytical results obtained in Refs. 13 and 14 are different from each other, and from the results of this paper (when taken in the short-range disorder limit). The theory of this paper at $\omega=0$ gives a consistent description of the MIS oscillations, valid for disorder potentials with arbitrary correlation lengths. A simple analytical expression for the oscillating dc resistivity in the limit of classically strong magnetic fields is derived.

When several subbands are involved into consideration, there exists the absorption of electromagnetic radiation polarized perpendicular to the quantum-well plane. This absorption demonstrates the intersubband resonance peaks extensively studied in quantum wells, both theoretically and experimentally, by means of transmission and reflection of electromagnetic radiation in the far-infrared and terahertz spectral regions.[18] The line shape of intersubband resonance in the presence of elastic scattering of electrons and a strong perpendicular magnetic field has been investigated by Ando.[19] Ando has shown that the line shape depends on the relative position of the Landau levels of two subbands and predicted quantum oscillations of absorption as a function of the applied magnetic field with the period determined by the condition $\Delta_{12}=k\omega_c$. Ando has considered the case of fully separated Landau levels. In this paper, the calculations are done for the case of overlapping Landau levels and the oscillations corresponding to combined resonances are described.

The paper is organized as follows. Section II contains the essentials of the linear-response formalism based on Kubo technique in application to the systems with two occupied 2D subbands. The calculation of the components of the ac conductivity tensor and investigation of some limiting cases are given in Sec. III. The static (dc) response is analyzed in Sec. IV. Magnetic oscillations of the absorption of radiation polarized perpendicular to the quantum-well plane are described in Sec. V. The conclusions and a discussion of the approximations are presented in Sec. VI.

II. FORMALISM

Consider a system of electrons interacting with a random static potential of impurities or other inhomogeneities. The conductivity tensor describing linear response of the system to the electric field of frequency ω is given by the Kubo formula:[20]

$$\sigma_{\alpha\beta}(\omega)=\frac{ie^2n_s}{m\omega}\delta_{\alpha\beta}+\frac{e^2}{2\pi\omega L^2}\int d\varepsilon f_\varepsilon Sp\langle\langle(\hat{G}^A_\varepsilon-\hat{G}^R_\varepsilon)\times\hat{v}_\alpha\hat{G}^R_{\varepsilon+\omega}\hat{v}_\beta+\hat{v}_\beta\hat{G}^A_{\varepsilon-\omega}\hat{v}_\alpha(\hat{G}^A_\varepsilon-\hat{G}^R_\varepsilon)\rangle\rangle, \tag{4}$$

where e is the electron charge, n_s is the electron density, f_ε is the equilibrium Fermi distribution, $\hat{G}^{R(A)}_\varepsilon$ is the retarded (advanced) Green's function in the operator form, $\hat{\mathbf{v}}$ is the velocity operator, L^2 is the normalization square, Sp denotes the trace over all quantum-mechanical variables including spin, and the double angular brackets denote the averaging over the random potential. The Hamiltonian of the system is written as

$$\hat{H}=\frac{(\hat{\mathbf{p}}-e\mathbf{A}/c)^2}{2m}+U(z)+V(\mathbf{r},z), \tag{5}$$

where $\hat{\mathbf{p}}$ is the momentum operator, $\mathbf{A}$ is the vector potential describing the magnetic field, $U(z)$ is the confinement potential, $V(\mathbf{r},z)$ is the random potential, $\mathbf{r}=(x,y)$ is the 2D coordinate vector, m is the effective mass of electrons, and c is the velocity of light. The magnetic field is assumed to be weak enough to neglect the Zeeman splitting.

Assuming that the magnetic field $\mathbf{H}$ is perpendicular to the 2D plane (xy), we use the gauge $\mathbf{A}=(0,Hx,0)$ and apply the basis of exact eigenstates in the absence of scattering potential, $|jnp_y\rangle$, where integers j and n number the subbands and the Landau levels, respectively. The corresponding free-

electron energy spectrum in each subband is given by the separate sets of equidistant Landau levels (see the right part of Fig. 1):

$$\varepsilon_{jn} = \varepsilon_j + \omega_c(n + 1/2), \tag{6}$$

where ε_j is the energy of the subband j describing electron confinement in the potential $U(z)$ and $\omega_c = |e|H/mc$ is the cyclotron frequency. This simple form of the energy spectrum reflects the fact that the motion in the quantum-well plane is separable from the motion in the z direction. For the same reason, the eigenstates can be written as products $|j\rangle|np_y\rangle$, where $|np_y\rangle$ are the Landau eigenstates in the chosen gauge and $|j\rangle$ corresponds to the envelope function of electrons in the subband j.

If the electric field $\mathbf{E}$ is directed in the 2D plane xy (the case of normally incident radiation), there are two components of the conductivity tensor: the dissipative and nondissipative ones, expressed as

$$\sigma_{xx} = \sigma_{yy} = \sigma_d(\omega) = \frac{\sigma_+(\omega) + \sigma_-(\omega)}{2},$$

$$\sigma_{yx} = -\sigma_{xy} = \sigma_\perp(\omega) = \frac{\sigma_+(\omega) - \sigma_-(\omega)}{2i}. \tag{7}$$

The absorption coefficient, defined as a ratio of absorbed power to incident power, for the linearly polarized electromagnetic radiation is written in terms of $\sigma_\pm(\omega)$ as[17,21]

$$\xi(\omega) = \sum_\pm \frac{2\pi\,\mathrm{Re}\,\sigma_\pm(\omega)/c\sqrt{\epsilon}}{|1 + 2\pi\sigma_\pm(\omega)/c\sqrt{\epsilon}|^2}, \tag{8}$$

where ϵ is the dielectric permittivity of the medium surrounding the quantum well. The denominator in Eq. (8) can be larger than unity for high-mobility and high-density electron layers.

Using expressions for the matrix elements of the velocity operator $\mathbf{v} = (\hat{\mathbf{p}} - e\mathbf{A}/c)/m$ in the chosen basis and taking into account double degeneracy in spin, one can rewrite the Kubo formula (4) as

$$\sigma_\pm(\omega) = i\frac{e^2 n_s}{m\omega} + \frac{e^2}{2\pi m\omega}\int d\varepsilon \sum_{jj'} [(f_\varepsilon - f_{\varepsilon+\omega})Q^{AR}_{jj'\pm}(\varepsilon,\omega) + f_{\varepsilon+\omega}Q^{AA}_{jj'\pm}(\varepsilon,\omega) - f_\varepsilon Q^{RR}_{jj'\pm}(\varepsilon,\omega)], \tag{9}$$

where

$$Q^{ss'}_{jj'+}(\varepsilon,\omega) = \frac{2\omega_c}{L^2}\sum_{nn'}\sqrt{(n+1)(n'+1)} \times \sum_{p_y p'_y} \langle\langle G^{jj',s}_\varepsilon(n+1p_y, n'+1p'_y) G^{j'j,s'}_{\varepsilon+\omega}(n'p'_y, np_y)\rangle\rangle, \tag{10}$$

$$Q^{ss'}_{jj'-}(\varepsilon,\omega) = \frac{2\omega_c}{L^2}\sum_{nn'}\sqrt{(n+1)(n'+1)}\sum_{p_y p'_y} \langle\langle G^{jj',s}_\varepsilon(np_y, n'p'_y) \times G^{j'j,s'}_{\varepsilon+\omega}(n'+1p'_y, n+1p_y)\rangle\rangle, \tag{11}$$

and $G^{jj',s}_\varepsilon(np_y, n'p'_y) \equiv \langle jnp_y|\hat{G}^s_\varepsilon|j'n'p'_y\rangle$.

The pair correlators standing in Eqs. (10) and (11) are related to the average of single Green's function. The averaged Green's function is given by $\langle\langle G^{jj',s}_\varepsilon(np_y, n'p'_y)\rangle\rangle = \delta_{nn'}\delta_{p_y p'_y} G^{jj',s}_{\varepsilon n}$, where $G^{jj',s}_{\varepsilon n}$ contains both intrasubband $(j = j')$ and intersubband $(j \neq j')$ components coupled by the system of equations $\Sigma_{j_1}[(\varepsilon - \varepsilon_{jn})\delta_{jj_1} - \Sigma^{jj_1,s}_{\varepsilon n}]G^{j_1 j',s}_{\varepsilon n} = \delta_{jj'}$. The self-energies $\Sigma^{jj',s}_{\varepsilon n}$ describe electron scattering by the disorder potential. Below we use the approximation of well-defined subbands when the scattering-induced broadening is small in comparison to subband separation. Then, the intersubband components of both averaged Green's functions and self-energies can be neglected, $G^{jj',s}_{\varepsilon n} = \delta_{jj'}G^{j,s}_{\varepsilon n}$ and $\Sigma^{jj',s}_{\varepsilon n} = \delta_{jj'}\Sigma^{j,s}_{\varepsilon n}$, and we obtain

$$G^{j,s}_{\varepsilon n} = \frac{1}{\varepsilon - \varepsilon_{jn} - \Sigma^{j,s}_{\varepsilon n}}. \tag{12}$$

In the self-consistent Born approximation (SCBA), the self-energy is given by[22]

$$\Sigma^{j,s}_{\varepsilon n} = \sum_{j'n'}\int_0^\infty \frac{dq^2}{4\pi} w_{jj'}(q)\Phi_{nn'}(q^2 l_H^2/2) G^{j',s}_{\varepsilon n'}, \tag{13}$$

where $w_{jj'}(q) = \int d\Delta\mathbf{r} e^{-i\mathbf{q}\cdot\Delta\mathbf{r}}\langle\langle V_{jj'}(\mathbf{r}+\Delta\mathbf{r})V_{j'j}(\mathbf{r})\rangle\rangle$ is the spatial Fourier transform of the correlation function of the effective random potentials defined as $V_{jj'}(\mathbf{r}) = \langle j|V(\mathbf{r},z)|j'\rangle$, q is the absolute value of the momentum transferred in the scattering, and $l_H = \sqrt{c/|e|H}$ is the magnetic length. The function $\Phi_{nn'}(u) = \frac{n!}{n'!}u^{n'-n}e^{-u}[L_n^{n'-n}(u)]^2$, where $L_n^\alpha(u)$ are the Laguerre polynomials, characterizes scattering between the Landau levels n and n'.

Calculation of $G^{j,s}_{\varepsilon n}$, and of the correlators in Eqs. (10) and (11) is considerably simplified for weak enough magnetic fields when the magnetic length is large in comparison to the correlation lengths l_{jj} characterizing the scale of the q dependence of $w_{jj}(q)$. In fact, one needs to satisfy the condition $l_{jj}^2\sqrt{n} \ll l_H^2$, where n is the relevant Landau-level number $[n \sim (\varepsilon - \varepsilon_j)/\omega_c \gg 1]$. Simultaneously, one should satisfy the condition $\ell_j \gg l_{jj}$, where ℓ_j is the mean-free path of electron in the subband j in the absence of magnetic field. Under these conditions, the Landau-level dependence of $\Sigma^{j,s}_{\varepsilon n}$ is weak enough to be neglected in the interval of small $|\varepsilon - \varepsilon_{jn}|$ where the contribution of $\Sigma^{j,s}_{\varepsilon n}$ into Green's function $G^{j,s}_{\varepsilon n}$ is essential. Moreover, Eq. (13) is reduced to

$$\Sigma^{j,s}_{\varepsilon n} \simeq \Sigma^{j,s}_\varepsilon = \sum_{j'}\nu_{jj'}(\varepsilon)S^{j',s}_\varepsilon, \quad S^{j,s}_\varepsilon = \frac{\omega_c}{2\pi}\sum_n G^{j,s}_{\varepsilon n}, \tag{14}$$

where

$$\nu_{jj'}(\varepsilon) = m\int_0^{2\pi}\frac{d\theta}{2\pi}w_{jj'}(\sqrt{p_{j\varepsilon}^2+p_{j'\varepsilon}^2-2p_{j\varepsilon}p_{j'\varepsilon}\cos\theta}) \tag{15}$$

are the partial elastic-scattering rates in the absence of magnetic field, $p_{j\varepsilon}=\sqrt{2m(\varepsilon-\varepsilon_j)}$ are the electron momenta in the subbands, and θ is the scattering angle. According to the definition [Eq. (15)], $\nu_{jj'}=\nu_{j'j}$. The dimensionless functions $S_\varepsilon^{j,s}$ become very simple in the case of zero magnetic field, $S_\varepsilon^{j,A}=i/2$. At a finite field, they acquire oscillating dependence on energy according to

$$S_\varepsilon^{j,A}=\frac{i}{2}-i\gamma_\varepsilon^j+\ldots, \quad \gamma_\varepsilon^j=e^{-\alpha_j}\exp\frac{2\pi i(\varepsilon_j-\varepsilon)}{\omega_c}, \tag{16}$$

and $S_\varepsilon^{j,R}=(S_\varepsilon^{j,A})^*$. Here and below, $e^{-\alpha_j}$ is the Dingle factor for subband j,

$$\alpha_j=\frac{\pi}{\omega_c\tau_j}, \quad \frac{1}{\tau_j}\equiv\nu_j=\sum_{j'}\nu_{jj'}, \tag{17}$$

and τ_j is the quantum lifetime of electron in the subband j. The terms denoted by the dots in Eq. (16) correspond to higher powers of Dingle factors. Basically, Eqs. (12) and (14)–(17) represent a many-subband generalization of the SCBA approach for the case of overlapping Landau levels.[23] According to the definition of $S_\varepsilon^{j,A}$, the density of states in the subband j is $(2m/\pi)\mathrm{Im}\,S_\varepsilon^{j,A}$, which is reduced to the 2D density of states, m/π, if the magnetic field is zero.

Now let us turn to the pair correlators. By defining the function

$$K_{\varepsilon\varepsilon'}^{jj',ss'}(n,n+1|n',n'+1)=\frac{1}{L^2}\sum_{p_yp_y'}\langle\langle G_\varepsilon^{jj',s}(np_y,n'p_y') \times G_{\varepsilon'}^{j'j,s'}(n'+1p_y',n+1p_y)\rangle\rangle, \tag{18}$$

one can write an equation for this function in the ladder approximation:

$$K_{\varepsilon\varepsilon'}^{jj',ss'}(n,n+1|n',n'+1) = \frac{1}{2\pi l_H^2}G_{\varepsilon n}^{j,s}G_{\varepsilon' n+1}^{j,s'}\left[\delta_{jj'}\delta_{nn'}+\sum_{j_1k}\int_0^\infty du w_{jj_1}(\sqrt{2u}/l_H) \times\Psi_{nk}(u)K_{\varepsilon\varepsilon'}^{j_1j',ss'}(n+k,n+k+1|n',n'+1)\right], \tag{19}$$

where the variable of integration is related to the transferred momentum q as $u=q^2l_H^2/2$, and

$$\Psi_{nk}(u)=\sqrt{\frac{n!\,(n+1)!}{(n+k)!\,(n+k+1)!}}e^{-u}u^kL_n^k(u)L_{n+1}^k(u). \tag{20}$$

To find $Q_{jj'-}^{ss'}(\varepsilon,\omega)$, one should multiply $K_{\varepsilon\varepsilon+\omega}^{jj',ss'}(n,n+1|n',n'+1)$ by $\sqrt{(n+1)(n'+1)}$ and take the sums over Landau levels. An equation similar to Eq. (19) can be written for the function $K_{\varepsilon\varepsilon+\omega}^{jj',ss'}(n+1,n|n'+1,n')$ defining $Q_{jj'+}^{ss'}(\varepsilon,\omega)$ in a similar way.

The second term on the right-hand side of Eq. (19) describes the vertex correction and vanishes in the limit of short-range scattering potentials when w_{jj_1} does not depend on the transferred momentum. In this case, the functions $Q_{jj'-}^{ss'}$ and $Q_{jj'+}^{ss'}$ are expressed through the "bare" correlators $K_{\varepsilon\varepsilon'}^{jj',ss'(b)}(n,n+1|n',n'+1)=(2\pi l_H^2)^{-1}G_{\varepsilon n}^{j,s}G_{\varepsilon' n+1}^{j,s'}\delta_{jj'}\delta_{nn'}$ and $K_{\varepsilon\varepsilon'}^{jj',ss'(b)}(n+1,n|n'+1,n')=(2\pi l_H^2)^{-1}G_{\varepsilon n+1}^{j,s}G_{\varepsilon' n}^{j,s'}\delta_{jj'}\delta_{nn'}$, respectively:

$$Q_{jj'-}^{ss'}=\delta_{jj'}2\omega_c\sum_n(n+1)K_{\varepsilon\varepsilon+\omega}^{jj,ss'(b)}(n,n+1|n,n+1), \tag{21}$$

and

$$Q_{jj'+}^{ss'}=\delta_{jj'}2\omega_c\sum_n(n+1)K_{\varepsilon\varepsilon+\omega}^{jj,ss'(b)}(n+1,n|n+1,n). \tag{22}$$

In the limits $(\varepsilon-\varepsilon_j)\gg\omega_c$, ω, one obtains

$$\sum_{jj'}Q_{jj'\pm}^{ss'}(\varepsilon,\omega)=\sum_j p_{j\varepsilon}^2\Pi_{j\pm}^{ss'}(\varepsilon,\omega). \tag{23}$$

The function $\Pi_{j\pm}^{ss'}$ has dimensionality of time and is expressed through the functions defined by Eq. (14):

$$\Pi_{j\pm}^{ss'}(\varepsilon,\omega)=\frac{\omega_c}{2\pi}\sum_n G_{\varepsilon,n\pm1}^{j,s}G_{\varepsilon+\omega,n}^{j,s'}=\frac{S_\varepsilon^{j,s}-S_{\varepsilon+\omega}^{j,s'}}{\omega\pm\omega_c+\Sigma_\varepsilon^{j,s}-\Sigma_{\varepsilon+\omega}^{j,s'}}. \tag{24}$$

If momentum dependence of w_{jj_1} in Eq. (19) is important, the vertex correction should be taken into account. However, using the fact that the products $G_{\varepsilon n}^{j,s}G_{\varepsilon' n+1}^{j,s'}$ are large only in the vicinity of $\varepsilon\simeq\varepsilon_{jn}$ and employing the assumed condition $l_{jj}^2\sqrt{n}\ll l_H^2$ together with $(\varepsilon-\varepsilon_j)\gg\omega_c$, ω, one gets the following system of algebraic equations:[24]

$$Q_{jj'\pm}^{ss'}=p_{j\varepsilon}^2\Pi_{j\pm}^{ss'}\delta_{jj'}+\Pi_{j\pm}^{ss'}\sum_{j_1}\tilde{\nu}_{jj_1}\frac{p_{j\varepsilon}}{p_{j_1\varepsilon}}Q_{j_1j'\pm}^{ss'}, \tag{25}$$

where

$$\tilde{\nu}_{jj'}(\varepsilon)=m\int_0^{2\pi}\frac{d\theta}{2\pi}\cos\theta\, w_{jj'}(\sqrt{p_{j\varepsilon}^2+p_{j'\varepsilon}^2-2p_{j\varepsilon}p_{j'\varepsilon}\cos\theta}). \tag{26}$$

In contrast to the case of short-range scattering potentials, the nondiagonal $Q_{jj'\pm}^{ss'}$ are nonzero.

In what follows, the subband basis is restricted to a pair of lowest subbands, $j=1,2$. Solution of Eq. (25) then gives

$$\sum_{jj'}Q_{jj'\pm}^{ss'}(\varepsilon,\omega)=\frac{p_{1\varepsilon}^2\Lambda_{2\pm}^{ss'}+p_{2\varepsilon}^2\Lambda_{1\pm}^{ss'}+2p_{1\varepsilon}p_{2\varepsilon}\tilde{\nu}_{12}}{\Lambda_{1\pm}^{ss'}\Lambda_{2\pm}^{ss'}-\tilde{\nu}_{12}^2},$$

$$\Lambda^{ss'}_{j\pm}(\varepsilon,\omega) \equiv [\Pi^{ss'}_{j\pm}(\varepsilon,\omega)]^{-1} - \tilde{\nu}_{jj}(\varepsilon), \tag{27}$$

where we have used the obvious symmetry property $\tilde{\nu}_{12}(\varepsilon) = \tilde{\nu}_{21}(\varepsilon)$. According to Eqs. (14) and (24),

$$\Lambda^{ss'}_{j\pm}(\varepsilon,\omega) = \frac{\omega \pm \omega_c + \nu_{12}(S^{j',s}_{\varepsilon} - S^{j',s'}_{\varepsilon+\omega})_{j'\neq j}}{S^{j,s}_{\varepsilon} - S^{j,s'}_{\varepsilon+\omega}} + \nu^{\mathrm{tr}}_{jj}, \tag{28}$$

where $\nu^{\mathrm{tr}}_{jj}(\varepsilon) \equiv \nu_{jj}(\varepsilon) - \tilde{\nu}_{jj}(\varepsilon)$, so $1/\nu^{\mathrm{tr}}_{jj}$ coincides with the standard definition of transport time for intrasubband scattering. Note that in Eq. (28) the difference between $\nu_{jj'}(\varepsilon)$ and $\nu_{jj'}(\varepsilon+\omega)$ is neglected because of the assumed smallness of ω. On the other hand, the difference between the functions $S^{j,s}_{\varepsilon}$ and $S^{j,s}_{\varepsilon+\omega}$ is essential because these functions rapidly oscillate with energy. Equation (27) is a generalization of Eq. (23) to the case of arbitrary correlation lengths. Equation (23) is obtained from Eq. (27) by formal substitutions $\tilde{\nu}_{jj'} \to 0$.

This part concludes calculation of the functions $\Sigma_{jj'}Q^{ss'}_{jj'\pm}(\varepsilon,\omega)$, which determine the components of the conductivity tensor and absorption of electromagnetic radiation according to Eqs. (7)–(9). The calculation of $\sigma_d(\omega)$ and $\sigma_\perp(\omega)$ is now reduced to evaluation of the integrals over energy in Eq. (9). This procedure and its results are given in the Sec. III.

III. AC CONDUCTIVITY AND ABSORPTION

We consider the case of degenerate electron gas when energy dependence of $p^2_{j\varepsilon}$, $\nu_{jj'}(\varepsilon)$, and $\tilde{\nu}_{jj'}(\varepsilon)$ is not essential within the region of thermal smearing, and within the region of width ω near the Fermi surface. Therefore, these quantities are replaced below by the constants corresponding to the position of chemical potential, $\varepsilon=\varepsilon_F$. In particular, one has $p^2_{j\varepsilon} \to p^2_{Fj}$ and the Fermi momenta p_{Fj} can be rewritten in terms of electron densities in the subbands $j=1,2$ according to $n_{sj}=p^2_{Fj}/2\pi$. The simple substitution $\varepsilon=\varepsilon_F$ cannot be done for the part $-i\gamma^j_\varepsilon$ of the function $S^{j,A}_\varepsilon$ because γ^j_ε is a rapidly oscillating function of energy [see Eq. (16)]. Nevertheless, analytical integration in Eq. (9) is possible and it is based on the smallness of γ^j_ε in the case of overlapping Landau levels when the Dingle factors are small. The expansion of the function $\Sigma_{jj'}Q^{AR}_{jj'\pm}$ [see Eqs. (27) and (28)] in series of γ^j_ε and $\gamma^{j*}_{\varepsilon+\omega}$ should be done up to the second order. The first-order corrections, linear in the Dingle factors, always give the contributions that oscillate with energy. These terms are suppressed if the temperature is high enough. Among the second-order corrections, quadratic in the Dingle factors, there are contributions that do not oscillate with energy because they come from the products $\gamma^j_\varepsilon\gamma^{j'*}_{\varepsilon+\omega}$ (such contributions do not appear in $Q^{AA}_{jj'\pm}$ terms). Only these contributions should be retained in the second-order corrections. This also means that there is no need to take into account subsequent terms in the expansion of $S^{j,A}_\varepsilon$ (in addition to $-i\gamma^j_\varepsilon$) because such terms would give only the oscillating corrections beyond the first order.

The rapidly oscillating contributions coming from $Q^{AR}_{jj'\pm}$ terms are integrated over energy as

$$\frac{1}{\omega}\int d\varepsilon(f_\varepsilon - f_{\varepsilon+\omega})[\gamma^j_\varepsilon + \gamma^{j*}_{\varepsilon+\omega}] = 2e^{-\alpha_j}\frac{\sin(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)}\mathcal{T}\exp\left(\frac{i\pi\omega}{\omega_c}\right)\cos\frac{2\pi(\varepsilon_F-\varepsilon_j)}{\omega_c}. \tag{29}$$

The expansions of $Q^{AA}_{jj'\pm}$ and $Q^{RR}_{jj'\pm}$ terms in powers of Dingle factors can be done in the region not too close to the cyclotron resonance when $|\omega-\omega_c| \gg \nu_j e^{-\alpha_j}$. Then, the rapidly oscillating contributions appearing from these terms are integrated as

$$\frac{1}{\omega}\int d\varepsilon(f_\varepsilon - f_{\varepsilon+\omega})[\gamma^j_\varepsilon - \gamma^j_{\varepsilon+\omega}] = 2ie^{-\alpha_j}\frac{\sin^2(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)} \times \mathcal{T}\exp\left[-i\frac{2\pi(\varepsilon_F-\varepsilon_j)}{\omega_c}\right], \tag{30}$$

and

$$\frac{1}{\omega}\int d\varepsilon(f_\varepsilon + f_{\varepsilon+\omega})[\gamma^j_\varepsilon - \gamma^j_{\varepsilon+\omega}] = -2e^{-\alpha_j}\frac{\sin(2\pi\omega/\omega_c)}{(2\pi\omega/\omega_c)} \times \mathcal{T}\exp\left[-i\frac{2\pi(\varepsilon_F-\varepsilon_j)}{\omega_c}\right]. \tag{31}$$

All these expressions are proportional to the factor describing thermal suppression:

$$\mathcal{T} = \frac{(2\pi^2 T/\omega_c)}{\sinh(2\pi^2 T/\omega_c)}. \tag{32}$$

On the other hand, the second-order contributions, which do not oscillate with energy, are calculated in the simplest way by using $\omega^{-1}\int d\varepsilon(f_\varepsilon - f_{\varepsilon+\omega}) = 1$.

After lengthy but straightforward transformations, one obtains the result

$$\sigma_\pm(\omega) = \frac{e^2}{2\pi m}\left[\frac{P_\pm}{D_\pm} - 2\mathcal{T}F_\pm + 2R_\pm\right], \tag{33}$$

which, together with Eq. (7), gives the ac conductivity tensor in relatively weak magnetic fields when the Landau levels are overlapping. The functions $F_\pm$ and $R_\pm$ are the contributions proportional to the first and second powers of Dingle factors, respectively. The function $P_\pm/D_\pm$, where

$$P_\pm = p^2_{F1}\lambda_{2\pm} + p^2_{F2}\lambda_{1\pm} + 2p_{F1}p_{F2}\tilde{\nu}_{12}, \tag{34}$$

$$D_\pm = \lambda_{1\pm}\lambda_{2\pm} - \tilde{\nu}^2_{12}, \tag{35}$$

and

$$\lambda_{j\pm} = \nu^{\mathrm{tr}}_{jj} + \nu_{12} - i(\omega \pm \omega_c), \tag{36}$$

describes the classical contribution to the components of the conductivity tensor of the two-subband system.

The first-order contribution is an oscillating function of the Fermi energy ε_F and frequency ω:

$$F_{\pm} = \frac{1-\exp(2\pi i\omega/\omega_c)}{(2\pi i\omega/\omega_c)} \sum_j \left[A_{j\pm} + \frac{ip_{Fj}^2}{\omega \pm \omega_c} \right] g_j(\varepsilon_F), \tag{37}$$

with

$$g_j(\varepsilon_F) = e^{-\alpha_j} \cos\frac{2\pi(\varepsilon_F - \varepsilon_j)}{\omega_c}, \tag{38}$$

$$A_{1\pm} = \frac{1}{D_{\pm}} \left[(a_{\pm} + b_{\pm}) - \frac{P_{\pm}}{D_{\pm}} (c_{\pm} + d_{\pm}) \right],$$

$$A_{2\pm} = \frac{1}{D_{\pm}} \left[(a_{\pm} - b_{\pm}) - \frac{P_{\pm}}{D_{\pm}} (c_{\pm} - d_{\pm}) \right], \tag{39}$$

$$a_{\pm} = -i(\omega \pm \omega_c) \frac{p_{F1}^2 + p_{F2}^2}{2},$$

$$b_{\pm} = [i(\omega \pm \omega_c) - 2\nu_{12}] \frac{p_{F1}^2 - p_{F2}^2}{2}, \tag{40}$$

and

$$c_{\pm} = -i(\omega \pm \omega_c) \frac{\lambda_{1\pm} + \lambda_{2\pm}}{2},$$

$$d_{\pm} = [i(\omega \pm \omega_c) - 2\nu_{12}] \frac{\lambda_{1\pm} - \lambda_{2\pm}}{2}. \tag{41}$$

In the static limit, $\omega \to 0$, this part describes the Shubnikov-de Haas oscillations. These oscillations are given by a superposition of two single-subband contributions.

Oscillations of the second-order contribution are not related to position of the Fermi energy, and are determined by the ratios ω/ω_c and Δ_{12}/ω_c, where $\Delta_{12}=\varepsilon_2-\varepsilon_1$ is the subband separation:

$$R_{\pm} = \exp\left(i\frac{2\pi\omega}{\omega_c} \right) \left[B_{1\pm} e^{-2\alpha_1} + B_{2\pm} e^{-2\alpha_2} + B_{12\pm} e^{-\alpha_1-\alpha_2} \cos\frac{2\pi\Delta_{12}}{\omega_c} \right], \tag{42}$$

where

$$B_{1\pm} = \frac{1}{D_{\pm}} [\nu_{12} - i(\omega \pm \omega_c)] \left\{ p_{F2}^2 - \frac{P_{\pm}}{D_{\pm}} [\nu_{22}^{\mathrm{tr}} - i(\omega \pm \omega_c)] \right\} - \frac{c_{\pm} + d_{\pm}}{D_{\pm}} A_{1\pm},$$

$$B_{2\pm} = \frac{1}{D_{\pm}} [\nu_{12} - i(\omega \pm \omega_c)] \left\{ p_{F1}^2 - \frac{P_{\pm}}{D_{\pm}} [\nu_{11}^{\mathrm{tr}} - i(\omega \pm \omega_c)] \right\} - \frac{c_{\pm} - d_{\pm}}{D_{\pm}} A_{2\pm}, \tag{43}$$

and

$$B_{12\pm} = -\nu_{12} \frac{p_{F1}^2 + p_{F2}^2}{D_{\pm}} + \frac{P_{\pm}}{D_{\pm}^2} [\nu_{12}(\nu_{11}^{\mathrm{tr}} + \nu_{22}^{\mathrm{tr}}) + (\omega \pm \omega_c)^2] - \frac{2(a_{\pm}c_{\pm} - b_{\pm}d_{\pm})}{D_{\pm}^2} + \frac{2P_{\pm}}{D_{\pm}^3} (c_{\pm}^2 - d_{\pm}^2). \tag{44}$$

As the temperature increases and the first-order contribution becomes exponentially small, oscillations of the conductivity are entirely determined by the second-order contribution [Eq. (42)]. At $\omega \to 0$ this contribution describes the MIS oscillations (Refs. 8–15). For the single-subband case and nonzero ω, the results of Ref. 5 can be restored.

The expression (42) is the central result of this paper. It shows that the mechanisms responsible for the MIS oscillations and for the ac magnetoconductivity oscillations described in Ref. 5 interfere with each other in the linear ac response of the systems with two occupied subbands. Formally, this is reflected by the presence of the product $[\exp(i2\pi\omega/\omega_c)\cos(2\pi\Delta_{12}/\omega_c)]$ of the corresponding oscillating factors. The real part of this product can also be written through the sum of the oscillating functions [Eq. (3)].

The expressions (33)–(44) present a complete solution of the dynamic conductivity problem in magnetic fields under the approximations listed above. Instead of the complex variables $\sigma_{\pm}(\omega)$, it is convenient to introduce real quantities,

$$\sigma_d^{(\pm)}(\omega) = \frac{1}{2} \mathrm{Re}\ \sigma_{\pm}(\omega), \quad \sigma_{\perp}^{(\pm)}(\omega) = \frac{1}{2} \mathrm{Im}\ \sigma_{\pm}(\omega), \tag{45}$$

so the real part of the conductivity tensor is defined by the expressions

$$\mathrm{Re}\ \sigma_d(\omega) = \sigma_d^{(+)}(\omega) + \sigma_d^{(-)}(\omega),$$

$$\mathrm{Re}\ \sigma_{\perp}(\omega) = \sigma_{\perp}^{(+)}(\omega) - \sigma_{\perp}^{(-)}(\omega). \tag{46}$$

In the general case, expressions for $\sigma_d^{(\pm)}$ and $\sigma_{\perp}^{(\pm)}$ are rather cumbersome. These expressions are simplified under some reasonable approximations considered below.

In the limit of short-range scattering potentials, when $\tilde{\nu}_{12}=0$ and $\nu_{jj}^{\mathrm{tr}}=\nu_{jj}$, the result is given as a sum of subband contributions:

$$\sigma_d^{(\pm)} = \frac{e^2}{2m} \sum_{j=1,2} \frac{n_{sj}\tau_j}{\kappa_{j\pm}^2 + 1} (1 - 2\mathcal{T}\mathcal{F}_{j\pm}^d + 2\mathcal{R}_{j\pm}^d), \tag{47}$$

and

$$\sigma_{\perp}^{(\pm)} = \frac{e^2}{2m} \sum_{j=1,2} \frac{n_{sj}\tau_j}{\kappa_{j\pm}^2 + 1} (\kappa_{j\pm} - 2\mathcal{T}\mathcal{F}_{j\pm}^{\perp} + 2\mathcal{R}_{j\pm}^{\perp}), \tag{48}$$

where $\kappa_{j\pm}=(\omega\pm\omega_c)\tau_j$. The first-order contributions in $\sigma_d^{(\pm)}$ and $\sigma_{\perp}^{(\pm)}$ are described by the following expressions:

$$\mathcal{F}_{j\pm}^{d(\perp)} = Y_{j\pm}^{d(\perp)} g_j(\varepsilon_F) + X_{j\pm}^{d(\perp)} \left[\frac{\nu_{jj}}{\nu_j} g_j(\varepsilon_F) + \frac{\nu_{12}}{\nu_j} g_{j'}(\varepsilon_F) \right], \tag{49}$$

where $j' \neq j$. In Eq. (49),

$$X_{j\pm}^d = \frac{\kappa_{j\pm}^2 - 1}{\kappa_{j\pm}^2 + 1} \frac{\sin(2\pi\omega/\omega_c)}{(2\pi\omega/\omega_c)} + \frac{2\kappa_{j\pm} \sin^2(\pi\omega/\omega_c)}{(\kappa_{j\pm}^2 + 1)(\pi\omega/\omega_c)},$$

$$X_{j\pm}^{\perp} = \frac{\kappa_{j\pm}^2 - 1}{\kappa_{j\pm}^2 + 1} \frac{\sin^2(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)} - \frac{2\kappa_{j\pm}\sin(2\pi\omega/\omega_c)}{(\kappa_{j\pm}^2 + 1)(2\pi\omega/\omega_c)}, \quad (50)$$

and

$$Y_{j\pm}^{d} = \frac{\sin(2\pi\omega/\omega_c)}{(2\pi\omega/\omega_c)} + \frac{1}{\kappa_{j\pm}} \frac{\sin^2(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)},$$

$$Y_{j\pm}^{\perp} = \frac{\sin^2(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)} - \frac{1}{\kappa_{j\pm}} \frac{\sin(2\pi\omega/\omega_c)}{(2\pi\omega/\omega_c)}. \quad (51)$$

The second-order contributions can be written as

$$\mathcal{R}_{j\pm}^{d} = \frac{Z_{j\pm}^{e}\cos(2\pi\omega/\omega_c) + \kappa_{j\pm}Z_{j\pm}^{o}\sin(2\pi\omega/\omega_c)}{\kappa_{j\pm}^2 + 1},$$

$$\mathcal{R}_{j\pm}^{\perp} = \frac{Z_{j\pm}^{e}\sin(2\pi\omega/\omega_c) - \kappa_{j\pm}Z_{j\pm}^{o}\cos(2\pi\omega/\omega_c)}{\kappa_{j\pm}^2 + 1}, \quad (52)$$

where

$$Z_{j\pm}^{e} = \left[\kappa_{j\pm}^2 - 1 + \frac{\nu_{jj}}{\nu_j}\eta_{j\pm}^{e}\right]\frac{\nu_{jj}}{\nu_j}e^{-2\alpha_j} + \eta_{j\pm}^{e}\frac{\nu_{12}^2}{\nu_j^2}e^{-2\alpha_{j'}} + \left[\kappa_{j\pm}^2 - 1 + 2\frac{\nu_{jj}}{\nu_j}\eta_{j\pm}^{e}\right]\frac{\nu_{12}}{\nu_j}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}, \quad (53)$$

and

$$Z_{j\pm}^{o} = \left[2 + \frac{\nu_{jj}}{\nu_j}\eta_{j\pm}^{o}\right]\frac{\nu_{jj}}{\nu_j}e^{-2\alpha_j} + \eta_{j\pm}^{o}\frac{\nu_{12}^2}{\nu_j^2}e^{-2\alpha_{j'}} + \left[2 + 2\frac{\nu_{jj}}{\nu_j}\eta_{j\pm}^{o}\right]\frac{\nu_{12}}{\nu_j}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}, \quad (54)$$

are the coefficients standing at the terms even and odd in $\kappa_{j\pm}$. Next, $\eta_{j\pm}^{e} = (1-3\kappa_{j\pm}^2)/(\kappa_{j\pm}^2+1)$ and $\eta_{j\pm}^{o} = (\kappa_{j\pm}^2 - 3)/(\kappa_{j\pm}^2+1)$. In Eqs. (53) and (54), $j' \neq j$. Since $\nu_j = \nu_{jj} + \nu_{12}$, the result depends on the three characteristic scattering rates: ν_1, ν_2, and ν_{12}. Equations (53) and (54) directly show that the terms containing the MIS oscillations [Eq. (2)] are proportional to the intersubband scattering rate ν_{12}.

Another important case, when expressions are simplified, is realized in DQWs where subband splitting is usually small in comparison to the Fermi energy, $\Delta_{12} \ll \varepsilon_F - \varepsilon_j$, so the difference in Fermi momenta is small, $p_{F1} \simeq p_{F2}$. Assuming also that the DQWs are symmetrically doped, one has $\nu_{11} \simeq \nu_{22}$ and $\nu_{11}^{\mathrm{tr}} \simeq \nu_{22}^{\mathrm{tr}}$. In summary, the approximations $|p_{F1}^2 - p_{F2}^2| \ll p_{F1}^2 + p_{F2}^2$ and $|\nu_{11}^{\mathrm{tr}} - \nu_{22}^{\mathrm{tr}}| \ll \nu_{11}^{\mathrm{tr}} + \nu_{22}^{\mathrm{tr}}$ are applied. However, one should keep in mind that a weak scattering asymmetry can produce a sizeable difference in the Dingle exponents. Therefore, the exponential factors $e^{-\alpha_1}$ and $e^{-\alpha_2}$ are still assumed to be different. One gets

$$\sigma_d^{(\pm)} = \frac{e^2 n_s \tau_{\mathrm{tr}}}{2m(1+\beta_{\pm}^2)}\left[1 - \mathcal{T}C_{\pm}^{d}\sum_j g_j(\varepsilon_F) + 2\mathcal{D}_{\pm}^{d}\right], \quad (55)$$

and

$$\sigma_{\perp}^{(\pm)} = \frac{e^2 n_s \tau_{\mathrm{tr}}}{2m(1+\beta_{\pm}^2)}\left[\beta_{\pm} - \mathcal{T}C_{\pm}^{\perp}\sum_j g_j(\varepsilon_F) + 2\mathcal{D}_{\pm}^{\perp}\right], \quad (56)$$

where $n_s = n_{s1} + n_{s2}$ is the total sheet density of electrons, $\beta_{\pm} = (\omega \pm \omega_c)\tau_{\mathrm{tr}}$, and τ_{tr} is the transport time defined as $1/\tau_{\mathrm{tr}} \equiv \nu_{\mathrm{tr}} = (\nu_{11}^{\mathrm{tr}} + \nu_{22}^{\mathrm{tr}})/2 + \nu_{12}^{\mathrm{tr}}$, where $\nu_{12}^{\mathrm{tr}} = \nu_{12} - \tilde{\nu}_{12}$ can be named as the intersubband transport scattering rate. The coefficients in the first-order contributions are given by

$$C_{\pm}^{d} = \frac{2\beta_{\pm}^2\sin(2\pi\omega/\omega_c)}{(1+\beta_{\pm}^2)(2\pi\omega/\omega_c)} + \frac{(1+3\beta_{\pm}^2)\sin^2(\pi\omega/\omega_c)}{\beta_{\pm}(1+\beta_{\pm}^2)(\pi\omega/\omega_c)},$$

$$C_{\pm}^{\perp} = \frac{2\beta_{\pm}^2\sin^2(\pi\omega/\omega_c)}{(1+\beta_{\pm}^2)(\pi\omega/\omega_c)} - \frac{(1+3\beta_{\pm}^2)\sin(2\pi\omega/\omega_c)}{\beta_{\pm}(1+\beta_{\pm}^2)(2\pi\omega/\omega_c)}. \quad (57)$$

The second-order contributions have the same structure as in Eq. (52):

$$\mathcal{D}_{\pm}^{d} = \frac{\mathcal{Z}_{\pm}^{e}\cos(2\pi\omega/\omega_c) + \beta_{\pm}\mathcal{Z}_{\pm}^{o}\sin(2\pi\omega/\omega_c)}{\beta_{\pm}^2 + 1},$$

$$\mathcal{D}_{\pm}^{\perp} = \frac{\mathcal{Z}_{\pm}^{e}\sin(2\pi\omega/\omega_c) - \beta_{\pm}\mathcal{Z}_{\pm}^{o}\cos(2\pi\omega/\omega_c)}{\beta_{\pm}^2 + 1}, \quad (58)$$

where

$$\mathcal{Z}_{\pm}^{e(o)} = h_{\pm}^{e(o)}\frac{1}{2}(e^{-2\alpha_1} + e^{-2\alpha_2}) + \tilde{h}_{\pm}^{e(o)}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}. \quad (59)$$

The coefficients in this expression are

$$h_{\pm}^{e} = (\beta_{\pm}^2 - 1)\left(1 - \frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right) + \frac{\zeta_{\pm}^{e}}{2} + \frac{\xi_{\pm}^{e}}{2}\left(1 - \frac{2\nu_{12}^{\mathrm{tr}}}{\nu_{tr}}\right)^2,$$

$$\tilde{h}_{\pm}^{e} = (\beta_{\pm}^2 - 1)\frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}} + \frac{\zeta_{\pm}^{e}}{2} - \frac{\xi_{\pm}^{e}}{2}\left(1 - \frac{2\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right)^2, \quad (60)$$

and

$$h_{\pm}^{o} = 2\left(1 - \frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right) + \frac{\zeta_{\pm}^{o}}{2} + \frac{\xi_{\pm}^{o}}{2}\left(1 - \frac{2\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right)^2,$$

$$\tilde{h}_{\pm}^{o} = 2\frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}} + \frac{\zeta_{\pm}^{o}}{2} - \frac{\xi_{\pm}^{o}}{2}\left(1 - \frac{2\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right)^2, \quad (61)$$

where

$$\xi_{\pm}^{e} = \frac{(1 + 2\tilde{\nu}_{12}/\nu_{\mathrm{tr}})(1-\beta_{\pm}^2) - 2\beta_{\pm}^2}{\beta_{\pm}^2 + (1 + 2\tilde{\nu}_{12}/\nu_{\mathrm{tr}})^2},$$

$$\xi_{\pm}^{o} = \frac{\beta_{\pm}^2 - 3 - 2\tilde{\nu}_{12}/\nu_{\mathrm{tr}}}{\beta_{\pm}^2 + (1 + 2\tilde{\nu}_{12}/\nu_{\mathrm{tr}})^2},$$

$$\zeta_{\pm}^{e} = (1 - 3\beta_{\pm}^2)/(\beta_{\pm}^2 + 1),$$

and

$$\zeta_{\pm}^{o} = (\beta_{\pm}^2 - 3)/(\beta_{\pm}^2 + 1).$$

In high-mobility modulation-doped heterostructures, the transport scattering times are large, and it is easy to realize the condition $\beta_{\pm}^2 \gg 1$ in a wide region of frequencies and magnetic fields, which are not too close to the cyclotron resonance. In this case, the expressions (55) and (56) have a simple form:

$$\frac{\sigma_d^{(\pm)}}{\sigma_{d(D)}^{(\pm)}} = 1 - 2\mathcal{T}\frac{\sin(2\pi\omega/\omega_c)}{(2\pi\omega/\omega_c)}\sum_j g_j(\varepsilon_F) + \cos\frac{2\pi\omega}{\omega_c}\left[\left(1 - \frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right)\right.$$
$$\left.\times(e^{-2\alpha_1} + e^{-2\alpha_2}) + 2\frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}\right], \quad (62)$$

and

$$\frac{\sigma_{\perp}^{(\pm)}}{\sigma_{\perp(D)}^{(\pm)}} = 1 - \frac{2}{\beta_{\pm}}\mathcal{T}\frac{\sin^2(\pi\omega/\omega_c)}{(\pi\omega/\omega_c)}\sum_j g_j(\varepsilon_F) + \frac{1}{\beta_{\pm}}\sin\frac{2\pi\omega}{\omega_c}$$
$$\times\left[\left(1 - \frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}\right)(e^{-2\alpha_1} + e^{-2\alpha_2}) + 2\frac{\nu_{12}^{\mathrm{tr}}}{\nu_{\mathrm{tr}}}e^{-\alpha_1-\alpha_2}\right.$$
$$\left.\times\cos\frac{2\pi\Delta_{12}}{\omega_c}\right], \quad (63)$$

where $\sigma_{d(D)}^{(\pm)} = (e^2 n_s \tau_{\mathrm{tr}}/2m)/(\beta_{\pm}^2 + 1)$ and $\sigma_{\perp(D)}^{(\pm)} = \beta_{\pm}\sigma_{d(D)}^{(\pm)}$ are the classical Drude expressions. Relative contribution of the oscillating part in the nondissipative component is small because of large $\beta_{\pm}$. However, since $\sigma_{\perp(D)}^{(\pm)} \gg \sigma_{d(D)}^{(\pm)}$, the amplitudes of the oscillations of $\sigma_d^{(\pm)}$ and $\sigma_{\perp}^{(\pm)}$ are comparable to each other.

The oscillating behavior of the absorption coefficient $\xi(\omega)$, given by Eq. (8), is demonstrated in Figs. 2 and 3. The calculation of the conductivities $\sigma_{\pm}$ is done according to the general expressions (33)–(44) applied to GaAs-based systems, where $m = 0.067m_0$, and the experimentally relevant case of long-range disorder potential is considered when the ratios of transport scattering rates to the scattering rates that enter the Dingle factors are small. For the chosen parameters $\nu_1 = \nu_2 = 0.18$ meV and $\nu_{jj}^{\mathrm{tr}}/\nu_{jj} = \nu_{12}^{\mathrm{tr}}/\nu_{12} = 0.1$, one has $\nu_{\mathrm{tr}} = 0.018$ meV, which approximately corresponds to the mobility of 10^6 cm^2/V s. The temperature is assumed to be high enough ($T = 10$ K) so the second term in Eq. (33) is completely suppressed and the oscillations are described by the third term, which is quadratic in the Dingle factors. The magnetic-field dependence demonstrates the cyclotron resonance (CR) peaks with superimposed oscillating contribution. The oscillations are better visible in the plots of the ratios ξ/ξ_0 where ξ_0 is the classical (nonoscillating) absorption coefficient calculated by using Eq. (33) with the second and the third terms omitted. These plots clearly show the interference of the slow component [Eq. (1)] and fast component [Eq. (2)] (note that the chosen ratio $\Delta_{12}/\omega = 8$ is large). When the relative contribution of intersubband scattering decreases (curve 2), the amplitude of the fast component decreases as well. Only the slow component [Eq. (1)] remains if the intersubband scattering is completely neglected (dashed line). In the last case, the magnetic-field dependence of the ratio ξ/ξ_0 is similar to that of 2D systems with a single populated subband.

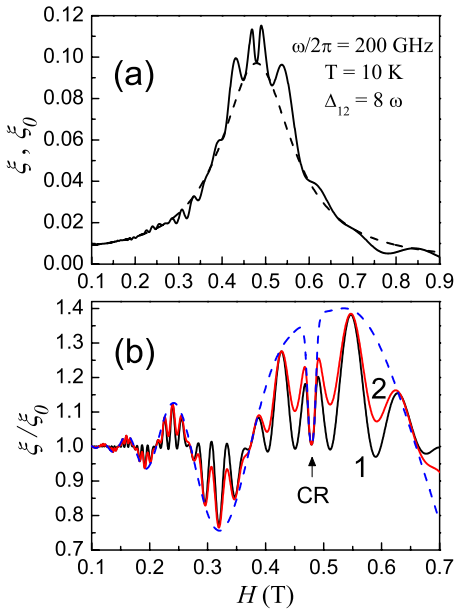

FIG. 2. (Color online) Magnetic-field dependence of the absorption coefficient $\xi(\omega)$ at $T = 10$ K and $\omega/2\pi = 200$ GHz for GaAs quantum wells with electron density $n_s = 10^{12}$ cm^{-2}. The ratio of subband separation to frequency is $\Delta_{12}/\omega = 8$. The scattering rates are chosen as $\nu_1 = \nu_2 = 0.18$ meV, giving the quantum lifetime of 3.66 ps, and the ratios of intrasubband and intersubband transport scattering rates to the corresponding quantum-scattering rates are $\nu_{jj}^{\mathrm{tr}}/\nu_{jj} = 0.1$ and $\nu_{12}^{\mathrm{tr}}/\nu_{12} = 0.1$. The part (a) shows both ξ (solid line) and classical absorption ξ_0 (dashed line) at $\nu_{11} = \nu_{22} = \nu_{12} = 0.09$ meV. The part (b) shows the ratio ξ/ξ_0 at $\nu_{11} = \nu_{22} = \nu_{12} = 0.09$ meV (curve 1), $\nu_{11} = \nu_{22} = 0.12$ meV, $\nu_{12} = 0.06$ meV (curve 2), and $\nu_{11} = \nu_{22} = 0.18$ meV and $\nu_{12} = 0$ (dashed line).

In the region of frequencies not too close to the cyclotron resonance, it is easy to satisfy the condition $|2\pi\sigma_{\pm}(\omega)/c\sqrt{\epsilon}| \ll 1$ when the absorption coefficient is en-

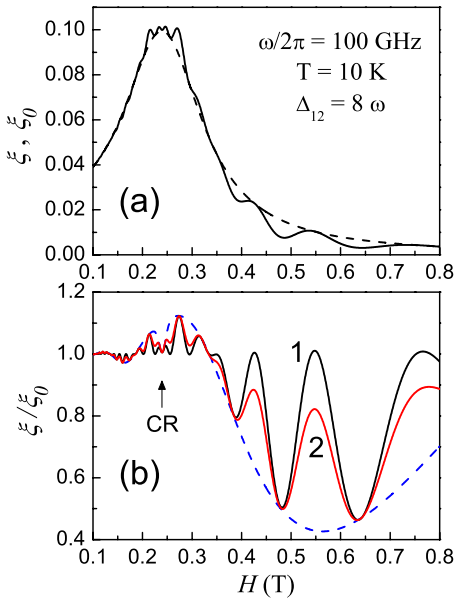

FIG. 3. (Color online) The same as in Fig. 2 at $\omega/2\pi = 100$ GHz.

tirely determined by the real part of the component $\sigma_d(\omega)$: $\xi(\omega)=4\pi\,\mathrm{Re}\,\sigma_d(\omega)/c\sqrt{\epsilon}$. The rotation angle of the polarization of incident radiation (Faraday angle) is described by $\sigma_\perp(\omega)$ in a similar way: $\theta(\omega)=2\pi\,\mathrm{Re}\,\sigma_\perp(\omega)/c\sqrt{\epsilon}$. The energy absorbed in unit time by the unit square of the system (absorption power) is determined by the dissipative component and is equal to $E^2\,\mathrm{Re}\,\sigma_d(\omega)/2$.

IV. STATIC RESISTIVITY

By taking the limit $\omega\to 0$ in the expressions obtained in the previous section, one finds the components of the static (dc) conductivity tensor, σ_d and $\sigma_\perp$. The result can be written as $\sigma_{d(\perp)}=\sigma_{d(\perp)}^{(0)}+\sigma_{d(\perp)}^{(1)}+\sigma_{d(\perp)}^{(2)}$, where $\sigma^{(0)}$ is the classical nonoscillating contribution while $\sigma^{(1)}$ and $\sigma^{(2)}$ describe first- and second-order corrections in Dingle factors. Similar to Sec. III, we consider the limiting cases when analytical expressions for $\sigma^{(1)}$ and $\sigma^{(2)}$ can be written in a simple explicit form. In particular, by applying the approximation of short-range disorder potential, one obtains

$$\begin{vmatrix} \sigma_d^{(0)} \\ \sigma_\perp^{(0)} \end{vmatrix} = \frac{e^2}{m}\sum_{j=1,2}\frac{n_{sj}\tau_j}{(\omega_c\tau_j)^2+1}\begin{vmatrix} 1 \\ \omega_c\tau_j \end{vmatrix}, \tag{64}$$

and

$$\sigma_d^{(1)} = -\mathcal{T}\frac{2e^2}{m}\sum_{j=1,2}\frac{n_{sj}\tau_j}{[(\omega_c\tau_j)^2+1]^2}\left\{2(\omega_c\tau_j)^2 g_j(\varepsilon_F)+[(\omega_c\tau_j)^2 - 1]\frac{\nu_{12}}{\nu_j}[g_{j'}(\varepsilon_F)-g_j(\varepsilon_F)]\right\}, \tag{65}$$

$$\sigma_\perp^{(1)} = \mathcal{T}\frac{2e^2}{m}\sum_{j=1,2}\frac{n_{sj}\omega_c\tau_j^2}{[(\omega_c\tau_j)^2+1]^2}\left\{[(\omega_c\tau_j)^{-2}+3]g_j(\varepsilon_F) + 2\frac{\nu_{12}}{\nu_j}[g_{j'}(\varepsilon_F)-g_j(\varepsilon_F)]\right\}, \tag{66}$$

where $j'\neq j$. The second-order corrections are

$$\begin{vmatrix} \sigma_d^{(2)} \\ \sigma_\perp^{(2)} \end{vmatrix} = \frac{2e^2}{m}\sum_{j=1,2}\frac{n_{sj}\tau_j}{[(\omega_c\tau_j)^2+1]^2}\begin{vmatrix} Z_j^e \\ -\omega_c\tau_j Z_j^o \end{vmatrix}, \tag{67}$$

where Z_j^e and Z_j^o are given by the right-hand sides of Eqs. (53) and (54) in the limit $\omega\to 0$, which is taken by straightforward substitutions $\kappa_{j\pm}^2\to(\omega_c\tau_j)^2$.

The problem of static magnetoconductivity oscillations in the quantum wells with two occupied subbands has been considered previously under the approximation of short-range disorder potential. Microscopic calculations of both components of the conductivity tensor have been carried out in Ref. 14 while the diagonal component also has been calculated in Ref. 13 with a different result. The expressions for the first-order corrections given in Ref. 14 are identical to the results presented by Eqs. (65) and (66). However, Eqs. (37) and (38) of Ref. 14, representing oscillating parts of $\sigma_d^{(2)}$ and $\sigma_\perp^{(2)}$, are different from the results given by Eq. (67) although the basic formalism used in Ref. 14 is the same as in this paper.

The expressions (64)–(67) can be applied for calculations of the resistivity $\rho_d=\sigma_d/(\sigma_d^2+\sigma_\perp^2)$ measured in the samples of Hall bar geometry. A simple analytical expression for ρ_d is obtained under the approximation of symmetric scattering, which is valid for single-quantum wells with uniformly distributed impurities or in DQWs with symmetric doping (see Appendix). Assuming that $|\nu_{11}-\nu_{22}|\ll\nu_{11}+\nu_{22}$, we introduce $(\nu_1+\nu_2)/2=\nu=1/\tau$, and obtain

$$\frac{\rho_d}{\rho_0} = 1-2\mathcal{T}[r_+ g_1(\varepsilon_F)+r_- g_2(\varepsilon_F)]+h_+e^{-2\alpha_1}+h_-e^{-2\alpha_2} + 2h_{12}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}, \tag{68}$$

where $\rho_0=(e^2\pi m_s/m)^{-1}$ is the zero-field Drude resistivity,

$$r_\pm = 1\pm\frac{n_{s1}-n_{s2}}{n_s}\left(1-\frac{\nu_{12}}{\nu}\right), \tag{69}$$

$$h_\pm = 1-\frac{\nu_{12}}{\nu}-\frac{1}{(\omega_c\tau)^2+1}\left(1-2\frac{\nu_{12}}{\nu}+2\frac{\nu_{12}^2}{\nu^2}\right)\pm\frac{n_{s1}-n_{s2}}{n_s} \times\left[1-\frac{\nu_{12}}{\nu}-\frac{1}{(\omega_c\tau)^2+1}\left(1-2\frac{\nu_{12}}{\nu}\right)\right], \tag{70}$$

and

$$h_{12} = \frac{\nu_{12}}{\nu}\left[1-\frac{2}{(\omega_c\tau)^2+1}\left(1-\frac{\nu_{12}}{\nu}\right)\right]. \tag{71}$$

The first-order contribution in Eq. (68) describes the Shubnikov-de Haas oscillations for two-subband systems. This contribution vanishes with increasing temperature. The second-order contribution contains the MIS oscillations term together with the terms responsible for positive magnetoresistance. The positive magnetoresistance, caused by localization of electrons in the magnetic field, of course remains in the case of single-subband occupation.[25] The coefficient h_{12} at the oscillating term does not depend on the difference in subband occupations. In DQWs this difference can be neglected, $|n_{s1}-n_{s2}|\ll n_s$, which leads to $r_+=r_-=1$ and $h_+=h_-=h$. Neglecting also the difference in Dingle exponents, $e^{-\alpha_1}\simeq e^{-\alpha_2}$, one gets[15]

$$\frac{\rho_d}{\rho_0}\simeq 1-4e^{-\alpha}\mathcal{T}\cos\frac{2\pi\varepsilon_F}{\omega_c}\cos\frac{\pi\Delta_{12}}{\omega_c} + e^{-2\alpha}\left[h+h_{12}\cos\frac{2\pi\Delta_{12}}{\omega_c}\right], \tag{72}$$

where $\alpha=\alpha_1=\alpha_2$ and

$$\begin{vmatrix} h \\ h_{12} \end{vmatrix} = 1\pm\delta^2-\frac{1\pm\delta^4}{(\omega_c\tau)^2+1}. \tag{73}$$

The Fermi energy in Eq. (72) is counted from $(\varepsilon_1+\varepsilon_2)/2$. The scattering rates are expressed through the parameter δ characterizing tunnel coupling in DQWs (see Appendix), according to $\nu_{12}/\nu=(1-\delta^2)/2$. In balanced DQWs ($\delta=0$), the probability of intersubband scattering is equal to the probability of intrasubband scattering. As the system is driven out

of the balance by the gates (δ^2 increases), the wave functions for subbands 1 and 2 become localized in the different wells. Therefore, the probability of intersubband scattering decreases so the amplitude of the MIS oscillations is reduced and the nonoscillating positive magnetoresistance becomes larger.

In the case of arbitrary disorder potential, a simple description of the resistivity is obtained under the approximations $|p_{F1}^2-p_{F2}^2| \ll p_{F1}^2+p_{F2}^2$ and $|\nu_{11}^{\rm tr}-\nu_{22}^{\rm tr}| \ll \nu_{11}^{\rm tr}+\nu_{22}^{E}$, relevant to the case of symmetrically doped DQWs. The components σ_d and $\sigma_\perp$ are easily obtained by taking the limit $\omega \to 0$ in Eqs. (55)–(61) of Sec. III. The resistivity takes the form of Eq. (68) where $\rho_0=(e^2\tau_{\rm tr}n_s/m)^{-1}$, $r_+=r_-=1$, and

$$h_+ = h_- = 1 - \frac{\nu_{12}^{\rm tr}}{\nu_{\rm tr}} - \frac{\mu_+}{2}, \quad h_{12} = \frac{\nu_{12}^{\rm tr}}{\nu_{\rm tr}} - \frac{\mu_-}{2}, \tag{74}$$

and

$$\mu_\pm = \frac{1}{(\omega_c\tau_{\rm tr})^2+1} \pm \frac{(1-2\nu_{12}^{\rm tr}/\nu_{\rm tr})^2(1+2\tilde{\nu}_{12}/\nu_{\rm tr})}{(\omega_c\tau_{\rm tr})^2+(1+2\tilde{\nu}_{12}/\nu_{\rm tr})^2}. \tag{75}$$

In the classically strong magnetic fields, $\omega_c\tau_{\rm tr} \gg 1$, the field-dependent corrections $\mu_\pm$ can be neglected.

In the systems with long-range disorder where the transport times are much greater than the quantum lifetimes, the oscillating terms in the resistivity should be considered in the limit of classically strong magnetic fields; otherwise these terms are exponentially small. In this approximation, one can derive a more general expression for the resistivity, valid for arbitrary ratios p_{F1}^2/p_{F2}^2 and $\nu_{11}^{\rm tr}/\nu_{22}^{\rm tr}$. It takes the form

$$\rho_d = \rho_d^{(0)} + \rho_d^{(1)} + \rho_d^{(2)},$$

$$\rho_d^{(1)} = -\mathcal{T}\frac{2m}{e^2 n_s}\sum_j \left[\frac{2n_{sj}}{n_s}\nu_{jj}^{\rm tr} + \nu_{12}^{\rm eff}\right] g_j(\varepsilon_F),$$

$$\rho_d^{(2)} = \frac{2m}{e^2 n_s}\left[\frac{n_{s1}}{n_s}\nu_{11}^{\rm tr}e^{-2\alpha_1} + \frac{n_{s2}}{n_s}\nu_{22}^{\rm tr}e^{-2\alpha_2} + \nu_{12}^{\rm eff}e^{-\alpha_1-\alpha_2}\cos\frac{2\pi\Delta_{12}}{\omega_c}\right], \tag{76}$$

where $\rho_d^{(0)}$ is the classical (nonoscillating) resistivity and $\nu_{12}^{\rm eff}=\nu_{12}-(2\sqrt{n_{s1}n_{s2}}/n_s)\tilde{\nu}_{12}$ is the effective intersubband scattering rate, which is close to the above-defined intersubband transport scattering rate $\nu_{12}^{\rm tr}$ at $n_{s1} \simeq n_{s2}$. Notice that one can consider the expression for $\nu_{12}^{\rm eff}$ as an alternative definition of the transport rate for intersubband scattering. The classical resistivity $\rho_d^{(0)}$ depends on the magnetic field in the case of two populated subbands[26] (as in the case of two groups of carriers with different mobilities). The expression (76) shows that the contribution of the MIS oscillations in comparison to the nonoscillating second-order terms is determined by the ratio of intersubband and intrasubband transport scattering rates. In DQWs this ratio can approach to unity.

The static magnetoresistance plots calculated according to the general expression [Eq. (33)] at $\omega=0$ are given in Fig. 4. The subband separation is chosen to be 4 meV, which is typical for DQWs. The resistivity is normalized to its zero-field value. The calculations correspond to $T=10$ K when the Shubnikov-de Haas oscillations are completely suppressed in the chosen interval of magnetic fields and only the MIS oscillations remain. The low-temperature resistivity, when the Shubnikov-de Haas oscillations are important, is shown in the inset. The plots demonstrate a decrease in the MIS oscillation amplitude when the relative contribution of intersubband scattering decreases (curve 2) and demonstrate an increase in the nonoscillating positive magnetoresistance for asymmetric scattering (curve 3). Application of the simple approximate expression [Eq. (76)] gives practically no difference in comparison to the results shown in Fig. 4, thereby confirming high reliability of Eq. (76) for description of the static magnetoresistance in the systems with two populated subbands.

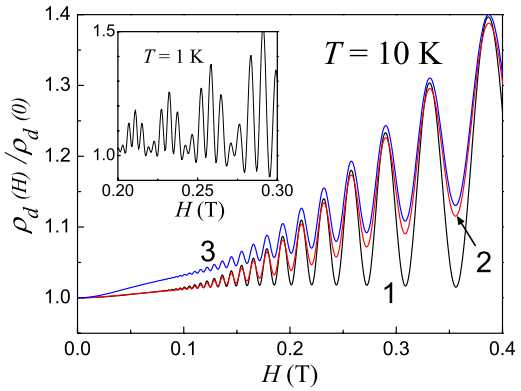

FIG. 4. (Color online) The MIS oscillations of resistivity for GaAs quantum wells with electron density $n_s=10^{12}$ cm^{-2}, subband separation $\Delta_{12}=4$ meV, and averaged scattering rate $(\nu_1+\nu_2)/2=0.18$ meV. The case of long-range scattering potentials is assumed: $\nu_{jj}^{\rm tr}/\nu_{jj}=0.1$ and $\nu_{12}^{\rm tr}/\nu_{12}=0.1$. The three different lines correspond to (1) $\nu_{11}=\nu_{22}=\nu_{12}=0.09$ meV, (2) $\nu_{11}=\nu_{22}=0.12$ meV, $\nu_{12}=0.06$ meV, and (3) $\nu_{11}=0.16$ meV, $\nu_{22}=0.08$ meV, and $\nu_{12}=0.06$ meV. The inset shows the low-temperature resistivity, when the Shubnikov-de Haas oscillations dominate, for $\nu_{11}=\nu_{22}=\nu_{12}=0.09$ meV.

V. AC RESPONSE FOR PERPENDICULAR POLARIZATION

If the vector of electric field has a component $\mathbf{E}_z$ perpendicular to the 2D plane, this component also induces absorption of electromagnetic radiation in quantum wells. The effect occurs because z component of the velocity operator $\hat{\mathbf{v}}$ has nondiagonal matrix elements in the subband basis $|j\rangle$. If only the lowest subband is populated, the radiation-induced intersubband transitions in the linear-response regime can take place when the frequency of electromagnetic radiation exceeds the energy distance between the second subband and position of the Fermi level in the first subband. When two subbands are populated, the transitions take place at arbitrary frequency ω although the absorption strongly depends on the difference between the subband separation Δ_{12} and ω.

The linear ac response to the field $\mathbf{E}_z$ is characterized by the real part of the component $\sigma_{zz}(\omega)$ of the conductivity tensor [Eq. (4)]. In the chosen basis $|jnp_y\rangle$, the z-velocity

operator has matrix elements $\langle jnp_y|\hat{v}_z|j'n'p_y'\rangle=\delta_{p_yp_y'}\delta_{nn'}\mathrm{v}_{jj'}$, where $\mathrm{v}_{jj'}$ is nonzero at $j\neq j'$. Therefore,

$$\mathrm{Re}\ \sigma_{zz}(\omega)=\frac{e^2}{\pi\omega}\sum_{jj_1(j\neq j_1)}\sum_{j'j_1'(j'\neq j_1')}\mathrm{v}_{j_1j}\mathrm{v}_{j'j_1'}\int d\varepsilon(f_\varepsilon-f_{\varepsilon+\omega})\times\mathrm{Re}[\tilde{Q}^{AR}_{jj_1,j'j_1'}(\varepsilon,\omega)-\tilde{Q}^{AA}_{jj_1,j'j_1'}(\varepsilon,\omega)],\tag{77}$$

where

$$\tilde{Q}^{ss'}_{jj_1,j'j_1'}(\varepsilon,\omega)=\frac{1}{L^2}\sum_{nn'}\sum_{p_yp_y'}\langle\langle G^{jj',s}_{\varepsilon}(np_y,n'p_y')\times G^{j_1'j_1,s'}_{\varepsilon+\omega}(n'p_y',np_y)\rangle\rangle.\tag{78}$$

Considering the correlators in Eq. (78), we use the resonance approximation, i.e., retain the contribution which is the largest under the intersubband resonance condition $|\omega-\Delta_{12}|\ll\Delta_{12}$. Therefore, only the terms with $j'=j=1$ and $j_1'=j_1=2$ should be taken into account in Eq. (77). Introducing the function

$$K^{12,ss'}_{\varepsilon\varepsilon+\omega}(n,n')=\frac{1}{L^2}\sum_{p_yp_y'}\langle\langle G^{11,s}_{\varepsilon}(np_y,n'p_y')G^{22,s'}_{\varepsilon+\omega}(n'p_y',np_y)\rangle\rangle,\tag{79}$$

we evaluate it in the ladder approximation leading to the following equation:

$$K^{12,ss'}_{\varepsilon\varepsilon+\omega}(n,n')=\frac{1}{2\pi l_H^2}G^{1,s}_{\varepsilon n}G^{2,s'}_{\varepsilon+\omega n}\left[\delta_{nn'}+\sum_{n_1}\int_0^\infty du\mathrm{w}_{12}\times(\sqrt{2u}/l_H)\Phi_{nn_1}(u)\mathrm{K}^{12,ss'}_{\varepsilon\varepsilon+\omega}(n_1,n')\right],\tag{80}$$

where $\mathrm{w}_{12}(q)=\int d\Delta\mathbf{r}e^{-i\mathbf{q}\cdot\Delta\mathbf{r}}\langle\langle V_{11}(\mathbf{r}+\Delta\mathbf{r})V_{22}(\mathbf{r})\rangle\rangle$. Finally,

$$\tilde{Q}^{ss'}_{12,12}(\varepsilon,\omega)=\frac{m}{[\tilde{\Pi}^{ss'}_{12}(\varepsilon,\omega)]^{-1}-\bar{\nu}_{12}(\varepsilon)},\tag{81}$$

where

$$\tilde{\Pi}^{ss'}_{12}(\varepsilon,\omega)=\frac{\omega_c}{2\pi}\sum_n G^{1,s}_{\varepsilon,n}G^{2,s'}_{\varepsilon+\omega,n}=\frac{S^{1,s}_\varepsilon-S^{2,s'}_{\varepsilon+\omega}}{\omega-\Delta_{12}+\Sigma^{1,s}_\varepsilon-\Sigma^{2,s'}_{\varepsilon+\omega}},\tag{82}$$

and

$$\bar{\nu}_{12}(\varepsilon)=m\int_0^{2\pi}\frac{d\theta}{2\pi}\mathrm{w}_{12}(\sqrt{p_{1\varepsilon}^2+p_{2\varepsilon+\omega}^2-2p_{1\varepsilon}p_{2\varepsilon+\omega}\cos\theta}).\tag{83}$$

The dimensionless sums $S^{j,s}_\varepsilon$ and the self-energies $\Sigma^{j,s}_\varepsilon$ are defined by Eq. (14).

According to Eq. (77), the remaining calculation of $\mathrm{Re}\ \sigma_{zz}(\omega)$ is reduced to integration of the factor $(f_\varepsilon-f_{\varepsilon+\omega})\mathrm{Re}[\tilde{Q}^{AR}_{12,12}(\varepsilon,\omega)-\tilde{Q}^{AA}_{12,12}(\varepsilon,\omega)]$ over energy. Similar as in Sec. III, we assume that the energy dependence of the scattering rates can be neglected in the interval ω near the Fermi energy (this approximation is always valid in the case of short-range disorder potential). Neglecting the oscillating terms proportional to the factor $\mathcal{T}$, one obtains

$$\frac{\mathrm{Re}\ \sigma_{zz}(\omega)}{\mathrm{Re}\ \sigma^{(0)}_{zz}(\omega)}=1+\mathcal{B}_1e^{-2\alpha_1}+\mathcal{B}_2e^{-2\alpha_2}+\mathcal{B}_{12}e^{-\alpha_1-\alpha_2},\tag{84}$$

where

$$\mathcal{B}_j=\frac{\nu_{12}}{\nu_r}\left[\frac{\beta_r^2-1}{\beta_r^2+1}-2\frac{3\beta_r^2-1}{(\beta_r^2+1)^2}\frac{\nu_{jj}-\bar{\nu}_{12}}{\nu_r}\right]\cos\frac{2\pi\omega}{\omega_c}+\frac{\nu_{12}}{\nu_r}\beta_r\left[\frac{2}{\beta_r^2+1}+2\frac{\beta_r^2-3}{(\beta_r^2+1)^2}\frac{\nu_{jj}-\bar{\nu}_{12}}{\nu_r}\right]\sin\frac{2\pi\omega}{\omega_c},\tag{85}$$

$$\begin{aligned}\mathcal{B}_{12}=2\Bigg[&\frac{\beta_r^2-1}{\beta_r^2+1}\left(1-\frac{\nu_{12}}{\nu_r}\right)-\frac{3\beta_r^2-1}{(\beta_r^2+1)^2}\times\frac{(\nu_{11}-\bar{\nu}_{12})(\nu_{22}-\bar{\nu}_{12})}{\nu_r^2}\Bigg]\cos\frac{2\pi(\omega-\Delta_{12})}{\omega_c}\\&+2\beta_r\Bigg[\frac{2}{\beta_r^2+1}\left(1-\frac{\nu_{12}}{\nu_r}\right)+\frac{\beta_r^2-3}{(\beta_r^2+1)^2}\times\frac{(\nu_{11}-\bar{\nu}_{12})(\nu_{22}-\bar{\nu}_{12})}{\nu_r^2}\Bigg]\sin\frac{2\pi(\omega-\Delta_{12})}{\omega_c}\\&-2\left(\frac{\nu_{12}}{\nu_r}\right)^2\frac{3\beta_r^2-1}{(\beta_r^2+1)^2}\cos\frac{2\pi(\omega+\Delta_{12})}{\omega_c}+2\left(\frac{\nu_{12}}{\nu_r}\right)^2\beta_r\frac{\beta_r^2-3}{(\beta_r^2+1)^2}\sin\frac{2\pi(\omega+\Delta_{12})}{\omega_c},\end{aligned}\tag{86}$$

and $\beta_r=(\omega-\Delta_{12})/\nu_r$. The scattering rate $\nu_r=(\nu_1+\nu_2)/2-\bar{\nu}_{12}$ characterizes the collision-induced broadening of the intersubband resonance.[19] The response at zero magnetic field is described by the expression

$$\mathrm{Re}\ \sigma^{(0)}_{zz}(\omega)=\frac{e^2m|\mathrm{v}_{12}|^2\nu_r}{\pi[(\omega-\Delta_{12})^2+\nu_r^2]}.\tag{87}$$

If only the lowest subband is populated, this expression should be multiplied by the factor $(\varepsilon_F-\varepsilon_1)/\Delta_{12}=\pi n_s/m\Delta_{12}$, and one obtains the commonly known result (see, for example, Ref. 19).

The absorption of electromagnetic radiation polarized perpendicular to the well plane is described by the absorption coefficient

$$\xi^<(\omega)=\frac{4\pi}{c\sqrt{\epsilon}}\mathrm{Re}\ \sigma_{zz}(\omega),\tag{88}$$

and the absorption power is $E_z^2\ \mathrm{Re}\ \sigma_{zz}(\omega)/2$. To estimate the absorption strength, we give expressions for the intersubband matrix elements of the velocity operator[27] for the systems shown in Fig. 1. For deep rectangular quantum well, $|\mathrm{v}_{12}|=8/3ma$, where a is the well width. For DQWs, $|\mathrm{v}_{12}|\simeq Z\sqrt{\Delta_{12}^2-\Delta^2}/2$, where Z is the distance between the centers of the wells and Δ is the energy defined in Appendix.

The structure of the expression for Re $\sigma_{zz}(\omega)$ is similar to that of $\sigma_d^{(\pm)}(\omega)$ and $\sigma_\perp^{(\pm)}(\omega)$ described in Sec. III. The oscillating response contains a combination of the terms [Eq. (1)] and interference terms [Eq. (3)]; all these terms are quadratic in Dingle factors. Note that, however, in weak (classical) magnetic fields when the oscillations of the density of states can be neglected, Re $\sigma_{zz}(\omega)$ is equal to Re $\sigma_{zz}^{(0)}(\omega)$ and does not depend on the magnetic field. In contrast, $\sigma_d^{(\pm)}(\omega)$ and $\sigma_\perp^{(\pm)}(\omega)$ remain sensitive to magnetic fields in the classical field region.

The coefficients in Eq. (84) are given by simple expressions in the frequency region not too close to the intersubband resonance, $\beta_r^2 \gg 1$. Assuming this condition in Eqs. (85) and (86), we rewrite them as

$$\mathcal{B}_1 = \mathcal{B}_2 \simeq \frac{\nu_{12}}{\nu_r}\cos\frac{2\pi\omega}{\omega_c}, \tag{89}$$

and

$$\mathcal{B}_{12} \simeq 2\left(1 - \frac{\nu_{12}}{\nu_r}\right)\cos\frac{2\pi(\omega - \Delta_{12})}{\omega_c}. \tag{90}$$

Since the condition $|\omega - \Delta_{12}| \ll \Delta_{12}$ is assumed, the oscillations of $\sigma_{zz}(\omega)$ include a fast-oscillating component $\cos(2\pi\omega/\omega_c)$ modulated by a slow interference component $\cos[2\pi(\omega-\Delta_{12})/\omega_c]$.

Far from the intersubband resonance (for example, when ω is much smaller than Δ_{12}), the absorption considered in this section is weak and the main absorption is caused by the parallel component of the radiation field (see Sec. III).

VI. CONCLUSIONS

The basic property that makes the quantum wells with two populated subbands different from the systems with single-subband occupation is the coupling between the subbands. The effects described in this paper are caused by the coupling owing to intersubband scattering of electrons.[28] The influence of the Landau quantization on the dynamic conductivity in the presence of intersubband scattering is a complicated problem, which is solved analytically in this paper for the case of sufficiently weak magnetic fields when the Landau levels are overlapping. Previous theoretical results, such as the oscillating ac magnetoconductivity of quantum wells with single populated subband (Ref. 5) and the classical dc magnetoresistance of quantum wells with two populated subbands (Ref. 26), follow from the presented theory as limiting cases. The analytical approach used in this paper implies expansion of the components of the conductivity tensor in powers of the Dingle factors. In the first order of this expansion, one obtains the magnetic oscillations caused by sequential passage of the Landau levels through the Fermi level. These oscillations are exponentially suppressed with increasing temperature owing to the thermal smearing of the Fermi surface. In the second order, there exist the magnetic oscillations that are irrelevant to the position of the Landau levels with respect to the Fermi surface and, therefore, survive at high temperatures. These important oscillations are caused by the resonances, which are classified into two groups. The first group, $\omega = k\omega_c$ (k is integer), is also relevant to the systems with single-subband occupation and can be viewed as the cyclotron resonance harmonics. The second group includes the combined resonances at $|\omega \pm \Delta_{12}| = k\omega_c$; they are specific for the systems with two populated subbands and depend on the subband separation Δ_{12}. In the static limit, $\omega = 0$, these resonance conditions are reduced to $\Delta_{12} = k\omega_c$, which correspond to the maxima of the MIS oscillations of resistivity, recently observed in DQWs.[15] The microscopic theory of these oscillations is developed in Sec. IV.

The combined resonances manifest themselves in oscillations of the absorption of electromagnetic radiation incident on the quantum-well layer. Such oscillations should exist both for parallel and perpendicular polarizations of the radiation field with respect to the layer. However, if the frequency ω is considerably smaller than the subband separation as in the case of microwave excitation, the absorption caused by the parallel component of the radiation field is much stronger. The magnetic oscillations of this absorption can be viewed as the MIS oscillations, $\propto \cos(2\pi\Delta_{12}/\omega_c)$, modulated by the component $\propto \cos(2\pi\omega/\omega_c)$, and the whole oscillation pattern is superimposed on the cyclotron resonance peak; see Figs. 2 and 3. The modulation looks like a periodic inversion of the groups of MIS oscillation peaks; if $\omega = \Delta_{12}/2$, each second peak is inverted. It is reasonable to presume that similar oscillatory patterns should appear in the dc resistivity under microwave photoexcitation because the oscillating dynamic conductivity determines the power of electromagnetic radiation absorbed by the electron system. On the other hand, the problem of microwave photoresistance implies consideration of nonequilibrium electron distribution and is essentially different from the problem of linear ac photoconductivity considered in this paper.

Among the systems with two populated subbands, the DQWs are the most convenient objects for experimental observation of the phenomena considered in this paper. First of all, the two-subband occupation in DQWs can be reached even at relatively small electron densities. Typically, the Fermi energy in DQWs is considerably larger than the subband separation so the oscillations associated with intersubband coupling have a large period and they are easily distinguishable from the Shubnikov-de Haas oscillations. Observation of these oscillations in weak magnetic fields requires large quantum lifetimes, which are attainable in the modulation-doped structures, and high probability of intersubband scattering. Both these conditions can be satisfied in DQWs because, owing to a small subband separation, the intersubband scattering does not require a large momentum transfer and its probability is comparable to the probability of intrasubband scattering. Another advantage of DQWs is a possibility to control both the intersubband scattering and subband separation via the gate voltage.

Finally, let us briefly discuss the approximations made in the paper. The analytical approach to the problem is valid in weak enough magnetic fields when many Landau levels are populated, $\omega_c \ll \varepsilon_F - \varepsilon_j$. Furthermore, it is assumed that $\omega_c\tau_j < \pi$ so the Dingle factors are small and that the magnetic length is large in comparison to the correlation lengths of the random potential. It is also assumed that $\Delta_{12}\tau_j \gg 1$, i.e., the collision-induced broadening of the energy spectrum is much

smaller than the subband separation. The case of degenerate electron gas, when the temperature T is much smaller than the Fermi energy, is assumed. The frequency ω is assumed to be small enough to neglect the energy dependence of the scattering rates in the interval ω around the Fermi surface. Next, only elastic scattering by random static potential has been taken into account because the scattering by phonons is much weaker at the temperatures of about 10 K and lower. The most serious approximation is the neglect of electron-electron interaction. This interaction leads to a shift in the position of the intersubband resonance (see Ref. 18 and references therein), causes a decrease in the quantum lifetime of electrons with increasing temperature (see Ref. 15 and references therein), and can give rise to an additional oscillating contribution to the ac conductivity.[29] Therefore, the effects of electron-electron interaction should be taken into account in analysis of experimental data. Nevertheless, these effects are not expected to cause a qualitative modification of the oscillatory phenomena studied in this paper.

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APPENDIX: RANDOM POTENTIAL CORRELATORS IN SINGLE- AND DOUBLE-QUANTUM WELLS

The correlation functions $w_{jj'}(q)$ depend on the symmetry of the envelope wave functions $\psi_j(z)$ describing confinement of electron states in the subbands (these functions always can be chosen real). According to the definition given after Eq. (13),

$$w_{jj'}(q) = \int dz \int dz' \psi_j(z)\psi_{j'}(z)\psi_{j'}(z')\psi_j(z') \times L^{-2}\langle\langle V_{\mathbf{q}}(z)V_{-\mathbf{q}}(z')\rangle\rangle, \tag{A1}$$

where $V_{\mathbf{q}}(z)$ are the 2D Fourier transforms of the random static potential $V(\mathbf{r},z)$.

The correlation function $\mathrm{w}_{12}(q)$, appearing in Sec. V, is written as

$$\mathrm{w}_{12}(q) = \int dz \int dz' \psi_1^2(z)\psi_2^2(z')L^{-2}\langle\langle V_{\mathbf{q}}(z)V_{-\mathbf{q}}(z')\rangle\rangle. \tag{A2}$$

If the potential $V(\mathbf{r},z)$ is created by impurities numbered by the index i, one has $V(\mathbf{r},z)=\Sigma_i v_i(\mathbf{r}-\mathbf{r}_i,z-z_i)$, where v_i is the single-impurity potential. Therefore,

$$V_{\mathbf{q}}(z) = \sum_i v_{i,\mathbf{q}}(z-z_i)e^{-i\mathbf{q}\cdot\mathbf{r}_i}. \tag{A3}$$

In the case of identical impurities, when $v_{i,\mathbf{q}}(z-z_i)=v_{\mathbf{q}}(z-z_i)$, the random potential correlators in Eqs. (A1) and (A2) are expressed through the integral over spatial distribution of impurities, $n_{\mathrm{im}}(z_i)$:

$$L^{-2}\langle\langle V_{\mathbf{q}}(z)V_{-\mathbf{q}}(z')\rangle\rangle = \int dz_i n_{\mathrm{im}}(z_i)v_{\mathbf{q}}(z-z_i)v_{-\mathbf{q}}(z'-z_i). \tag{A4}$$

In the limit of short-range impurity potential, one can apply the δ-function approximation: $v_{\mathbf{q}}(z-z_i)=v_0\delta(z-z_i)$. Thus,

$$w_{jj'} = \int dz \psi_j^2(z)\psi_{j'}^2(z)n_{\mathrm{im}}(z)|v_0|^2, \quad \mathrm{w}_{12}=w_{12}. \tag{A5}$$

These correlators are independent of q but still determined by the envelope wave functions and impurity distribution. For a deep single-quantum well of rectangular shape [Fig. 1(a)], one can use $\psi_1(z)=\sqrt{2/a}\sin(\pi z/a)$ and $\psi_2(z)=\sqrt{2/a}\sin(2\pi z/a)$, where a is the well width $(0<z<a)$. Therefore, if the short-range impurities are homogeneously distributed inside the well, which means that $n_{\mathrm{im}}(z)=n_{\mathrm{im}}$ is z independent, a straightforward integration in Eq. (A5) leads to

$$w_{11}=w_{22}=\frac{3}{2a}w_0, \quad w_{12}=\frac{1}{a}w_0, \tag{A6}$$

where $w_0=n_{\mathrm{im}}|v_0|^2$.

In DQWs, which consist of two quantum wells separated by a narrow barrier, the symmetry of electron states is determined by the tunnel coupling. The envelope wave functions of two subbands are represented by superpositions of normalized single-well ground-state envelope wave functions $\varphi_l(z)$ and $\varphi_r(z)$, where the indices l and r denotes left and right quantum-well layers:

$$\psi_1(z) = \eta_-\varphi_l(z) + \eta_+\varphi_r(z),$$

$$\psi_2(z) = \eta_+\varphi_l(z) - \eta_-\varphi_r(z). \tag{A7}$$

The coefficients in these expressions are given by

$$\eta_\pm = \frac{1}{\sqrt{2}}\sqrt{1 \pm \frac{\Delta}{\Delta_{12}}}. \tag{A8}$$

The physical meaning of the energy Δ is the subband separation in the absence of tunnel coupling. This energy can be continuously varied by the bias applied to external gates (if present). If $\Delta=0$, the subband separation Δ_{12} is entirely determined by tunneling and is usually denoted by Δ_{SAS} [symmetric-antisymmetric splitting (SAS)] because it has direct meaning of the energy gap between the states with symmetric and antisymmetric wave functions, $\psi_1(z)=[\varphi_l(z)+\varphi_r(z)]/\sqrt{2}$ and $\psi_2(z)=[\varphi_l(z)-\varphi_r(z)]/\sqrt{2}$. In the general case, $\Delta_{12}=\sqrt{\Delta_{SAS}^2+\Delta^2}$.

It is often convenient to represent the random potential correlators $w_{jj'}(q)$ for DQWs in terms of intralayer and interlayer correlators $w_{kk'}(q)=L^{-2}\langle\langle V_{k,\mathbf{q}}V_{k',-\mathbf{q}}\rangle\rangle$, where $k=l,r$, by introducing the effective random 2D potentials in the layers, $V_{k,\mathbf{q}}=\int dz\varphi_k^2(z)V_{\mathbf{q}}(z)$:

$$w_{11}(q) = w_{ll}(q)\eta_-^4 + w_{rr}(q)\eta_+^4 + 2w_{lr}(q)\eta_-^2\eta_+^2,$$

$$w_{22}(q) = w_{ll}(q)\eta_+^4 + w_{rr}(q)\eta_-^4 + 2w_{lr}(q)\eta_-^2\eta_+^2,$$

$$w_{12}(q) = [w_{ll}(q) + w_{rr}(q) - 2w_{lr}(q)]\eta_-^2\eta_+^2. \quad \text{(A9)}$$

For an arbitrary random potential $V(\mathbf{r},z)$, there exists the case when $w_{11}(q)=w_{22}(q)$. This occurs at $\Delta=0$ (balanced DQWs) when $\eta_+^2=\eta_-^2=1/2$. For any random potential satisfying the property $w_{ll}(q)=w_{rr}(q)$ (for example, created by impurities symmetrically distributed with respect to the DQW's symmetry plane), the property $w_{11}(q)=w_{22}(q)$ is always valid. If the interlayer correlators $w_{lr}(q)$ are zero and the scattering is symmetric, $w_{ll}(q)=w_{rr}(q)\equiv w(q)$, there exist simple relations following from Eq. (A9):

$$w_{jj}(q) = w(q)\frac{1+\delta^2}{2}, \quad w_{12}(q) = w(q)\frac{1-\delta^2}{2}, \quad \text{(A10)}$$

where $\delta=\Delta/\Delta_{12}$. The interlayer correlators can be neglected in the case of short-range scattering potential or when the scattering is caused by interface roughness. According to Eq. (A9), all the correlation functions $w_{jj'}(q)$ are equal to each other at $\Delta=0$ and $w_{lr}(q)=0$. An increase in Δ leads to a decrease in $w_{12}(q)$ because of suppression of interlayer coupling. As a result, the intersubband scattering is also suppressed.

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