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Partial focusing by indefinite complementary metamaterials

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We have experimentally realized a two-dimensional partial focusing within a planar waveguide using complementary indefinite metamaterials. When the electric fields emitted from the dipole are TE polarized, the focusing condition requires negative magnetic response in the propagation direction of the waveguide, which can be achieved by the complementary electric resonator (CELC) structures. We have carefully designed the experimental configurations and the dimensions for the CELC structures. The experimental result is consistent with the theoretical prediction, which validates the partial focusing phenomenon.

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In the past few years, indefinite metamaterials have aroused more and more interest within the scientific community. Similar to the isotropic metamaterials, indefinite metamaterials are also composed of electric or magnetic resonant units. However, their effective permittivity or permeability usually take the form of tensors.[1–3] The indefinite metamaterials can find more exotic characteristics than the isotropic ones due to their complex dispersion relations.[3] Some interesting applications have been proposed based on these artificial materials, such as the invisible cloaks and energy localization devices.[4,5] Indefinite metamaterials can also be applied in planar microwave circuits, which are usually made up of two-dimensional (2D) periodic units such as the inductor-capacitor grids.[6]

We should note that in the forementioned two-dimensional indefinite metamaterial (TIM), waves or signals can only be guided along the circuit components and they cannot penetrate the areas within the grids. Therefore this kind of TIM is actually only valid from the circuit point of view. However, there is an alternative way to realize TIM: the complementary resonant structures, which were first investigated by Falcone *et al.*[7] in 2003. The complementary structures can be regarded as the TIM units when their dimensions are much smaller than the wavelength of the TEM mode in the waveguide.[8,9] In this Rapid Communication, we will use the complementary TIM to realize partial focusing, which was investigated in Ref. 10 using the three-dimensional split-ring resonators. The focusing phenomena in the indefinite metamaterials have also been investigated by other groups with different techniques.[11–14]

The problem geometry under consideration is shown in Fig. 1, where the planar waveguide is divided into three regions. The left and the right regions are occupied by air, with the permittivity ϵ_0 and the permeability μ_0. The middle region is covered by an indefinite slab, with the permittivity tensor $\bar{\bar{\epsilon}}=\hat{x}\hat{x}\epsilon_x+\hat{y}\hat{y}\epsilon_y+\hat{z}\hat{z}\epsilon_z$ and the permeability tensor $\bar{\bar{\mu}}=\hat{x}\hat{x}\mu_x+\hat{y}\hat{y}\mu_y+\hat{z}\hat{z}\mu_z$. An electric dipole is placed before the slab, whose ends are connected with the top and bottom perfectly electric conducting (PEC) plates, respectively, yielding TE to y waves within the planar waveguide.

The dispersion relation for the TE wave in the indefinite medium can be written as[3]

$$\frac{k_x^2}{\mu_y}+\frac{k_y^2}{\mu_x}=\left(\frac{\omega}{c}\right)^2\epsilon_z. \quad (1)$$

When $\mu_y<0$, $\mu_z>0$, and $\epsilon_x>0$, it is obvious that the dispersion curve for Eq. (1) is a hyperbola. In such a case, it can be easily shown that the phase velocity of the incident waves will undergo a positive refraction, while the group (or energy) velocity will undergo a negative refraction at the boundary between air and the indefinite medium, which will help refocus the incident waves inside or outside the slab.[10] In Ref. 10, a ray-tracing diagram was given to describe the propagation of the waves emitted from a source in front of the indefinite slab, showing the occurrence of negative refraction at the interface between air and the indefinite medium and also the existence of partial focusing for incident waves.

We have selected the complementary electric resonator (CELC) structure as the basic unit of the artificial indefinite medium, as illustrated in Fig. 2. The CELC structure refers to the planar-waveguide unit with the ELC pattern etched on the bottom metallic plate. When the working frequency is selected to be lower than the cut-off frequency for the second-order mode (TE mode), only the dominant TEM mode could be supported in the waveguide, the corresponding electric field is just parallel to the axis of the CELC unit in the z direction. From Babinet's principle, the magnetic response may be produced under the excitation of the external electric field along the y direction.

We remark that the components of the permeability tensor in the x and y directions are different since the shape of the

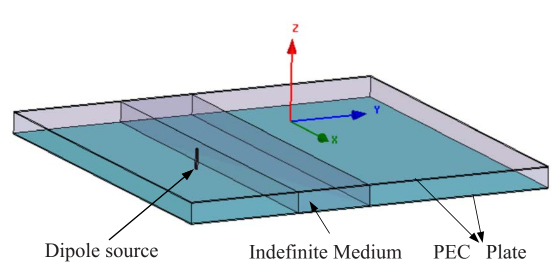


FIG. 1. (Color online) Theoretical model for partial focusing in a planar waveguide. An indefinite metamaterial slab is inserted into the waveguide, with a dipole source excitation.

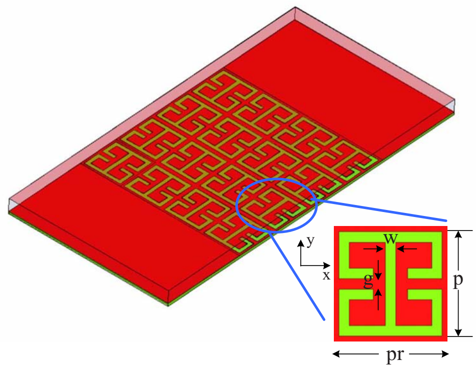


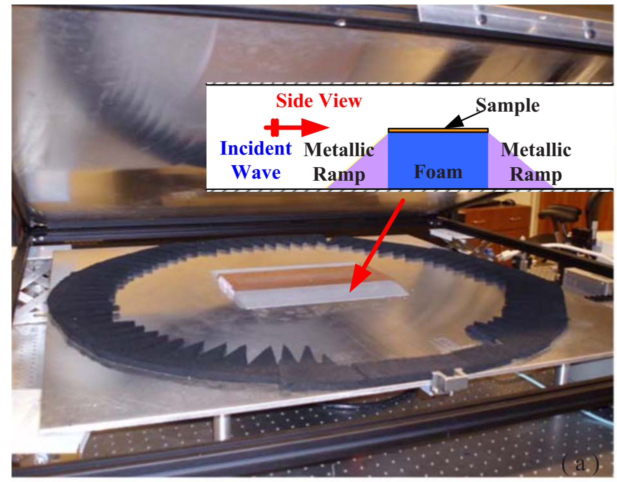
FIG. 2. (Color online) The CELC structure is chosen as the unit cell to realize the indefinite metamaterial.

CELC unit is not identical in these two directions. Therefore the effective medium, which is composed of the CELC particles, is actually indefinite and it is suitable for the realization of the partial focusing as mentioned earlier. In Fig. 2, the CELC region can be regarded as the indefinite slab in the theoretical model shown in Fig. 1. In order to get the effective permittivity and permeability of the CELC unit, we have made full-wave numerical simulations by a commercial software HFSS to obtain the S parameters for the whole structure. Then, we followed the standard retrieval procedure to get the effective-medium parameters.[15] The detailed description for the simulation set up could be found in Ref. 8.

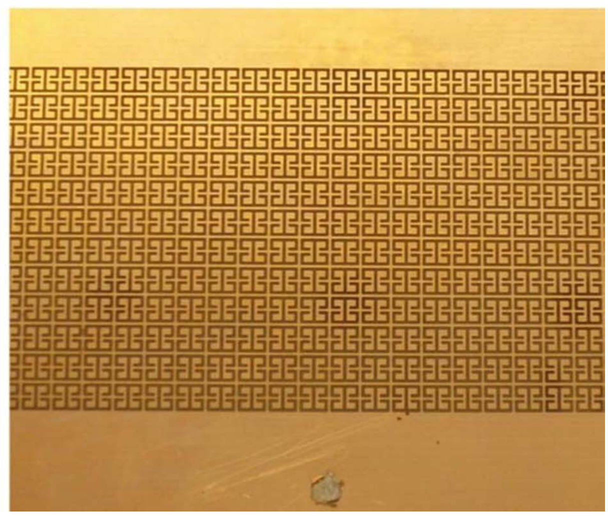
In our experiment, we have used the 2D near-field microwave scanning apparatus (2D mapper) for observation of the field distributions in the planar waveguide and within the CELC region. The 2D mapper is a parallel-plate waveguide apparatus in the X-band frequencies[16] as shown in Fig. 3(a). There is a probe in the upper plate of the waveguide, which can measure the electric field at different positions with the network analyzer. The gap between the top and bottom plates is 11 mm. In Fig. 3(b), we have shown the partial focusing sample, where the CELC patterns are formed from copper-clad FR4 circuit board with the thickness of 0.2 mm. The dimensions for the CELC unit shown in Fig. 2 are selected as $pr=3.333$ mm, $p=3$ mm, and $w=g=0.3$ mm, and the thickness of the copper layer is 0.018 mm.

In our design, the gap between the patterned circuit board and the top PEC plate of the waveguide is kept as 1 mm and the sample is placed upon a cubic Styrofoam. The CELC units are fabricated using the standard photolithography, and there are altogether 12 units in the longitudinal direction and 60 units in the transverse direction. There is a hole below the CELC patterns as shown in Fig. 3(b), where the excitation antenna could protrude into the waveguide after penetrating the Styrofoam.

Since the height of the 2D mapper is much larger than the gap in the CELC region, two metallic ramps are placed on each side of the sample in order to avoid the severe impedance mismatch due to the change in geometry [see the inset in Fig. 3(a)]. There are two copper regions beside the CELC patterns on the circuit board, which forms a planar waveguide together with the top PEC plate. Thus the experimental configurations are consistent with the theoretical model described in Fig. 1. A ring of microwave absorber with saw-



(a)

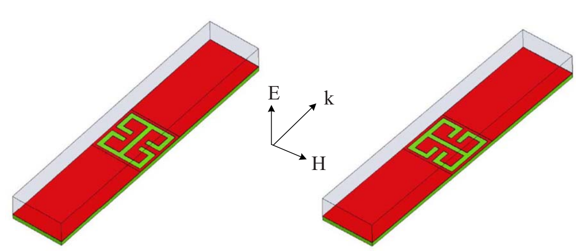


(b)

FIG. 3. (Color online) (a) The experimental set up for the partial focusing. (Inset) Side view of the experimental set up. (b) Details of the fabricated CELC sample.

toothed pattern has been placed near the boundary of the 2D mapper so as to reduce the reflection of electromagnetic waves at the edge of plates.

In order to retrieve the effective-medium parameters, it is not sufficient to simulate only one CELC due to the strong



(a) (b)

FIG. 4. (Color online) Simulation set ups for the anisotropic CELC unit when the plane waves are incident from two directions.

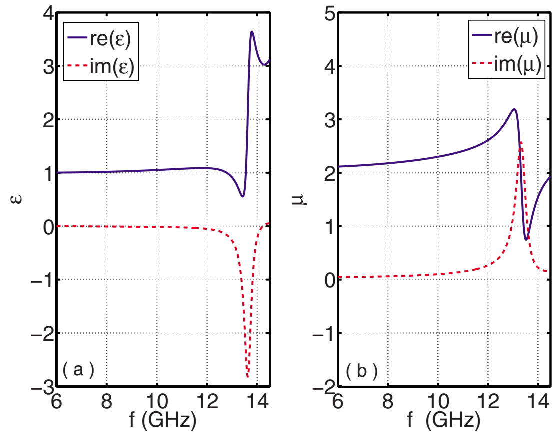


FIG. 5. (Color online) The effective permittivity and permeability curves for the simulation set up in Fig. 4(a). (a) ϵ_z. (b) μ_x.

mutual coupling of CELCs in our slab. Instead, in our design, we have adapted the advanced parameter retrieval method, which is quite efficient for the resonant structures with strong coupling among the neighbors.[17] We need to make two different simulations to get the components of the permittivity and permeability tensors when the magnetic field of the TEM mode is along the x and y directions, respectively, as shown in Fig. 4. After the standard retrieval procedure, we obtain the effective permittivity and permeability curves for the two kinds of simulation set ups shown in Figs. 4(a) and 4(b). The effective ϵ_z and μ_x from Fig. 4(a) are plotted in Fig. 5, while the effective ϵ_z and μ_y from Fig. 4(b) are demonstrated in Fig. 6.

By comparing Fig. 5 with Fig. 6, we observe that the effective ϵ_z varies a lot in most of the frequency band in the two cases due to the particle response and the coupling between adjacent units. However, at our desired frequency, $f=11.5$ GHz, both ϵ_z are quite close. In Fig. 5(a), we have $\epsilon_z=1.085-i0.1123$; in Fig. 6(a), we have $\epsilon_z=1.047-i0.1338$. Hence we can assume that at 11.5 GHz, the effective permittivity in the z direction does not change for waves incident from the x and y directions. Also we obtain from Figs. 5 and 6 that $\mu_x=2.489+i0.193$ and $\mu_y=-0.970+i0.122$ at 11.5 GHz.

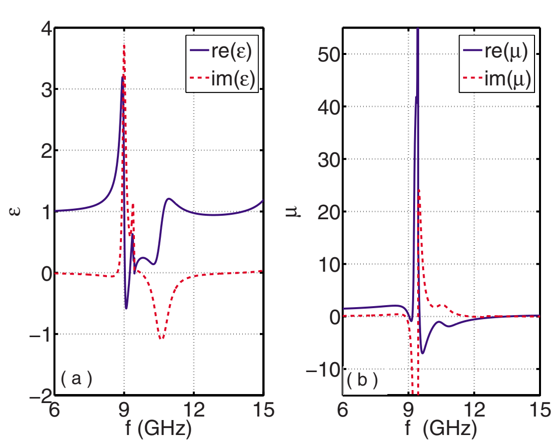


FIG. 6. (Color online) The effective permittivity and permeability curves for the simulation set up in Fig. 4(b). (a) ϵ_z. (b) μ_y.

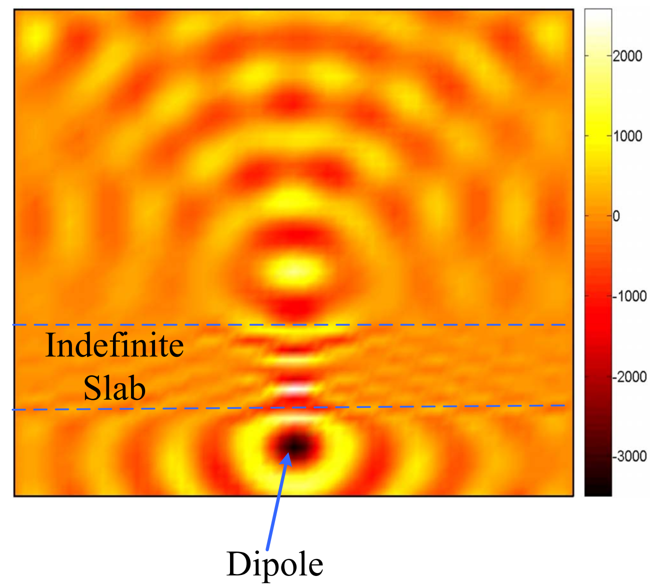


FIG. 7. (Color online) The distribution of simulated electric fields in a section of the planar waveguide at 11.5 GHz.

We have made numerical simulations for the structure shown in Fig. 1 by using the software package HFSS at $f=11.5$ GHz based on the extracted permittivity and permeability mentioned above. The distribution of the real part of the electric field at a section of the planar waveguide is illustrated in Fig. 7. It is obvious that there exist several foci inside and outside the indefinite slab. The waves continue to propagate radially behind the focus above the slab just like the cylindrical waves radiated from a 2D point source. The corresponding experimental result for the distribution of the real part of the electric field at 11.5 GHz is shown in Fig. 8, where the sign "X" stands for the location of the excitation

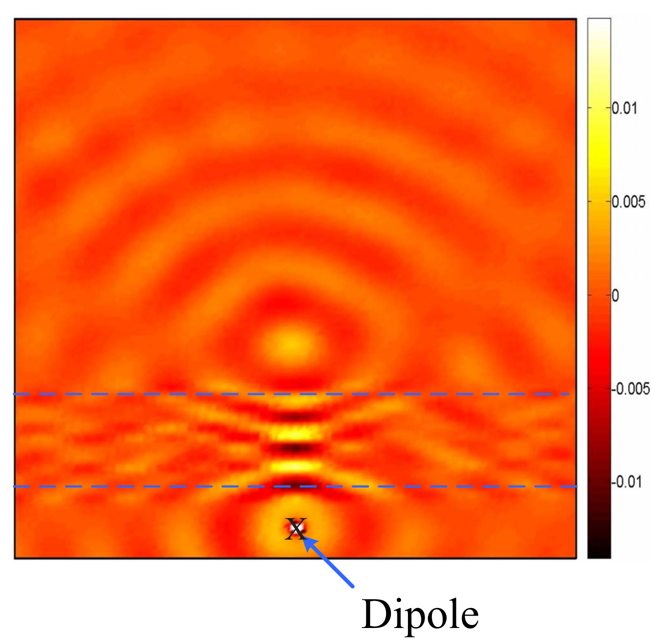


FIG. 8. (Color online) The experimental result for the electric-field distributions inside the 2D mapper at 11.5 GHz.

antenna, and the region between the two dashed lines are covered with the CELC structures. We can see that the experimental result has excellent agreement with the numerical simulation and the partial focusing phenomenon is quite obvious. Note that the field amplitudes in Figs. 7 and 8 are not consistent since the excitation current in the simulation is not the same as that in the experiment.

In conclusion, we have realized the partial focusing by using the complementary indefinite medium. We have carefully designed the effective-medium parameters and the experimental configurations. The experimental result shows excellent agreement with the numerical simulation. We believe that the complementary indefinite medium will find more potential applications in the planar microwave circuits in the future.

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