Compton limit of the Josephson supercurrent

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(Received 1 August 2008; published 10 September 2008)

Coulomb-blockade suppressed electron transport through tunnel junctions is a well-understood phenomenon. The complementary effect appears in Josephson point contacts due to the restricted horizon which equals the Compton wavelength of the Josephson junction. This leads to a strongly reduced critical current as well as a residual contact resistance due to quantum tunneling of the phase. When the lead capacitance becomes smaller than its natural unit $e^2/\hbar c \approx 0.81$ pF/m, the Josephson phase transforms into the Bloch lattice.

DOI: 10.1103/PhysRevB.78.092501 PACS number(s): 74.50.+r, 73.23.−b, 73.63.Rt, 74.25.Sv

I. INTRODUCTION

The horizon is a rather universal concept: on astronomical scales, the largest object we know of is our visible universe, given by its age and the speed of light. Less well known is the fact that the horizon also matters for processes on the nanoscale. For the smallest possible objects, pointlike contacts between two electrodes, the relevant time scale that defines the horizon is the inverse rate of the charge-transfer processes between the electrodes. A zero-bias anomaly observed at normal tunnel junctions can be attributed to the Coulomb-blockade effect. Since electrical current flows in discrete quanta of the electron charge and the junction has a certain capacitance C, the charging energy $E_C = e^2/2C$ has to be paid by the supply voltage which is therefore reduced with respect to the tunneling process itself. For the classical parallel-plate geometry, C depends on the electrode area, the distance of the dielectric barrier, and its dielectric constant. The problem is more involved when the contact consists of a sharp tip that approaches another electrode, either a plate or another tip. In this case capacitances of even less than 1 aF can be expected.[1] With such very small capacitances, huge anomalies should be observed in the current-voltage $I(V)$ characteristics. This is usually not the case because of the so-called lead capacitances.

According to the Heisenberg uncertainty principle a tunneling event takes place within a certain time span $\tau \approx \hbar/e|V|$, depending on the bias voltage V. All interactions of the tunneling electron with the environment spread with the speed of light c and, therefore, are confined to within the horizon $c \cdot \tau \approx \hbar c/e|V|$. A popular interpretation is that the tunneling electron can probe its surroundings only within this horizon, which is around 0.2 μm at $V=1$ V. The world beyond does not matter at all for the tunneling process. This concept was first proposed and derived theoretically by Nazarov,[2] and it is at present well established.[3] We can then assume that the capacitance also derives from this restricted spatial volume. The electrodes that form the point contact can be considered as part of the leads which have a certain capacitance per length κ. The total capacitance of the point contact depends then on the applied bias voltage like $C(V) \approx C_0 + \kappa \hbar c/e|V|$, where C_0 is the static part of the capacitance. Typical microstructured samples have lead capacitances of around 10–30 pF/m.[4,5] In an electrical circuit the horizon of a point contact can be cut off by inserting into the leads a large resistor or a small capacitor. On the other hand, for point contacts like a sharp tip above a metallic surface, large zero-bias anomalies have been reported[6] which could be described by $\kappa \approx 1$ pF/m. This is near the natural unit of the lead capacitance $\kappa_0 = e^2/\hbar c$.

II. RESISTIVELY AND CAPACITIVELY SHUNTED JUNCTION MODEL OF JOSEPHSON JUNCTIONS

The analog situation arises for a Josephson junction. In the so-called resistively and capacitively shunted junction (RCSJ) model the junction itself, defined by its intrinsic critical current I_0, is short circuited by a capacitance C which allows the high-frequency displacement current to flow, as well as a quasiparticle damping resistor R_{qp} which describes losses in the sample and in the environment.[7,8] This Josephson-junction circuit represents a "particle" of "mass" C oscillating in the washboard potential $E_{WB}(I,\phi) = -(\hbar/2e)I\phi - E_{JE}\cos\phi$, where ϕ is the phase difference across the junction and $E_{JE} = \hbar I_0/2e$ the Josephson coupling energy (the Coulomb charging energy is assumed to be negligible). Its zero-point energy $\hbar\omega_p/2$ is defined by the plasma frequency $\omega_p = \sqrt{2eI_0/\hbar C}$. The intrinsic critical current has theoretically three limiting cases, and at low temperatures it depends only on the normal contact resistance R_N and the superconducting energy gap 2Δ: a tunneling type contact has[9] $I_0 = \pi\Delta/2eR_N$, a direct ballistic contact in the dirty limit has[10] $I_0 = 1.32\pi\Delta/2eR_N$, and in the clean limit[11] $I_0 = \pi\Delta/eR_N$. The injected current I tilts the washboard potential, affecting the depth of the potential minima as well as the eigenfrequency. In terms of the reduced current $x = I/I_0$ the maximum depth of the potential becomes $2E(x) = E_{JE}[x(2\arcsin x - \pi) + 2\cos\arcsin x]$ and the eigenfrequency $\omega(x) = \omega_p(1-x^2)^{1/4}$. The properties of those junctions, in case of planar contacts with an oxide tunneling barrier between the two superconductors that defines the capacitance, have been investigated and their behavior found to agree with the RCSJ model.[12–15]

III. DYNAMIC CAPACITANCE IN THE COMPTON LIMIT

A direct metallic contact has no static but a dynamic capacitance. For its estimate we note that each classical particle

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of mass m is associated with the Compton wavelength $l_C=\hbar/mc$. The particle can interact with a quantum relativistic field, such as the electrodynamic field, only within a radius l_C.[16] Otherwise new particles could be created at energy mc^2 or the original one annihilated, as described by quantum field theory. We interpret the internal energy mc^2 of the classical particle as the energy difference $\hbar\omega$ to the first-excited level of the particle in the washboard potential to obtain

$$l_C(\omega) \approx \frac{c}{\omega} \tag{1}$$

as horizon of the Josephson junction.[17] Without this spatial restriction, the particle could be excited out of its ground state, which in turn would correspond to an elevated temperature, contradicting the basic assumptions. It is then only reasonable to assume that the contact capacitance origins from the same volume element around the contact defined by l_C, analogous to the case of normal tunnel junctions, and to consider the point contact as part of the leads to the contact which have a certain capacitance per length κ.

Replacing the capacitance by the lead capacitance is more than a simple variable substitution because it takes into account the frequency dependence of the dynamic capacitance,[17]

$$C \approx \frac{\kappa c}{\omega(C)}. \tag{2}$$

The plasma frequency $\omega_p \approx 2eI_0/\kappa c\hbar = 2I_0\kappa_0/e\kappa$ increases now linearly with the critical current. The eigenfrequency of the tilted potential $\omega(x) \approx \omega_p\sqrt{1-x^2}$ depends differently on the reduced current. Since the washboard potential itself is not affected by the capacitance or the horizon, we obtain the identities (at $I \to 0$)

$$\frac{\kappa}{\kappa_0} \approx 4\frac{E_{\mathrm{JE}}}{\hbar\omega_p} = \sqrt{\frac{2E_{\mathrm{JE}}}{E_C}} = \frac{Q}{2\pi}\frac{R_K}{R_{\mathrm{qp}}}, \tag{3}$$

which relate the lead capacitance κ to the ratio between barrier height and zero-point energy as well as between $Q=\omega R_{\mathrm{qp}}C$ and R_{qp}. The von Klitzing resistance is $R_K = h/e^2 \approx 25.8$ kΩ.

The properties of a real Josephson junction depend on many parameters that are not easy to control, such as stress in the contact region or the size and the purity of the materials used. Those effects are not considered here. We also neglect all those corrections of the discrete energy levels and the eigenfrequencies that arise from the finite height of the washboard potential or from the nonsinusoidal $I(\phi)$ relationship at a large transmission coefficient of the contact (see Ref. 18). We assume to have a point contact like Josephson junction with a certain intrinsic critical current I_0, and we want to understand the behavior of this junction at temperatures $T \to 0$ as a function of the lead capacitance.

IV. REDUCED CRITICAL CURRENT

The effects of a small capacitance are quite dramatic. First of all, the actual critical current of a Josephson junction is *always* smaller than its theoretical I_0 value. At least three different mechanisms can limit the supercurrent. (i) At large κ, a supercurrent exists, but it is already reduced due to quantum fluctuations of the phase. To ensure that at least one discrete level exists in the well of the washboard potential requires $2E(x) \geq \hbar\omega(x)/2$. The left- and the right-hand sides coincide at the reduced critical current $x_c = I_c/I_0$, implicitly described by

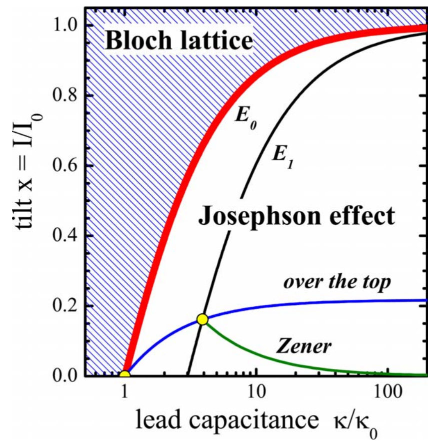

FIG. 1. (Color online) Tilt of the washboard potential in terms of the injected current I normalized to the theoretical critical current I_0 versus normalized lead capacitance κ/κ_0. The shaded area indicates the forbidden region without any supercurrent (Bloch lattice). At larger lead capacitances a supercurrent can flow, but its magnitude is strongly suppressed. E_0 and E_1 indicate the current at which the first and the second discrete energy levels cease to exist. The line E_0 therefore marks the maximum possible supercurrent. Over the top indicates the current at which the tunneling particles arrive in the neighboring well above the potential barrier. Zener is the current at which the particle starting at E_0 arrives at E_1 in the neighboring well.

$$\frac{\kappa_0}{\kappa} \approx \frac{\omega_p}{E_{\mathrm{JE}}}\frac{E(x_c)}{\omega(x_c)}. \tag{4}$$

Figure 1 shows that the actual I_c approaches the expected theoretical value I_0 only at very large $\kappa \gg 10\kappa_0$. The supercurrent is completely suppressed ($x_c=0$) at $\kappa \leq \kappa_0$ when the zero-point energy $\hbar\omega_p/2$ exceeds the depth $2E_{\mathrm{JE}}$ of the untilted washboard potential. (ii) The supercurrent will also be suppressed when the tunneling particle arrives in the neighboring potential well with an energy above the minimum height of this well so that the particle can escape immediately. This condition is met for $2E(x) \leq \hbar\omega(x)/2 + 2\pi x E_{\mathrm{JE}}$. According to

$$\frac{\kappa_0}{\kappa} \approx \frac{\omega_p}{E_{\mathrm{JE}}}\frac{E(x_c)}{\omega(x_c)} - \frac{\pi\omega_p}{\omega(x_c)}x_c, \tag{5}$$

the critical current also vanishes at $\kappa \leq \kappa_0$, but it saturates at $x_c = E(x_c)/\pi E_{\mathrm{JE}} \approx 0.22$ at large κ. (iii) The third mechanism that could reduce the critical supercurrent is Zener tunneling of the phase. When the injected current exceeds the Zener current $I_Z = e\omega/\pi$, the washboard potential is tilted by at least $\hbar\omega$ per phase difference 2π, and the tunneling particle arrives in the neighboring well at, or above, the first-excited state (if there exists one). From there the tunneling probability is so strongly enhanced that like in a chain reaction the

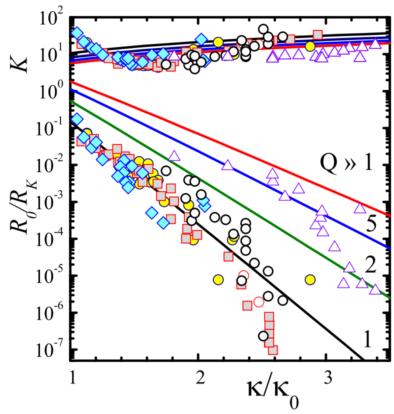

FIG. 2. (Color online) Residual resistance R_0 in units of the von Klitzing resistance R_K as well as the parameter $K=1.8\pi(\kappa/\kappa_0)[1+\beta]$ at $x\approx 0$ of a Josephson junction versus normalized lead capacitance κ/κ_0 at $Q=1$, 2, 5, and ∞. The symbols are the experimental results of In-In junctions at $T=0.1$ K.

phase slips faster and faster. The reduced critical current $x_c=I_Z/I_0=e\omega/\pi I_0$ due to Zener tunneling can be obtained by plugging in the above derived eigenfrequency ω of the tilted potential. This results in

$$\frac{\kappa_0}{\kappa}\approx\frac{2}{\pi}\frac{x_c}{\sqrt{1-x_c^2}} \tag{6}$$

and a critical current that decreases with increasing κ since more and more discrete levels become available with smaller spacing.

V. ZERO-BIAS PROPERTIES

A small capacitance also causes a finite contact resistance due to quantum tunneling of the phase. The tunneling rate is[19,20]

$$\Gamma=\gamma\frac{\omega_p}{2\pi}\exp\left(-\frac{14.4E_{\mathrm{JE}}}{\hbar\omega_p}[1+\beta]\right), \tag{7}$$

where $\gamma\approx\sqrt{120\pi(14.4E_{\mathrm{JE}}/\hbar\omega_p)}=\sqrt{432\pi\kappa/\kappa_0}$. It also includes a correction term $\beta=0.87/Q+\cdots$ due to damping, which depends on $Q=\omega R_{\mathrm{qp}}C\approx 2\pi(\kappa_0/\kappa)(R_{\mathrm{qp}}/R_K)$. Injecting a current into the junction tilts the washboard potential, introducing an imbalance of forward and backward rates. The resulting drift of the phase leads, according to the Josephson equation $V=(\hbar/2e)\dot{\phi}$, to a voltage drop, thus to a differential resistance around zero bias $x\approx 0$,

$$\frac{dV}{dI}\approx R_0\cosh(Kx), \tag{8}$$

and a residual contact resistance,

$$R_0\approx R_K\sqrt{\frac{432}{\pi}\frac{\kappa_0}{\kappa}}K\exp(-2K/\pi), \tag{9}$$

with the parameter $K=1.8\pi(\kappa/\kappa_0)[1+\beta]$ (see Fig. 2). Both R_0 and K depend only on the lead capacitance κ and the Q factor in the $I\to 0$ limit. Figure 3 displays the relationship between K and R_0, which barely depends on the Q factor, except for the lower bound of K which is $1.8\pi\approx 5.7$ at $Q\to\infty$. R_0 does not directly depend on the normal-state contact resistance, which means in principle we could get even $R_0>R_N$. This would be typical for a Josephson tunnel junction but not for a direct metallic contact without tunneling barrier and weak damping.

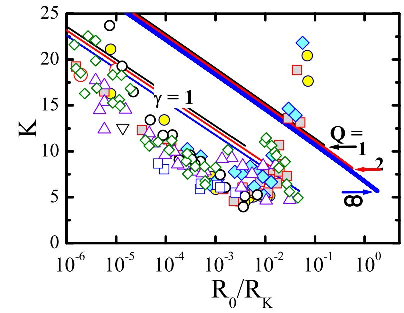

FIG. 3. (Color online) Parameter $K=1.8\pi(\kappa/\kappa_0)[1+\beta]$ of the differential resistance at $x\approx 0$ versus normalized residual resistance R_0/R_K. The arrows mark the minimum K values for the indicated Q factors. Also shown in thin solid lines the expected data when the tunneling prefactor $\gamma=1$. The symbols are typical experimental data of our indium-indium Josephson junctions at low temperatures $T=0.1$ K.

VI. COMPARISON WITH EXPERIMENTS

Few reports exist on the systematic variation as function of contact size of the critical current and the finite residual resistance of Josephson point contacts of classical superconductors.[17,21–23] Those experiments were carried out on mechanically controllable break junctions over a wide range of contact sizes. We have repeated those experiments on indium junctions, based on 1–2 mm diameter indium wire, and found that the experimental critical currents agree rather well with those earlier data (see Fig. 4). We are therefore led to believe that these results are not artifacts but are related to intrinsic properties of Josephson junctions.

We note that the experimental critical current is usually larger than expected from mechanisms (ii) “over the top” and (iii) “Zener” discussed above and for which $x_c\leq 0.22$. This surprises a bit since for these two mechanisms a single tunneling event would drive the contact onto its normal branch. This indicates a very efficient damping mechanism that quickly removes the excess energy of the quasiparticle. In our experiments I_c approaches the clean-limit (KO2) I_0 at small R_N. There are no anomalies indicating a preference of the dirty (KO1) or the tunneling limit (AB) value. Our main assumption is therefore that all contacts are in the clean limit (KO2) and that the reduction with respect to the theoretical value I_0 is due to a small lead capacitance. By comparing the measured I_c with the phase diagram in Fig. 1 we can then estimate the lead capacitance κ, which seems to vary systematically with R_N. This means that the lead capacitance changes either with the size of the contact or with the critical current I_0. The latter case could be attributed to the frequency dependence of the horizon.

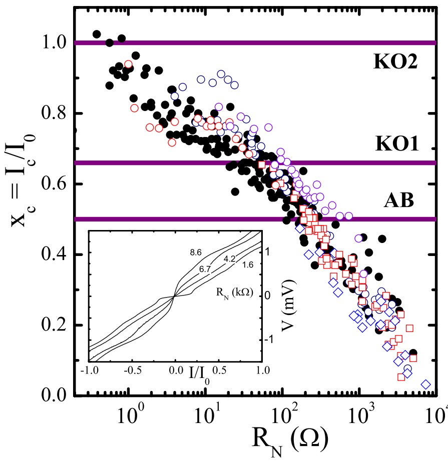

FIG. 4. (Color online) Experimental critical current I_c normalized to the theoretical clean-limit value (KO2) versus normal contact resistance R_N. Closed circles are from Ref. 17 and all open symbols are the results of the present experiments, all taken at $T=0.1$ K. Indicated are the expected values for the clean (KO2) and the dirty (KO1) limits as well as for tunneling contacts (AB). The contact diameter varies from about 50 nm at 0.4 Ω down to about 0.4 nm at 6 kΩ. The inset shows typical $I(V)$ characteristics at the indicated R_N in the transition region from Josephson effect with reduced critical current to the Bloch lattice. The current is normalized to the theoretical I_0.

Figure 2 shows that the residual resistance R_0 and the parameter K fit the predictions reasonably well and indicate $Q\approx 1$. $K(\kappa)$ has a shallow minimum at $\kappa\approx 1.5\kappa_0$. Since at this lead capacitance the Josephson coupling energy equals the Coulomb charging energy $E_{JE}=E_C$, deviations from the used theory[19,20] have to be expected. Figure 3 shows that at the same time $K(R_0)$ has a minimum of around five to ten at $R_0\approx R_K/200\approx 100$ Ω. This would mean that $Q>1$, that is, the contacts are only weakly damped. At larger R_0 the Coulomb charging dominates and possibly causes the steep upturn of $K(R_0)$. At smaller R_0 the $K(R_0)$ data are offset on the R_0 axis by a factor of order 10^3. This could be due to the fact that the theory assumes tunneling into a continuum while for our contacts tunneling into the discrete levels of the neighboring well is more appropriate. A tunneling prefactor $\gamma=1$ would better fit the data.

VII. CONCLUSIONS

Although our experimental data support the suggested model, there are other examples where a noticeable reduction in I_c has not been observed. This applies especially to point contacts fabricated by nanolithography, like in Ref. 24 (see also the review Ref. 25). This discrepancy could be easily explained by the fact that in the latter type of experiments the junction itself sits very close to a (conducting) substrate and therefore has a rather large lead capacitance. The effects discussed here do not depend on the type of superconductor. Strongly reduced critical currents are often found for contacts with unconventional superconductors, such as the heavy-fermion ones.[26,27] In those cases it would be rather challenging to separate the intrinsic material-dependent properties that originate from the superconducting order parameter and its symmetry from device-dependent effects due to the lead capacitance.

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[1] J. Wang, H. Guo, J. L. Mozos, C. C. Wan, G. Taraschi, and Q. Zheng, Phys. Rev. Lett. **80**, 4277 (1998).
[2] Yu. Nazarov, Zh. Eksp. Teor. Fiz. **95**, 975 (1989) [Sov. Phys. JETP **68**, 561 (1989)].
[3] M. H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990).
[4] P. Delsing, K. K. Likharev, L. S. Kuzmin, and T. Claeson, Phys. Rev. Lett. **63**, 1180 (1989).
[5] P. Wahlgren, P. Delsing, and D. B. Haviland, Phys. Rev. B **52**, R2293 (1995).
[6] P. J. M. van Bentum, H. van Kempen, L. E. C. van de Leemput, and P. A. A. Teunissen, Phys. Rev. Lett. **60**, 369 (1988).
[7] E. D. McCumber, J. Appl. Phys. **39**, 3113 (1968).
[8] W. C. Stewart, Appl. Phys. Lett. **12**, 277 (1968).
[9] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **11**, 104 (1963).
[10] I. O. Kulik and A. N. Omel'yanchuk, JETP Lett. **21**, 96 (1975).
[11] I. O. Kulik and A. N. Omel'yanchuk, Sov. J. Low Temp. Phys. **4**, 142 (1979).
[12] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B **9**, 4760 (1974).
[13] R. F. Voss and R. A. Webb, Phys. Rev. Lett. **47**, 265 (1981).
[14] S. Washburn, R. A. Webb, R. F. Voss, and S. M. Faris, Phys. Rev. Lett. **54**, 2712 (1985).
[15] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. Lett. **55**, 1543 (1985).
[16] Albert Messiah, *Quantenmechanik* (Walter de Gruyter, Berlin, 1979), Vol. 2, p. 429; *Méchanique Quantique* (Dunod Editeur, Paris, 1964), Vol. 2.
[17] K. Gloos and F. Anders, J. Low Temp. Phys. **116**, 21 (1999).
[18] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[19] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[20] A. I. Larkin, K. K. Likharev, and Yu. N. Ovchinnikov, Physica B & C **126**, 414 (1984).
[21] C. J. Muller, J. M. van Ruitenbeek, and L. J. de Jongh, Physica C **191**, 485 (1992).
[22] C. J. Muller, M. C. Koops, B. J. Vleeming, R. de Bruyn Ouboter, and A. N. Omelyanchouk, Physica C **220**, 258 (1994).
[23] K. Gloos and F. Anders, Physica B (Amsterdam) **284-288**, 1854 (2000).
[24] E. Scheer, P. Joyez, D. Esteve, C. Urbina, and M. H. Devoret, Phys. Rev. Lett. **78**, 3535 (1997).
[25] N. Agraït, A. L. Yeyati, and J. M. van Ruitenbeek, Phys. Rep. **377**, 81 (2003).
[26] K. Gloos, F. B. Anders, W. Assmus, B. Buschinger, C. Geibel, J. S. Kim, A. A. Menovsky, R. Müller-Reisener, S. Nuettgens, C. Schank, G. R. Stewart, and Yu. G. Naidyuk, J. Low Temp. Phys. **110**, 873 (1998).
[27] K. Gloos, F. Anders, S. Nuettgens, and W. Assmus, Physica B **259-261**, 674 (1999).