Some unique magnetic properties of nanoscale quantum rings subjected to a Rashba spin-orbit interaction

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Quantum rings subjected to a Rashba spin-orbit coupling and an external magnetic field possess interesting magnetic properties. Specifically, the spectral linewidth of magnetic susceptibility is found to be independent of the ring geometry. For zero spin-orbit coupling it varies linearly with temperature and the slope is proportional to a scaling factor of material parameters. When the spin-orbit interaction is switched on, there is a *turning temperature* below which the spin-orbit coupling strength is directly measurable. The upper bound of the turning temperature is also investigated.

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Semiconductor nanostructures have witnessed phenomenal developments in recent years due to their promising potential applications in optical and electronic devices. They are also ideal for exploration of fundamental physics at the nanoscale. A few-electron quantum ring (QR) with its unique optical and electronic properties is a brilliant example of such a structure. Recent important advances in fabricating nanoscale quantum rings,[1] where the topology and geometrical properties can be externally controlled, have generated a lot of attention on the studies of electronic states in a ring geometry.[2–6] The magnetization and the magnetic susceptibility at zero temperature of a metallic ring subjected to an external magnetic field exhibit periodic oscillations, reflecting the behavior of the ground-state energy.[3,4,7] The Rashba spin-orbit (SO) interaction,[8–12] which provides a means for coupling the electron spin and its orbital motion, does not influence the period of oscillations. It couples different spin states and generates anticrossings of the energy levels.[11,12]

The spin-orbit coupling is an important component for applications in spintronics. Although there are many experiments that can confirm this coupling in nanostructures, two interesting issues still remain unresolved: the temperature range where one would detect the spin-orbit interaction, and if the temperature range could be expanded to arbitrary large values. We show below that the magnetic susceptibility at finite temperature exhibits interesting properties that would be very useful for a wide range of studies at the nanoscale. In particular, it provides an intriguing possibility to determine directly the SO coupling strength, which (as yet) can only be determined indirectly from the Shubnikov–de Haas oscillations in a planar electron gas confined in a narrow-gap quantum well.[10] Manipulation of electron spin via the Rashba SO interaction in a quantum ring would be a promising avenue for quantum information processing,[13] as well as for spintronics in reduced dimensions,[10,14,15] where the properties reported below would be important for determining and tuning the SO coupling strength.

The Hamiltonian of interacting electrons in a parabolically confined nanoscopic ring of radius R, subjected to a perpendicular magnetic field, and Rashba SO interaction is

$$\mathcal{H} = \sum_{i=1}^{N} \left\{ \frac{1}{2m^*}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right)^2 + \frac{1}{2}m^*\omega_0^2(r_i - R)^2 + g\mu_B \mathbf{B}\cdot\mathbf{S}_i + \frac{\alpha}{\hbar}\left[\boldsymbol{\sigma}\times\left(\mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}_i)\right)\right]_z \right\} + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{\epsilon|\mathbf{r}_i - \mathbf{r}_j|}, \quad (1)$$

where $\mathbf{A} = 1/2B(-y,x,0)$ is the symmetric gauge vector potential, α is the spin-orbit coupling constant and $\boldsymbol{\sigma}$ denotes the vector of the Pauli matrices, $\hbar\omega_0$ is the confinement potential strength, and ϵ is the background dielectric constant. The confinement potential corresponding to a QR is depicted in Fig. 1(a).[2] The SO coupling strength α can be tuned by the external gate voltages or by asymmetric doping.[10] Figure 1(b) shows the schematic of the energy levels of a noninteracting two-electron quantum ring. The degenerate $\ell=-1$ states at $B=0$ are split into three states by the Zeeman interaction. In the absence of the SO coupling, the two lowest energy states cross at a finite magnetic field evoking the transition of the ground state from the state with $l=0$ and $s_z=0$ to the state with $l=-1$ and $s_z=1$. Since these crossing states have the same total angular momentum $j_z=\ell+s_z=0$, the SO interaction mixes them, causing an anticrossing, which lifts the degeneracy at the crossing point.[16] The Hamiltonian above cannot be solved exactly, so we first consider the case where an electron is confined to a strictly one-dimensional (1D) circle (δ-function confinement) with eigenenergies[17]

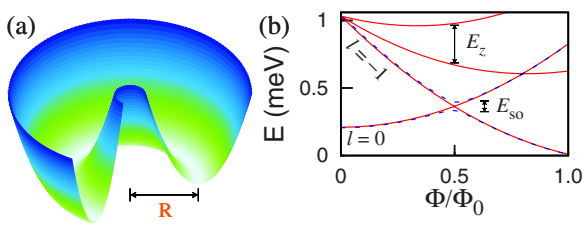

FIG. 1. (Color online) (a) The shape of the confining potential of a quantum ring of radius R. (b) A schematic of energy levels for two electrons in a quantum ring. Solid (dashed) lines correspond to states without (with) Rashba SO coupling.

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$$E = \frac{1}{2}\varepsilon_0(1+2t+2t^2) \pm \frac{1}{2}\varepsilon_0 \times \sqrt{\left(2t+1-2\frac{\varepsilon_z}{\varepsilon_0}\Theta\right)^2 + \frac{\varepsilon_R^2}{\varepsilon_0^2}(2t+1)^2}, \quad (2)$$

where $t=\ell+\Theta$, $\Theta=\Phi/\Phi_0$, $\Phi=\pi R^2 B$ is the magnetic flux through the ring and $\Phi_0=hc/e$ is the flux quanta, $\varepsilon_0=\hbar^2/2m^*R^2$, $\varepsilon_z=s_z g\mu_B\Phi_0/\pi R^2$ is the Zeeman energy with spin s_z, and $\varepsilon_R=\alpha/R$ is the SO coupling energy.

Around the crossing point, the Zeeman energy is estimated to be 0.1 and 0.01 meV for InAs and GaAs QRs, respectively. Therefore, at low temperatures ($T \simeq 0.01$ meV), the energy states that contribute to the thermodynamic properties correspond to $\ell=0$ and -1 states. For the InAs QRs, since its Zeeman energy is greater than T, the $\ell=0$ state and the lowest energy state of $\ell=-1$ are enough to account for the properties at low temperatures. InAs QR is then essentially a two-level system with the partition function without the SO coupling ($\varepsilon_R=0$),

$$\mathcal{Z} = \sum_n e^{-\beta E_n} = 2e^{-\beta\varepsilon_0[\Theta^2-(1-\varepsilon_z/2\varepsilon_0)\Theta+1/2]} \times \cosh\beta\varepsilon_0\left[\frac{1}{2}-\left(1-\frac{\varepsilon_z}{2\varepsilon_0}\right)\Theta\right]. \quad (3)$$

The susceptibility as a function of temperature and magnetic flux is

$$\chi = \frac{1}{\beta}\frac{\partial^2}{\partial B^2}\ln\mathcal{Z} \propto -2\varepsilon_0 + \beta\varepsilon_0^2\,\mathrm{sech}^2\,\beta\varepsilon_0\left[\frac{1}{2}-\left(1-\frac{\varepsilon_z}{2\varepsilon_0}\right)\Theta\right]. \quad (4)$$

The spectral line shape is determined by the hyperbolic function in Eq. (4), yielding a linewidth

$$\Delta B = \frac{\eta}{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)}T, \quad (5)$$

where $m_{\mathrm{eff}}=m^*/m_e$ is the effective mass and η is a constant. For GaAs, the Zeeman energy is around 0.01 meV, which is of the same order as T. The partition function here includes the $\ell=0$ state and three $\ell=-1$ states, i.e., for a four-level system. The linewidth relation is the same as in Eq. (5).

In the absence of the SO coupling, the ground-state energy has a cusp at the level crossing, where the energy states $\ell=0$, $s_z=0$ and $\ell=-1$, $s_z=1$ are degenerate [dashed lines in Fig. 1(b)]. Magnetization (the first derivative of the free energy) is discontinuous at this crossing where, as a consequence, the susceptibility diverges. At finite temperatures, thermal excitations will make the magnetization a continuous and a smooth function and the susceptibility then has the *spectral line shape* with a finite linewidth. Figure 2(a) shows the numerical results [for the Hamiltonian (1)] of the magnetic susceptibility of an interacting two-electron QR for two different materials (InAs and GaAs). In Fig. 2(b), we show the full width at half maximum (FWHM) of the spectral line of the susceptibility. The spectral linewidth varies linearly with temperature. In addition, the ratio of the slope of the linewidth between a GaAs QR and an InAs QR is around 2.5. These can be understood as follows: We consider a system of a free-electron gas moving along a 1D QR. The linear behavior is clearly given by the linewidth Eq. (5), while the ratio of $\Delta B/T$ for the GaAs and InAs rings is

$$\frac{\left(\frac{\Delta B}{T}\right)_{\mathrm{GaAs}}}{\left(\frac{\Delta B}{T}\right)_{\mathrm{InAs}}} = \frac{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)_{\mathrm{InAs}}}{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)_{\mathrm{GaAs}}} \simeq 2.5,$$

i.e., identical to the numerical results presented above. The reason for the success of the free-electron model can be explained as due to the fact that, at low temperatures, two electrons would tend to stay at the opposite sides of the QR and move in such a way that the center of mass (CM) is fixed. As a consequence, the CM energy is identical to the single-particle energy. A somewhat similar situation also occurs for a parabolic quantum dot (QD).[18] Electron-electron interactions do not affect the spectral linewidth for a QR of fixed radius. The scaling factor $(1/m_{\mathrm{eff}}-g)$ depends only on the material parameters and is *independent of the geometry of the quantum ring*. It can be used to determine the spectral linewidth for different materials. For an InAs QR and a GaAs QR, it is about 38 and 15, respectively.

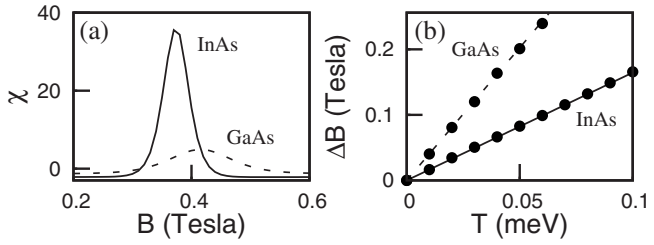

FIG. 2. (a) The magnetic susceptibility as a function of the magnetic field of a two-electron QR with $R=30$ nm at $T=0.03$ meV. (b) The FWHM of the spectral linewidth of the susceptibility as a function of temperature. The solid line and the dashed line correspond to the materials InAs and GaAs, respectively. The material parameters for the InAs ring are $m^*=0.042m_e$, $g=-14$, and $\epsilon=14.62$, while for the GaAs ring $m^*=0.067m_e$, $g=-0.44$, and $\epsilon=13.1$.

In the presence of the SO coupling, the level crossing will turn into an anticrossing at the crossing point.[11] The anticrossing gap due to the SO coupling [as shown in Fig. 1(b)] is

$$E_{\mathrm{SO}} = \varepsilon_0\sqrt{\left(2\Theta_0-1-2\frac{\varepsilon_z}{\varepsilon_0}\Theta_0\right)^2+\frac{\varepsilon_R^2}{\varepsilon_0^2}(2\Theta_0-1)^2} = \frac{|g|}{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)}\frac{\alpha}{R}, \quad (6)$$

where $\Theta_0=\varepsilon_0/2(\varepsilon_0-\varepsilon_z)$. Equation (6) indicates that the SO gap is proportional to the Zeeman energy, i.e., it vanishes at vanishing Zeeman interaction. A similar behavior was also found by Loss *et al.*[19] for a QD. It is worth noting however that one can use fully spin-polarized (FSP) states with no

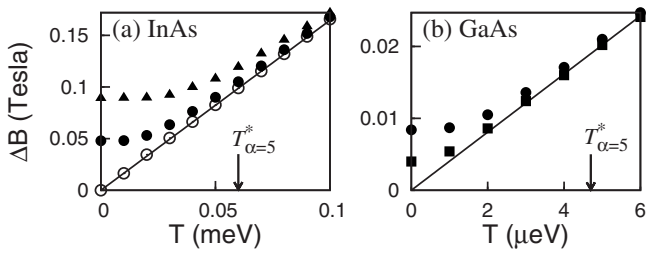

FIG. 3. (a) The spectral linewidth as a function of temperature for the InAs QR with Rashba spin-orbit coupling $\alpha=10$ (triangle) and 5 (circle) and without the spin-orbit coupling $\alpha=0$ (open circle). (b) The spectral linewidth as a function of temperature of GaAs QR with Rashba spin-orbit coupling $\alpha=5$ (circle) and 2.5 (square) and without spin-orbit coupling $\alpha=0$ (line).

Zeeman term to create a SO gap.[20] This is the case of the quantum-Hall ferromagnet at filling factor unity, which has the FSP ground state (singlet, $\ell=0$, $s_z=0$) and the Skyrmion-type first-excited state (triplet, $\ell=-1$, $s_z=1$). The SO interaction couples the states having $j_z=0$ and forms an anticrossing gap.

The magnetization as a function of the magnetic field becomes a continuous and a smooth function. As a result, the susceptibility exhibits a finite spectral linewidth even at zero temperature. In Fig. 3(a) we show the numerical results of the spectral linewidth as a function of temperature for an InAs QR. At zero temperature, the spectral linewidth is zero for $\alpha=0$ and is finite for $\alpha\neq 0$. The value of the spectral linewidth is proportional to α; at $\alpha=10$ it is twice that at $\alpha=5$. As the temperature is increased, the spectral linewidths for finite α slowly increase and the deviation of the linewidth between $\alpha\neq 0$ and $\alpha=0$ decreases. At a temperature T^*, the spectral linewidth satisfies the condition[21]

$$\frac{\Delta B(T^*)_{\alpha\neq 0}-\Delta B(T^*)_{\alpha=0}}{\Delta B(T^*)_{\alpha=0}} \ll 1. \tag{7}$$

The *turning temperature* T^* represents the highest temperature where the SO coupling effect is directly measurable. As the temperature goes beyond T^*, the spectral linewidth attributed to thermal excitation dominates and smears out the contribution by the SO coupling. Therefore, the curves corresponding to zero and nonzero SO couplings merge for $T>T^*$. The turning temperature can thus be used to distinguish the SO and the temperature effects. Figure 3(a) reveals that the turning temperature is proportional to α.

Figure 3(b) shows the spectral linewidth as a function of temperature for a GaAs QR. It is worth noting that the temperature scale for the GaAs case is in μeV. The large difference of the temperature scale between the InAs and the GaAs results can be attributed to the size of the SO gap, which is large for InAs and small for GaAs. Therefore, the turning temperature for InAs is much larger than that for GaAs. The ratio $T^*_{\mathrm{InAs}}/T^*_{\mathrm{GaAs}}$ determined from numerical evaluation of the interacting Hamiltonian is $\sim$13. These results suggest that the turning temperature is proportional to the size of the SO gap, which is described as

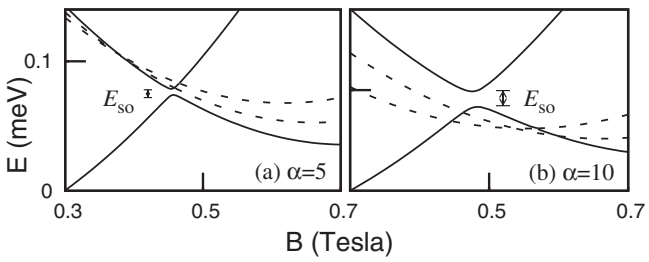

FIG. 4. The energy levels as functions of the magnetic field of a GaAs QR for (a) $\alpha=5$ and (b) $\alpha=10$. The solid (dashed) lines correspond to $j_z=s_z+\ell=0$ ($\neq 0$). The energy levels have been vertically shifted for clarity.

$$T^*=\eta_R\frac{|g|}{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)}\frac{\alpha}{R}, \tag{8}$$

where η_R ($\simeq 0.96$) is a constant. From the turning temperature equation, we find that

$$\frac{T^*_{\mathrm{InAs}}}{T^*_{\mathrm{GaAs}}}=\frac{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)_{\mathrm{GaAs}} g_{\mathrm{InAs}}}{\left(\frac{1}{m_{\mathrm{eff}}}-g\right)_{\mathrm{InAs}} g_{\mathrm{GaAs}}}\simeq 13$$

in accordance with the numerical results obtained above. Equations (5) and (8) are perhaps the most important properties found here for a quantum ring in a magnetic field and a Rashba SO interaction.

In Eq. (8) the turning temperature depends on the value of the SO coupling constant α, which can be controlled by the external gate voltage. Ideally, as we apply a large gate voltage, we can have a jump in the turning temperature. To discuss this, we first look at the zero-temperature case. At zero temperature, the existence of the SO coupling will affect the magnetic-field dependence of the ground state. It will smoothen the cusp for $\alpha=0$ at the crossing point and generate an energy gap E_{SO}. Figure 4(a) shows the energy levels of GaAs QR as a function of the magnetic field for $\alpha=5$. Magnetization is now a smooth function while the susceptibility is nonzero and has a finite linewidth. With increasing SO coupling, the size of the SO gap increases and the smoothing effect is enhanced. At the same time, the gap position moves to a higher magnetic field. The smoothing effect accompanies an increase in the spectral linewidth and the turning temperature. However, when α is greater than a critical value, the movement of the SO gap position results in a level crossing of the ground state and the cusp of the ground state reappears. In Fig. 4(b) ($\alpha=10$) the level crossing of the ground state is observed. The magnetization is a continuous function but not a smooth function around the crossing points. Consequently, the susceptibility will diverge at these points. As a result, the spectral linewidth cannot be well defined. Therefore, there is an *upper bound* of α where magnetization remains a smooth function and the susceptibility has a well-defined line shape. From our numerical results we deduce that to be $\alpha_{\mathrm{GaAs}}\leq 7$ and $\alpha_{\mathrm{InAs}}\leq 60$.

In quantum rings the effect of the mutual Coulomb interactions on magnetization is known[2] to be vanishingly small. As a result, the linewidth of the susceptibility peak retains its noninteracting temperature dependence and, as expected, a careful analysis of the numerical data shows only an infinitesimal change in the slope of the linear part. Therefore, for all practical purposes, the electrons in the ring behave as independent particles when it comes to the linewidth of the susceptibility. In all other respects, such as in the energy spectrum, the interactions still (of course) play a remarkable role.[4]

The linewidth of the susceptibility shows that it is independent of the geometry of the ring. Any deviations from this behavior must then be attributed to nongeometrical properties, most notably the SO coupling, and—to a much lesser extent—the mutual Coulomb interactions. For a nonzero SO coupling, the deviations from the linear behavior of the linewidth versus temperature (found at $\alpha=0$) are directly proportional to the strength of the spin-orbit coupling. This feature could be used to measure the spin-orbit coupling strength, which is an important element for semiconductor spintronics.[10] Measurement of susceptibility, in particular the temperature dependence of the linewidth and the position of the peak, would provide direct information about the material parameters.[22] Finally, the upper bound of α determined here would provide guidelines for its experimental determination in a material.

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[22] The hyperbolic function in Eq. (4) provides a maximum value of the susceptibility at $(1-\frac{1}{2}gm_{\mathrm{eff}})\Theta=\frac{1}{2}$, where Θ is the position of the peak.