Generation of traveling solitons in one-dimensional monatomic quartic lattices

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Generation of traveling soliton waves on a one-dimensional monatomic quartic lattice is investigated using numerical techniques. Solitons are generated by the application of an external forcing function to the end atom of a free chain of monatomic atoms. Multiple traveling solitons are observed to flow down the chain when the strength of the forcing function, applied over a period of time, is beyond a threshold value. Total number of traveling solitons in the chain increases rapidly as the strength of the forcing function is increased beyond this critical value. The amplitudes and velocities of these multiple solitons saturate with the increasing strength of the forcing function. The frequencies and wave vectors of all the traveling solitons on the quartic lattice are independent of forcing functions and lie within a narrow range of values. These results are compared with those for solitons on a one-dimensional monatomic Toda lattice generated using similar forcing functions.

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I. INTRODUCTION

Generation of traveling solitons on a one-dimensional lattice of monatomic atoms is discussed. The discussion is limited to the type of lattice, whose potential energy between adjacent atoms contains a nonlinear quartic term,

$$V(Q_{j+1},Q_j) = +\frac{K_4}{4}(Q_{j+1}-Q_j)^4, \tag{1}$$

where $Q_j(t)$ is the displacement of an atom at site j at time t and K_4 is the quartic spring constant. The interest in solitons arises from their role in thermal transport in one-dimensional systems. Toda[1] first proposed that energy is mainly transported by solitons in nonlinear lattices and showed that, in the case of a lattice with exponential interaction, the heat flux in the isotopically disordered lattice is enhanced by the introduction of the nonlinearity. In recent years, heat conductivity in low-dimensional systems has attracted increasing attention[2–4] due to the discovery of nanotubes and nanowires. In these studies,[2] it has been speculated that soliton-like propagation is generically favored in one-dimensional systems. The study of solitons in one-dimensional nonlinear lattices would help to better understand the heat transport in one-dimensional systems.

There have been numerous investigations on the lattice dynamics of quartic and other nonlinear lattices using different analytical[5–16] and numerical approaches.[17–24] In these studies, the potential contains quadratic terms along with nonlinear cubic or quartic terms (Fermi-Pasta-Ulam or FPU problem). The quadratic term describes phonons on the lattices, which affect the existence and stability of solitons. Phonons do not exist in a pure quartic lattice and this would help us better understand the nature of solitons in one-dimensional nonlinear lattices. Approximate analytical techniques to analyze the traveling solitons with this potential are discussed in an earlier work by Mahan.[25] Here, the aim is to numerically generate traveling solitons on a quartic lattice with the application of various forcing functions and determine the properties of the solitons thus obtained.

In this paper, the generation of traveling soliton waves with the application of various external forcing functions is studied. These forcing functions would also be referred to as pulses in this article. This method of numerically generating solitons has been referred to as "sharp-pulse method" in literature.[24] Multiple solitons are observed to flow down the free chain of atoms when the strength of the forcing function is beyond a threshold value. Observation of multiple solitons is reported[24] for the FPU problem with sinusoidal forcing function, but the article does not mention anything about the strength of the forcing function needed to generate multiple solitons. In this paper, the generation of solitons with different forcing functions is analyzed and multiple solitons are found to appear only for the forcing functions with finite duration with strength beyond a threshold value. The analysis is then extended to characterize the nature of these multiple solitons. The amplitude values of the multiple solitons in quartic lattice are different from each other at low pulse strengths but tend to saturate with the increase in the pulse strength. A similar phenomenon has been reported for a Toda lattice of capacitors[26] but has not been reported so far for quartic lattices.

II. EQUATIONS OF MOTION

The interactions between the first neighbors are sufficient for a general description of the classical vibrations on a one-dimensional lattice. Therefore, the general equation of motion for the jth atom in a quartic lattice consisting of N atoms is

$$m\frac{d^2}{dt^2}Q_j = -K_4[(Q_j-Q_{j-1})^3+(Q_j-Q_{j+1})^3], \quad 2 \le j \le N. \tag{2}$$

The one-dimensional chain is considered to have free boundaries. The equations of motion for the end atoms of the chain are given by

$$m\frac{d^2}{dt^2}Q_1 = -K_4[(Q_1-Q_2)^3], \tag{3}$$

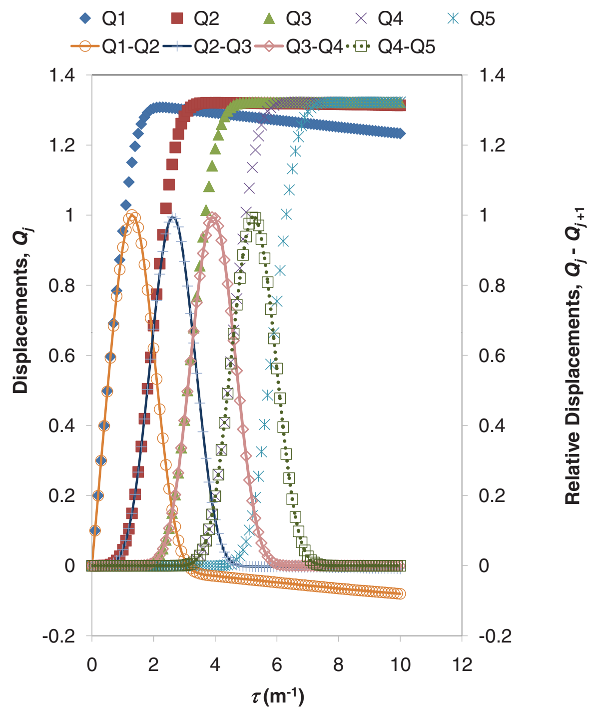

FIG. 1. (Color online) Generation of solitons on quartic lattice with delta function pulse; impulse strength $v_0=1$ units; lattice parameters $K_4=1$ and $m=1$; and number of atoms in the chain $N=50$.

$$m\frac{d^2}{dt^2}Q_N = -K_4[(Q_N - Q_{N-1})^3]. \tag{4}$$

In order to write the above equations in dimensionless forms, a parameter τ is defined, which plays the role of time but has the dimensional unit of inverse distance $\tau = t\sqrt{\frac{K_4}{m}}$. The equations of motion (2)–(4) become

$$\frac{d^2}{d\tau^2}Q_1 = -(Q_1 - Q_2)^3, \tag{5}$$

$$\frac{d^2}{d\tau^2}Q_N = -(Q_N - Q_{N-1})^3, \tag{6}$$

and

$$\frac{d^2}{d\tau^2}Q_j = -(Q_j - Q_{j-1})^3 - (Q_j - Q_{j+1})^3, \quad 2 \leq j \leq N. \tag{7}$$

The above equations are the primary starting points of further calculations.

III. TRAVELING SOLITONS

A forcing function of zero duration or an impulse is applied to one of the free ends of the chain of atoms to generate traveling solitons on the quartic lattice. The ends remain unclamped (are free to move) at all times, even after the application of the impulse. The impulse can be mathematically represented by a delta function pulse $f(\tau) = v_0\delta(\tau)$; here v_0 is the impulse strength and has the dimension of $[L]^3$. The application of the impulse is numerically modeled as follows: at $\tau=0$, the displacements of the atoms are zero ($Q_j=0$, $1 \leq j \leq N$) and all the atoms, except the one at the end, are at rest ($\dot{Q}_j=0$, $2 \leq j \leq N$). The end atom has an initial velocity ($\dot{Q}_1 = v_0$) due to the application of the impulse. Mass of an atom m and quartic spring constant K_4 are chosen to have unit values throughout this investigation. The method of finite differences is used to solve the equations of motion (5)–(7) numerically. Figure 1 shows the displacements of the atoms in time. They all move to the right by a distance a_0 and stop. An interesting feature noted from the equations of motion is that if the Eqs. (5)–(7) for all the atoms are added together, some sum rules can be generated,

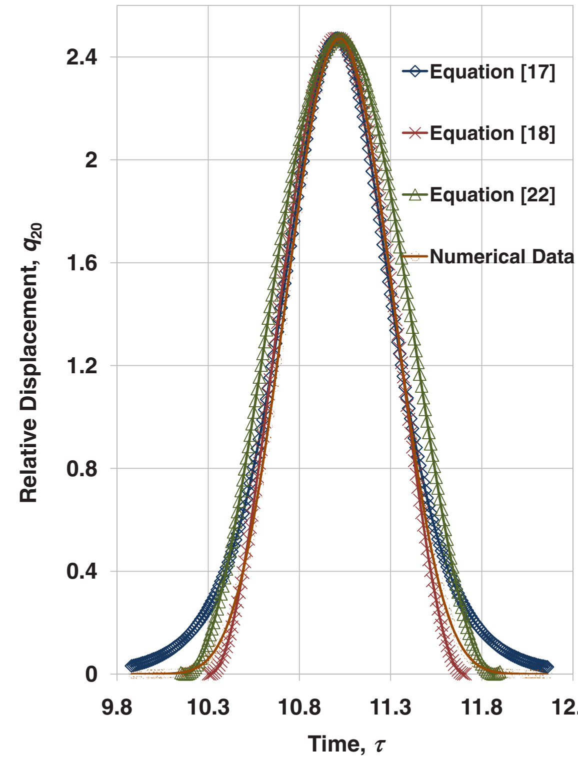

FIG. 2. (Color online) Comparison between the numerically and analytically obtained values of the relative displacements of the 20th atom.

$$\sum_{j=0}^{\infty} \ddot{Q}_j(\tau) = 0, \tag{8}$$

$$\sum_{j=0}^{\infty} \dot{Q}_j(\tau) = v_0, \tag{9}$$

$$\sum_{j=0}^{\infty} Q_j(\tau) = v_0\tau. \tag{10}$$

The sum rules mentioned in Eqs. (8)–(10) are obeyed in Fig. 1.

In order to find out the number of atoms in motion while the soliton propagates through the chain, the number of atoms that have relative displacements greater than a chosen

TABLE I. Percentage of times different number of atoms in motion during the propagation of the soliton through the chain.

	Times different number of atoms in motion (%)				
Min rel. displ. (order)	1	2	3	4	5
10^{-1}	2.12	46.73	51.045	0.00	0.00
10^{-2}	1.37	2.965	80.15	15.50	0.00
10^{-3}	0.87	2.645	39.6	56.88	0.00
10^{-4}	0.545	2.465	7.69	89.295	0.00
10^{-5}	0.345	2.25	4.865	69.405	23.13

minimum value is counted. Reference 24 reports that only three atoms are in motion at all times. It is found here that the number of atoms in motion depends on the choice of the cut-off value for the minimum relative displacement. Table I shows the percentage of times the number of atoms are in motion for different choices of the minimum relative displacement. As can be seen from the table, for the choice of a lower minimum relative displacement, a larger number of atoms are in motion and vice versa. For most values of the cutoff, initially, only one or two atoms are in motion while the soliton is in formation; once the soliton starts propagating, three to four atoms are in motion at all times.

Traveling solitons are found to exist in chains of atoms with free boundaries. If the atom farthest from the point of application of the impulse is clamped in its position or is bound to a wall, soliton generation is not affected. If the atom at the point of application is clamped in its position after the application of the impulse, a soliton is still generated but its amplitude is comparable to the unclamped one only for a high-impulse strength. On the other hand, if the atom at the point of application of the impulse is bound to a wall, the bound atom has an equation of motion,

$$\dot{Q}_1 = v_0, \tag{11}$$

$$\ddot{Q}_1 = -(Q_1 - Q_2)^3 - Q_1^3, \tag{12}$$

and

$$\ddot{Q}_j = -(Q_j - Q_{j+1})^3 - (Q_j - Q_{j-1})^3, \ \ 2 \le j \le N. \tag{13}$$

A soliton can never be generated in this case for any value of v_0. Instead, the atoms at the other end of the chain all continue to vibrate and the effect of the impulse decreases after a few atoms. Similar results have been reported in the literature. However, a soliton is obtained for every value of v_0 for a chain with free boundaries.

IV. APPROXIMATE WAVE FORM

The general solution for a traveling soliton can be written as

$$Q_j = a_0 f[vj - \omega(v)\tau], \tag{14}$$

where a_0 is the maximum displacement of the atoms in the chain or the amplitude of the soliton wave. The two parameters v and $\omega(v)$ can be treated as the "wave vector" and the "frequency" of the soliton wave, respectively. It is a characteristic of quartic lattice that "frequency" parameter of the soliton wave depends on the amplitude a_0. Hence $\omega(v)$ can be written as $\beta(v)a_0$. Henceforth, the parameters v and β would be referred as the "wave vector" and "frequency" of the soliton solutions. It can be noted from Fig. 1 that the atoms displace smoothly to their final value with no ringing (except for the end atom). The wave form shown in Fig. 1 is approximately described by the formula,

$$Q_j(\tau) = \frac{a_0}{2}[1 - \tanh(\phi_j)], \tag{15}$$

with

$$\phi_j = vj - \beta a_0 \tau. \tag{16}$$

This formula is a good but not perfect description of the soliton motion. Using the above form for the displacements of the atoms, the relative displacements between the atoms can be written as

$$q_j(\tau) = \frac{a}{[\cosh(vj - \beta a_0 \tau)]^2}, \tag{17}$$

where a is the maximum relative displacements of the atoms.

Reference 24 has given a formula for the soliton solutions of the FPU problem assuming that only three atoms are in motion at all times during the propagation of the soliton,

$$q_j = \pm \frac{a}{2}\left[1 + \cos\left(\frac{2\pi}{3}j - \omega\tau\right)\right] \quad \text{if} \quad -\pi < \frac{2\pi}{3}j - \omega\tau < \pi, \tag{18}$$

$$q_j = 0, \quad \text{otherwise},$$

and the relations for the frequency ω and the velocity v_S are given as functions of a:

$$\omega = \sqrt{3 + (45/16)a^2},$$
$$v_S = \omega/(2\pi/3) = 3\sqrt{3 + (45/16)a^2}/(2\pi). \tag{19}$$

Reference 5 has reported an exact solution for the lattice waves in a quartic lattice with period of three lattice sites. The lattice wave has a cosine solution,

$$q_j = a\cos(\theta_j), \ \ \theta_j \equiv \frac{2\pi}{3}j - \omega_3\tau, \tag{20}$$

with

$$\omega_3^2 = \frac{9}{4m}a^2 K_4 \tag{21}$$

for quartic lattice. These formulas can be rewritten in the same way as mentioned in Ref. 24 to satisfy the soliton solutions in quartic lattice,

$$q_j = \frac{a}{2}\left[1 + \cos\left(\frac{2\pi}{3}j - \omega_3\tau\right)\right] \quad \text{if} \quad -\pi < \frac{2\pi}{3}j - \omega\tau < \pi, \tag{22}$$

with

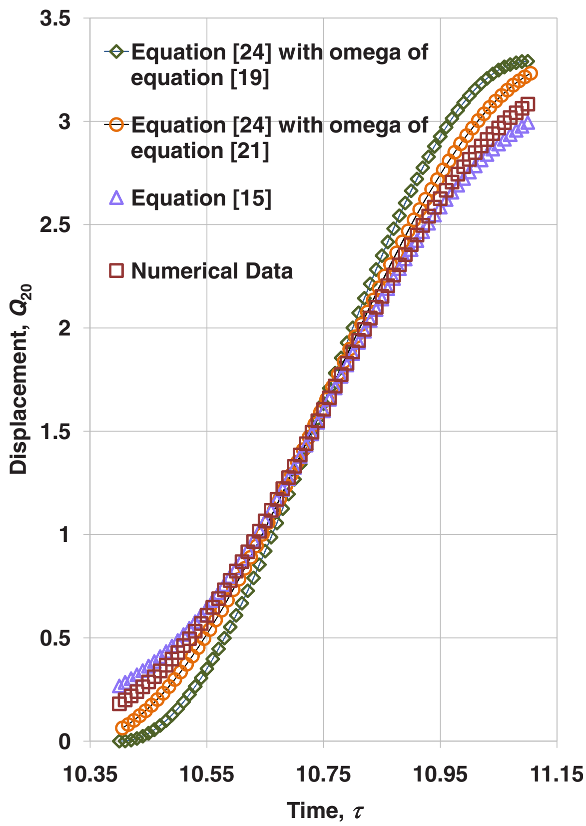

FIG. 3. (Color online) Comparison between the numerically and analytically obtained values of the displacements of the 20th atom.

$$\omega_3^2 = \frac{9}{4m} a^2 K_4 .$$

A comparison between these analytical solutions (17), (18), and (22) and our numerical data is shown in Fig. 2. The numerical values of the relative displacements of the 20th atom in the chain are plotted against time along with the analytical values obtained using various formulas [Eqs. (17), (18), and (22)] in Fig. 2.

Now, an analytical solution for the displacements of the atoms of the period three lattice wave is given in Ref. 5,

$$Q_j = a_0 \sin\left(\frac{2\pi}{3} j - \omega_3 \tau - \frac{\pi}{3}\right). \tag{23}$$

This formula is modified in a similar fashion as in Eq. (22),

$$Q_j = \frac{a_0}{2}\left[1 - \sin\left(\frac{2\pi}{3} j - \omega_3 \tau - \frac{\pi}{3}\right)\right],$$
$$-\frac{\pi}{2} < \frac{2\pi}{3} j - \omega_3 \tau - \frac{\pi}{3} < \frac{\pi}{2}. \tag{24}$$

Analytical values are obtained using the above formula for atomic displacements for both $\omega = \omega_3$ and $\omega = \sqrt{3 + (45/16)a^2}$. The analytical values obtained using the above analytical expressions [Eqs. (15) and (24)] are compared with numerical data of the displacements of the 20th atom in the chain and are shown in Fig. 3.

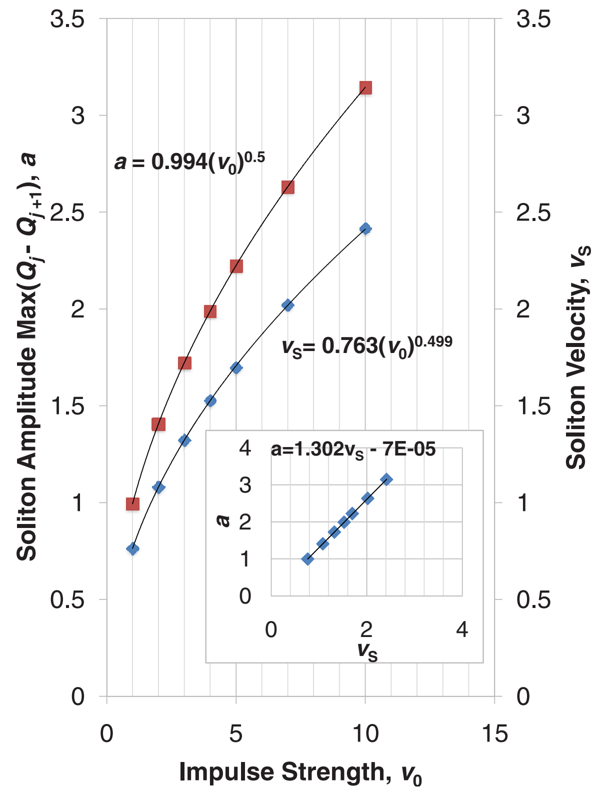

FIG. 4. (Color online) Power-law relationship of soliton amplitude and soliton velocity with applied impulse strength v_0. Inset: Soliton amplitude varies linearly with soliton velocity.

As can be noticed from both Figs. 2 and 3 that the form of the analytical solutions reported in Refs. 5 and 24, as well as our proposed solution, matches with that of the numerical data. The values obtained using the *tanh* solution match closely with the data for the atomic displacements but it does not match that well with the tails of the numerical relative displacements data.

V. PARAMETERS OF SOLITON

Using the approximate wave form given in Eqs. (15) and (16), the soliton parameters (β, v) can be deduced from the numerical data in the following way. First, τ_j is defined as the value of time when the displacement of the jth atom is half its maximum amplitude and $\phi_j(\tau_j)=0, \tau_j = vj/\beta a_0$. A numerical derivative of $Q_j(\tau)$ is taken at $\tau = \tau_j$,

$$\frac{d}{d\tau} Q_j(\tau) = \frac{a_0^2 \beta}{2 \cosh(\phi_j)^2}, \tag{25}$$

$$\left[\frac{d}{d\tau} Q_j(\tau)\right]_{\tau_j} = \frac{a_0^2 \beta}{2}. \tag{26}$$

The slope is proportional to β. The other parameter v is obtained from the displacement of the $(j+1)$th atom at $\tau = \tau_j$,

$$Q_{j+1}(\tau_j) = \frac{a_0}{2}[1 - \tanh(v)], \tag{27}$$

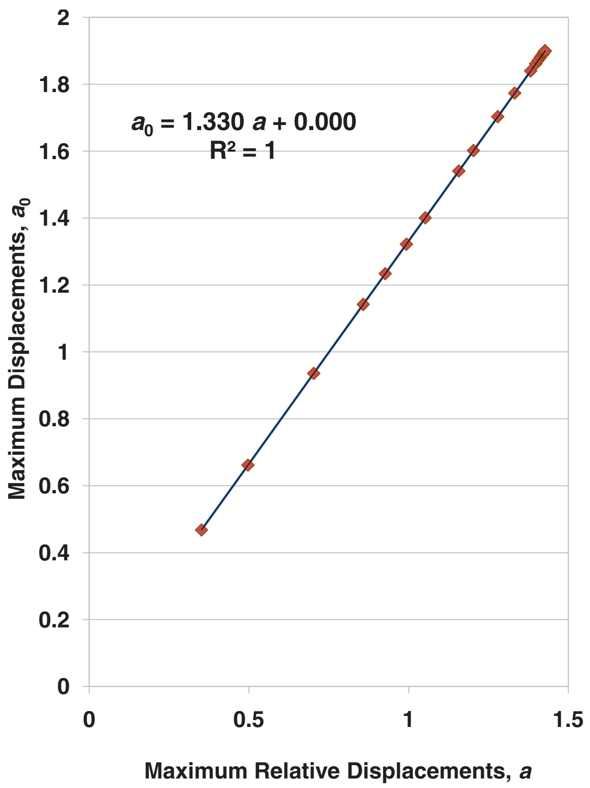

FIG. 5. (Color online) Linear relationship between maximum atom displacements and maximum relative displacements $a_0 \approx (4/3)a$.

$$v = \frac{1}{2}\ln\left[\frac{a_0}{Q_{j+1}(\tau_j)} - 1\right]. \tag{28}$$

The values of these parameters, obtained using the above Eqs. (26) and (28) for some of the solitons, are shown in Table II.

The "frequency" parameters of the solitons generated above are proportional to a_0 or a ($\omega = \beta a_0$) similar to the cases described in Refs. 5 and 24 but the proportionality constant is different. The values of "wave-vector" parameters are constant, as in Refs. 5 and 24, but they are not equal to the "magic" wave number $2\pi/3$.

Now, the strength of a pulse is defined as the area under the curve of $f(\tau)$ vs τ. In case of an impulse, the strength is simply v_0. The interesting part of the results shown in Table II is that β and v are independent of v_0, the impulse strength. The only parameter that depends upon v_0 is a_0, the soliton amplitude. The relationship between the soliton amplitude

TABLE II. Parameters of a soliton on the quartic lattice generated by an impulse v_0 on a free end.

v_0	a_0	β	v
0.5	0.933	1.03	2.45
1.0	1.322	1.03	2.45
1.5	1.618	1.03	2.45
2.0	1.867	1.03	2.45

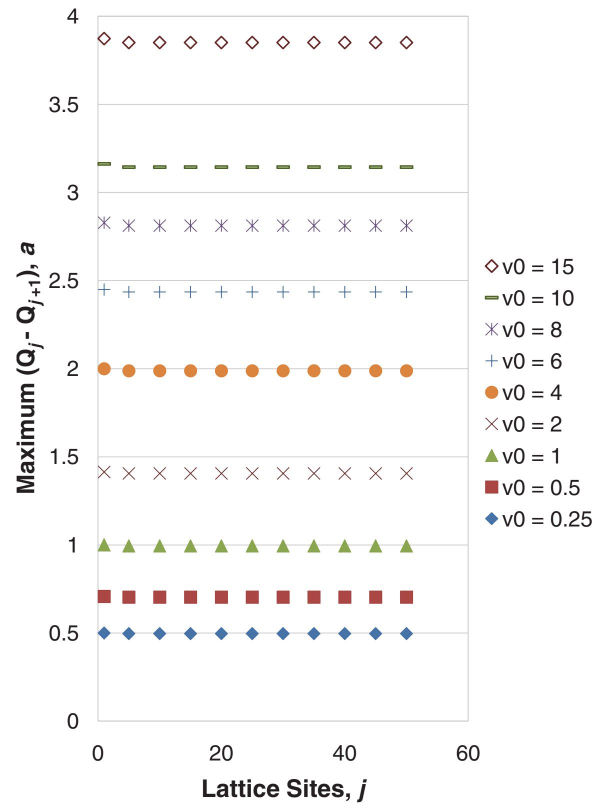

FIG. 6. (Color online) Peak relative displacements, a at different lattice sites for various impulse strengths v_0. Soliton amplitudes are all the same for different j showing the existence of stable solitons for all impulse strengths.

and impulse strength can be understood in the following manner: energy supplied to the lattice by an impulse is $E_{\text{initial}} = \frac{1}{2}mv_0^2$. Now, atoms gain energy as a result of the propagation of a soliton, both KE and PE vary as $\approx K_4 a_0^4$. Hence, parameter a_0 is expected to vary as $a_0^4 \propto v_0^2$ or $a_0 \propto v_0^{1/2}$ from dimensional analysis. As can be noted from Table II,

$$a_0 = 1.322\sqrt{v_0}, \tag{29}$$

which matches the expectation that soliton amplitude should have a square-root relationship with the impulse strength.

Now, the maximum relative displacements between adjacent atoms a also behave in a similar manner $a \sim \sqrt{v_0}$ (Fig. 4). Henceforth, the peak relative displacement a would be referred to as soliton amplitude. The maximum relative displacements a scale linearly with the maximum displacements a_0; the relationship, as shown in Fig. 5, can be approximately described as $a_0 \approx (4/3)a$. The times, when the relative displacements reach peak value, a increase linearly as the soliton propagates through lattice sites. This relationship is utilized to obtain the velocity of soliton v_S. Soliton velocities v_S are also observed to have a square-root relationship with impulse strength $v_S \sim \sqrt{v_0}$. And the velocities scale linearly with the soliton amplitudes as shown in the inset of Fig. 4.

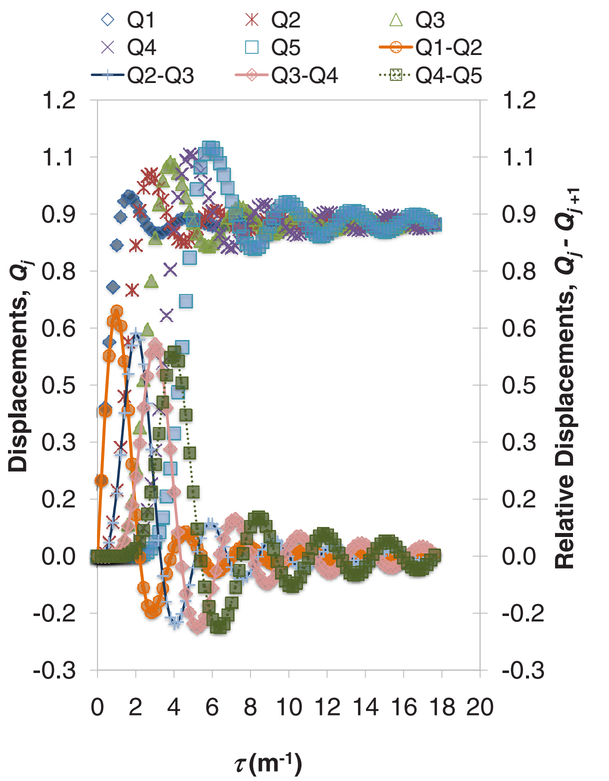

FIG. 7. (Color online) Generation of solitons on Toda lattice with constant impulse. Impulse strength $v_0=1$ units; Toda lattice parameters $a=1$, $b=1$; and number of atoms in the chain $N=50$.

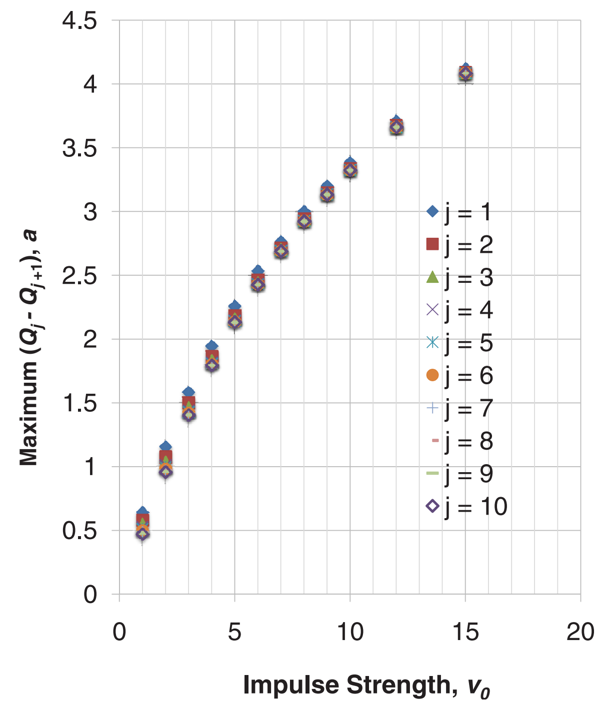

FIG. 8. (Color online) Variation of peak relative displacements of the soliton wave form in Toda lattice for different impulse strengths. For $v_0 \leq 10$, amplitude of soliton decreases as j increases but it becomes constant for stronger impulses. A stable soliton is generated only when $v_0 \geq 10$.

VI. STABILITY OF SOLITON

Stability of a soliton refers to its ability to maintain its width and amplitude throughout the propagation. The observation that only three to four atoms are in motion during soliton propagation points toward the fact that the soliton is able to maintain its width. Information about the amplitude can be obtained by comparing the peak values of the relative displacements, a at different lattice sites. Figure 6 shows the soliton amplitudes at different lattice sites for various impulse strengths v_0. The constant value of the amplitudes indicates the formation of stable solitons on the quartic lattice irrespective of impulse strengths.

This result is compared with the solitons generated on a one-dimensional monatomic Toda lattice $\{V(Q_{j+1},Q_j) = +\frac{a}{b}\exp[-b(Q_{j+1}-Q_j)]+a(Q_{j+1}-Q_j)\}$ with the application of external impulses as shown in Fig. 7. Soliton amplitudes at different lattice sites are shown in Fig. 8 for varied impulse strengths. For weak impulses, the amplitude of soliton decreases as the wave moves along the chain, but the amplitude remains almost constant for impulses beyond a critical strength. A stable soliton is produced on the Toda lattice only when the input impulse is beyond a critical strength. Similar behavior has been reported in literature for a chain of capacitors modeled as Toda lattice.[26] The explanation for this behavior is linked to the presence of phonons in Toda lattice, which can be perceived from the oscillatory tail of the displacement-time plot (Fig. 7). The energy supplied to the Toda lattice by the impulse is partly spent in the generation of phonons. Therefore, higher impulse strength is required to produce a stable soliton. In comparison, the energy supplied to the quartic lattice is used entirely for the generation of solitons. Hence, a stable soliton is generated on the quartic lattice for all impulse strengths.

VII. MULTIPLE SOLITONS

A. Pulse of constant height

The parameters (β,v) of solitons generated using an impulse are found to be constants independent of impulse strengths. This observation motivated the study of generation of solitons using a different forcing function. A pulse of constant height is applied to the free end of the chain for a finite amount of time $f(\tau)=A\Theta(\tau)\Theta(T-\tau)$. The equations of motion can be written as

$$\dot{Q}_1 = A\tau, \tag{30}$$

$$\ddot{Q}_1 = -(Q_1-Q_2)^3 + A\Theta(\tau)\Theta(T-\tau), \tag{31}$$

$$\ddot{Q}_j = -(Q_j-Q_{j+1})^3 - (Q_j-Q_{j-1})^3, \quad 2 \leq j \leq N. \tag{32}$$

Here, the dimension of A is $[L]^3$ and that of T is same as τ. Numerical solutions of the above equations for $A=1.5$, $T=1$ are shown in Fig. 9. The presence of a second soliton can be discerned from the graph. It is interesting to note that when a delta function pulse is applied to the end atom, only one soliton travels down the chain for all pulse strengths. In

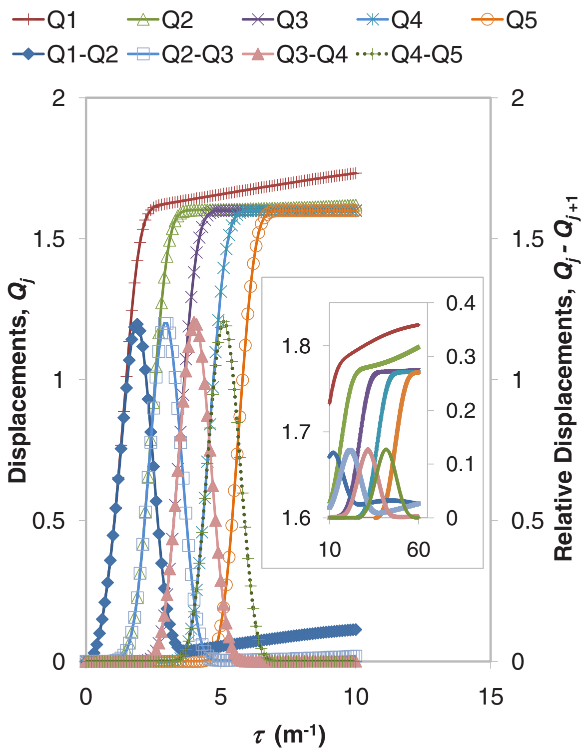

FIG. 9. (Color online) Multiple solitons generated on quartic lattice with pulse of constant height. Pulse strength $AT=1.5$ units; lattice parameters $K_4=1$, $m=1$; and number of atoms in the chain $N=50$. Inset: Second soliton in the chain.

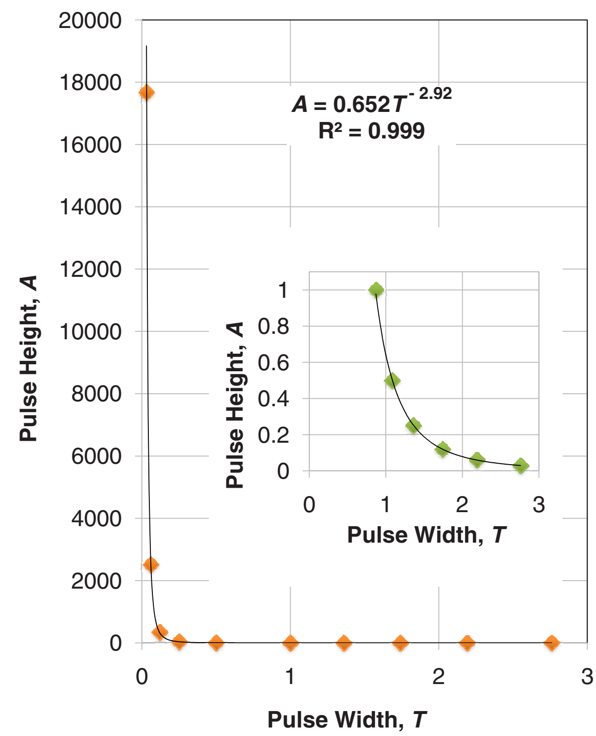

FIG. 10. (Color online) Minimum pulse height required to generate a second soliton as a function of pulse width. Inset: minimum pulse height required for pulse width $T \gtrsim 1$ units and minimum pulse width does not increase significantly for small pulse heights.

contrast, a second soliton is observed to appear whenever the strength of the constant-height pulse, AT exceeds a threshold value. In an attempt to make sure that this phenomenon does not arise due to the finite size of the lattice, this numerical calculation is repeated for various chain lengths ($N=50$, $N=100$, $N=200$). The critical pulse strengths are found to be the same for all chain lengths.

Now, pulse strength can be increased in two ways: (i) increasing pulse height keeping the width constant, (ii) increasing pulse width keeping the height constant. Figure 10 shows the minimum pulse height required to produce a second soliton in the chain as the width varied. The minimum pulse height required increases rapidly with a decrease in pulse width. In fact, the relationship is observed to be $A \approx 1/T^3$. Hence, as $T \to 0$, $A \to \infty$, which explains why multiple solitons are never observed with a delta function pulse. On the other hand, the minimum pulse width required to generate a second soliton increases slowly as the pulse height is reduced as shown in the inset of Fig. 10. It can be concluded that it is easier to produce a second soliton by varying the width of forcing function keeping the height constant rather than varying the height keeping the width constant. Once the pulse strength is increased beyond the threshold value, more and more solitons are generated for increments of T, keeping a constant pulse height.

Multiple soliton formation has also been observed on the Toda lattice (Fig. 11). The critical pulse strength required to produce a second soliton is higher than that of the quartic lattice, which can be attributed to the presence of phonons on the Toda lattice. For the same reason, more energy is required to generate a third soliton and the increment in energy has to be a fixed amount to generate successive solitons. Similar results were reported in Ref. 26 for a chain of capacitors modeled as the Toda lattice.

B. Sinusoidal pulse

The discovery of multiple solitons with pulse of constant height inspired the investigation of the effect of pulse shape on generation of solitons. A sinusoidal pulse $[f(\tau) = A\Theta(\tau)\Theta(\frac{T}{2}-\tau)\sin(\frac{2\pi\tau}{T})]$ is applied to the end atom for half the period $T/2$,

$$\dot{Q}_1 = 0, \tag{33}$$

$$\ddot{Q}_1 = -(Q_1 - Q_2)^3 + A\sin\left(\frac{2\pi\tau}{T}\right)\Theta(\tau)\Theta\left(\frac{T}{2}-\tau\right), \tag{34}$$

$$\ddot{Q}_j = -(Q_j - Q_{j+1})^3 - (Q_j - Q_{j-1})^3, \quad 2 \le j \le N. \tag{35}$$

Here, A has dimension of $[L]^3$ and T that of τ. Strength of the pulse is given by $\frac{AT}{\pi}$. As the pulse strength is increased, multiple solitons are generated on the quartic lattice similar to the earlier case of pulse of constant height. It is also observed that a weaker pulse is capable of producing multiple solitons if the pulse strength is increased by broadening it rather than increasing the height. Once the second soliton appears in the chain for a critical pulse strength, more number of solitons appear for increments in pulse width as can be noted from

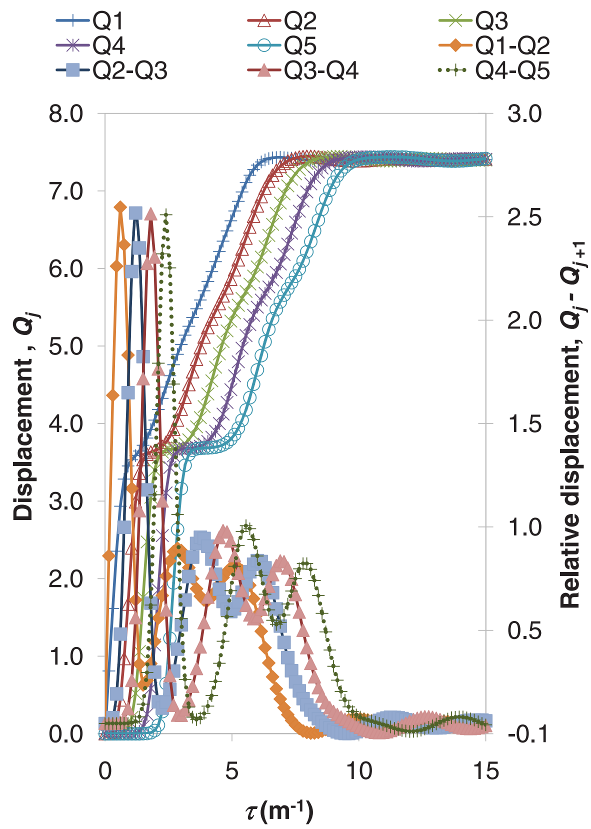

FIG. 11. (Color online) Multiple solitons generated in Toda lattice with pulse of constant height. Pulse strength $AT=5.4$ units; lattice parameters $a=1$, $b=1$; and number of atoms in chain $N=50$.

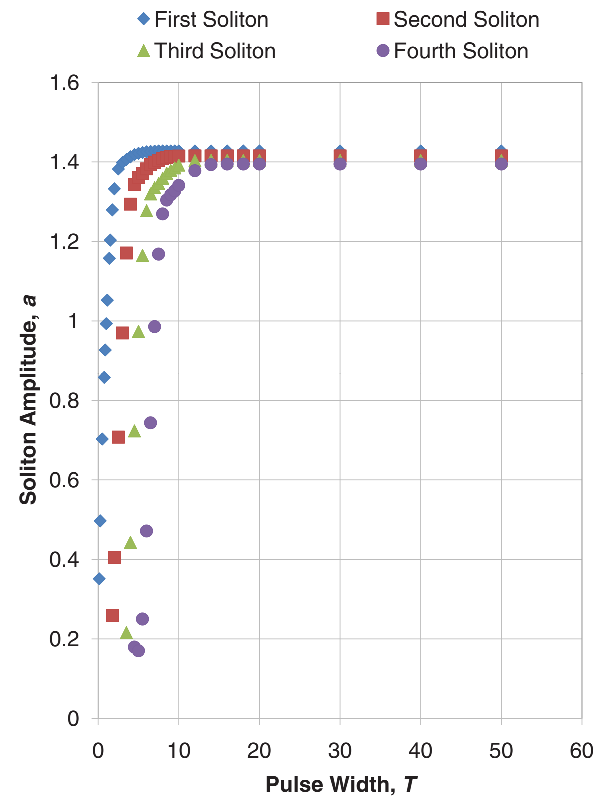

FIG. 12. (Color online) Multiple soliton amplitudes as a function of pulse width. Amplitudes tend to saturate with increase in pulse strength.

Fig. 12. Therefore, it can be concluded that the generation of multiple solitons is independent of the pulse shape. Multiple solitons are always generated on the quartic lattice for pulse strength beyond a threshold value, given the pulse is applied for a finite amount of time.

C. Parameters of multiple solitons

The parameters (β, v) are calculated for the multiple solitons, generated using both the sinusoidal pulse and the pulse of constant height. All of these multiple solitons are found to have parameters confined in a very narrow range of values, $\beta \rightarrow (1.02,\ 1.03)$ and $v \rightarrow (2.45,\ 2.50)$. A similar result is also obtained using a delta function pulse as shown in Table II. Hence, it can be concluded that the parameters (β, v) of solitons generated on one-dimensional monatomic quartic lattice are independent of forcing functions.

The other parameter, soliton amplitude a, increases with the increase in pulse strength. The plot of a against pulse width is shown in Fig. 12. The interesting feature to note here is that the amplitudes of all the solitons smoothly increase toward a saturation value. The saturation of the soliton amplitudes with the increase in pulse width and the narrow range of (β, v) imply that the traveling solitons generated on the quartic lattice using a forcing function is very specific in nature. It can be predicted from this finding that the energy of the solitons generated would have an upper bound, although this investigation is not included in this paper. The distribution of energy among the multiple solitons is also left for future investigation.

VIII. SUMMARY

A method to generate traveling solitons on a one-dimensional monatomic lattice with quartic interatomic potential is presented here. The method includes application of an external forcing function to one of the ends of a chain of atoms with free boundaries. As a result, the end atom attains a finite velocity and this movement causes a soliton to move down the chain. The solitons are observed to maintain their amplitudes and widths throughout propagation for all pulse strengths. The effect of varying pulse width and height on the parameters of solitons (amplitude a_0 or a and β, v) are investigated. The soliton amplitudes are found to have a square-root relationship with the pulse strength for a delta function pulse. As the pulse width increased, keeping the pulse height constant, a different phenomenon is observed when the pulse width is beyond a threshold value. Multiple stable solitons are observed to flow down the chain, each maintaining its individual identity as they cross through one another. It is found that the pulse height needs to be excessively large in order to produce multiple solitons for smaller pulse widths. This explains the absence of multiple solitons when a pulse of zero width is used. The appearance of a second soliton in the chain is the onset of generation of a larger number of solitons. All of these solitons share a very small parameter space in β and v. The multiple solitons have the special property that the amplitudes saturate with the increase in pulse strength. The generation and properties of multiple solitons on the one-dimensional quartic lattice are found to be independent of the forcing function.

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