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Josephson effects in condensates of excitons and exciton polaritons

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We analyze theoretically the phenomena related to the Josephson effect for exciton and polariton condensates, taking into account their specific spin degrees of freedom. We distinguish between two types of Josephson effects: the extrinsic effect, related to the coherent tunneling of particles with the same spin between two spatially separated potential traps, and the intrinsic effect, related to the "tunneling" between different spinor components of the condensate within the same trap. We show that the Josephson effect in the nonlinear regime can lead to nontrivial polarization dynamics and produce spontaneous separation of the condensates with opposite polarization in real space.

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I. INTRODUCTION

The experimental observation of Bose-Einstein condensation (BEC) of trapped cold atoms has stimulated research on the properties of this new state of matter[1] and opened a way to study the interplay between macroscopic phenomena and quantum coherence. Indeed, several interesting physical phenomena in this domain, such as superfluidity and the Josephson effect, are connected with the appearance of a macroscopic order parameter (playing a role of a single particle wave function) $\Psi(\mathbf{r},t)=\sqrt{\rho}e^{-i\phi}$, with ρ being a condensate density and ϕ a well-defined phase.

In the field of condensed matter physics, BEC of quasiparticles of various types has also been proposed. The formation of exciton condensates was theoretically considered more than 40 years ago[2] but appeared to be difficult to realize experimentally. In the recent years, however, other solid-state systems were proposed as possible candidates for the achievement of high-temperature BEC. It was claimed that BEC was experimentally observed in quantum Hall bilayers,[3] in the systems of magnons[4] and cavity exciton-polaritons (polaritons).[5,6] The latter will be in the focus of the present paper.

Polaritons are elementary excitations of microcavities in the strong coupling regime. Being a combination of quantum-well (QW) excitons and cavity photons, they retain the properties of both. The photonic component results in the small effective mass (10^{-4}–10^{-5} of the electron mass) and the short lifetime (several picoseconds), while the excitonic component makes possible effective polariton-polariton interactions. These factors are crucial for polariton BEC, whose critical temperature is extremely high (in Ref. 5 BEC of polaritons was reported at about 20 K). Except the high critical temperature, the condensates of polaritons and of QW excitons [in particular indirect excitons in double QWs (Ref. 7)] have many similarities and their theoretical description can be carried out in the same way.

The important peculiarity of optically active two-dimensional (2D) excitons and polaritons is linked with their spin structure: they have two allowed spin projections on the structure growth axis, related to the right-circular and left-circular polarization of the counterpart photons. Moreover, due to the effects of exchange, the interparticle interactions are strongly spin anisotropic:[8,9] particles with similar spin projections strongly repel, while particles with opposite spin projections almost do not interact with each other.[10] All this makes exciton and polariton condensates behave differently from atomic condensates or superfluids even in the thermodynamic limit. The real-space dynamics of polariton droplets is qualitatively different from those of atomic superfluids and reveal strong polarization effects.[11]

Strictly speaking, being 2D objects, polaritons and QW excitons can undergo BEC only when confined in a potential trap. The latter can appear due to the intrinsic lateral photonic disorder in a cavity (as it was the case in Ref. 5) or can be created in a controllable way by external laser beams,[12] by application of stress,[6,13] or by using photolithographic techniques.[14] The possibility of engineering of a spatial confinement opens a way to the investigation of the Josephson effects for excitons and polaritons, related to the tunneling between two condensates possessing macroscopic phase coherence. Being first predicted[15] and experimentally observed[16] for two superconductors separated by a thin insulator layer, the Josephson effect was later discovered for two reservoirs of superfluid helium connected by nanoscale aperture[17] and very recently for interacting atomic condensates.[18]

The crucial property of the condensates of cold atoms and the condensates of excitons and polaritons, as compared to the superconductors, is the interaction between the tunneling particles. This leads to the striking nonlinear effects in Josephson dynamics, such as anharmonicity of the Josephson oscillations[19] and macroscopic quantum self-trapping (MQST) in the case, when the initial imbalance between the two condensates exceeds some critical value.[20] In this Rapid Communication we consider these effects applied to the condensates of excitons and polaritons. With respect to the recent works published on a similar topic,[21–23] the novelty is that we take into account the polarization degrees of freedom, which gives rise to much richer and original phenomenology. A related problem has also been addressed theoretically for a two-species atomic condensate in a double well but without direct interconversion between the species.[24,25]

II. MODEL

The starting point for the description of the Josephson effects is a model Hamiltonian for interacting bosons with pseudospin confined in two traps, R and L. In the basis of circular polarized states $\uparrow,\downarrow$ it reads

$$\hat{H} = E \sum_{j=L,R;\sigma=\uparrow,\downarrow} c_{j\sigma}^{+} c_{j\sigma} + J \sum_{\sigma=\uparrow,\downarrow} (c_{L\sigma}^{+} c_{R\sigma} + c_{R\sigma}^{+} c_{L\sigma}) + \Omega \sum_{j=L,R} (c_{j\uparrow}^{+} c_{j\downarrow} + c_{j\downarrow}^{+} c_{j\uparrow}) + \frac{U}{2} \sum_{j=L,R;\sigma=\uparrow,\downarrow} c_{j\sigma}^{+} c_{j\sigma}^{+} c_{j\sigma} c_{j\sigma}, \tag{1}$$

where the first term corresponds to free particles, the second term describes the spin-conservative tunneling of particles between the two traps, and the third term describes the possibly existing anisotropy of the QW in the direction of the structure growth-axis[26], which is equivalent to the application of an effective in-plane magnetic field able to provoke spin-flip processes. The last term of the Hamiltonian corresponds to the interactions between particles (we neglected the interactions between particles situated in different traps and particles having opposite circular polarizations). The geometry of the trap being known, the parameter J can be estimated as follows: $J \approx 4Ve^{-\hbar^{-1}\sqrt{2mV}L}$, where V is the depth of the trap and L is the distance between the traps. This estimation can be easily obtained by calculating the energy splitting between the symmetric and the antisymmetric wave function resulting from the coupling between two QWs separated by a distance much greater than the well width. We assume that V and, therefore, J are independent of the particle density, which is not exact, since repulsive interaction leads to the blueshift and to the reduction in the trap effective potential. The effects of the blueshift, which has always been neglected in the consideration of the nonlinear Josephson oscillations, will be discussed later, together with the Gross-Pitaevskii equation approach.

The interaction constant is $U \approx E_B a_B^2 / S$, with E_B and a_B being the binding energy and Bohr radius of the 2D exciton, and S being an area of a trap.[9] Ω is not easy to calculate (see Ref. 26 for details) but it has been measured in the range 50–100 μeV (Ref. 27).

Using the Heisenberg equations of motion for the operators $c_{j\sigma}$ and neglecting the effects of dephasing due to the finite number of the particles,[28] one obtains the following set of four coupled kinetic equations for a condensate order parameter $\psi_{j\sigma} = \langle c_{j\sigma} \rangle$:

$$i\hbar \frac{d\psi_{L\sigma}}{dt} = (E + U|\psi_{L\sigma}|^2)\psi_{L\sigma} + J\psi_{R\sigma} + \Omega\psi_{L,-\sigma}, \tag{2}$$

$$i\hbar \frac{d\psi_{R\sigma}}{dt} = (E + U|\psi_{R\sigma}|^2)\psi_{R\sigma} + J\psi_{L\sigma} + \Omega\psi_{R,-\sigma}. \tag{3}$$

If one of the tunneling parameters J or Ω is zero, the four coupled equations (2) and (3) separate into two independent pairs. In the case of the absence of the effective in-plane magnetic field $\Omega = 0$, one has independent coherent tunneling of the condensates with opposite circular polarizations between two traps, completely analogical to the conventional Josephson effect for atomic condensates. For excitons and polaritons this will be referred to as the *extrinsic Josephson effect*. On the other hand, if different traps are uncoupled $J = 0$ but $\Omega \neq 0$ in each of the traps, we have coherent exchange of particles between the condensates with different polarizations. This will be referred to as the *intrinsic Josephson effect*. This latter effect can be related with the oscillations of the circular polarization degree, which have been observed in the recent years[27] and which have been successfully described within a semiclassical approach based on the description of the polariton pseudospin dynamics.[29]

The nonlinear term plays a crucial role in Josephson dynamics. Once nonlinearity is neglected, Eqs. (2) and (3) give a well-known expression for the Josephson current for both extrinsic and intrinsic Josephson effects: $I_{i,e} = I_{i,e}^{(0)} \sin \phi$, where ϕ is the difference between the phases of the two condensates; $I_e^{(0)} = N_T J \hbar^{-1}, I_i^{(0)} = N_T \Omega \hbar^{-1}$, with N_T being the total number of particles. In this regime the occupancies of the coupled condensates exhibit harmonic oscillations with periods given by J and Ω for extrinsic and intrinsic Josephson effects, respectively. The situation changes drastically if nonlinear terms are taken into account. The oscillations of the occupation numbers become anharmonic and their period depends on N_T (Refs. 19 and 20). Besides, if the initial imbalance between the occupation numbers of the two coupled condensates exceeds some critical value N_c, the effect of the MQST occurs.[20] In this regime the tunneling between the condensates is suppressed and the particles remain in the state where they have been created as was recently observed for atomic condensates.[18]

III. RESULTS

We consider polaritons in a GaAs microcavity. The exciton binding energy is taken as $E_b = 12$ meV and the Bohr radius $a_B = 100$ A. The detuning between the cavity photon and the QWs exciton is zero and the Rabi splitting is taken as 7 meV. We consider either an ideal case of very long lifetime for the polariton or a more realistic one where it is taken as 16 ps.

Figure 1(a) illustrates the intrinsic Josephson effect ($J = 0$). It shows the time oscillations of the circular polarization degree $\rho_c(t)$ for two different initial values $\rho_c(0) = 0.58$ and $\rho_c(0) = 0.71$, respectively, the critical value for the MQST effect being 0.63. In that case the MQST leads to the suppression of beats of ρ_c (and by the onset of the self-induced Larmor precession, which is an oscillation of the linear part of the polarization about the effective magnetic field created by the circular polarization degree).[29,30] The black curve is calculated considering a pulsed resonant excitation and tak-

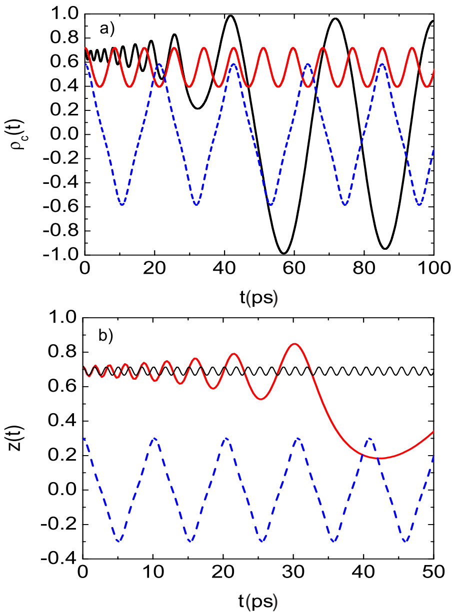

FIG. 1. (Color online) Intrinsic and extrinsic Josephson effects: (a) ρ_c for the intrinsic Josephson effect ($J=0$ and $\Omega=50$ μeV) (blue/dark gray—nonlinear oscillations, red/gray—MQST, and black—finite lifetime $\tau=16$ ps); (b) population imbalance for the extrinsic Josephson effect ($J=50$ μeV and $\Omega=0$) illustrating nonlinear oscillations (blue/dark gray), MQST (black), and finite lifetime effects (red/gray).

ing into account the decay of polaritons ($\tau=16$ ps). At short times, the polarization oscillations are suppressed. However, the decay of the number of particles leads to the increase in the critical value ρ_c and after 40 ps the oscillatory regime is recovered.

In Fig. 1(b) we consider the extrinsic Josephson effect taking place between two potential wells. We consider $\Omega=0$ so that the two circular polarizations are uncoupled. We consider the pure circular excitation of one of the coupled wells and we show the time behavior of the normalized population imbalance $z(t)$. Both linear Josephson oscillations and MQST can be achieved, as well as the transition between the two regimes when the finite lifetime of particles is taken into account.

Figure 2 shows the more complex and original situation of the dynamics of an elliptically polarized condensate, still assuming $\Omega=0$ for simplicity. The right potential well is populated at $t=0$ with a given circular polarization degree ρ_c^R. In Figs. 2(a) and 2(c) the two condensates of opposite polarization are oscillating regularly but with two independent frequencies, which gives a very specific oscillation pattern of the population imbalance [Fig. 2(a)] and of the circular polarization degree ρ_c^L and ρ_c^R [Fig. 2(c)] in the left and right well, respectively. For Figs. 2(b) and 2(d), the initial imbalance is higher and the MQST effect occurs for the spin-up component but not for the spin-down component. In this case, the former remains confined to the initial trap, while the latter exhibits Josephson oscillations between two traps. The dynamic spatial separation of two circular polarizations thus occurs [Fig. 2(d)]. The inset of Fig. 2 summarizes the range of parameters where the effect of spatial separation of polarization occurs.

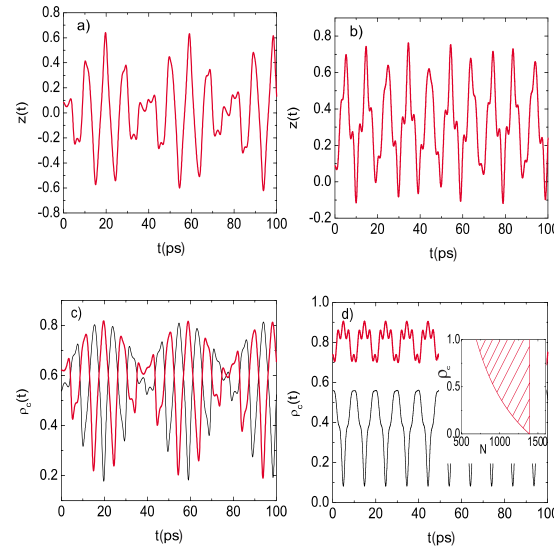

FIG. 2. (Color online) Extrinsic Josephson effect and spatial separation of polarizations: (a) Population imbalance for the case of normal Josephson oscillations; (b) population imbalance for the case of spatial separation of polarizations; (c) ρ_c^L (black) and ρ_c^R (red/gray) without spatial separation of polarizations; and (d) ρ_c^L and ρ_c^R for the case of spatial separation of polarizations. Inset: Phase diagram for polarization separation condition versus initial polarization degree ρ_c and the number of particles N.

To illustrate the spatial separation of polarizations, we used the description of the spinor polariton condensate by a system of coupled Gross-Pitaevskii equations for excitons and Schrödinger equations for photons,[11] which account for both realistic trapping potential and interparticle interactions,

$$i\hbar\frac{\partial\psi_\sigma(\mathbf{r},t)}{\partial t}=\left[\epsilon_{\mathrm{ex}}{}^0-\frac{\hbar^2}{2m_{\mathrm{ex}}}\Delta+V(\mathbf{r})+\alpha|\psi_\sigma(\mathbf{r},t)|^2\right]\psi_\sigma(\mathbf{r},t)+V_R\chi_\sigma(\mathbf{r},t), \tag{4}$$

$$i\hbar\frac{\partial\chi_\sigma(\mathbf{r},t)}{\partial t}=\epsilon_{\mathrm{ph}}{}^0-\frac{\hbar^2}{2m_{\mathrm{ph}}}\Delta\chi_\sigma(\mathbf{r},t)+V_R\psi_\sigma(\mathbf{r},t)+P_\sigma(\mathbf{r},t), \tag{5}$$

where $\psi_\sigma(\mathbf{r},t)$ is the exciton wave function with circular polarization σ, $\chi_\sigma(\mathbf{r},t)$ is the photon wave function with the same polarization, V_R is the Rabi splitting, $V(\mathbf{r})$ is an external potential of the two traps, which we introduce only for excitons (as in Ref. 6), and the parameter α is the interaction constant. Equation (5) is an obvious generalization of the system of Eqs. (2) and (3). The imaginary part of energy allows us to take into account the finite particle lifetime, and the pumping is introduced as a separate term $P_\sigma(\mathbf{r},t)$ for the photonic component.

We have considered a realistic trapping potential for polaritons with a barrier of 1 meV in height and 1 μm in width between the two minima. For such a barrier, a tunneling period of about 10 ps is expected from the approximative formula. From simulations, we find a period of 25 ps for this potential. However, in the nonlinear regime the period of oscillations can be strongly reduced, which should allow ex-

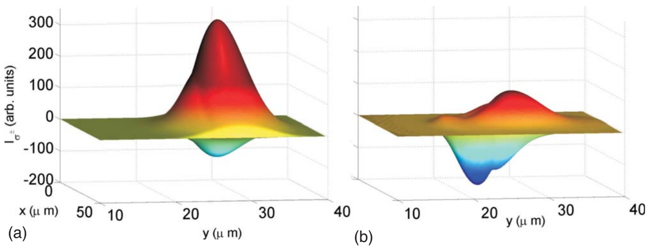

FIG. 3. (Color online) Spatial images showing the intensity of emission for two circular-polarized components. The σ^{-} component is plotted with negative sign: (a) No spatial separation; (b) Spatial separation of polarization during nonlinear oscillations.

perimental observation of the effects under study for exciton polaritons with a lifetime of 16 ps.

Figure 3 shows the spatial distribution of the two circular-polarized components σ^{+} and σ^{-}. The latter is plotted with negative sign, in order to allow direct comparison of both distributions. At $t=0$ [Fig. 3(a)], the maxima of the two polarizations coincide in real space, whereas at some later moment of time $t=6$ ps [Fig. 3(b)] the maxima are separated because of the different periods of oscillations for two components.

We conclude that the Josephson oscillation of polariton condensates created by resonant excitation is experimentally achievable but it requires very high quality samples and good design of the in-plane potential. As for the bare excitons, they possess a longer lifetime, but their heavier mass makes the period of oscillations longer and is at the end not easier to observe. Increasing the particle density leads to a blueshift of the energy, which gives rise to two competing effects: the MQST but also the decrease in the effective height of the barrier, which increases the tunneling constant. This change decreases the period of the Josephson oscillations and may eventually yield a complete delocalization of the condensate.[31] It reduces the possibility to observe MQST, which requires high occupation numbers in the condensates and which, to be observable, requires deep potential traps with steep walls.

IV. CONCLUSIONS

We analyzed the Josephson-type effects in condensates of spinor excitons and polaritons. We distinguish the extrinsic effect related to coherent tunneling of particles with the same spin between two spatially separated potential traps and the intrinsic effect related to tunneling between different spinor components of the condensate within the same trap. The Josephson effects in the nonlinear regime lead to nontrivial polarization dynamics and produce a spontaneous separation of the condensates with opposite polarizations in the real space.

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