Theory of quantum noise detectors based on resonant tunneling

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We propose to use the phenomenon of resonant tunneling for the detection of noise. The main idea of this method relies on the effect of homogeneous broadening of the resonant tunneling peak induced by the emission and absorption of collective charge excitations in the measurement circuit. In thermal equilibrium, the signal-to-noise ratio of the detector as a function of the detector bandwidth (the detector function) is given by the universal hyperbolic tangent, which is the statement of the fluctuation-dissipation theorem. The universality breaks down if nonequilibrium processes take place in the measurement circuit. We propose a theory of this phenomenon and make predictions for the detector function in the case when nonequilibrium noise is created by a mesoscopic conductor. We investigate measurement circuit effects and prove the universality of the classical noise detection. Finally, we evaluate the contribution of the third cumulant of current and make suggestions of how it can be measured.

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I. INTRODUCTION

Universalities play an important role in physics because they point to fundamental laws and properties such as symmetries, topology, scaling behavior, and others. Moreover, when broken, they open a door to new physics. Here we wish to consider one example that is important in the context of this paper. Recently, following the suggestion of Kane and Fisher,[1] experiments on shot noise in quantum Hall systems[2,3] directly measured fractional charge of Laughlin quasiparticles. The interpretation of these experiments invokes a simple argument that weak quasiparticle tunneling is an uncorrelated Poisson process, which is described by the Schottky formula $S=q\langle I\rangle$, where $\langle I\rangle$ is the average tunneling current, S is the zero-frequency noise power of the tunneling current, and q is the fractional charge of quasiparticles.

More rigorously, the Schottky formula follows from the fluctuation-dissipation theorem (FDT), which states that when a tunnel junction weakly connects two metallic reservoirs, the following relation generally holds:[4,5]

$$q\langle I\rangle/S = \tanh(\Delta\mu/2k_BT), \tag{1}$$

where $\Delta\mu$ is the electrochemical potential difference applied to the barrier. This relation is a generalization of the well-known Callen-Welton FDT, which connects the noise power and the linear-response coefficient[8] and follows from the argument similar to the one used in the linear response theory. This implies the universality of the relation (1), i.e., it holds independently of the character of the interaction, spectrum of quasiparticles, the geometry of a tunnel junction, etc. It is easy to see that the Callen-Welton theorem and the Schottky formula are the two limits of the relation (1).

Here we present a simplified derivation of Eq. (1) based on the "golden rule."[9] Quantum mechanical transition rate between the energy states E_n and E_m is given by $W_{mn}=2\pi\delta(E_n-E_m)|A_{mn}|^2$, where $A_{mn}\equiv\langle E_m|A|E_n\rangle$ is the matrix element of the tunneling amplitude A. Then the average current can be evaluated as $\langle I\rangle=q\Sigma_{mn}W_{mn}(\rho_n-\rho_m)$, where $\rho_n\equiv\langle E_n|\rho|E_n\rangle$ is the diagonal matrix element of the density operator, i.e., the probability to find the system in the state E_n. When tunneling is weak, forward and backward tunneling transitions are independent Poisson processes with the dispersions of fluctuations equal to mean currents. Therefore, the total noise power is equal to $S=q^2\Sigma_{mn}W_{mn}(\rho_n+\rho_m)$.

In equilibrium $\rho_n\propto\exp(-E_n/k_BT)$, so that $\rho_n=\rho_m$ and the current vanishes. If the potential difference $\Delta\mu$ is applied between the leads, which are locally at equilibrium, then the density matrix acquires the grand canonical form $\rho(\Delta\mu)=\rho(0)\exp(\Delta\mu N/k_BT)$, where N is the number of electrons in one of the leads. Since the tunneling amplitude changes the number of particles in this lead by one, then obviously one can write $\rho_m=\exp(\Delta\mu/k_BT)\rho_n$, which immediately gives the relation (1).

From the derivation of the FDT it is obvious that nonequilibrium processes in reservoirs play a special role, since they may lead to a deviation from the universal relation (1). The goal of this paper is to investigate this phenomenon in the case when nonequilibrium processes take place in the electrical (measurement) circuit to which the tunnel junction is attached. In Fig. 1 we draw its simplified version that contains essential parts; the mesoscopic system which creates nonequilibrium noise and has the conductance G_M, the de-

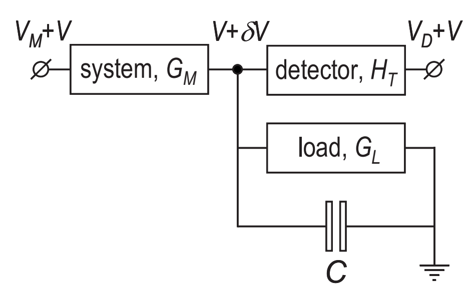

FIG. 1. The measurement circuit contains a mesoscopic system, which creates nonequilibrium noise and has the conductance G_M, the detector consisting of a tunnel junction shunted by the load resistor G_L, and the capacitor C. The voltage biases V_M and V_D are applied to the system and to the tunnel junction, respectively. Fluctuating current through a mesoscopic system is accumulated on the capacitor and creates the fluctuating potential δV across the tunneling barrier.

tector consisting of a tunnel junction shunted by the load resistor G_L, and the capacitor C. One of the important results of this paper is that the FDT breaks down in a minimal way so that some properties of the current-to-noise ratio, which contain an important information about nonequilibrium processes in the leads, retain their universality. This leads to the idea of using tunnel junctions as on-chip detectors of nonequilibrium noise, which we investigate below in details.

The measurement circuit has been proven to play an important role in the physics of the noise detection with the standard measurement technique[10–12] and with the help of on-chip noise detectors.[13–16] It has been established[10–12,16] that in the long-time (Markovian) limit, the backaction of the measurement circuit on the system leads to "cascade corrections" to statistics of noise.[17,18] In order to quantify the circuit effects, one solves the Kirhgoff (the current conservation) law for the fluctuations of the current through the mesoscopic system δI_M and through the load resistor δI_D, and the voltage fluctuations δV on the capacitor,

$$\delta V(\omega) = Z(\omega)[\delta I_M(\omega) + \delta I_L(\omega)]. \quad (2)$$

The circuit impedance is given by

$$Z(\omega) = R/(1 - i\omega\tau_C), \quad (3)$$

where $R=1/(G_M+G_L)$ is the differential circuit resistance and $\tau_C=RC$ is the circuit response time. Equation (2) describes the effect of the system current fluctuations via the circuit on the tunnel junction, which directly detects potential fluctuations. The normalized circuit resistance $\mathcal{R}=G_0R$, where $G_0=e^2/2\pi$ is the conductance quantum and we set $\hbar=1$, plays a role of the dimensionless coupling constant, which parametrizes the strength of the circuit effects. In this paper, we assume that coupling is weak, $\mathcal{R}\ll 1$.

Quantum noise detectors—the main operating principle of which is based on the resonant tunneling in a two-level system—were investigated experimentally and theoretically in a number of previous works.[19–24] Here we consider two different detectors of this type. The first one, the double-dot (DD) detector of quantum noise that is theoretically analyzed by Aguado and Kouwenhoven[20] is shown in Fig. 2. It consists of two quantum dots, which are strongly coupled to leads and weakly coupled to each other. To the lowest order in interdot coupling, the electron transport in the DD detector occurs via inelastic transitions between nearest energy levels of two dots. These transitions are assisted by the emission (absorption) of the energy ε to (from) the circuit, where ε is the interdot level distance. Away from the resonance, the average current through the DD detector is given by $\langle I\rangle \sim S_M(\varepsilon)/\varepsilon^2$ (Ref. 20), where $S_M(\varepsilon)$ is the spectral density of the nonsymmetrized correlator of the system current. It is easy to see that the parameter ε plays a role of the bandwidth of the detector.

The operating principle of the second detector, based on the telegraph process (TP detector, see Fig. 3), is slightly different. It contains two weakly coupled quantum dots, which are electrically isolated from the circuit but capacitively coupled to it. Fluctuations of the potential on the capacitor, caused by the current noise in the mesoscopic system, lead to rare electron transitions between two dots, which change the electrical charge of, say, the left dot randomly in time. When the left dot interacts with a nearby quantum point contact (QPC), it randomly switches the QPC current between two levels, I_d and I_u, leading to the telegraph process. The average QPC (detector) current $\langle I_D\rangle$ is a monotonic function of the interdot level distance ε, which changes from one current level to the other depending on the average occupation of the left dot. Thus the QPC acts as a sensitive electrometer of the occupation of the quantum dot—the principle demonstrated in an early work[25] and subsequently elaborated in recent experiments—where the real time detection of single-electron tunneling,[26–31] the measurement of counting statistics,[32–34] and the information backaction of a detector[35] were also shown.

We denote with $D(\varepsilon,\Delta)$ the current-to-noise ratio for the DD detector and call it the detector function. Although D depends on the bias voltage V_D, we choose to represent it as a function of the energy $\Delta=eV_D-\varepsilon$ of the electron-hole pair created in the leads by the elementary tunneling event. This energy parametrizes the asymmetry of the detector because in the case $\Delta=0$, or equivalently $\varepsilon=eV_D$, there is no differ-

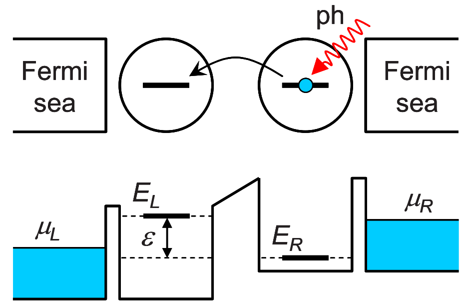

FIG. 2. (Color online) The DD detector operates as shown on the upper panel. The absorption of the quantum of the collective charge excitation of the circuit leads to the inelastic electron transition between two weakly connected quantum dots. Because dots are strongly connected to two metallic reservoirs, multiple random transitions generate current through the detector and the current noise. Lower panel shows the energy diagram of the detector and its most important parameters.

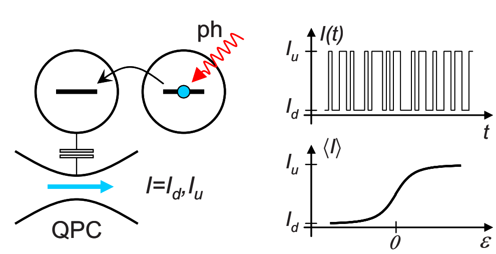

FIG. 3. (Color online) The TP detector consists of a double-dot system, which is capacitively coupled to a QPC. When one of the dots is charged, it pinches off the QPC and thus changes the current through it from the upper level I_u to the lower level I_d. Random interdot transitions, caused by the emission and absorption of the collective charge excitations of the circuit, lead to random switching of the QPC current. The resulting telegraph process is shown on the upper right panel. The average current through the QPC, shown on the lower right panel, develops a smooth crossover between two current levels as a function of the DD level distance ε.

ence between the left and the right dot of the DD system. Below we prove an important fact that the average current through the QPC of the TP detector, after a proper normalization [see Eqs. (26) and (27)], is given by the symmetric detector function $D(\varepsilon,0)$. When the circuit is at thermal equilibrium, $D(\varepsilon,0)=\tanh(\varepsilon/2k_BT)$ according to the FDT.

The physics of quantum noise detection is quite rich, thanks to a number of energy scales that determine the dynamics of entire system. While these energy scales are not important in the case of equilibrium circuit because the FDT holds and leads to the universality, they start to play an important role away from equilibrium. First of all, it is an effective temperature of the noise source Ω, which is formally defined by Eq. (18). It has a meaning of the energy provided by the system and the load. Alternatively, one can think of the correlation time $1/\Omega$ of the noise source. Second important parameter is the detector bandwidth ε, which is introduced earlier. Third, the circuit itself is characterized by the response time τ_C and corresponding energy scale $1/\tau_C$. Finally, the asymmetry of the DD detector is characterized by the energy Δ.

In the weak-coupling limit, which we consider throughout the paper, where the dimensionless circuit impedance $\mathcal{R}$ is small, the detector operates at the *Gaussian point*, i.e., the contribution of high-order cumulants (irreducible moments of noise) is small. The physical reason for this is that in the limit $\mathcal{R}\ll 1$, the detector only weakly interacts with the noise source; therefore, it has to operate for a relatively long time interval of the order of $1/\mathcal{R}\Omega$ in order to accumulate sufficient information about the noise. During this time interval, many fluctuations contribute to the detector signal so that by virtue of the central limit theorem, the resulting noise becomes Gaussian. In Sec. VII we show how the third cumulant, which is the simplest characteristics of the non-Gaussianity, nevertheless can be extracted from the detector output signal.

The new energy scale Γ_Ω arises in the weak-coupling limit due to the effect of *homogeneous level broadening*. Close to the resonance, $\varepsilon\to 0$, the interaction of the detector with the circuit becomes effectively strong and inelastic transitions in the detector are assisted by multiple photon absorption and emission processes. As a result, the detector signal at this point acquires a peak as a function of ε of the width $\Gamma_\Omega\ll\Omega$. The shape of the peak depends on the circuit details. We distinguish two limiting cases depending on the circuit response time τ_C. In the "fast" circuit limit $\mathcal{R}\Omega\tau_C\ll 1$, the peak has a Lorentzian shape and the width $\Gamma_\Omega=2\pi\mathcal{R}\Omega$ [see Eq. (21)]. In the "slow" circuit limit $\mathcal{R}\Omega\tau_C\gg 1$, the peak acquires the Gaussian shape (23) with the width $\Gamma_\Omega=2\sqrt{E_C\Omega}$, where $E_C=e^2/2C$ is the Coulomb charging energy of the circuit.

Depending on energy scales, the following regimes can be distinguished. In the quantum noise detection regime, $\varepsilon\sim\Omega$, the detector signal is due to the inelastic tunneling with the absorption or emission of a single photon of the energy ε. The probability of this process is given by Eq. (16). In the case when the circuit is driven away from equilibrium by a coherent mesoscopic conductor, the symmetric detector function is given by Eq. (31). For a low circuit impedance, $G_MR\ll 1$, this expression simplifies and we obtain the result (32). The results are summarized in Fig. 4. In the case of strongly asymmetric detector, $\Delta\gg\Omega$, the detector function takes the equilibrium tangent form (33), which is however shifted by the energy E_M given by Eq. (34), which can be viewed as the noise rectification effect.

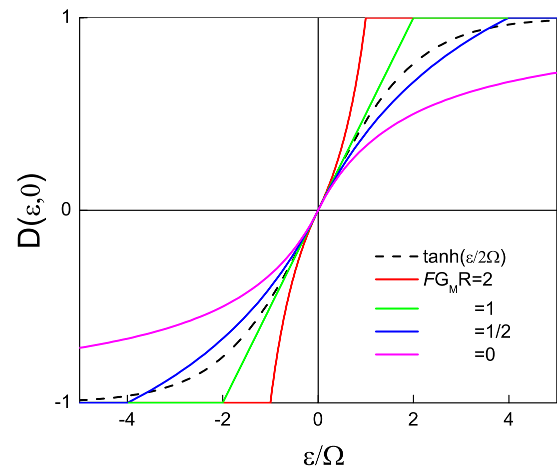

FIG. 4. (Color online) The symmetric detector function $D(\varepsilon,0)$ is plotted versus the normalized level spacing ε/Ω for different values of the parameter $\mathcal{F}G_MR$. Typically, D is concave function of ε, although for a super-Poissonian noise, $\mathcal{F}>1$, it may become convex. Note that in the limit $\mathcal{F}G_MR\ll 1$, the detector function has a power-law behavior as compared to the exponential behavior of the equilibrium $D=\tanh(\varepsilon/2\Omega)$ (shown by the dashed line).

In the classical noise detection regime, $\varepsilon\ll\Omega$, the detector function is simply linear in ε [see Eq. (35)], with the slope determined by the effective noise temperature Ω. Thanks to this universality, there is no need to specify the mesoscopic system that is measured. Close to the resonance, $\varepsilon\sim\Gamma_\Omega$, the inelastic tunneling becomes nonperturbative despite the small parameter $\mathcal{R}$, and the $P(E)$ function acquires a peak of the width Γ_Ω. The shape of the peak depends on the circuit details [see Eqs. (21) and (23)]. Nevertheless, the detector function retains its universal form (35), so it can be used to extract the noise temperature.

We evaluate the small contribution of the third cumulant of the system current in the classical (Markovian) limit and find that it slightly shifts the zero of the detector function (43) by the energy E_3, which is proportional to the third cumulant. The coefficient depends on the circuit response time τ_C and is evaluated in the case of fast and slow circuit [see Eqs. (46) and (49)]. The total third cumulant of the system current contains cascade corrections, which depend on the circuit response time. In the case of fast circuit, the cascade corrections are given by Eq. (45), i.e., they are those introduced by Nagaev in Ref. 17. In the slow circuit case, the detector measures equal-time fluctuations of the potential on the capacitor, and the cascade corrections in this case are given by Eq. (48) as predicted in Ref. 16 and measured in Ref. 12. We finally note that the third cumulant of current may be extracted from the shift of the detector function using the technique recently introduced in experiments[36,37] on the mesoscopic threshold detectors.[16,38,39] The universality of the detector function in the classical noise detection regime, proven in Sec. IV, may become crucial for the success of this procedure.

The rest of the paper is organized as follows. After reviewing the $P(E)$ theory of tunneling in Sec. II, we focus on the Gaussian noise case in Sec. III and classify the measurement circuit effects according to the circuit response time. In Sec. IV we analyze quantum noise detectors based on the resonant tunneling effect and connect the detector function $D(\varepsilon,\Delta)$ to the current-to-noise relation for tunnel junctions. We use the results of the $P(E)$ theory in Sec. V to calculate the detector function in quantum and classical noise detection regimes. In Sec. VI we prove that the detector function is universal in the classical noise detection regime, i.e., it is independent of the measurement circuit details. Finally, in Sec. VII we investigate the third cumulant contribution to the detector function including the circuit cascade corrections. The Sec. VIII outlines further directions of research.

II. REMINDER ON $P(E)$ THEORY OF TUNNELING

The purpose of this section is to remind the essential steps of the $P(E)$ theory of photon-assisted tunneling.[40,41] In addition, we extend the theory in order to take into account the weak non-Gaussian effects in noise. The tunnel junction, attached to two metallic leads, is described by the Hamiltonian

$$H=\sum_{k}\varepsilon_k(c_k^\dagger c_k+d_k^\dagger d_k)+H_T, \tag{4}$$

where c_k and d_k are the electron operators in the left and right lead, respectively, and H_T is the tunneling Hamiltonian. It can be written as[40]

$$H_T=A+A^\dagger,\quad A=e^{i\phi}\sum_{pk}T_{pk}d_p^\dagger c_k, \tag{5}$$

where the amplitude A transfers the electron from left to right and the phase factor $e^{i\phi}$ changes the charge on the capacitor by $-e$. The last fact follows from the charge quantization $[\phi,Q]=ei$. Then the charge Hamiltonian $H_C=Q^2/2C$ generates the equation of motion for the phase operator $\dot{\phi}=e\delta V$. We thus assume that the interaction of electrons with the collective charge excitations in the electrical circuit is generated solely via tunneling.

Next we evaluate the average tunneling current $\langle I_D\rangle$ and the zero-frequency noise power $S_D=\int dt\langle\delta I_D(t)\delta I_D(0)\rangle$. We define the tunneling current operator as $I_D\equiv edN_L/dt=ie(A-A^\dagger)$, where $N_L=\Sigma_k c_k^\dagger c_k$ is the number of electrons in the left lead. To leading order in the tunneling Hamiltonian (5), we can write,

$$\langle I_D\rangle=e\int dt\langle[A(t),A^\dagger(0)]\rangle, \tag{6a}$$

$$S_D=e^2\int dt\langle\{A(t),A^\dagger(0)\}\rangle. \tag{6b}$$

Substituting A from Eq. (5) to Eqs. (6) and tracing out electronic operators, we finally obtain,

$$\langle I_D\rangle=2\pi e\int\int dE\,dE'\ \nu_R(E)\nu_L(E')[P_{LR}(E-E'+eV_D)\times f(1-f')-P_{RL}(E'-E-eV_D)f'(1-f)], \tag{7a}$$

$$S_D=2\pi e^2\int\int dE\,dE'\ \nu_R(E)\nu_L(E')[P_{RL}(E-E'+eV_D)\times f(1-f')+P_{LR}(E'-E-eV_D)f'(1-f)], \tag{7b}$$

where $f=f_F(E)$ and $f'=f_F(E')$ are the equilibrium distributions in the leads and ν_L and ν_R are the electronic densities of states. Here $P_{LR}(E)$ and $P_{RL}(E)$ are the probability distributions of the emission (absorption) of a collective charge excitation of energy E caused by inelastic tunneling of an electron from the right to the left lead and vice versa.

$$P_{LR}(E)=\frac{1}{2\pi}\int dte^{iEt}\langle e^{i\phi(t)}e^{-i\phi(0)}\rangle, \tag{8a}$$

$$P_{RL}(E)=\frac{1}{2\pi}\int dte^{iEt}\langle e^{-i\phi(t)}e^{i\phi(0)}\rangle. \tag{8b}$$

In general, the phase correlation functions in Eqs. (8a) and (8b) can be expanded in terms of the noise cumulants. However, every cumulant comes with an extra coupling constant $\mathcal{R}\ll 1$. Therefore, we keep only the first two nonvanishing cumulants

$$J_2(t)=\frac{1}{2}\langle\phi^2(t)-2\phi(t)\phi(0)+\phi^2(0)\rangle, \tag{9}$$

$$J_3(t)=\frac{1}{6}\langle\phi^3(t)-3\phi^2(t)\phi(0)+3\phi(t)\phi^2(0)-\phi^3(0)\rangle, \tag{10}$$

and write

$$P_{LR}(E)=\frac{1}{2\pi}\int dte^{iEt-J_2(t)-iJ_3(t)}, \tag{11a}$$

$$P_{RL}(E)=\frac{1}{2\pi}\int dte^{iEt-J_2(t)+iJ_3(t)}. \tag{11b}$$

We postpone the discussion of the third cumulant effect until Sec. VII and for a moment assume that the noise is Gaussian.

III. GAUSSIAN NOISE

We now set $J_3=0$ and write $P_{LR}=P_{RL}\equiv P$, where

$$P(E)=\frac{1}{2\pi}\int dte^{iEt-J_2(t)}. \tag{12}$$

Note that in Eq. (9) each term of the form $\langle\phi^2\rangle$ contains a classical contribution, which in the long-time limit is proportional to time.[42] This is a consequence of the Brownian motion of the phase "pushed" by a fluctuating potential. However, these potentially dangerous terms cancel and Eq. (9) can be rewritten in the form

$$J_2=\frac{1}{2}\langle(\Delta\phi)^2\rangle+\frac{1}{2}\langle[\phi,\Delta\phi]\rangle,\quad \Delta\phi\equiv\phi(t)-\phi(0), \tag{13}$$

so that it does not contain divergences. In this equation the first term can be interpreted as a classical contribution, which is proportional to time in the long-time limit, and the second term is pure quantum. Using Eq. (2), we obtain

$$J_2(t) = G_0 \int \frac{d\omega S(\omega)}{\omega^2 + \eta^2} |Z(\omega)|^2 (1 - e^{-i\omega t}), \quad \eta \to 0, \quad (14)$$

where $S(\omega)=S_M(\omega)+S_L(\omega)$ is the power of the total noise created in the circuit, and S_M and S_L are the nonsymmetrized correlators of the mesoscopic system and of the load resistor,

$$S_M(\omega) = \int dt e^{i\omega t} \langle \delta I_M(t) \delta I_M(0) \rangle, \quad (15a)$$

$$S_L(\omega) = \int dt e^{i\omega t} \langle \delta I_L(t) \delta I_L(0) \rangle. \quad (15b)$$

Next we note that in the weak-interaction case $\mathcal{R}=G_0 R \ll 1$ considered here, $J_2(t)$ is usually small. For instance, in equilibrium $RS(\omega)=2k_B T$ so that for $t \sim 1/k_B T$, the correlator given by Eq. (14) can be roughly estimated as $J_2 \sim \mathcal{R}$. Therefore, we expand the exponential on the right-hand side of Eq. (12) and obtain,

$$P(E) = P_0 \delta(E) + G_0 |Z(E)|^2 S(E)/E^2, \quad (16)$$

where P_0 is the probability of the elastic process, fixed by the normalization $\int dE\, P(E)=1$. The probability of the inelastic process is proportional to the nonsymmetrized correlator $S(E)$ (Refs. 20 and 43–45) and at relatively large energies, it is sensitive to quantum fluctuations.

Special care, however, has to be taken about the long-time limit in Eq. (14) since growing with time classical contribution to J_2 may compensate smallness of $\mathcal{R}$. The Fourier integral cuts off a small region around $\omega=0$, where the noise is classical, and the noise power can be approximately replaced with $S(0)$. The important note is in order; quantum effects, which lead to the interaction-induced suppression of tunneling (i.e., to the so-called dynamical Coulomb blockade effect)[40,46] are not neglected. They are fully taken into account in Eq. (16) and, subsequently, in Sec. V. However, at the energy scale of interest here, their contribution to the long-time asymptotic is small. We now focus on the long-time limit and consider the cases of fast and slow circuit, depending on the circuit response time τ_C.

A. Fast circuit

We first assume that the relevant time scale is longer than τ_C and, therefore, set $Z(\omega)=R$. From Eq. (14) we find,

$$J_2(t) = 2\pi \mathcal{R}\Omega [|t| + i\partial_\omega S(0)/S(0)\mathrm{sign}(t)], \quad (17)$$

where the energy scale Ω is the circuit noise temperature,

$$\Omega \equiv (1/2) R S(0). \quad (18)$$

Note that although the interaction is weak, $\mathcal{R} \ll 1$, in the long-time limit $|t| \sim 1/\mathcal{R}\Omega$, the exponential in Eq. (8) cannot be expanded. We then use the result (17) and obtain,

$$P(E) = \frac{2\mathcal{R}\Omega}{E^2 + (2\pi \mathcal{R}\Omega)^2} [1 + E\partial_\omega S(0)/S(0)], \quad (19)$$

which is consistent with the result (16) in the limit $E \gg \mathcal{R}\Omega$.

Thus we find that in the limit $|t|\Omega \sim 1/\mathcal{R} \gg 1$, the multiple photon processes lead to the broadening of the δ function in Eq. (16), so that it is replaced with the Lorentzian peak with the width $\Gamma_\Omega = 2\pi \mathcal{R}\Omega$. One can now use Eq. (3) to check that the assumption $Z(\omega)=R$ is justified if $\mathcal{R}\Omega\tau_C \ll 1$. This means that the response of the circuit to current fluctuations is instantaneous, and the phase fluctuations are Markovian on the time scale of interest.

The asymmetry of $P(E)$ given by the second term in Eq. (19) is weak, $E\partial_\omega S/S \sim \mathcal{R}$. Interestingly, the expression $\partial_\omega S(0) = \int dt (it/2) \langle [I(t), I(0)] \rangle$ coincides with the Kubo formula for the differential conductance $1/R = \partial_V \langle I \rangle$. Therefore we obtain $\partial_\omega S(0) = 1/R$, and alternatively,

$$\frac{\partial_\omega S(0)}{S(0)} = \frac{1}{2\Omega}. \quad (20)$$

Thus one can express the asymmetry in Eq. (19) in terms of the noise temperature alone,

$$P(E) = \frac{\mathcal{R}(2\Omega + E)}{E^2 + (2\pi\mathcal{R}\Omega)^2}, \quad \mathcal{R}\Omega\tau_C \ll 1. \quad (21)$$

B. Slow circuit

Next we consider the opposite limit $\mathcal{R}\Omega\tau_C \gg 1$, when the circuit responds slowly to current fluctuations. In this case it is the singularity in $Z(\omega)$ that cuts off the integral in Eq. (14) at small frequencies $\omega \sim 1/\tau_C$. Using the impedance (3) and the relations (18) and (20), we obtain,

$$J_2(t) = \pi \mathcal{R}(\Omega/\tau_C) t^2 + i\pi \mathcal{R}(1/\tau_C) t. \quad (22)$$

The first term in this equation has a simple interpretation. We note that it can also be obtained by considering the phase ϕ as classical variable and writing $\phi(t) - \phi(0) = e\delta V t$, because the variation of the potential is slow. Then, Eq. (13) leads to $J_2(t) = (e^2/2)\langle(\delta V)^2\rangle t^2$, which [together with Eq. (3)] gives the first term in Eq. (22). Thus the phase correlator is determined by the equal-time correlator of the potential. We will rely on this interpretation in Sec. VII.

The second term in the Eq. (22) has a quantum nature. It slightly shifts the energy in $P(E)$, given by Eq. (12), and leads to the asymmetry of the distribution. This term is small; therefore, the Fourier transform in Eq. (8) can be written as

$$P(E) = \frac{1 + E/2\Omega}{\sqrt{4\pi E_C \Omega}} \exp\left(-\frac{E^2}{4E_C\Omega}\right), \quad \mathcal{R}\Omega\tau_C \gg 1, \quad (23)$$

where $E_C = e^2/2C$ and the resonance width is $\Gamma_\Omega = 2\sqrt{E_C \Omega}$. Thus we see that the dissipative properties of the circuit, determined by the resistance R, do not enter the final result. This is related to the fact that slow fluctuations of charge on the capacitor obey the law of the equipartition of energy, $(1/2C)\langle(\delta Q)^2\rangle = \Omega/2$, as if it were in equilibrium.

We remark that in the intermediate regime $\mathcal{R}\Omega\tau_C \sim 1$, the exact shape of the zero-energy peak in $P(E)$ is more complex and depends on details of the circuit. Nevertheless, as we show in Sec. VI, the fluctuation-dissipation relations remain insensitive to these details. Finally, we also note that these results on the asymmetry in $P(E)$ were published in Ref. 47. Recently the asymmetry was found in the experiment[48] and theoretically discussed in Ref. 49.

IV. QUANTUM NOISE DETECTORS

We have briefly discussed two types of quantum noise detectors in Sec. I. Here we analyze them in details and show that their properties are determined by the $P(E)$ function, obtained in Sec. III. Starting with the double-dot (DD) detector, we first assume that tunneling between two dots is the weakest process. In this simple case, the transport can be described by lowest order in tunneling, so that the result (7) of Sec. II fully applies. Moreover, a weak coupling of the dots to the reservoirs leads to the broadening of the dot levels, so that the densities of states acquire a Breit-Wigner form $\nu_\alpha=(\Gamma_\alpha/\pi)/[(E-E_\alpha)^2+\Gamma_\alpha^2]$; $\alpha=L$ and R.

If the noise temperature is small, so that classical contribution Γ_Ω to the resonance width satisfies $\Gamma_\Omega<\Gamma_\alpha$, then the elastic transport dominates the photon-assisted inelastic transitions. In this case the left and the right leads are approximately at thermal equilibrium and the fluctuation-dissipation theorem (FDT) holds. The most efficient noise detection takes place for a relatively strong noise in the circuit, $\Gamma_\Omega>\Gamma_\alpha$, when the homogeneous level broadening dominates the quantum effect. In this case Breit-Wigner resonances can be replaced by delta functions $\nu_\alpha=\delta(E-E_\alpha)$, $\alpha=L$ and R, where E_L and E_R are the energies of dot levels counted from the local Fermi level in the left and the right leads (see Fig. 2). Substituting delta functions to Eqs. (7) and using $f'(1-f)=f(1-f')e^{(E_R-E_L)/k_BT}$ for the current-to-noise ratio, we obtain the following function:

$$\frac{e\langle I_D\rangle}{S_D}\equiv D(\varepsilon,\Delta)=\frac{P(\varepsilon)e^{\Delta/k_BT}-P(-\varepsilon)}{P(\varepsilon)e^{\Delta/k_BT}+P(-\varepsilon)}. \tag{24}$$

Here the tunable level distance $\varepsilon\equiv E_R-E_L+eV_D$ is the detector bandwidth, and the energy of the electron-hole pair $\Delta\equiv E_L-E_R$ parametrizes the asymmetry of the detector. The detector function $D(\varepsilon,\Delta)$ will be analyzed in details in Sec. V. Below we show that the properties of the telegraph process (TP) detector are determined by the symmetric variant of this function with $\Delta=0$.

We evaluate the average current through the quantum point contact (QPC) that is capacitively coupled to the DD (see Fig. 3). Switching of the DD from one state to another changes the current I_D through the QPC from the low level I_d to the high level I_u. The probabilities of finding the DD in the lower and upper states are given by $P_d=\gamma_d/(\gamma_u+\gamma_d)$ and $P_u=\gamma_u/(\gamma_u+\gamma_d)$, where γ_d and γ_u are the switching rates. Then the average current is given by $\langle I_D\rangle=I_uP_u+I_dP_d$ and is equal to

$$\langle I_D\rangle=\frac{I_u\gamma_u+I_d\gamma_d}{\gamma_u+\gamma_d}. \tag{25}$$

It is convenient to rewrite the detector current $\langle I_D\rangle$ in the dimensionless form,

$$\mathcal{I}_D\equiv\frac{2\langle I_D\rangle-(I_u+I_d)}{I_u-I_d}=\frac{\gamma_u-\gamma_d}{\gamma_u+\gamma_d}, \tag{26}$$

so that it acquires the maximum value $\mathcal{I}_D=1$ when $\gamma_u\gg\gamma_d$ and the upper level is occupied, $P_u=1$, and $\mathcal{I}_D=-1$ in the opposite case, when mostly the lower level is occupied.

Next we assume that one of the dots is strongly coupled to the circuit capacitor. Then with a good approximation, switching of the DD changes the charge of the capacitor by the value e, so that the interdot coupling is proportional to $e^{i\phi}$ (Ref. 50). Assuming that the interdot coupling is weak compared to the width of levels, one can evaluate the switching rate using the golden rule approximation with the results $\gamma_u\propto P(\varepsilon)$ and $\gamma_d\propto P(-\varepsilon)$, where ε is the DD level distance. Therefore, using the result (26) we obtain,

$$\mathcal{I}_D(\varepsilon)=\frac{P(\varepsilon)-P(-\varepsilon)}{P(\varepsilon)+P(-\varepsilon)}=D(\varepsilon,0), \tag{27}$$

i.e., the normalized average current through the QPC as a function of the tunable level distance is given by the symmetric variant of the detector function.

V. FLUCTUATION-DISSIPATION RELATIONS

In this section we investigate how nonequilibrium processes in the circuit lead to a breakdown of the FDT. We first focus on the inelastic regime $\varepsilon>\mathcal{R}\Omega$, where Eq. (16) applies, and later consider the classical regime described by Eqs. (21) and (23). Substituting Eq. (16) to the definition (24), we obtain,

$$D(\varepsilon,\Delta)=\frac{S(\varepsilon)e^{\Delta/k_BT}-S(-\varepsilon)}{S(\varepsilon)e^{\Delta/k_BT}+S(-\varepsilon)}, \tag{28}$$

where, we remind, $\varepsilon=eV_D-\Delta$ is the interdot level distance. Thus all the circuit details cancel from the final result, and the exact form of the function D is determined solely by a nonsymmetrized correlator of the current fluctuations in the circuit.

In order to make further progress, we have to specify the model of the current source. The load resistor may be considered as a macroscopic system that creates an equilibrium current noise. Nonequilibrium processes are generated by the mesoscopic system alone. An interesting and experimentally important example of the mesoscopic system is a coherent mesoscopic conductor, which is fully characterized by a set of transmission eigenvalues T_n with $n=1,\ldots,N$. Using the scattering theory,[51] one obtains the following expression for the nonsymmetrized current correlator:

$$S_M(\omega)=G_0\sum_n\{2T_n^2F(\omega)+T_n(1-T_n)\times[F(\omega+eV_M)+F(\omega-eV_M)]\}, \tag{29}$$

where

$$F(\omega)\equiv\frac{\omega}{1-e^{-\omega/k_BT}}, \tag{30}$$

and we assumed that transmission eigenvalues T_n are energy independent. Using $F(\omega)-F(-\omega)=2\omega$, we now check that indeed $S_M(\omega)-S_M(-\omega)=2\omega G_M$, where the conductance $G_M=G_0\Sigma_n T_n$. The same relation obviously holds for the macroscopic resistor.

In equilibrium Eq. (29) gives $S=S_M+S_L=2(G_M+G_L)F(\omega)$, where, we remind, S_M and S_L are the nonsymmetrized correlators of the mesoscopic system and of the load resistor, respectively. This is a well-known result for the nonsymmetrized noise power. It satisfies the detailed balance relation $S(-\omega)=e^{-\omega/k_BT}S(\omega)$. Substituting this relation to Eq. (28), we arrive at the equilibrium function $D=\tanh(eV_D/2k_BT)$, which is in agreement with the FDT. If the load conductance is large, $G_L \gg G_M$, the equilibrium noise of the load resistor may dominate in the circuit noise. In this case the function D may retain its equilibrium form even if the mesoscopic conductor is biased. It is therefore interesting to consider the strong bias regime, $eV_M > k_BT/R$, so that the equilibrium noise contribution can be neglected. Three important cases, which deserve special consideration, are discussed below.

A. Symmetric detector $\Delta=0$

This case is most relevant for the TP detector, which is symmetric detector. Using the zero-temperature limit $F(\omega)=\omega$ in Eq. (29) and substituting the result to the Eq. (28), we obtain an important result,

$$D(\varepsilon,0)=\frac{\varepsilon}{\mathcal{F}G_M R(eV_M-|\varepsilon|)+|\varepsilon|},\quad \text{for}\quad |\varepsilon|<eV_M, \tag{31}$$

and $D=\pm 1$, otherwise. Here $\mathcal{F}\equiv\Sigma_n T_n(1-T_n)/\Sigma_n T_n$ is the Fano factor of the system noise. Note that the slope of D at $\varepsilon=0$ is equal to $1/(e\mathcal{F}G_M RV_M)=1/(2\Omega)$, where Ω, we remind, is the circuit noise temperature. Interestingly, as we show below, this slope is universal and the same for an arbitrary mesoscopic conductor and arbitrary circuit.

In the case of a very low load impedance, $G_M R \ll 1$, the result (31) can be written in the dimensionless form

$$D(\varepsilon,0)=\frac{\varepsilon/\Omega}{2+|\varepsilon/\Omega|}. \tag{32}$$

It is plotted on Fig. 4 together with the equilibrium $D(\varepsilon,0)=\tanh(\varepsilon/2\Omega)$. The nonequilibrium D has a power-low asymptotic at $|\varepsilon|\to\infty$, while the equilibrium one shows an exponential behavior. Note also that for $\mathcal{F}G_M R>1$, the detector function $D(\varepsilon,0)$ is convex, which could be considered a signature of a super-Poissonian noise.

B. Asymmetric detector

We consider a circuit far away from equilibrium, $eV_M > k_BT/R$. We expect that it might be difficult to adjust the DD detector precisely to cancel the asymmetry; therefore, we first assume that the asymmetry is strong, $|\Delta| \gg eV_M$. Then, looking at the result (28), we expect that $D=1$, i.e., the noise of the DD detector is Poissonian. In fact, more careful analysis shows that the strong asymmetry simply shifts the zero of the function D. This is because small or large value of the exponential e^{Δ/k_BT} may be compensated by the opposite effect in S due to the activation processes.

Looking at the results (29) and (30), we find that for positive ω, the dominant contribution is $S_M(-\omega)=\mathcal{F}G_M(\omega-eV_M)e^{(eV_M-\omega)/k_BT}$ due to such activation processes. The contribution of the load resistor $S_L(-\omega)$ is small. On the other hand, both the load resistor and the mesoscopic system contribute to the term in Eq. (28), $S(\omega)=2\omega/R$. Neglecting the terms eV_M and eV_D compared to Δ, we finally obtain,

$$D(\varepsilon,\Delta)=\tanh\left[\frac{eV_D+\text{sgn}(\Delta)E_M}{2k_BT}\right], \tag{33}$$

where the energy shift E_M is given by

$$E_M=eV_M+k_BT\log(\mathcal{F}G_M R/2). \tag{34}$$

Thus we arrive at the remarkable result that the only role of Δ is to fix the sign of the energy shift in Eq. (33). This fact is easily understood when we notice that the energy shift can be viewed as a *drag* or noise *rectification* effect, the direction of which depends on the sign of Δ.

Interestingly, there is an additional contribution to E_M in the form of the logarithm, which contains the system Fano factor. It is exactly the same parameter that also appears in the symmetric case (31). This additional shift may be interpreted as originating from high-energy excitations that create the shot noise in mesoscopic system. Its explicit form depends on the assumption we made that the system is a coherent mesoscopic conductor. Therefore, it would be interesting to consider other examples of mesoscopic systems which may change the results (33) and (34).

C. Classical noise regime

So far we have discussed an essentially quantum regime of the noise detection, where specific form of the function D depends on the choice of the system. In the rest of the paper, we concentrate on the classical Markovian limit, which corresponds to a small detector bandwidth ε, and demonstrate a number of universalities.

We note that although the function D in Eqs. (31) and (32) behaves regularly at $\varepsilon=0$, it has been obtained in the limit $\varepsilon > \mathcal{R}\Omega$ using the result (16). If the detector bandwidth ε tends to zero, $P(\varepsilon)$ that is given by Eq. (16) diverges and has to be replaced with the resummed version (21). The natural question that arises is whether a considerable change in $P(\varepsilon)$, including appearance of the peak at $\varepsilon=0$, affects the symmetric detector function (31). The answer is no. We first check this for a fast and slow circuit limit and prove the universality in Sec. VI.

Indeed, substituting either the function (21) or the function (23) into Eq. (24) for $\Delta=0$, we obtain

$$D(\varepsilon,0)=\frac{P(\varepsilon)-P(-\varepsilon)}{P(\varepsilon)+P(-\varepsilon)}=\frac{\varepsilon}{2\Omega},\quad \text{if}\quad \varepsilon \ll \Omega, \tag{35}$$

where, we remind, Ω is the circuit noise temperature (18). This result agrees with Eq. (31) as $\varepsilon\to 0$. We stress, however, that the result (35) is more general, since its derivation does not relay on the scattering theory[51] for a mesoscopic coherent conductor.

We are now in the position to investigate the effect of asymmetry. Restricting ourselves to the classical regime $\varepsilon \ll \Omega$, we write that generally $P(\varepsilon)=P_0(\varepsilon)(1+\varepsilon/2\Omega)$, where $P_0(\varepsilon)=P_0(-\varepsilon)$ is the classical contribution. Substituting this expression to the Eq. (24), we arrive at

$$D(\varepsilon,\Delta) = \tanh(\delta) + \frac{\varepsilon}{2\Omega \cosh^2(\delta)}, \quad \delta = \frac{\Delta}{2k_B T}. \quad (36)$$

Note that this result does not contradict the strongly asymmetric case (33), because here we assume $\varepsilon \ll \Omega$. It implies that at small asymmetry, the zero of the detector function is shifted,

$$D(\varepsilon,\Delta) = \frac{\varepsilon + E_2}{2\Omega}, \quad E_2 = (\Omega/k_B T)\Delta. \quad (37)$$

Again, this shift is solely due to the second cumulant of current noise, and it can be interpreted as an asymmetry-induced noise rectification effect. This fact is important for the discussion in Sec. VII.

VI. UNIVERSALITY OF CLASSICAL NOISE DETECTION REGIME

In order to arrive at the result (35), we used Eqs. (21) and (23) for a fast and slow circuit, respectively. In general, the shape of the peak in $P(E)$ depends on the circuit response time τ_C and circuit details via the impedance function $Z(\omega)$. In this section we show that, surprisingly, in the classical limit $\varepsilon \ll \Omega$, the detector function retains its forms (35) and (36).

We return now to Eqs. (12) and (14) and assume that the circuit impedance $Z(\omega)$ is arbitrary. The only requirement that we impose is that the interaction is weak, $\mathcal{R}=G_0 Z(0) \ll 1$. We focus on the long-time limit of $J_2(t)$, so that the integral in Eq. (14) comes from small frequencies $\omega \sim \mathcal{R}\Omega$, where the noise power can be approximately expanded as $S(\omega)=S(0)+\partial_\omega S(0)\omega$. Consequently, $J_2(t)$ acquires two contributions that can be written as

$$J_2(t) = G_0 S(0) H(t) + i G_0 \partial_\omega S(0) \partial_t H(t), \quad (38)$$

where

$$H(t) = \int \frac{d\omega}{\omega^2} |Z(\omega)|^2 [1 - \cos(\omega t)]. \quad (39)$$

We are interested in time scales $t \sim 1/(\mathcal{R}\Omega)$, where the first term in Eq. (38) is of the order of one, and the peak of the function $P(E)$ is formed. Then the second term in Eq. (38) is of the order of $\mathcal{R}$, i.e., it is always small. Therefore, its contribution to the exponential in Eq. (12) should be expanded, giving the odd part of the $P(E)$ function. Thus we obtain the following result:

$$P(E) + P(-E) = \pi^{-1} \int dt \exp[-G_0 S(0) H(t)] \cos(Et), \quad (40a)$$

$$P(E) - P(-E) = \pi^{-1} \int dt \exp[-G_0 S(0) H(t)] \times G_0 \partial_\omega S(0) \partial_t H(t) \sin(Et). \quad (40b)$$

It is easy to see that by the integration by parts, Eq. (40a) can be presented in the same form as Eq. (40b). Thereby, independent of the exact function $Z(\omega)$, we arrive at the most general result for the classical noise regime, $E \ll \Omega$,

$$\frac{P(E) - P(-E)}{P(E) + P(-E)} = \frac{\partial_\omega S(0)}{S(0)} E. \quad (41)$$

Using again the result (20), we arrive at Eq. (35), which therefore holds for an arbitrary circuit.

VII. THIRD CUMULANT CONTRIBUTION

We have shown in Sec. II that in the long-time limit, the quantum noise contribution to the correlator $J_2(t)$ is small. The same remains true for the third cumulant $J_3(t)$. Since the third cumulant contribution is small by the parameter $\mathcal{R}$, right from the beginning, we focus on its classical part and rewrite Eq. (10) as follows:

$$J_3(t) = (1/6)\langle [\phi(t) - \phi(0)]^3 \rangle. \quad (42)$$

Thus, we see that $J_3(-t)=-J_3(t)$. This breaks the symmetry between the right and the left lead, $P_{LR} \neq P_{RL}$, and the third cumulant adds to the potential difference across the tunnel junction. In the classical limit $E \ll \Omega$, where $P(E)$ has a peak, this additional potential simply shifts the energy by a small amount E_3 that depends on the third cumulant of current, $P_{LR}(E)=P(E-E_3)$ and $P_{RL}(E)=P(E+E_3)$. Therefore, the function of the symmetric detector (35) has to be replaced with

$$D(\varepsilon,0) = \frac{P_{LR}(\varepsilon) - P_{RL}(-\varepsilon)}{P_{LR}(\varepsilon) + P_{RL}(-\varepsilon)} = \frac{\varepsilon - E_3}{2\Omega}. \quad (43)$$

The same shift obviously takes place in the asymmetric case. However, there the shift E_3 adds to the shift E_2 due to the noise rectification effect (see the discussion in the end of Sec. V C). Fortunately, in contrast to the rectification shift, the energy E_3 depends on the direction of current in a mesoscopic system. Therefore, experimentally the third cumulant contribution can be extracted by changing the direction of the current through the mesoscopic system. This experimental technique has been recently used to measure the third cumulant with the help of Josephson-junction threshold detectors.[36,37] In addition, an important role in this context plays the universality of the Gaussian noise effect on the detector function $D(\varepsilon,0)$ that is proven in Sec. VI. In what follows we evaluate the shift E_3 for the cases of fast and slow circuit depending on the circuit response time τ_C.

A. Fast circuit

In the case of fast circuit, $\mathcal{R}\Omega\tau_C \ll 1$, the potential fluctuations are Markovian, so that Eq. (42) gives

$$J_3(t) = (e^3/6)\langle\langle V^3\rangle\rangle t, \tag{44}$$

where $\langle\langle V^3\rangle\rangle$ is the Markovian cumulant of the potential. According to Refs. 10, 17, and 18, it is given by $\langle\langle V^3\rangle\rangle = R^3\langle\langle I^3\rangle\rangle$, where the total third cumulant of the current is equal to

$$\langle\langle I^3\rangle\rangle = \langle\langle I_M^3\rangle\rangle + 6\Omega\partial_V S(0) + 12(\Omega/R)^2\partial_V R. \tag{45}$$

Here $\langle\langle I_M^3\rangle\rangle$ is the intrinsic third cumulant of the system current and the second and third terms are the "environmental" and nonlinear cascade corrections, respectively. They originate from the circuit backaction.

It is useful to write J_3 in the form that explicitly shows the coupling constant $J_3(t)=(1/6)(2\pi\mathcal{R}/e)^3\langle\langle I^3\rangle\rangle t$. We see that indeed, such a contribution to the correlator simply shifts the energy in the Fourier transform (11) for the probability distribution functions by the amount,

$$E_3 = (1/6)(2\pi\mathcal{R}/e)^3\langle\langle I^3\rangle\rangle, \quad \mathcal{R}\Omega\tau_C \ll 1. \tag{46}$$

In order to estimate the relative effect of the third cumulant, we note that the width of the peak in $P(E)$, where the detector signal is maximum, is of the order of $\mathcal{R}\Omega$, so that $D \sim \mathcal{R}$. The energy shift can be estimated as $(\mathcal{R}/e)^3\langle\langle I^3\rangle\rangle \sim \mathcal{R}^2 G_M R\Omega$. Therefore, the relative contribution of the third cumulant is of the order of $\mathcal{R}G_M/(G_M+G_L)\ll 1$.

B. Slow circuit

In the case of slow circuit, $\mathcal{R}\Omega\tau_C \gg 1$, the detector "feels" slow fluctuations of the potential. Therefore, one can approximate $\phi(t)-\phi(0)=e\delta V t$ according to exact calculations in Sec. III B. Then the Eq. (42) gives

$$J_3(t) = (e^3/6)\langle(\delta V)^3\rangle t^3, \tag{47}$$

where $\langle(\delta V)^3\rangle$ is the third cumulant of equal-time fluctuations of the potential V. In Refs. 12 and 16 it has been shown that $\langle(\delta V)^3\rangle = (R/3C^2)\langle\langle I^3\rangle\rangle$, where the total current cumulant in this case is given by

$$\langle\langle I^3\rangle\rangle = \langle\langle I_M^3\rangle\rangle + 3\Omega\partial_V S(0) + 3(\Omega/R)^2\partial_V R. \tag{48}$$

It contains the intrinsic cumulant of the system current $\langle\langle I_M^3\rangle\rangle$ and the cascade corrections. Note that in this case, $\mathcal{R}\Omega\tau_C \gg 1$, the cascade corrections are smaller compared to those for a fast circuit [see Eq. (45)]. This fact was recently experimentally verified in Ref. 12.

We now substitute the small term (47) to the definition (11). This gives $P_{LR}=[1+(e^3/6)\langle(\delta V)^3\rangle\partial_E^3]P(E)$ and $P_{RL}=[1-(e^3/6)\langle(\delta V)^3\rangle\partial_E^3]P(E)$. Using Eqs. (23) and (43) we obtain,

$$E_3 = \frac{(2\pi\mathcal{R})^2}{6\tau_C\Omega e^3}\langle\langle I^3\rangle\rangle, \quad \mathcal{R}\Omega\tau_C \gg 1. \tag{49}$$

Note that this energy shift is smaller than the one for the case of fast circuit [Eq. (46)] by the parameter $1/(\mathcal{R}\Omega\tau_C)\ll 1$. Since the width of the distribution $P(E)$ is of the order of $\sqrt{\mathcal{R}\Omega/\tau_C}$, the relative contribution of the third cumulant to the function D can be estimated as $\mathcal{R}G_M/[(G_M+G_L)(\mathcal{R}\Omega\tau_C)^{1/2}]\ll 1$.

VIII. OUTLOOK

We have presented the theory of quantum noise detectors based on the resonant tunneling phenomenon. It is summarized in Sec. I, which can also be used as a guide to most important results. Here we briefly discuss related problems which are yet to be solved. First of all, it would be interesting to relax the condition of a weak coupling. In the case $\mathcal{R}\sim 1$, fluctuation-dissipation relations may contain an information about the full distribution of the fluctuating potential. Interestingly, it has been shown in Ref. 52 that the double-dot system in the adiabatic regime, $\tau_C\langle I_D\rangle \gg 1$, may serve as a nonlinear element, which generates an instability in the mesoscopic circuit. It then may be used as an on-chip threshold detector of rare event in transport. The difficulty of this problem is that the dynamical Coulomb blockade effect in this case is not generally negligible.

In the case of a TP detector the excitation of an electron-hole pair in the QPC may cause a transition in the DD system. This competing quasiparticle process reduces the precision of the detection of collective charge excitations in the measurement circuit. Intuitively, one should keep the current through the QPC on a very low level. However, this will reduce the rate of the measurement. Moreover, the quasiparticle process is interesting in itself and should be investigated theoretically.

We think that the physics of double-dot systems described here is rather universal and should be the same in various two-level systems of different nature. Nevertheless, it is important to consider other systems too. Moreover, it would be interesting to generalize the present results to the case of a quantum detector with many levels with the energies ε_n with $n=1,2,\ \ldots$. There is a hope that such system will be able to detect high-order correlators of current at finite frequencies equal to the energies ε_n.

Concerning specific results presented in this paper, two problems remain to be solved. First, we have shown that in the classical noise detection regime, the effect of the second cumulant of the system current is universal, i.e., it does not depend on the circuit details. On the contrary, the third cumulant contribution depends on the circuit response time and has been found here in the limit of fast and slow circuit. An interesting problem, which may also be experimentally very relevant, is to find the third cumulant contribution including cascade correction for arbitrary circuit.

Second, the most dramatic effect of a nonequilibrium system noise on the detector function is that the exponential behavior (1) is replaced with the power-law functions (31) and (32). Thus the power-law behavior is a signature of nonequilibrium processes. However, this result has been obtained by considering a coherent noninteracting mesoscopic conductor as an example of the system. Therefore, it would be interesting to consider other systems in order to check the generality of our conclusion.

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