Spin and angular momentum transitions in few-electron quantum dots and rings

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The studies on quantum dots and rings containing $4\sim 6$ electrons reveal that the shapes of the systems have great impact on the spin transitions of the ground states. For five-electron case, the different shapes lead to the different transition processes in quantum dots and rings. For six-electron case, the ring confinement forbids the spin transition, which really exists in dot confinement. The investigations on the angular momentum transitions in magnetic fields show that the sensibility of the transitions to the size of the system depends remarkably on the particle number. The four-electron case is almost size insensitive but the five-electron case has obvious size dependence. It is also found that the behaviors of the few-electron entanglement in quantum dots and rings are quite different. The entanglement entropies in quantum dots increase with the increase in the magnetic field but exhibit Aharonov-Bohm oscillation patterns in narrow rings. The studies of the shape, size, and particle number effects are important for understanding the liquid-to-crystal transitions, and may be helpful for the manipulation of electronic states in quantum dots and rings.

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I. INTRODUCTION

The electronic structure of two-dimensional quantum dots (QDs) has been a topic extensively studied in recent years due to the great application potential of QDs in future quantum electronics, spintronics, and quantum information devices. The specific spin states of electrons and their manipulations by magnetic and electric fields have been proposed for basic qubit schemes[1,2] for future quantum computation. The particle number in QDs can be reduced precisely down to a few electrons with highly controllable confinements, interactions, and external fields. Even the Zeeman splitting of electrons in magnetic fields can be precisely controlled in recent experiments.[3,4] Benefiting from the developments of manufacturing and experimental techniques, the quantum dots with different size and shape can be fabricated, and exhibit various characters. Especially the ones with ringlike geometry, namely quantum rings (QRs) (Refs. 5–7), have shown unique electronic and optical properties. The ringlike geometry also makes the system suitable to investigate the Aharonov-Bohm effect of the threading magnetic flux.[8,9]

Within the accurate manipulation of the electronic states in future applications, the understanding of the size and shape effects of QDs and QRs are important. Previous investigations have revealed that changing the size of the dots and rings can lead to the transitions of the electronic states, and the spin configurations of the systems. With the decrease in the confinement, the electrons transform from delocalized states with short-range liquidlike correlation to the localized ones with crystal-like correlation.[10–12] The spin of the ground state and the electronic structures of low-lying states of both one-dimensional (1D) and two-dimensional quantum rings within the liquid-to-crystal transitions were also studied by theoretical and numerical methods.[13,14] The magnetic fields can also lead to the transitions of electronic states in QDs and QRs. For the two-dimensional parabolic QDs with rotational symmetry, the crystal-like states have no broken symmetry in charge density so they are also referred to as rotating Wigner molecules (RWMs).[15–17] The trial wave functions for RWMs were proposed based on the ideas of localized Hartree-Fock orbits[15,18] or composite fermion crystallization.[19–21] The few-electron spectra of QDs and QRs with spins have also been understood with the idea of rotational-vibrational states.[22–24]

For the system with small particle number, the exact diagonalization (ED) (Refs. 25–27) of the many-body Hamiltonian with exact interactions and the full consideration of the spin degree of freedom is feasible, and it can give accurate information of the wave functions. The spin configurations of the ground state[26] and the structures of excitation spectra[27] of few-electron QDs around some filling factors have been analyzed in depth. In this work, we employ the ED method to extend the previous studies of the ground-state transitions of both QDs and QRs to five- and six-electron systems, and also investigate the angular momentum transitions in magnetic fields. We will mainly focus on the size and shape effects of the systems with different particle numbers to understand their behaviors in the liquid-to-crystal transitions.

The correlation in many-particle states are interesting issues in condensed-matter theory and are especially important for the understanding of quantum phase transitions[28] in multiparticle systems. The studies of entanglement, which means nonlocal correlation among particles, are meaningful to future quantum information and quantum computation devices. The essential role of the entanglement in strong correlation system such as fractional quantum Hall system has been discussed. Especially the entanglement properties of the Laughlin wave functions were studied in detail.[29] In large QDs and QRs, the electronic states are strongly correlated. However, the quantitative studies of the entanglement in few-electron QDs and QRs are still limited. So in this work, we focus on the size and shape effect of dots and rings on the entanglement properties of different spin states in magnetic fields.

The remainder of the paper is organized as follows. In Sec. II, we explain the model Hamiltonian of the two-dimensional few-electron QDs and QRs, and introduce the formula of the von Neumann entropy, which is used for measuring the entanglement between identical particles. The shape effect on the spin transitions of five-electron and six-electron QDs and QRs, the characters of the angular momentum transitions of four-electron and five-electron states, and their entanglement properties, are discussed in Sec. III, followed by a summary in Sec. IV.

II. MODEL AND FORMULA

The Hamiltonian of a N-electron parabolic quantum dot or quantum ring in a perpendicular magnetic field without the Zeeman splitting is written as

$$H = \sum_{i=1}^{N}\left[\frac{1}{2m}(\hat{P}_i + e\vec{A})^2 + \frac{1}{2}m_e^*\omega_0^2(r_i - R_0)^2\right] + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0\varepsilon_r|\vec{r}_i - \vec{r}_j|}. \tag{1}$$

The first and second parts of Eq. (1) are the single-particle and interaction energies of the electrons, respectively. ω_0 is the characteristic frequency of the radial confinement. $\vec{A}$ is the vector potential of the magnetic field. m_e^* and ε_r are the effective mass and relative static dielectric constant, which are $0.067m_e$ and 12.4 for GaAs, respectively. R_0 is the mean radius of the ring. For parabolic quantum dots, R_0 is set to zero and the radius of the dot is defined as $R = (2\hbar/m_e^*\omega_0)^{1/2}$, which is just $\sqrt{2}$ times of characteristic length $l = (\hbar/m_e^*\omega_0)^{1/2}$ of QDs.

The general form of the single-particle eigenstates of Eq. (1) is

$$\varphi_{nm}(\vec{r}) = R_{nm}(\vec{r})\exp(im\varphi), \tag{2}$$

where $n=0,1,2\ldots$ and $m=0,\pm1,\pm2\ldots$ are the radial and the orbital angular momentum quantum numbers, respectively. The radial part $R_{nm}(\vec{r})$ can be obtained by the series-expansion method.[30,31] Then we can use the obtained single-particle eigenstates to construct the antisymmetric many-particle bases of the Hamiltonian. In the second quantization scheme, the Hamiltonian [Eq. (1)] can be written as

$$H = \sum_{\alpha}\epsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{2}\sum_{\alpha\beta\alpha'\beta'} g_{\alpha'\beta'\beta\alpha}a_{\alpha'}^\dagger a_{\beta'}^\dagger a_\beta a_\alpha, \tag{3}$$

with

$$g_{\alpha'\beta'\beta\alpha} = \int d\vec{r}_1 d\vec{r}_2 \varphi_{\alpha'}^*(\vec{r}_1)\varphi_{\beta'}^*(\vec{r}_2)\frac{e^2}{4\pi\epsilon|\vec{r}_1 - \vec{r}_2|}\varphi_\beta(\vec{r}_2)\varphi_\alpha(\vec{r}_1),$$

where $a_\alpha^\dagger(a_\alpha)$ is the creation (annihilation) operator of the single-particle state φ_α and ϵ_α is its energy. α represents the set of the radial quantum number n and orbital angular momentum quantum number m. Then Eq. (3) can be diagonalized in the space spanned by the many-particle bases to obtain the energies $E_{L,S}$ and the corresponding wave functions Ψ of the few-electron states. Without spin-orbit coupling, the total angular momentum L, total spin S, and its z component S_z are good quantum numbers. Of course, the states with same L and S but different S_z are degenerate due to the absence of the Zeeman splitting. So we can set S_z to the allowable smallest value without losing generality. It is worthwhile to point out that the series-expansion method is applicable not only to the parabolic confinement but also to other confinement forms, for which analytic single-particle bases cannot be obtained. In our calculations, the single-particle bases are not restricted in the lowest Landau level; we can then achieve exact few-electron states within the whole range of the magnetic field.

In this paper, we also investigate the behaviors of entanglement of the few-electron states. The "correlation entropy" (Refs. 32 and 33) has been suggested to be the correlation measure. We employ the von Neumann entropy[34] to quantify the nonlocal correlation, namely entanglement among particles. Although it is still an open problem on how to quantify all the entanglement properties of an identical-particle state, the von Neumann entropy can indeed give the entanglement information between one particle and another part of the system. In the following discussions, we employ a modified form of the von Neumann entropy as[29]

$$S_{ent} = -\operatorname{tr}[\rho^f \ln \rho^f] - \ln N, \tag{4}$$

where N is the particle number of the system and ρ^f is the single-particle reduced density matrix

$$\rho_{\alpha\beta}^f = \langle\Psi|a_\alpha^\dagger a_\beta|\Psi\rangle. \tag{5}$$

With such modification, the entropy, due to the indistinguishability of the particles, is subtracted. The lower limit values of entropies for the system with different particle numbers are all equal to zero, which corresponds to the unentangled states. Here, the term "unentangled states" means that the states can be expressed as a single Slater determinant.

III. RESULTS AND DISCUSSIONS

For convenience, we define the ratio $\beta = W/R_0$ to describe the shape of quantum rings, where R_0 and W are the mean radius and the width of the ring. For a parabolic confinement, the width of the ring[13] is defined as $W = (2\hbar/m_e^*\omega_0)^{1/2}$. Then the smaller β is, the narrower the ring becomes. It should be pointed out that the definition of W here is actually the same as the R defined above but slightly differ from the definition of mean width in some other literatures.[35]

A. Spin transition of the ground state

In the spin transitions of few-electron quantum dots and rings, the shape effects are important. In Fig. 1 we show the shape and size dependences of the energy differences between the four lowest states in five-electron quantum dots and rings. The spin transition in two-dimensional quantum rings is demonstrated by our studies, as shown in Fig. 1. For quantum rings with small radii, the ground state is $L=1$, $S=0.5$. With the increase of the radius, the angular momentum of the ground state will change to zero, namely the ground state will become the one with $L=0$, $S=0.5$. If the radius increases further, the spin transition will take place, the ground state will become the one with $L=0$, $S=2.5$. For the two-dimensional rings with different shapes, the patterns of the spin transitions are different. As shown in Fig. 1, with the

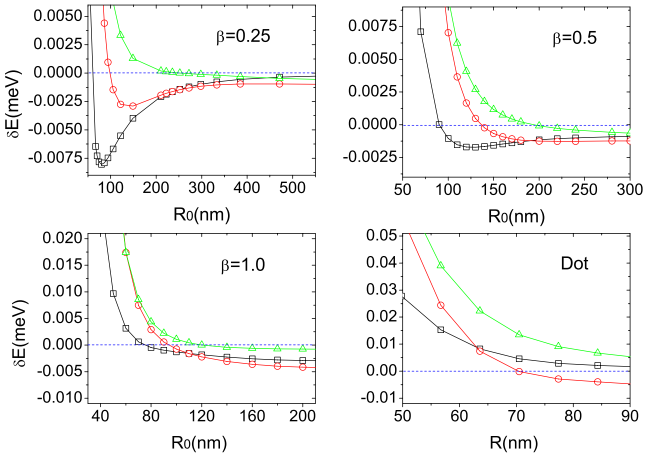

FIG. 1. (Color online) Size dependence of the energy differences between the four lowest states of the five-electron quantum rings with [(a)–(c)] different shapes and (d) the quantum dot. The energy of the state $(L,S)=(1,0.5)$ is set to be zero (dashed lines), and the lines with □, ○, and △ correspond to the states $(L,S)=(0,0.5)$, $(0,2.5)$, and $(1,1.5)$, respectively.

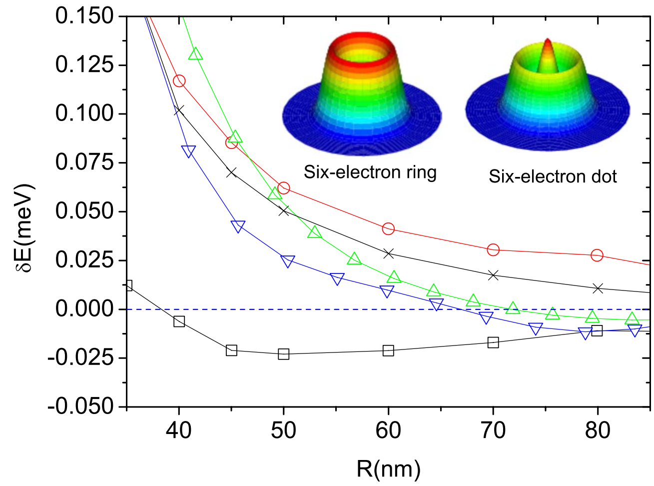

FIG. 2. (Color online) For quantum dots, the energy differences of three-electron $E_{0,3/2}-E_{1,1/2}$ (□), four-electron $E_{0,0}-E_{0,1}$ (○), five-electron $E_{0,5/2}-E_{1,1/2}$ (△), and six-electron $E_{0,2}-E_{0,0}$ (▽) as functions of the radius of the dots. For quantum rings, the energy differences of six-electron $E_{0,2}-E_{0,0}$ (×) as function of the mean radius of the rings with $\beta=1.0$. The electronic density distributions of the six-electron state with $L=0$, $S=2$ in the quantum dot with $R=80$ nm, and the quantum ring with $R_0=80$ nm, $\beta=1.0$ are shown in the inset.

increase of β, the region where the state $L=0$, $S=0.5$ becomes the ground state is reduced. Previous study on the 1D quantum rings has suggested the similar spin transition of the five-electron ground state and has also convinced the angular momentum transition in the subspace with certain total spin, with the increasing radius. It has given an understanding to the spin rules of large 1D rings within the multispin-exchange model.[14]

Unlike the two-step transition of the ground state in quantum rings, the spin and angular momentum transitions are simultaneous in the quantum dot, namely the ground state changes from $L=1$, $S=0.5$ to $L=0$, $S=2.5$ directly. The state $L=0$, $S=0.5$ in quantum dots has higher energy all along. It means that the energy of the fully-polarized five-electron state is lowered more quickly in quantum dots than that in quantum rings. The reason for such difference lies in the following fact: The radial distribution of the wave function in the quantum dot changes dramatically when the radius increases. On the contrary, the change is smaller in quantum rings, especially in narrow ones, because of the strong radial confinement. So for quantum dots, the fully-polarized state can easily lower its interaction energy by occupying the single-particle orbits with higher radial quantum numbers, as well as the angular momentum quantum numbers, when the radius changes. However, for narrow quantum rings, such decrease of the energy can be achieved almost only by occupying the higher angle orbits.

In Fig. 2 we show the spin transitions of the quantum dots with different particle numbers. For three-electron case, there is the transition of the ground state from $L=1$, $S=0.5$ to $L=0$, $S=1.5$ with the increase of the radius. For four-electron case, there is no spin transition caused by the change of the dot's size. For three-electron and four-electron cases, the situations in quantum rings are similar to that in dots. The situation of the five-electron case has been discussed above.

For six-electron quantum dots, we find the interesting result that there is similar spin transition as that of the five-electron case, namely $(L,S)=(0,0)\rightarrow(0,2)$ vs $(1,0.5)\rightarrow(0,2.5)$. On the contrary, there is no such correspondence between six-electron and five-electron quantum rings. For the six-electron rings with $\beta=1.0$, the energy of the state $S=0$ is lower than that of the state $S=2$, even when the radius is larger than 200 nm. Such difference originates from the characters of the spatial distributions of the electrons in quantum dots and rings, or more generally speaking, the topology of the two nanostructures. For six-electron quantum dots, it is known that spatial distribution of the electrons can form a shell structure with a core inside (see the inset of Fig. 2 as an example). In the shell, there are nearly five electrons with pentagonal correlation. Such distribution makes the character of the six-electron dots similar to the five-electron ones since the effect of the core on the shell also becomes weak with the increase of the dot's size. It is worthwhile to point out that the variational quantum Monte Carlo study[10] has also revealed a polarization effect in quantum dots. The effect occurs with the confinement strength $\hbar\omega_0 \approx 0.28$ meV (with our definition of R, it corresponds to $R \approx 64$ nm), which is very close to the characteristic size of the spin transition obtained by our ED results.

For quantum rings, the structure with core inside is forbidden by the ringlike confinement or the topology (also see the typical electronic density distribution of the quantum ring in Fig. 2). So there is no similarity between the spin transition of the six-electron and five-electron quantum rings. Indeed, for a small ring with the parabolic confinement, the weak confinement may allow electrons to enter the central area. Some more elaborate forms of confinement[36,37] may greatly depress such phenomenon. However, it will not affect the above conclusion because the six-electron-spin transition mainly takes place when the radius of the ring becomes large enough. It is also worthwhile to note that the diffusion Monte Carlo simulation also demonstrated that there are no spin transitions in quasi-one-dimensional six-electron quantum rings. However the variational quantum Monte Carlo method suggested a spin transition for purely one-dimensional quantum rings (see Refs. 11 and 12 for details).

The spin configuration of the ground state in few-electron QDs and QRs can be studied experimentally by electron-transport measurements. A study on a small Si quantum dot[38] has revealed that there is interesting spin transition between the five-electron and six-electron ground states with $\Delta S \equiv S(N)-S(N-1)=3/2$. It is different from the one between four-electron and five-electron cases, which has $\Delta S=1/2$. The similar observation can be also anticipated for the GaAs dots since the spins of four-electron, five-electron, and six-electron ground states can be $S=1$, 1/2, and 2, respectively, with appropriate sizes (R is about $65\sim75$ nm for ideal parabolic dot, see Fig. 2).

B. Angular momentum transitions and entanglement in magnetic field

The shape and size of quantum dots and rings affect not only the spin of the ground state but also the characters of the few-electron states in magnetic field.

In Fig. 3, we show two typical energy-level structures of the many-particle states of the four-electron quantum dot and ring in magnetic fields. For the quantum dot with $R=40$ nm, the states with different spins are clearly separated and the energy spectra have no clear structures when the field is small. The increase of the magnetic field makes the energies of the lowest states with different total spins increase rapidly and form a narrow band. The emergence of such structure in magnetic fields is a characteristic of few-electron quantum dots and can be attributed to the formation of the rotating Wigner molecules. The field leads to the crystallization of the electrons. With the decreased overlapping of electrons, the exchange energies of different spin states are depressed totally. Then the energy differences between lowest states with different total spins gradually disappear. For a narrow quantum ring, as the one shown in Fig. 3(b), the formation of the narrow band in energy spectra is much more notable and should be attributed to the strong confinement of the ring. The increase of the energy caused by the field is small since the confinement of the magnetic field is much smaller than that of the ring.

Next we step into the angular momentum transitions in the narrow band. The studies on the angular momentum transition rules of few-electron quantum dots in strong magnetic fields have been made based on both theoretical and numerical methods. For four-electron case, the so-called "magic number" (Refs. 22 and 23) of angular momenta of the states $S=0$ are $L=4k$ and $4k+2$, those of the states $S=1$ are $L=4k$ and $4k\pm1$, and those of the fully-polarized states are $L=4k+2$, where k are arbitrary integers. The state whose angular momentum and spin obey those rules can take place in the transition sequences of the lowest states. Our calculations demonstrate that the angular momentum transitions in four-electron quantum rings are in accordance with the

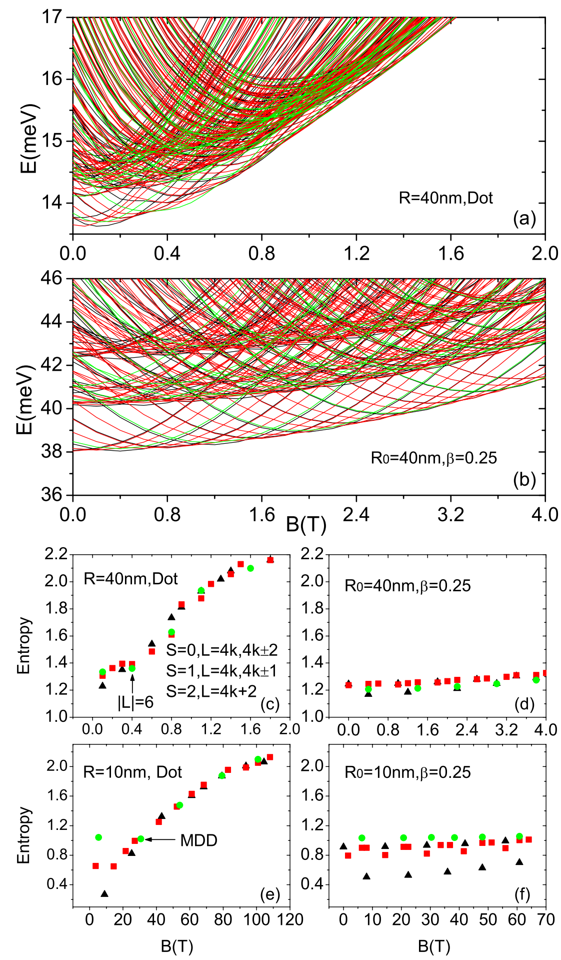

FIG. 3. (Color online) Four-electron energy spectra of (a) the quantum dot with $R=40$ nm and (b) the ring with $R_0=40$ nm, $\beta=0.25$ in magnetic fields. The black, red, and green lines correspond to the energies of the states with total spin $S=0$, 1, and 2. Field dependence of entanglement entropies of the lowest states with different spins of [(c) and (e)] the quantum dots and [(d) and (f)] the quantum rings with $R(R_0)=40$ and 10nm. For all rings, $\beta=0.25$. The black ▲, red ■, and green ● correspond to the states with $S=0$, 1, and 2, respectively. The correspondence of the magic number angular momentum and the total spin of four-electron states are shown in the blank of subfigure (c). The MDD state for the smaller dot and the state with same $L=6$ for the larger dot are indicated in subfigure (e) and (c), respectively.

magic number rules introduced above. And for the rings with $\beta=0.25$, the transitions are almost size independent while the results of the rings with $R_0=40$ nm and $R_0=10$ nm are same. For quantum dots in strong magnetic fields, the transitions also obey the rules of the magic number but a few states with certain angular momenta may be absent from the transition sequences when the field is weak.

It should be emphasized that in order to study the magic number of different spin states, we have ignored the Zeeman splitting in Eq. (1). There have been literatures discussing the angular momentum transitions of ground states with the Zeeman splitting. Then in strong magnetic fields, the angular momentum transition of ground states will have a period of $\Delta L=N$ ($N\leq5$), which is in accordance with our results of the fully-polarized states. For the case $N>5$, the transition will have more complicated behaviors.[39] (See also Refs. 16, 22, and 39 for detailed theoretical and numerical analyses.)

Although the angular momentum transitions in QDs and QRs are very similar, the characters of the entanglement entropies of the four-electron quantum dots and rings are quite different. In Fig. 3 we show the typical transitions of the entanglement entropies of the lowest states with different spins in the quantum dots and rings as functions of the magnetic field. For the quantum dots in strong magnetic fields (strictly speaking, when the flux excess one quanta of magnetic flux), no matter what the total spin is, the entanglement entropies of the lowest states increase rapidly with the increase of the field.

For a small dot such as the one in Fig. 3(e), the change of the entropies in small fields is not monotonic. The states with different S have their own minimal values in their respective fields. Taking the transition sequence of fully-polarized states as an example, we find that the state with $|L|=6$, whose filling factor is just equal to one, has the smallest entropy. This state is also referred to the maximum density droplet state (MDD) (Refs. 16, 40, and 41) in quantum dots and very similar to the Laughlin wave function with filling factor $\nu=1$. If the single-particle bases are restricted in the lowest Landau level, the MDD with maximum S_z is just the Laughlin state.[27] Then it must be unentangled since the Laughlin wave function with $\nu=1$ is just a single Slater determinant.[29] It should be noticed that the entropy of the MDD state does not equal to zero but almost equals to one in our calculation because we take $S_z=0$ in our calculations for convenience. For a large dot [see Fig. 3(c) as an example], the increase of the entropies are monotonic even in the small field. For fully-polarized states, the state with $|L|=6$ no longer has the smallest entropy. It is due to this reason that the crystallization of the electrons in the large dot is much more easier than that in the small one. Then the crystallization makes the state deviate remarkably from the MDD one.

For quantum rings, the increase of the entanglement entropies caused by the field is much smaller than that in quantum dots. Instead, the entropies exhibit apparent Aharonov-Bohm oscillations [see Figs. 3(d) and 3(f)]. It is due to this reason that the effect of the magnetic field on quantum dots is to make the electrons crystallized, which leads to the increase of the entanglement. However, in narrow quantum rings, the confinement of the ring is much stronger than that of the magnetic field. Then the field can only contribute the effect of the magnetic flux. It means that the states with angular momenta L and $L+N$ can almost have the same single-particle orbital occupation except for a translation of all occupied single-particle angular momentum orbits from m to $m+1$. Such two states must have same entanglement entropy since they have the same single-particle reduced density matrix.

We take the five-electron case as an example to discuss the size effect of the ring on the angular momentum transitions and entanglement of the lowest states with different spins. In Figs. 4(c)–4(f) we show the magic number of angular momenta and the entanglement of the corresponding states of two quantum rings with different radii $R_0=10$ and 40nm but same shape $\beta=0.25$.

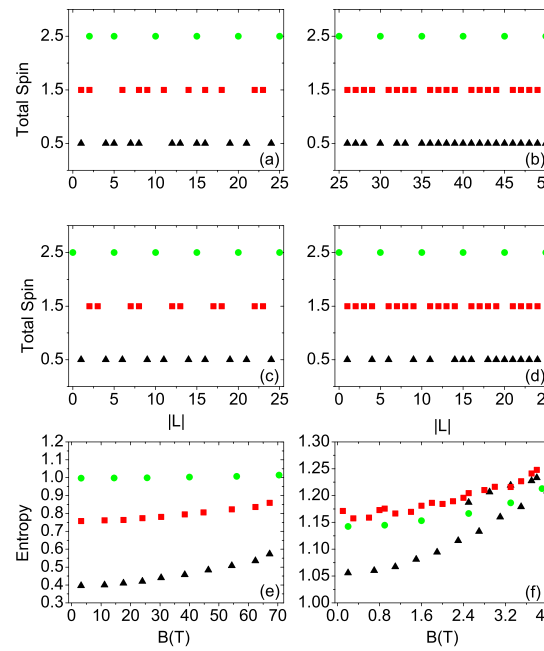

FIG. 4. (Color online) [(a) and (b)] Magic number angular momenta of the lowest five-electron states with different spins in quantum dots with $R=24$ nm. Magic number angular momenta of the lowest five-electron states with different spins and their field dependence of the entanglement entropies in the quantum rings with (c) and (e): $R_0=10$ nm, $\beta=0.25$, and (d) and (f): $R_0=40$ nm, $\beta=0.25$. The black ▲, red ■, and green ● correspond to the states with $S=0.5$, 1.5, and 2.5, respectively.

As discussed above, the angular momentum transitions of the four-electron quantum rings are almost size independent and the transition rules are in accordance with the results of quantum dots. On the contrary, the five-electron angular momentum transitions depend on not only the shape but also the size of the ring. For quantum dots, we find that, in strong magnetic fields, the magic number of the five-electron states $S=1.5$ are $L\neq 5k$, where k are arbitrary integers. However, as shown in Fig. 4(c), our studies suggest that the angular momenta of the lowest states with $S=1.5$ in the quantum ring with $R_0=10$ nm and $\beta=0.25$ are only those $L=5n\pm 2$. Only for the rings with larger radius (see the result of the one with $R_0=40$ nm as an example), the states $L=5k\pm 1$ can take place in the transition sequence. It is known that for the ring with small radius, the confinement energy is important. Then only the states whose single-particle orbital occupations have lower orbital energy can be presented in the transition sequence of the lowest states. In small rings, for $S=1.5$, the states $L=5k\pm 2$ can have almost two electrons with opposite spins occupy the single-particle orbit $m=k$ [see Fig. 5(a) as a sketch]. Then those states have lower orbital energy than the states $L=5k\pm 1$, where the two electrons with opposite spins mainly occupy $m=k\mp 1$, as shown in Fig. 5(b). For the ring with larger radius, the weaker confinement makes the difference of the single-particle orbital energies decrease. So for the ring with $R_0=40$ nm, the states with $L=5k\pm 1$ are no longer absent [see Fig. 4(d)].

For the five-electron states with $S=0.5$ in strong magnetic fields, the magic numbers for quantum dots can be succes-

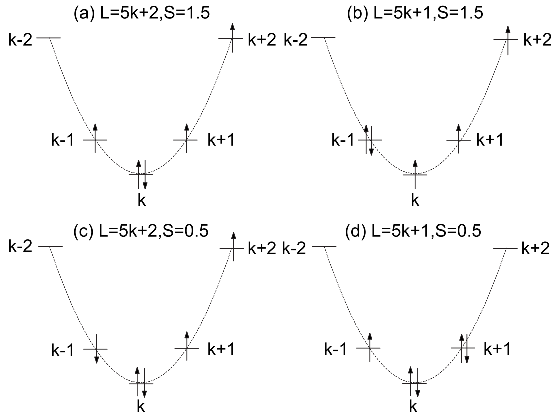

FIG. 5. Sketches of the single-particle angular momentum occupations of five-electron states (a) $L=5k+2$, $S=1.5$, (b) $L=5k+1$, $S=1.5$, (c) $L=5k+2$, $S=0.5$, and (d) $L=5k+1$, $S=0.5$ in a narrow quantum ring. In the subfigures (a) and (b), we select the states with $S_z=1.5$ for simplicity. ↑ and ↓ represent the spin-up and spin-down electrons.

sive integers. Similar to the case of $S=1.5$, there are also regular absences of the magic number states in quantum rings. However, at this time, the phenomenon of the absence is just contrary to the case of $S=1.5$; the absent states are those with $L=5k\pm2$ rather than $L=5k\pm1$. It is due to this reason that the four electrons with two opposite spins in the states $L=5k\pm1$ and $S=0.5$ are almost paired in two orbits $m=k$ and $k\pm1$, and the residual electron occupies $m=k\mp1$ [see Fig. 5(d)]. In the states $L=5k\pm2$, only two electrons are paired in the orbit $m=k$, and the other three electrons occupy $m=k+1$, $k-1$, and $k\pm2$, respectively. So the states $L=5k\pm2$ have higher orbital energy and may be absent from the transition sequence of small rings, as also shown in Fig. 4(c). However, in the case of $S=0.5$, the orbital energy difference between the states $L=5k\pm1$ and $5k\pm2$ is larger than that in the case of $S=1.5$. It can be found that for $S=0.5$, the absence of the states $L=5k\pm2$ is also true in small magnetic fields even for the ring with $R_0=40$ nm. Unlike the case of $S=1.5$, the absence of the states $L=5k\pm2$ in the case of $S=0.5$ only disappears in larger magnetic fields.

As mentioned above, for $S=0.5$, the states $L=5k$ are also the magic number states in quantum dots. In the quantum ring with $R_0=40$ nm, they are also absent from the transition sequence when the field is small and gradually take place with the increase of the field, which is similar to the situation of the states $L=5k\pm2$.

It is worthwhile to point out that the phenomenon of the absence of some magic number states from the transition sequences is also true in quantum dots. For example, as shown in Figs. 4(a) and 4(b) for the five-electron dot with $R=24$ nm, the states with $|L|=2,3,6,9,10,11,14,17,18,20,22,23,25,29$ are absent from the transition sequence of $S=0.5$, and the states with $|L|=3,4,7,12,13,17,19,21,24,30$ are absent from the sequence of $S=1.5$. For the ring with smaller radius, the absences are even more notable. There have been studies on the phase diagram of QDs with different sizes in magnetic fields. Our results, presented above, are in accordance with those in Ref. 26, and the size of the QD in our Figs. 4(a) and 4(b) corresponds to the dimensionless parameter $\lambda\approx1.7$ there. However, the relation between the absent magic number states and the size of the dot is still an open problem.

We also show the entanglement entropies of the lowest states of the five-electron quantum rings in Fig. 4. The oscillations of the entanglement entropies in quantum rings with small β are clear. And the absences of certain angular momentum states in quantum rings with different size lead to the different patterns of the entropies. It is found that the absent states in small fields have higher entanglement since the absent states that have higher orbital energies also have more complex single-particle orbit occupations.

IV. SUMMARY

To conclude, we have employed the exact diagonalization and series-expansion method to investigate the shape and size effects on the angular momentum and spin transitions, and the entanglement properties in few-electron quantum dots and rings.

Without the magnetic fields, changing the size of the five-electron dots or rings can lead to the transition of the ground state. For quantum dots, the spin transition ($S:0.5\rightarrow2.5$) is simultaneous with the angular momentum transition ($L:1\rightarrow0$). However, for quantum rings, the transition takes two steps: the angular momentum transition is completed first and followed by the spin transition. For six-electron quantum dots, there is a similar spin transition as the five-electron case but it is not found for quantum rings. All those differences originate from the difference of the shapes or the topology of the two systems.

With magnetic fields for the narrow ring, the angular momentum transitions of four-electron lowest states are almost size independent. However, the five-electron case is size sensitive and the absence of certain angular momentum states from the transition sequences in small rings can be understood by inspecting the single-particle orbital occupation. The shape and size of the system affect not only the spin and angular momentum transitions but also the entanglement behaviors of the lowest states. In quantum dots, the crystallization of the electrons is mainly caused by the field, which results in the increasing entropies in strong magnetic fields. However, for narrow quantum rings, the magnetic field can almost only cause the AB oscillations of the entropies.

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