

Nonlinear dynamics of two-dimensional electromagnetic solitons in a ferromagnetic slab

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Line solitons of magnetic polaritons can propagate in a ferromagnetic slab. For certain values of the soliton velocity, they are unstable, and decay into stable two-dimensional solitary waves called lumps. The latter is investigated both numerically and by means of a variational approach.

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I. INTRODUCTION

The nonlinear propagation of waves in ferromagnetic media has been widely studied in the magnetostatic range. Envelope solitons have been theoretically described^{1,2} in the *slowly varying envelope approximation* (SVEA) framework and experimentally observed.^{3–5} Dark solitons have also been predicted⁶ and observed.⁷ In (2+1) dimensions, it has been seen that the damping may stop the collapse of the wave packet.⁸ Transverse self-focusing is inhibited if the wave packet propagates in a narrow strip.⁹

The electromagnetic or polariton modes are well known from the linear point of view.^{10–14} Envelope soliton propagation has been predicted.^{15,16} Nonlinear solitary wave propagation and Korteweg–de Vries (KdV)-type solitons have also been theoretically described.^{17–19} From the point of view of mathematical physics, solitons or stable solitary waves localized in more than (1+1) dimensions are of great interest. In both the long-wave approximation and the SVEA, the universal (1+1) model, which is either the KdV equation or the nonlinear Schrödinger (NLS) one, has been generalized to (2+1) dimensions: Either the Kadomtsev–Petviashvili²⁰ (KP) equation or the Davey–Stewartson²¹ (DS) model is obtained. Both models possess localized solutions and lumps²² and dromions in the case of the DS model.²³

All these studies are based on long-wave type approximations: Envelope solitons have wavelengths larger than the associated carrier's wavelengths and solitary waves have wavelengths larger than some typical space scale of the sample. Consequently, these models and their solutions predict behaviors of large-scale phenomena. However, the experimental observation of such structures often requires sample sizes that exceed the ones available at present. Studies on short-range phenomena via the multiscale or reductive perturbation method (adapted to describe short-wave dynamics) started in the rather different context of hydrodynamics^{24,25} and have been subsequently used in ferromagnetic media.^{26–28}

The short-wave approximation has evidenced the possibility of propagating single-oscillation solitons²⁶ in (1+1) dimensions. The propagation obeys the sine-Gordon equation. A (2+1)-dimensional model, in the form of a generalization of the sine-Gordon equation, has been derived in Ref. 27 and two-dimensional single-oscillation solitons propagating in a ferromagnetic slab were studied in Ref. 28. An unstable line soliton breaks into several parts, which evolve as stable lo-

calized structures. Since the latter presents some shape analogy with the lump solutions of the KP equation, we call them lumps. We give here a detailed analysis of these entities. Numerical experiences allow us to determine their dynamics beyond the threshold of the linear stability analysis.

II. (2+1)-DIMENSIONAL GENERALIZATION OF THE SINE-GORDON EQUATION

A. Equations in the short-wave approximation

Approximate equations are derived from the Maxwell–Landau model, which consists of the Maxwell equations, which reduces to

$$-\nabla(\nabla \cdot \mathbf{H}) + \Delta \mathbf{H} = \frac{1}{c^2} \partial_t^2 (\mathbf{H} + \mathbf{M}), \quad (1)$$

and governs the evolution of the magnetic field $\mathbf{H}$, and the Landau–Lifschitz equation,

$$\partial_t \mathbf{M} = -\gamma \mu_0 \mathbf{M} \wedge \mathbf{H}_{\text{eff}} + \frac{\sigma}{M_s} \mathbf{M} \wedge (\mathbf{M} \wedge \mathbf{H}_{\text{eff}}), \quad (2)$$

which is satisfied by the magnetization density $\mathbf{M}$. The velocity $c = 1/\sqrt{\mu_0 \tilde{\epsilon}_0}$ is the speed of light where $\tilde{\epsilon} = \tilde{\epsilon}_0 \tilde{\epsilon}_r$ is the scalar permittivity of the medium, γ is the gyromagnetic ratio, μ_0 is the magnetic permeability of the vacuum, σ is the damping constant, and M_s is the saturation magnetization. We consider bulk polaritons with typical wavelengths ranging from 10 to 100 μm . Hence, the wavelengths are large with regard to the exchange length, and inhomogeneous exchange can be neglected. We assume that the crystalline and surface anisotropy of the sample are also negligible.

The demagnetizing field is accounted for by using the effective magnetic field $\mathbf{H}_{\text{eff}} = \mathbf{H} - N \cdot \mathbf{M}$, where N is diagonal with $(N_x, N_y, N_z) = (0, 0, 1)$. We consider indeed a ferromagnetic film lying in the xy plane, as shown in Fig. 1. The

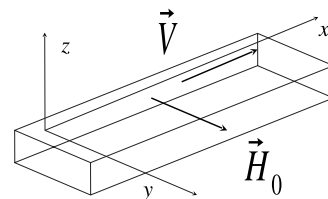


FIG. 1. The configuration considered.

pinning boundary conditions are not relevant as soon as inhomogeneous exchange is neglected. The effect of the electromagnetic boundary conditions on the dispersion properties and transverse profile of the waves in the slab is essential for surface modes and remains very important in thin films, or when the wavelengths are close to the film thickness. We consider here only volume modes and assume that the propagation occurs in a thick slab. The typical value of the thickness is about 0.5 mm, which is large with respect to the wavelength and is small with respect to the slab width, which is about 1 cm. In this situation, the approximation of replacing the exact boundary conditions by a demagnetizing tensor N is justified.

An external field is applied to magnetize the sample to saturation as

$$\mathbf{M}_0 = (0, M_s, 0), \quad \mathbf{H}_0 = \alpha \mathbf{M}_0. \quad (3)$$

The static field $\mathbf{H}_0$ lies in the xy plane, which is the plane of the film, and is thus collinear to the magnetization $\mathbf{M}_0$.

We introduce a small parameter ε and apply the reductive perturbation method in the short-wave approximation.^{24,25} The process assumes that the wave amplitude is weak, of order ε , that the length of the solitary wave is short, of order ε too, and that the propagation distance is large, of order $1/\varepsilon$. The reference length $L_0 = c/(\gamma\mu_0 M_s)$ (order ε^0) is characterized by the magnetization saturation and light velocity in the medium. Using values corresponding to yttrium iron garnet (YIG), $\gamma\mu_0 = 1.759 \times 10^7$ rad s⁻¹ Oe⁻¹, $M_s = 1800$ Oe, and $\tilde{\varepsilon}_r = 12$, we get $L_0 \approx 5$ mm. Assuming a perturbative parameter $\varepsilon \approx 10^{-2}$, the characteristic scale for X , i.e., the typical wavelength, is $\varepsilon L_0 \approx 50$ μ m, while the propagation distance could be, in principle, as large as $L_0/\varepsilon \approx 50$ cm and will be less in practice due to available sample sizes.

We further assume that the damping is weak. In YIG films, envelope solitons have been observed.^{2,3} It has been shown that the observations could be accounted for using a NLS-type model including the damping term. Such a model can be derived from the Landau–Lifschitz and Maxwell equations [Eqs. (2) and (1), respectively], assuming that the dimensionless damping constant $\tilde{\sigma} = \sigma/\mu_0\gamma$ is small, of order ε^2 (with ε still being the perturbation parameter measuring the wave amplitude).²⁹ In YIG films, $\tilde{\sigma}$ can be as small as 10^{-4} (cf. Ref. 30), which would correspond to a perturbation parameter $\varepsilon \approx 0.01$. It is shown in Refs. 28 and 31 that under this assumption, the effect of the damping can be completely neglected within the short-wave approximation, in contrast to the case of envelopes.²⁹ Since the velocity is close to that of light, and the propagation distance of a few cm (tens of cm in principle), the propagation time is very short and therefore, the damping does not have enough time to operate in an appreciable way.

This way, the equations are

$$C_{XT} = -BB_X + C_{YY} + B_Y, \quad (4)$$

$$B_{XT} = BC_X + B_{YY} - C_Y, \quad (5)$$

where the subscripts denote partial derivatives (i.e., $C_Y = \partial_Y C$, and so on), are derived. The variables in Eqs. (4) and (5) are expressed in terms of the physical quantities as

$$C_X = \frac{-H^y}{M_s} - 1, \quad B = \frac{1}{\varepsilon} \frac{M^x}{2M_s}, \quad (6)$$

$$X = \frac{-1}{\varepsilon} \frac{\gamma\mu_0 M_s}{2c} (x - ct), \quad (7)$$

$$Y = \frac{\gamma\mu_0 M_s}{c} y, \quad T = \varepsilon \gamma\mu_0 M_s t. \quad (8)$$

The perturbation parameter ε can be removed from expressions (6)–(8); however, keeping them presents several advantages: (i) It recalls the validity conditions of Eqs. (4) and (5); (ii) It allows us to perform the numerical computations using values of the order unity only; (iii) The results obtained either analytically or numerically can be straightforwardly generalized using the scale invariance related to a change in the value of ε .

The magnetic field components can be computed from the dynamical variables B and C according to

$$H^y = -M_s(1 + C_X), \quad H^z = -M_s B_X. \quad (9)$$

B. Line soliton

Comparing systems (4) and (5) to the system derived in bulk media²⁷ shows that the effect of the demagnetizing field is negligible too. Hence, the results established in bulk media apply.

In (1+1) dimensions, setting

$$B_X = A \sin \theta, \quad C_X = \sin \theta, \quad (10)$$

systems (4) and (5) reduce to

$$\theta_{XT} = A \sin \theta, \quad (11)$$

where A is a constant. Equation (11) is the sine-Gordon equation, which is completely integrable by means of the IST method^{32,33} and was first derived in the frame of electromagnetic waves in ferromagnets in Ref. 26. It admits the kink solution

$$B = 2w \operatorname{sech} \zeta, \quad C = w(2 \tanh \zeta - \zeta), \quad (12)$$

where $\zeta = (X - wT)$, the velocity w of the kink being an arbitrary real parameter. Solutions (12) and (11) obviously yields a solution of systems (4) and (5) in the form of a line soliton: a solitary wave invariant in the transverse direction. A more general plane solitary wave can be deduced from solution (12) as

$$B = p + 2w \operatorname{sech} \zeta, \quad C = w(2 \tanh \zeta - \zeta), \quad (13)$$

$$\zeta = X + pY - (w - p^2)T,$$

where p is an arbitrary real parameter. For nonzero p , w does not represent the soliton velocity anymore. The stability of line soliton (12) with respect to slow transverse perturbations was studied in Ref. 27. It is shown that the line soliton is stable if its velocity w is less than

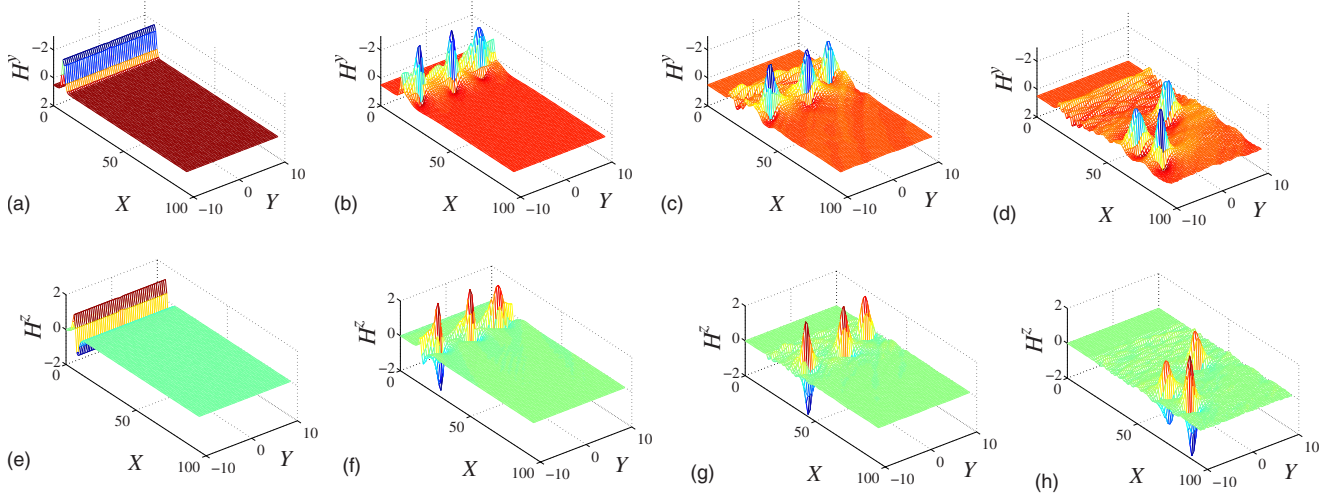


FIG. 2. (Color online) Emission of three localized pulses from an unstable line soliton. Parameter of the line soliton: $w=1.5$; parameters of the perturbation: $X_0=7$, $b=0.1$, $Y_r=0.5$, and $X_r=1.5$.

$$w_{\text{th}} = \frac{\pi^2}{8} - 1, \quad (14)$$

and is unstable for $w > w_{\text{th}}$. This has been confirmed by numerical analysis. The question now is, what happens when the line soliton is unstable?

III. NUMERICAL INVESTIGATION

A. Line soliton breaks up into lumps

The numerical resolution of the system, starting from an unstable line soliton transversely perturbed as an initial data, shows the breaking of the line soliton into several parts, which continue to evolve as stable two-dimensionally localized entities. A characteristic example is given in Fig. 2. The initial data is line soliton (12), with the value of the parameter $w=1.5$, for which it is unstable, and which is initially perturbed by setting $\zeta(T=0)=X-X_1$, where

$$X_1 = X_0 + b \exp[-Y^2/Y_r^2 - (X-X_0)^2/X_r^2], \quad (15)$$

The numerical scheme we use is the same as described in Ref. 27. The size of the numerical box is defined as $0 < X < 96$, $-10 < Y < 10$, $0 < T < 30$, with a number of points $(n_X, n_Y, n_T) = (2000, 50, 4800)$. Both H^y and H^z components are involved. The residual energy, which is not transformed into lumps through the interaction, is less important for the H^z components than for the H^y one.

The stability of the pulses that arise from the destruction of the line soliton depend on the parameter w , as shown by numerical computations. Notice that w is the velocity of the line soliton but is also linked to its amplitude (which is proportional to w) and to the background [through $H^y = m(w-1)$]. Whether the value of the background field or that of the amplitude of the localized structures is responsible for this observation cannot be decided from these computations. Precisely, for $w=1.05$ or less, the fragments are not stable, while stable structures are always formed for $w=1.1$ and larger. However, for $1.1 < w < 1.5$, the fragments are not all

stable and can undergo inelastic interaction and fusion, while three stable localized pulses are formed for $w=2$.

The shape of the formed structure can be isolated from the propagation computations; a typical case is shown in Fig. 3. The shape of the structure looks like a KP lump, especially for the H^y component, which presents a large negative peak between two smaller positive ones. The shape of the other component, H^z , is quite different, since it presents two peaks with opposite signs. The profile of H^y along the propagation direction is even, while that of H^z is odd.

However, a characteristic of the lump is that it algebraically decays in all directions,³⁴ while a two-dimensional soliton is expected to be exponentially localized. The present structures algebraically decay in the X direction, and exponentially in the Y one. This can be seen by plotting B against the distance from the pulse center in logarithmic and semi-logarithmic scales (see Fig. 4).

After fitting the curves, we find that

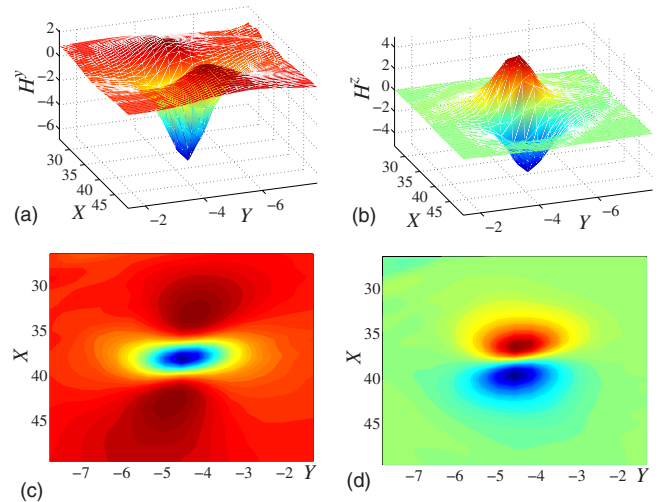


FIG. 3. (Color online) The shape of the two-dimensional lump. The figure is obtained from the evolution of a variational lump and corresponds to the example no. 4 in Table I.

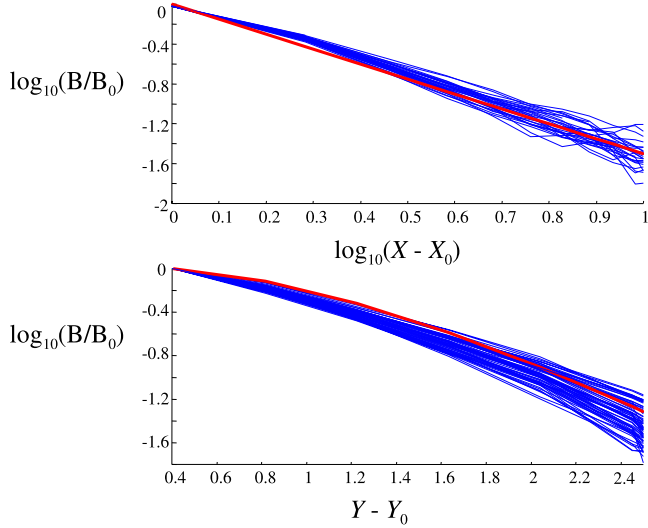


FIG. 4. (Color online) The decay of the localized structure vs X (top) and Y (bottom). The thick red curves are a fit by $X\sqrt{X}$ (top) or $\exp(-Y\sqrt{Y})$ (bottom) [the field values are normalized to yield $B = B_0 [= \max(B)]$ at the first computed point, which is not exactly the center (X_0, Y_0) of the pulse but shifted by one numerical step δX or δY].

$$B \sim \frac{1}{X\sqrt{X}}, \quad B \sim e^{-Y\sqrt{Y}}. \quad (16)$$

The breaking up of a line soliton into lumps has already been described in the case of the KPI equation³⁵ [and also its generalization to (3+1) dimensions³⁶]. We observe here the same phenomenon except that (i) the lumps have a transverse velocity component and (ii) the various lumps have different parameters. Indeed, in the case of KPI, all emitted lumps are identical and travel along the X axis. The asymmetry of the present model, which is induced by the application of an external field to the ferromagnetic film, and the chiral properties of the magnetic force, are evidently at the origin of this feature.

B. Few examples of interactions

We did not perform a systematic study of the interactions between lumps, but some observations can be drawn from the simulations of lump emission by the unstable line soliton. Two lumps can merge together, or one be absorbed by another. After that, three situations may happen: (i) The absorbed lump is re-emitted. In this case, the re-emitted lump is shifted forward for an appreciable distance (see Fig. 5). (ii) The absorbed lump is not re-emitted. It is, properly speaking, a merging of the two lumps (see Fig. 6). (iii) The two lumps annihilate together, and their energy is dispersed and diffracted. However, numerical evidence for this latter scenario to occur is not decisive.

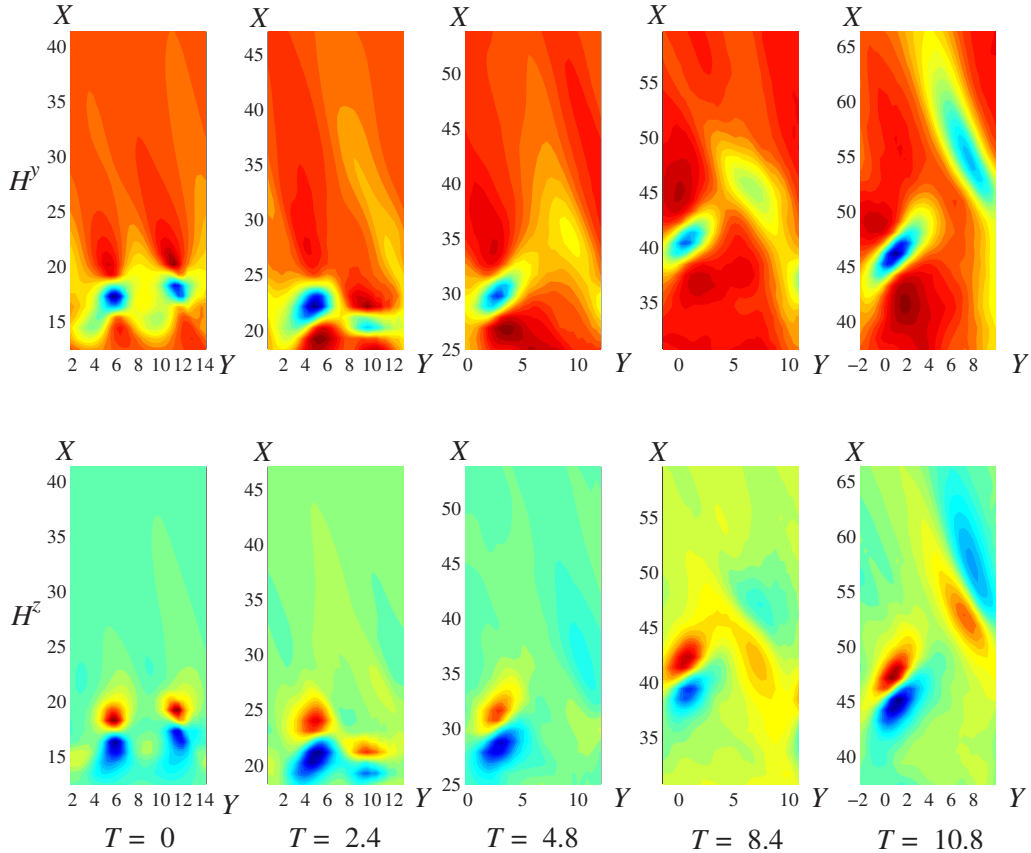


FIG. 5. (Color online) Interaction of two lumps: One lump is absorbed by the other, and then re-emitted forward. Notice that the picture frame moves with the lumps. The time is indicated below the frame; it is arbitrarily set to zero for the first picture.

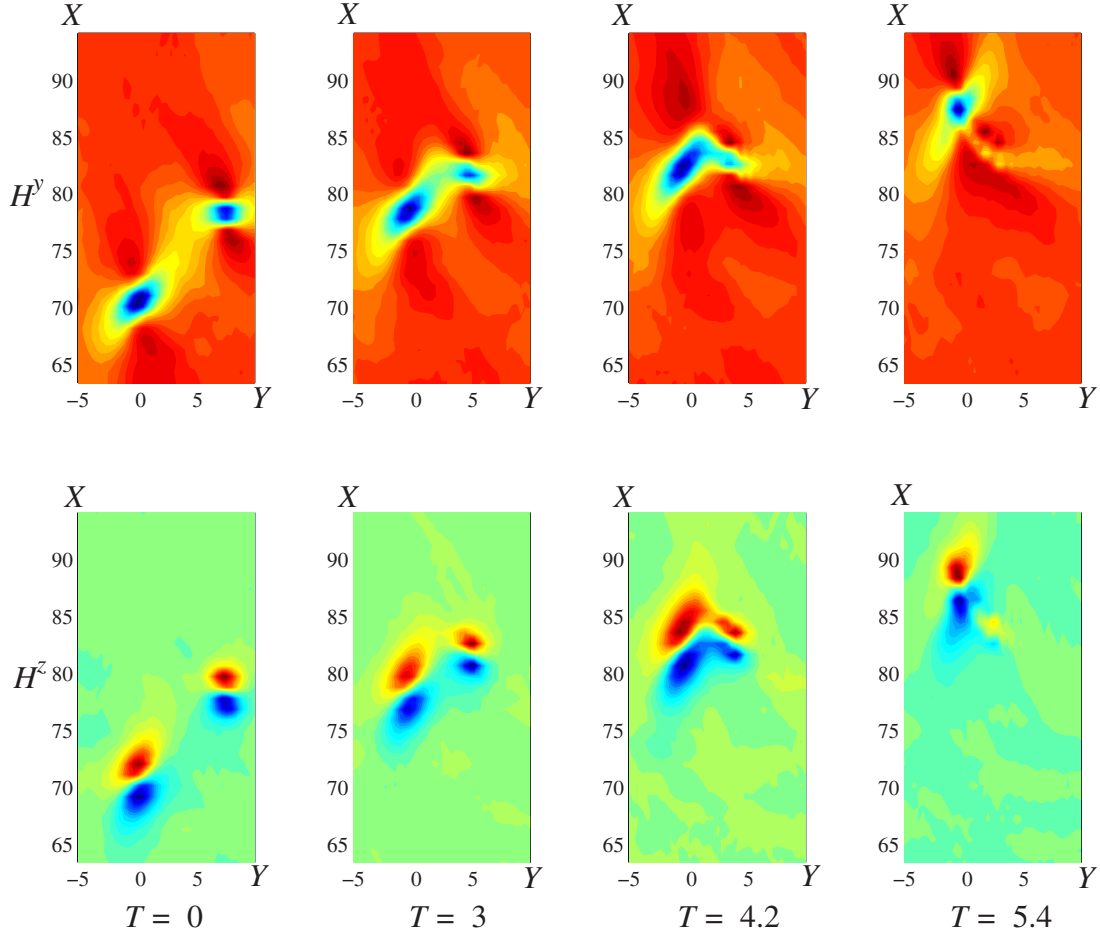


FIG. 6. (Color online) Merging of two lumps. The time is indicated below the frame; it is arbitrarily set to zero for the first picture.

IV. VARIATIONAL APPROACH

A. Variational lump solution

Systems (4) and (5) derive from the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}C_X C_T + \frac{1}{2}B_X B_T - \frac{1}{2}(C_Y)^2 - \frac{1}{2}(B_Y)^2 + CB_Y + \frac{1}{2}C_X B^2, \quad (17)$$

through

$$\frac{\delta \mathcal{L}}{\delta C} = 0, \quad \frac{\delta \mathcal{L}}{\delta B} = 0. \quad (18)$$

We seek for traveling solutions of systems (4) and (5), including a background field $\vec{H}_0 = (0, am, 0)$. Therefore, we transform B and C to have the form $B = B(X - vT, Y - wT, T)$, $C = -aX + C'(X - vT, Y - wT, T)$. The equations become (dropping the primes)

$$C_{XT} - vC_{XX} - wC_{XY} = -BB_X + C_{YY} + B_Y, \quad (19)$$

$$B_{XT} - vB_{XX} - wB_{XY} = -aB + BC_X + B_{YY} - C_Y, \quad (20)$$

and the effective Lagrangian density is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}C_X(C_T - vC_X - wC_Y) + \frac{1}{2}B_X(B_T - vB_X - wB_Y) - \frac{1}{2}(C_Y)^2 - \frac{1}{2}(B_Y)^2 + CB_Y + \frac{1}{2}C_X B^2 - \frac{a}{2}B^2. \quad (21)$$

We make use of the variational approximation method with the ansatz

$$B = p \exp\left(-\frac{X^2}{f^2} - \frac{Y^2}{g^2}\right), \quad (22)$$

$$C = -(\mu X + \rho Y) \exp\left(-\frac{X^2}{f^2} - \frac{Y^2}{g^2}\right). \quad (23)$$

The Gaussian shape in Eqs. (22) and (23) does not match the numerical results mentioned above; it is used for reasons of tractability. The Lagrangian $L = \int_{\mathbb{R}^2} \mathcal{L}_{\text{eff}} dX dY$ is computed by standard methods and is

$$L = \frac{-\pi}{144fg} \{9f^4\mu^2 + 9g^2(4p^2 + g^2\rho^2)v + f^2[4(9 + 9ag^2 + 4g^2\mu)p^2 - 36g^2p\rho + 9g^2(3\rho^2 + 3\mu^2v + 2\mu\rho w)]\}. \quad (24)$$

Five equations are obtained by deriving the Lagrangian L

with respect to the dynamical variables p , μ , ρ , f^2 , and g^2 . The quantities a , v , and w are treated as parameters, and we seek for a stationary solution in the moving frame. After reduction and taking the sum and difference of the two latter equations, we get the following set of equations:

$$18f^2p + 18af^2g^2p + 8f^2g^2\mu p - 9f^2g^2\rho + 18g^2pv = 0, \quad (25)$$

$$9f^2\mu + 8g^2p^2 + 27g^2\mu v + 9g^2\rho w = 0, \quad (26)$$

$$-2f^2p + 3f^2\rho + g^2\rho v + f^2\mu w = 0, \quad (27)$$

$$9f^4\mu^2 + 36af^2g^2p^2 + 16f^2g^2\mu p^2 - 36f^2g^2\rho p + 27f^2g^2\rho^2 + 27f^2g^2\mu^2v + 9g^4\rho^2v + 18f^2g^2\mu\rho w = 0, \quad (28)$$

$$f^4\mu^2 + 2f^2p^2 - 2g^2p^2v - g^4\rho^2v = 0. \quad (29)$$

It can be solved to yield a single-valued expression of the variables as follows: First, we introduce two parameters q and ς defined by

$$g^2 = qf^2, \quad \rho = \mu\varsigma, \quad (30)$$

then p is computed from Eq. (27), μ from Eq. (26), and f from Eq. (29). The two other variables q and ς cannot be explicitly computed, but the parameters a and v can be, from Eqs. (28) and (25), respectively.

Finally, we get

$$p = \frac{9\varsigma(-1 - qv + q^2\varsigma^2v + q^3\varsigma^2v^2)}{2(-3 + 3q\varsigma^2 - 4qv + 3q^2v^2 + q^3\varsigma^2v^2)}, \quad (31)$$

$$\mu = \frac{9q\varsigma^2(qv - 1)(-1 - qv + q^2\varsigma^2v + q^3\varsigma^2v^2)}{(-3 + 3q\varsigma^2 - 4qv + 3q^2v^2 + q^3\varsigma^2v^2)^2}, \quad (32)$$

$$\rho = \frac{9q\varsigma^3(qv - 1)(-1 - qv + q^2\varsigma^2v + q^3\varsigma^2v^2)}{(-3 + 3q\varsigma^2 - 4qv + 3q^2v^2 + q^3\varsigma^2v^2)^2}, \quad (33)$$

$$f^2 = \frac{(-3 + 3q\varsigma^2 - 4qv + 3q^2v^2 + q^3\varsigma^2v^2)^2}{2q^2\varsigma^2(qv - 1)(1 - q^2\varsigma^2v)}, \quad (34)$$

$$g^2 = qf^2, \quad v = v, \quad (35)$$

$$w = \frac{3 + 4qv - 2q^3\varsigma^2v^2 - q^2v(2\varsigma^2 + 3v)}{q\varsigma(qv - 1)}, \quad (36)$$

$$a = \frac{q\varsigma^2(qv - 1)[-5 + 3q(\varsigma^2 - 2v) + 3q^3\varsigma^2v^2 + q^2v(2\varsigma^2 + 3v)]}{-3 + 3q\varsigma^2 - 4qv + 3q^2v^2 + q^3\varsigma^2v^2}. \quad (37)$$

f and g must be real, hence, q and f^2 must be positive, and the velocity v must lie between $1/q$ and $1/(q^2\varsigma^2)$. We have thus obtained a three parameter family of lumps, which corresponds to the numerical observation. Indeed, the applied field must belong to the parameter set, and the lumps appear to vary in amplitude, velocity, and direction of propagation, while it is unlikely that the three parameters are independent.

B. Comparison to numerical simulation

We fix a , ς and v ; q is computed by solving Eq. (37), and then the variational solution is completely known. We nu-

merically compute the evolution of the pulse, taking the variational solution as the initial data. Most of the energy is propagated as a lump, but its speed differs from the one expected. Eventually, two lumps can be obtained. For an adequate choice of the parameters, the agreement can be very good (see Figs. 3 and 7). Our variational analysis is expected to yield an approximation of the stationary states, but it cannot predict their stability. In order to achieve this task, the variational analysis should be time dependent. However, due to the particular time dependency of the Lagrangian density (17), any kind of symmetry in the ansatz results in the can-

TABLE I. Comparison between numerical and variational results.

		p	μ	ρ	f^2	g^2	v	w	a
No. 1	Numeric	17.05	-7.81	1.35	8.67	1.57	10.92	-1.87	2.00
	Variational	19.75	-8.82	1.53	8.58	1.45	10.92	-3.64	2.00
No. 2	Numeric	14.07	-7.32	1.52	7.10	1.77	8.06	-1.84	2.00
	Variational	16.96	-8.76	1.82	6.40	1.46	8.06	-2.86	2.00
No. 3	Numeric	20.35	-7.74	1.12	14.81	1.73	15.79	-2.37	2.00
	Variational	23.76	-8.88	1.29	12.31	1.45	15.79	-4.65	2.00
No. 4	Numeric	19.36	-7.87	1.26	10.53	1.48	14.16	-2.21	2.00
	Variational	22.50	-8.86	1.42	11.07	1.45	14.16	-4.30	2.00

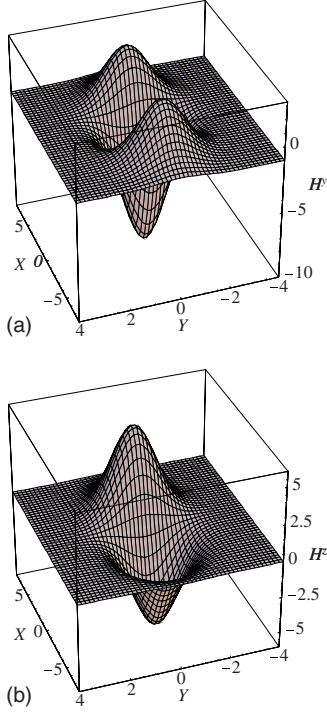


FIG. 7. (Color online) The lump profile, according to the variational approximation. The parameters are the ones listed in Table I as example no. 4. They correspond to the numerically computed lump drawn in Fig. 3. Quite good agreement between the variational and numerical solution is observed.

cellation of the time-depending term in the effective Lagrangian L . Therefore, the time-dependent variational analysis is not affordable. Hence, the variational solution can be unstable; in this case, it will, in general, decay into a stable lump, which can be rather different from the initial unstable one.

On the other hand, we can prove that the variational expressions are in quite good agreement with the numerical results, for the lumps whose stability has been numerically shown. Indeed, the parameters involved by the variational approximation that correspond to the numerical results can be determined. p is the maximum of v . a can be computed from the boundary value of H^y at $x=0$. The parameter μ is computed from the extremal values of H^y from which we have removed the background $a-1$; μ is either $\max(H^y)+1-a$ or $\min(H^y)+1-a$. We define a mean value by $\langle X \rangle = \int X B^4 dX dY / \int B^4 dX dY$, and then $f^2 = \langle (X - \langle X \rangle)^2 \rangle$ and $g^2 = \langle (Y - \langle Y \rangle)^2 \rangle$. Let us call (X_p, Y_p) the value of (X, Y) for which $C+aX$ is maximal and (X_v, Y_v) the value for which this quantity is minimal. Then, we can compute ρ as $\rho = \mu(Y_p - Y_v) / (X_p - X_v)$. The velocities are computed by a linear fit of $\langle X \rangle$ and $\langle Y \rangle$ versus T .

Then, s , v , and a are set as the numerically obtained values and all parameters computed from the variational approximation (to get q , we numerically solve the equation for a). This way, we get Table I (the run for which two lumps have been emitted has been omitted). The agreement is quite good, especially if we notice that the parameters of the variational solution vary very quickly with s , v , and q . Notice that the direction of X is opposite to that of x and that the lumps travel slower than the unstable line solitons, which are themselves slower than light in the medium.

C. Back to physical units

The above values are dimensionless, the space scales being normalized with respect to $L_0 = c / (\gamma \mu_0 M_s)$ and the field amplitudes to the saturation magnetization M_s . In physical units, the width and length of the lump are

$$y_l = \frac{c}{\gamma \mu_0 M_s} g, \quad x_l = \frac{2\epsilon c}{\gamma \mu_0 M_s} f, \quad (38)$$

and the lump velocity components are

$$\mathcal{V} = c(1 - 2\epsilon^2 v), \quad \mathcal{W} = \epsilon c w. \quad (39)$$

Assuming values typical for YIG ($\gamma \mu_0 = 1.759 \times 10^7 \text{ rad s}^{-1} \text{ Oe}^{-1}$, $M_s = 1800 \text{ Oe}$, and $\epsilon_r = 12$, as above) and taking $\epsilon = 10^{-2}$, we get from the data of Table I the following: $x_l \approx 0.14\text{--}0.21 \text{ mm}$, $y_l \approx 3.3\text{--}3.7 \text{ mm}$, $\mathcal{V} \approx 8.66 \times 10^7 (1 - 2 \times 10^{-4} v) = 8.633 \times 10^7 \text{ to } 8.646 \times 10^7 \text{ ms}^{-1}$, and $\mathcal{W} \approx -4.1 \times 10^6 \text{ to } -1.6 \times 10^6 \text{ ms}^{-1}$.

V. CONCLUSION

We have shown that the unstable line solitons, belonging to the magnetic polariton modes, which can propagate in ferromagnetic slabs, can decay into stable two-dimensional solitary waves. The process is very close to the decay of the line soliton of KPI into lumps. The characteristics of the lumps have been specified using both numerical and variational approaches. The agreement between the two approaches is quite good. The size of the lumps, which is less than 0.2 mm in length and about 3–4 mm in width, is quite reasonable compared to the size of the ferromagnetic slabs used in the experiments, so that the observations of these objects should be possible.

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