

Topological order and non-Abelian statistics in noncentrosymmetric s -wave superconductors

Satoshi Fujimoto

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

(Received 3 April 2008; published 5 June 2008)

We demonstrate that in two-dimensional noncentrosymmetric s -wave superconductors under applied magnetic fields for a particular electron density, topological order emerges, and there exists a zero-energy Majorana fermion mode in a vortex core, which obeys non-Abelian statistics, in analogy with p_x+ip_y superconductors, the Moore–Read Pfaffian quantum Hall state, and the gapped non-Abelian spin liquid phase of the Kitaev model.

DOI: 10.1103/PhysRevB.77.220501

PACS number(s): 74.20.-z, 71.10.Pm, 73.43.-f, 74.25.Ha

Recently, there is considerable interest in emergent topological states of many-body quantum systems characterized by a topologically nontrivial structure of the Hilbert space, i.e., “topological order.”¹ In certain classes of topological states in 2+1 dimension, quasiparticles are non-Abelian anyons.^{2–10} The essential feature of the non-Abelian statistics is that the exchange of particles is described by unitary operations in the multidimensional space, which is spanned by the basis of the degenerate many-body ground state. Thus, the state depends on the order of the multiple exchange processes of particles. The possible realization of non-Abelian statistics in real systems has been extensively studied so far in connection with the $\nu=5/2$ fractional quantum Hall (FQH) state and the vortex state of chiral p_x+ip_y superconductors (superfluids).^{2–7,11,12} These topological states are of interest also in the context of quantum computing, since the non-Abelian anyon can be utilized as a decoherence-free qubit and can be potentially applied to the construction of fault-tolerant topological quantum computers.^{8,13–15} In this Rapid Communication, we present another candidate of a topological phase, allowing the existence of non-Abelian anyons, which can be realized in strongly noncentrosymmetric (NC) s -wave superconductors. This topological phase belongs to the same class as those of the Moore–Read (MR) Pfaffian FQH state,² p_x+ip_y superconductors,⁶ and the gapped non-Abelian spin liquid phase of the Kitaev model.^{9,10,16} In NC superconductors, the asymmetric spin-orbit (SO) interaction, which breaks inversion symmetry, plays important roles in various exotic superconducting properties.^{17–21} In our proposal, the asymmetric SO interaction combined with an external magnetic field yields the nontrivial topological state for a particular electron filling.

We consider type II NC s -wave superconductors with the Rashba-type SO interaction in two dimension. We neglect the parity mixing of triplet components of Cooper pairs due to the asymmetric SO interaction¹⁷ because the inclusion of this effect does not change the essential part of our argument. For concreteness, we define our model on the square lattice, although the following consideration does not rely on the particular choice of the crystal structure. Then the model Hamiltonian is

$$\mathcal{H} = \sum_{k,\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \alpha \sum_{k,\sigma\sigma'} \mathcal{L}_0(k) \cdot \boldsymbol{\sigma}_{\sigma\sigma'} c_{k\sigma}^\dagger c_{k\sigma'} - \sum_k [\Delta c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \text{h.c.}] \quad (1)$$

$$= \sum_{\nu=\pm} \left[\sum_k \varepsilon_{k\nu} a_{k\nu}^\dagger a_{k\nu} - \sum_k \{ \Delta_\nu(k) a_{k\nu}^\dagger a_{-k\nu}^\dagger + \text{h.c.} \} \right]. \quad (2)$$

Here, $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is a creation (an annihilation) operator for an electron with momentum k , spin σ . The energy band dispersion is $\varepsilon_k = -2t(\cos k_x + \cos k_y) - \mu$. The second term of Eq. (1) is the Rashba SO interaction with $\mathcal{L}_0(k) = (\sin k_y, -\sin k_x, 0)$. Equation (2) is expressed in terms of the chirality basis, which diagonalizes the SO term. The energy band is split into two parts by the SO interaction: $\varepsilon_{k\pm} = \varepsilon_k \pm \alpha |\mathcal{L}_0(k)|$. The gap function in this basis is odd parity and possesses the momentum dependence $\Delta_\pm(k) = \Delta \eta_\mp(k)$ with $\eta_\pm(k) = -(\mathcal{L}_{0x} \pm i\mathcal{L}_{0y}) / \sqrt{\mathcal{L}_{0x}^2 + \mathcal{L}_{0y}^2}$, which is, importantly, similar to that of p_x+ip_y superconductors. Thus, for $\Delta \ll E_F$, we can exploit the same argument as that applied to p -wave superconductors,^{22,23} and find that in the mixed state with vortices parallel to the z axis, there is a zero-energy quasiparticle state of a vortex core, which is described by a Majorana fermion. However, in this case, the existence of Majorana fermions in vortices does not directly lead to the non-Abelian statistics of them because there are two bands ($\varepsilon_{k\mu}, \mu = \pm$), each of which contributes to a zero-energy mode with a different band index. The existence of two different species of Majorana fermions in a single vortex implies that the sign change of the fermion operators under the braiding of two vortices, which is a hallmark of the non-Abelian statistics (see below),^{2,6} is canceled. To eliminate this unwanted multiplicity of Majorana fermions, we tune the chemical potential as $\mu = -4t$ for which the Fermi level crosses the Γ point in the Brillouin Zone (BZ). In this situation, there are still two bands near the Fermi level in the model [Eq. (2)]: one from ε_{k-} with a finite Fermi momentum, and the other in the vicinity of the Γ point, which is given by the Dirac cone. To generate the mass gap in the Dirac cone, we introduce the Zeeman coupling $\mu_B H_z \sum_k (c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow})$. The magnitude of the gap is of the order $\mu_B H_z$. Then, there is only a single energy band ε_{k-} , which crosses the Fermi level. To make our argument more precise, let us assume that H_z is sufficiently smaller than the orbital depairing field H_{orb} . The Pauli depairing effect due to H_z is negligible for $\alpha \gg \mu_B H_z, \Delta$.¹⁹ Also, for this condition, pairings between electrons with finite Fermi momenta and electrons around the Γ point are strongly suppressed because the Cooper pairs with such large center-of-mass momenta are energetically unstable. The Zeeman coupling generally induces interband pairings, in addition to the intraband pairings, resulting in the

Hamiltonian expressed in terms of the chirality basis,

$$\mathcal{H} = \sum_{\nu=\pm} \left[\sum_k \varepsilon'_{k\nu} a_{k\nu}^\dagger a_{k\nu} - \sum_k \{ \Delta'_\nu(k) a_{k\nu}^\dagger a_{-k\nu}^\dagger + \text{h.c.} \} \right] - \sum_k [\Delta_2(k) a_{k+}^\dagger a_{-k-}^\dagger + \text{h.c.}] \quad (3)$$

Here $\varepsilon'_{k\nu} = \varepsilon_k + \nu\alpha|\mathcal{L}(k)|$, $\Delta'_\nu(k) = \Delta \tilde{\eta}_{-\nu}(k)$, and $\Delta_2(k) = -\mu_B H_z \Delta / |\mathcal{L}(k)|$ with $|\mathcal{L}(k)| = \sqrt{\mathcal{L}_0^2 + \mathcal{L}_y^2 + \mu_B^2 H_z^2 / \alpha^2}$ and $\tilde{\eta}_\nu(k) = \eta_\nu(k) |\mathcal{L}_0(k)| / |\mathcal{L}(k)|$. The single-particle energy of Eq. (3) is

$$E_{k\nu} = \pm [\varepsilon_k^2 + \alpha^2 |\mathcal{L}(k)|^2 + \Delta^2 + 2\nu \sqrt{\varepsilon_k^2 \alpha^2 |\mathcal{L}(k)|^2 + \mu_B^2 H_z^2 \Delta^2}]^{1/2}. \quad (4)$$

For $\mu = -4t$, low energy excitations of the Hamiltonian (3) consist of two parts: one is the contribution from quasiparticles of the band E_{k-} with the finite Fermi momentum k_F defined by $\varepsilon'_{k_F-} = 0$, and the other from quasiparticles of both two bands E_{k+} and E_{k-} around the Γ point. We first consider the latter. As seen from Eq. (4), the quasiparticles around the Γ point have energy gaps $\sim |\Delta \pm \mu_B H_z|$, which implies that in the vortex core state, a quasiparticle energy gap $\sim \mu_B H_z$ exists, and there is no zero-energy state associated with a Majorana fermion mode in the vicinity of the Γ point. Therefore, we can neglect the contributions from the quasiparticles around the Γ point for the discussion on the Majorana fermion modes in the vortex cores. We now turn to the quasiparticles with the finite Fermi momentum k_F . For $k \sim k_F$, the band ε'_{k+} is separated from the Fermi level by an energy gap $\sim \alpha$. In the case of $\alpha \gg \mu_B H_z, \Delta$ with which we are mainly concerned, the intraband pairs in the band ε'_{k+} are completely negligible, and the interband pairs are strongly suppressed, i.e., $|\Delta_2(k_F)| \ll |\Delta'_-(k_F)|$. We can exactly integrate out the high-energy contributions from the band ε'_{k+} , which merely renormalize slightly the energy dispersion and the gap function of the band ε'_{k-} , and obtain the low energy effective model

$$\mathcal{H}_{\text{eff}} = \sum_k \tilde{\varepsilon}_{k-} a_{k-}^\dagger a_{k-} - \sum_k [\tilde{\Delta}_-(k) a_{k-}^\dagger a_{-k-}^\dagger + \text{h.c.}] \quad (5)$$

Here the renormalized energy band is $\tilde{\varepsilon}_{k-} = \varepsilon_{k-} + \varepsilon_0$ with $\varepsilon_0 = H_z^2 \Delta^2 / \alpha^2 |\mathcal{L}(k_F)|^2 m_0$ and $m_0 = (4\alpha^2 |\mathcal{L}(k_F)|^4 + \Delta^2 |\mathcal{L}_0(k_F)|^2) / 2\alpha |\mathcal{L}(k_F)|^3$. m_0 is an energy gap of quasiparticles of the band ε'_{k+} for $k \sim k_F$. The gap function is $\tilde{\Delta}_-(k) = a\Delta \tilde{\eta}_+(k)$, where $a = 1 + H_z^2 \Delta^2 / 2\alpha^3 |\mathcal{L}(k_F)|^3 m_0$. Note that the gap function $\tilde{\Delta}_-(k)$ still possesses the same momentum dependence as that of $p_x + ip_y$ superconductors. For the detection of non-Abelian anyons and the potential application to topological quantum computing, it is crucial that no other particles are excited in the processes of braiding them. Therefore, the above argument is applicable only to the low energy scale $\ll \mu_B H_z, |\Delta \pm \mu_B H_z|, \Delta^2 / E_F$.

The topological order of the effective model [Eq. (5)] clearly manifests in the Chern number, which is for the Hamiltonian of the form $\mathcal{H}_{\text{eff}} = \sum_{\mu=x,y,z} \sum_k (a_{k-}^\dagger, a_{-k-}) \sigma_\mu E_\mu(k) (a_{k-}, a_{-k-}^\dagger)^t$, defined as^{5,16}

$$\mathcal{N} = \int \frac{d^2 k}{8\pi} \epsilon_{ij} \hat{\mathbf{E}} \cdot \left(\frac{\partial \hat{\mathbf{E}}}{\partial k_i} \times \frac{\partial \hat{\mathbf{E}}}{\partial k_j} \right), \quad (6)$$

where $\hat{\mathbf{E}} = (E_x(k), E_y(k), E_z(k)) / |\mathbf{E}(k)|$. The integral of Eq. (6) is taken over the whole BZ, while Eq. (5) is derived for k in the vicinity of k_F . Nevertheless, we can consider the Chern number of model (5) by reinterpreting Eq. (5) as a lattice regularized version of the low-energy effective theory and extending the k -space in which the model [Eq. (5)] is defined to the entire BZ. Then, the numerical evaluation of $\mathcal{N}$ for the Hamiltonian (5) gives $\mathcal{N} = 1$. Therefore, model (5) is classified as the same topological class as those of the MR state, spinless $p_x + ip_y$ superconductors, and the gapped non-Abelian phase of the Kitaev model. The existence of the Zeeman field H_z in model (5) is important for this topological characterization, because it does not only break time-reversal symmetry, but also ensures the differentiability of $E_{x,y}(k)$ for Eq. (5), which is singular at $k=0$ for $H_z=0$.

The Chern number $\mathcal{N}=1$ implies the existence of zero-energy Majorana fermion modes in vortices, which obey the non-Abelian statistics, as in the case of $p_x + ip_y$ superconductors.^{5,6,16} To demonstrate this, we proceed to solve the Bogoliubov de-Gennes (BdG) equations for model (5) with a single vortex inserted parallel to the z axis. For simplicity, we switch to the continuum model, replacing the energy band ε_k of Eq. (1) with $\varepsilon'_k = k^2 / 2m - \mu$, and $\mathcal{L}_0$ with $\mathcal{L}'_0 = (k_y, -k_x, 0)$. Furthermore, we assume that the gap amplitude $\Delta(\mathbf{r})$ vanishes inside of the vortex core and is equal to a constant Δ outside of the core, and $\Delta \ll E_F$. Then, in the vicinity of the Fermi surface, the BdG equations corresponding to model (5) with a single vortex are

$$\begin{pmatrix} -i\mathbf{v}_F \cdot \nabla + \varepsilon_0 & \Delta_0 e^{i\phi/2} \hat{P} e^{i\phi/2} \\ \Delta_0 e^{-i\phi/2} \hat{P}^\dagger e^{-i\phi/2} & i\mathbf{v}_F \cdot \nabla - \varepsilon_0 \end{pmatrix} \Psi = \varepsilon \Psi, \quad (7)$$

where $\Psi^t = [u(\mathbf{r}), v(\mathbf{r})]$, $\hat{P} = -(\partial_x + i\partial_y)$, $\hat{P}^\dagger = -\hat{P}^*$, and $\Delta_0 = a\Delta(\mathbf{r}) / |\mathcal{L}(k_F)|$. The BdG [Eq. (7)] are equivalent to those of spinless $p_x + ip_y$ superconductors except that there are the ε_0 terms in the diagonal components, which can be formally absorbed into the shift of the Fermi momentum $k_F \rightarrow k_F - \varepsilon_0 / v_F$. Thus, the solution of Eq. (7) is given by $\Psi = e^{-i\varepsilon_0 v_F} \Psi_{p+ip}$ with Ψ_{p+ip} the eigenfunction of the BdG equations for spinless $p_x + ip_y$ superconductors, and there exists a zero-energy mode inside the vortex core, which is separated from the first excited state by a gap of energy size Δ^2 / E_F .^{22,23} The Bogoliubov quasiparticles for this zero-energy state are described by a Majorana fermion field $\gamma = \int d\mathbf{r} [u(\mathbf{r}) a_-(\mathbf{r}) + v(\mathbf{r}) a_-(\mathbf{r})]$, since $[v^*(\mathbf{r}), u^*(\mathbf{r})] = [u(\mathbf{r}), v(\mathbf{r})]$ for $\varepsilon=0$. Here $a_-^\dagger(\mathbf{r}) = \sum_k a_{k-}^\dagger e^{-ik\mathbf{r}}$.

To confirm the above prediction, we apply numerical analysis directly to the BdG equations for the tight-binding model [Eq. (1)] under an applied magnetic field without referring to the low-energy effective theory [Eq. (5)]. The energy spectrum and the eigenfunctions of the BdG equations were calculated for model (1) with a vortex located at the center of the system on the square lattice with open boundaries. In this calculation, we assume that the GL parameter is so large that the Zeeman field H_z is approximated to be uni-

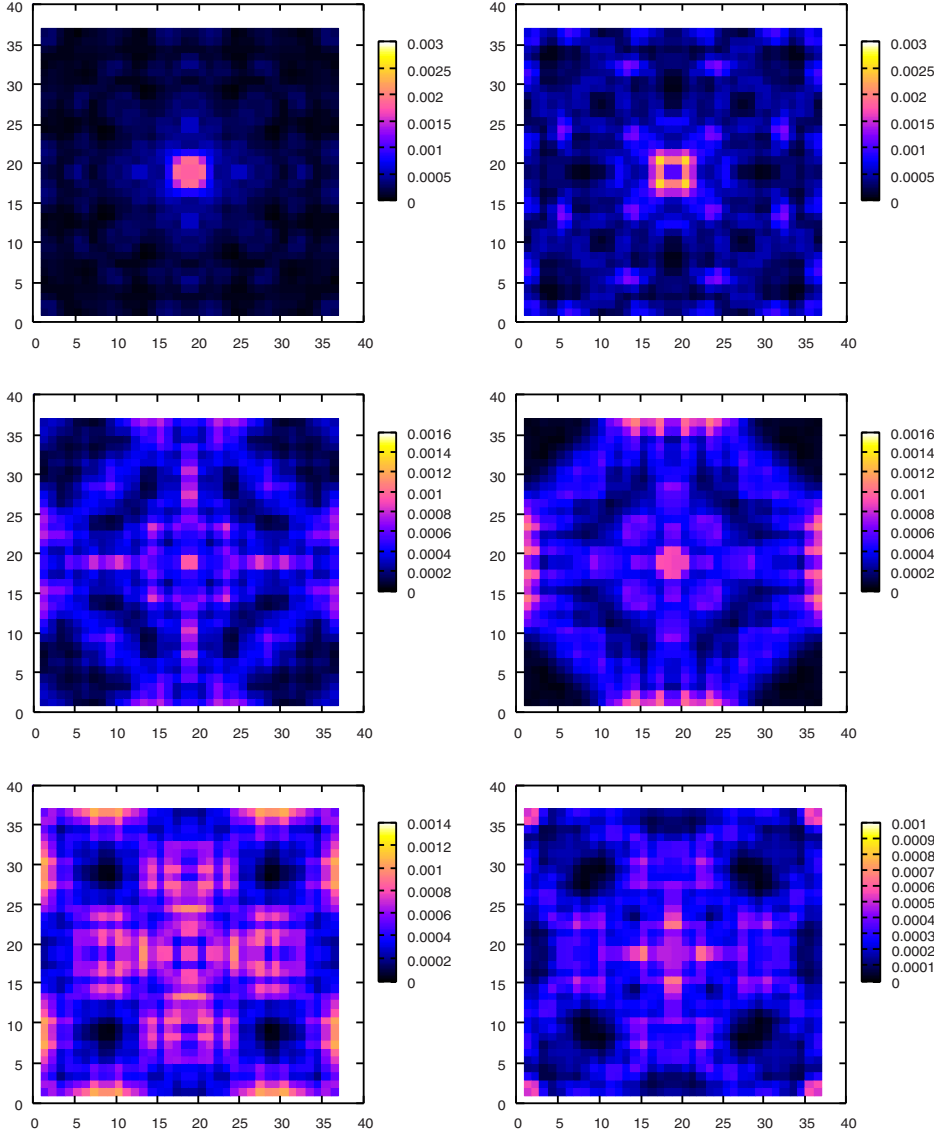


FIG. 1. (Color online) The density of quasiparticles on the 37×37 xy plane for $\varepsilon = 4.136 \times 10^{-4}t$ (top), $\varepsilon = 1.289 \times 10^{-3}t$ (middle), and $\varepsilon = 1.579 \times 10^{-3}t$ (bottom). The left and right panels are, respectively, the plots of $|u(\mathbf{r})|^2$ and $|v(\mathbf{r})|^2$.

form, and the spatial dependence of the superconducting gap function due to the vortex is taken into account only in its phase for simplicity. The topological properties with which we are concerned are not sensitive to these approximations. We set parameters as $\mu = -4t$, $\alpha = t$, $\Delta = 0.05t$, and $\mu_B H_z = 0.04$. Figure 1 shows the spatial distributions of the density of Bogoliubov quasiparticles for several low-energy states calculated for the lattice size 37×37 . The lowest energy state with $\varepsilon = 4.136 \times 10^{-4}t$ is dominated by a vortex core state, which cannot be the Caroli-de Gennes-Matricorn mode of the conventional s -wave superconductors, because for our choice of the parameters, the Fermi energy is $E_F = 0.25t$, and $\Delta^2/E_F = 0.01t$. Also the lowest energy level decreases toward zero as the system size increases. Thus, we identify the lowest energy state with the zero-energy mode. Furthermore, we find the low-energy edge states at the boundaries, e.g., for $\varepsilon = 1.289 \times 10^{-3}$, 1.579×10^{-3} . The edge state with energy $\ll \Delta$ is a concomitant of the zero-energy vortex core state, which is in accordance with the Chern number $\mathcal{N} = 1$.⁹ Taking these observations into account, we can conclude that the NS s -wave superconductor with a magnetic field for the particular electron filling is in the topological state.

As was proved by Ivanov, vortices with zero-energy Majorana modes obey non-Abelian statistics.^{3,6,8,9} Two Majorana fermion fields γ_i and γ_j in two vortices can be fused into a complex fermion field $\psi = (\gamma_i + i\gamma_j)/2$. The fermionic state described by ψ is occupied or unoccupied. The fusion rules for three particle states—a vortex (denoted as σ), a fermion occupied state (ψ), and an unoccupied vacuum state $\mathbf{1}$ —are analogous to the operator product expansions of the Ising conformal field theory: $\sigma \times \sigma = \mathbf{1} + \psi$, $\psi \times \sigma = \sigma$, and $\psi \times \psi = \mathbf{1}$. The braiding of vortices at i and j is described by the unitary operation $R^{\sigma\sigma} = \theta_\sigma \exp(\frac{\pi}{4} \gamma_j \gamma_i)$. Here θ_σ is a phase factor called a topological spin, which is associated with the conformal spin of the chiral Ising spin field $\theta_\sigma = e^{i\pi/8}$. Under the exchange of two vortices, the Majorana fermion operators are transformed as $\gamma_i \rightarrow R^{\sigma\sigma} \gamma_i (R^{\sigma\sigma})^\dagger = \gamma_j$ and $\gamma_j \rightarrow R^{\sigma\sigma} \gamma_j (R^{\sigma\sigma})^\dagger = -\gamma_i$. The minus sign in the second transformation rule is essential for non-Abelian statistics. When two vortices are fused into ψ , the braiding of them yields the phase factor $R^{\sigma\sigma} = e^{i3/8\pi}$, while when the fusion gives the topological charge $\mathbf{1}$, the phase factor due to the braiding is $R_1^{\sigma\sigma} = e^{-i\pi/8}$. The braiding rules for the other particle types are $R^{\sigma\psi} = -i$ and $R^{\psi\psi} = -1$.

We now discuss the feasibility of the experimental detection of the non-Abelian statistics. One promising approach is to use the two-point-contact interferometer proposed in the context of the FQH state.^{4,11,12} In the superconducting state considered here, this experiment is applicable only to the thermal transport. According to Refs. 11 and 12, the interference term of the edge heat current J^{int} depends on the parity of the total number of vortices n in the bulk. J^{int} for odd n is much smaller than J^{int} for even n although both of them do not exhibit the dependence on a magnetic flux Φ because the σ particle is neutral. This parity dependence characterizes the non-Abelian statistics. Another possible experiment is a bit indirect but simpler. It uses a disk-shaped system with which two heat baths with different temperatures are attached at the boundary. For this geometry, as in the case of $p_x + ip_y$ superconductors,²⁴ the energy spectrum of the edge state depends on the parity of the total number of vortices n in the bulk. For even n , the lowest energy state has a gap of the order $\Delta/k_F L$, where L is the length of the boundary. Although the gap is small for a sufficiently large system size, it is nonzero, and thus the quasiparticle corresponding to this edge mode is a complex fermion interacting with Φ . This Bogoliubov quasiparticle is categorized as the same particle type as the ψ fermion in the bulk because in the limit that the two vortices merge together at a position $\mathbf{r}$, the resulting ψ particle is nothing but the Bogoliubov quasiparticle with a nonzero energy.⁸ For odd n , the low-energy edge state is a Majorana fermion mode and can be fused with an unpaired Majorana fermion in the bulk, resulting in the ψ state or the 1 state. The phase accumulated by the current flow of the edge σ or ψ particles encircling the bulk n vortices are obtained from the square of the braiding operator $(R^{ab})^2$ ($a, b = \sigma, \psi$). We denote the shorter path between two heat baths on the closed boundary as C_1 and the longer path as C_2 ; we assume that the path C_2 encloses all vortices in the bulk and the path C_1 encircles no vortex. When the temperature difference between two heat baths is such that the chiral-edge heat current flows mainly in the path C_2 , the edge heat current carried by the ψ particles exhibits a usual dependence on Φ for even n , i.e., $J_{\text{even}}^{\text{int}} \sim \sum_{m=1}^{\infty} A_m \cos(2\pi m e \Phi / hc)$, while for odd n , using the fusion rules and the braiding rules mentioned above, we obtain $J_{\text{odd}}^{\text{int}} \sim \sum_{m=1}^{\infty} B_{4m} \cos \pi m$, where the B_{4m} term corresponds to a trajectory winding around the boundary loop $4m$ times. Thus $J_{\text{odd}}^{\text{int}}$ is much suppressed.

These observable effects can be utilized for the detection of the non-Abelian statistics.

An advantage of NC s -wave superconductors over $p_x + ip_y$ -wave superconductors and the $\nu=5/2$ FQH state is that the gap energy scale of the former can be typically much larger than that of the latter. Note that the superconductivity in NC systems does not need to be a bulk phenomenon. Let us consider the junction between an s -wave superconductor and a metallic thin film placed on an insulating substrate. The thin film must be sufficiently clean so that the mean free path is larger than its thickness. In this system, inversion symmetry is broken, and an asymmetric potential gradient perpendicular to the interface is introduced. We can use a material with a high transition temperature such as MgB_2 ($T_c \sim 39$ K) for the superconductor.²⁵ Then, the proximity effect induces s -wave superconductivity in the two-dimensional NC system realized in the thin film. If the Fermi energy of the film E_F is much smaller than that of the bulk superconductor, the energy gap in the vortex core Δ^2/E_F for the proximity-induced NC superconductor can be considerably large. The strength of the asymmetric SO interaction can be controlled by changing the substrate or applying a perpendicular voltage on the film. Although electrons should experience strong SO scatterings at the interface, the transition temperature and the gap of the s -wave pairing state are not affected by them. Also, note that the Majorana fermions in vortices of the NC superconductors do not require the existence of half quantum vortices, i.e., textures of the $\mathbf{d}$ vector, because our system is essentially regarded as spinless. In this sense, zero-energy Majorana states in NC s -wave superconductors are more realizable than in spinful p -wave superconductors.

In conclusion, NC s -wave superconductors under magnetic field exhibit a topological order for a particular electron filling and can be playgrounds for the non-Abelian anyons. Although we consider only the Rashba SO interaction here, our argument can be easily generalized to other asymmetric SO interactions.

The author thanks N. Kawakami and R. Ikeda for invaluable discussions. The numerical calculations were performed on SX8 at YITP in Kyoto University. This work was supported by a Grant-in-Aid from the Ministry of Education, Science, Sports and Culture, Japan.

¹X.-G. Wen and Q. Niu, Phys. Rev. B **41**, 9377 (1990).

²G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).

³C. Nayak and F. Wilczek, Nucl. Phys. B **479**, 529 (1996).

⁴E. Fradkin *et al.*, Nucl. Phys. B **516**, 704 (1998).

⁵N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).

⁶D. A. Ivanov, Phys. Rev. Lett. **86**, 268 (2001).

⁷A. Stern *et al.*, Phys. Rev. B **70**, 205338 (2004).

⁸M. Stone and S.-B. Chung, Phys. Rev. B **73**, 014505 (2006).

⁹A. Kitaev, Ann. Phys. (N.Y.) **321**, 2 (2006).

¹⁰H. Yao and S. A. Kivelson, Phys. Rev. Lett. **99**, 247203 (2007).

¹¹P. Bonderson *et al.*, Phys. Rev. Lett. **96**, 016803 (2006).

¹²A. Stern and B. I. Halperin, Phys. Rev. Lett. **96**, 016802 (2006).

¹³M. Freedman *et al.*, Bull., New Ser., Am. Math. Soc. **40**, 31 (2003).

¹⁴S. Das Sarma *et al.*, Phys. Rev. Lett. **94**, 166802 (2005).

¹⁵S. Tewari *et al.*, Phys. Rev. Lett. **98**, 010506 (2007).

¹⁶D.-H. Lee *et al.*, Phys. Rev. Lett. **99**, 196805 (2007).

¹⁷V. M. Edelstein, Sov. Phys. JETP **68**, 1244 (1989).

¹⁸L. P. Gor'kov and E. I. Rashba, Phys. Rev. Lett. **87**, 037004 (2001).

¹⁹P. A. Frigeri *et al.*, Phys. Rev. Lett. **92**, 097001 (2004).

²⁰S. Fujimoto, Phys. Rev. B **72**, 024515 (2005).

²¹S. Fujimoto, Phys. Rev. B **76**, 184504 (2007).

²²G. E. Volovik, JETP Lett. **70**, 609 (1999).

²³N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).

²⁴M. Stone and R. Roy, Phys. Rev. B **69**, 184511 (2004).

²⁵J. Nagamatsu *et al.*, Nature (London) **410**, 63 (2001).