Comment on "Renormalization group analysis and numerical simulation of propagation and localization of acoustic waves in heterogeneous media"

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(Received 23 April 2007; revised manuscript received 28 July 2007; published 18 June 2008)

We make comments on the paper of Bahraminasab *et al.* [Phys. Rev. B **75**, 064301 (2007)] by pointing out that the high-frequency result of one-dimensional localization length (Fig. 8 therein) is different from previous results of many research groups. We point out that the approximate approaches (using both perturbation and numerical methods) that Bahraminasab *et al.* adopted can be valid only for the low-frequency regime.

DOI: 10.1103/PhysRevB.77.216301 PACS number(s): 62.65.+k, 71.23.An, 91.60.Lj, 47.56.+r

Bahraminasab *et al.*[1] recently presented a study of the propagation of acoustic waves in strongly heterogeneous elastic media using renormalization group analysis and extensive numerical simulations. The heterogeneities are characterized by a broad distribution of the local elastic constants.[1,2] They claimed that the problem that they formulated and the results were, however, applicable to acoustic wave propagation in any disordered elastic material that contains the types of heterogeneities. Finally, from the results of their numerical simulations, they claimed the possibility of the existence of a regime of superlocalization in which the waves' amplitudes decay as $\exp[-(|x|/\xi)^{\gamma}]$, with $\gamma>1$, where ξ is the localization length.

Based on a similar analysis,[3–5] considering the localization of classical (acoustical) waves, the present author would like to point out that there are doubtable differences shown in Fig. 8 of Ref. 1, especially for the high-frequency (say, larger than $\omega\sim 0.08$) regime, compared to previous results[4,6,7] for similar problems (Fig. 1 in Ref. 7 is effectively one-dimensional).

We first remind the readers of the notations adopted in Ref. 1 for the comparison shown in Fig. 8 of Ref. 1 with Refs. 4, 6, and 7. $\lambda(x)$ is the ratio of the elastic stiffness and the medium's mean density. D_0 represents the strength of the disorder due to the delta-correlated parts of the disorder. For the one-dimensional case, Bahraminasab *et al.* used $\lambda_0=10$ and took its random component, $\eta(x)$ $[\lambda(x)=\lambda_0(\equiv\langle\lambda(x)\rangle)+\eta(x)]$, to be a white noise with variance, $2D_0=\sigma$. For every realization of the disorder they computed ψ_N and, hence, $\xi(\omega)$.

To check what happens when the frequency ω is higher, we first examine Eqs. (23)–(26). The finite difference equation is essentially using a central difference (considering three points at sites $n-1,n,n+1$), which might produce large errors in solving the wave-propagation problems[8] as the information at site $n+1$ is unknown [only the information at site $n-1$ is *ad hoc* known (by initial guess or upstream boundary condition) once we are considering the site n]. To be precise, during one (numerical) iteration, when a plane wave propagates downstream $(n+1)$, the upstream $(n-1)$ information is known to the present site (n) and it can be used to solve for the present site information with less errors (accumulated during previous iterations or solving the relevant differenced equations). However, once the downstream site information (which uses the previous iterated value or is not exact, especially when the physical information is still not approaching) was also used to solve the present site information, e.g., by taking the average for the downstream and upstream, the numerical errors will be larger. In fact, the above-mentioned technique results in difficulty in calculating the length-dependent physical parameters[9] [higher frequency corresponds to smaller wavelength for fixed phase speed; thus the spatial (numerical) resolution is better much smaller].

Meanwhile, we cite the explanation of Bahraminasab *et al.*,[1] "We used, $\psi_0=\psi_1=1/\sqrt{2}$, and carried out computations at selected values of ω. Every computed value of ξ represents an average over 6000 realizations for a fixed system size N and frequency ω." They illustrated in Fig. 8 the frequency dependence of $\xi(\omega)$ for $N=6\times 10^6$ ($\sigma_c\sim 2.34$) and three regimes, $\sigma<\sigma_c$, $\sigma=\sigma_c$, and $\sigma>\sigma_c$. In the limit, $\omega=0$ (and $N\to\infty$), and the localization length ξ diverges. Finally, in the Summary of Ref. 1, Bahraminasab *et al.* mentioned, "… The determination of ξ (i.e., localization length) remains a major numerical task." In fact, to reduce the numerical errors, Bahraminasab *et al.*[1] also mentioned that the calculated frequency range $\omega\le\omega_{co}=2\pi\sqrt{\lambda_0}/N^{1/2}$, which is of low frequency considering N is $O(10^6)$ and λ_0 is $O(10)$. Thus, their numerical approach is limited to low-frequency regime.

Next, we recall the similar results presented before, e.g., (i) Fig. 5 of Ref. 4 (which used a quantum kinetic model; note that in this figure $h\propto f_{\text{collision}}/f_{\text{sound}}$; f means the frequency[5]); (ii) Fig. 5(a) of Ref. 6 (which used a diagrammatic technique and Green's functions by Kirkpatrick to study the localization of acoustical waves where the energy is just the frequency considering the acoustic analog[3]); and quite recently (iii) Fig. 1(a) of Ref. 7 (the inverse of localization length was illustrated therein and the approach or result was effectively one dimensional). The apparent differences, to stress it again, in the latter three presentations are that the localization length increases even as the frequency is rather large and there is a minimum of the localization length with respect to the frequency. These did not appear in Fig. 8 of Ref. 1. As the theoretical approach and the physical problem are almost the same for Refs. 1, 2, 4, 6, and 7, the illustration of Fig. 8 in Ref. 1 (similar to that in Ref. 2) is thus doubtful, especially for the high-frequency regime.

From another point of view, Bahraminasab *et al.* stated (in Sec. III of Ref. 1, p. 6 therein, below Fig. 4), "Therefore, a localized state may be defined as follows: we have localized states if, under the RG rescaling, at least either g_0 or g diverges." Therefore, compared to Fig. 5 (the flow is periodic or nonmonotonic) and the rather high-frequency part of Fig. 8 in Ref. 1 (which is neither periodic nor nonmonotonic) at the high-frequency side, the inconsistencies made in Ref. 1 as the present author just pointed out above are very clear (without comparing with Refs. 4, 6, and 7).

To check the approach of Bahraminasab *et al.*,[1] we shall again cite their statements below. First, as shown in the governing Eq. (1) therein (cf. Ref. 1), it is a wave equation. Next, Bahraminasab *et al.* claimed (p. 2 of Ref. 1), "We…show that in any case there is a disorder-induced transition from delocalized to localized states for any spatial dimension d." This is obvious based on the problem [p. 3, after Eq. (1)]: "Eq. (1) is valid in any dimension d ($d=1$, 2, and 3)." Thus, there are no differences between Ref. 1 and previous approaches.[4,6,7] It means the results of these cannot be qualitatively too different.

The last important point is that Bahraminasab *et al.* mentioned their theoretical approach [p. 4, after Eq. (12)], which was "…carrying out a RG analysis in the critical limit, $\omega^2/\lambda_0 \to 0$,…in the perturbative evaluation of the rescaling in the RG analysis..." Thus, we can understand the limitations presented in Ref. 1: It is a perturbation-based (RG) analysis ($\omega^2/\lambda_0 \to 0$) and ω cannot be too large (as λ_0 being finite). The numerical presentation in Fig. 8 of Ref. 1 cannot present similar rather high-frequency regime (results) as explained above and evidenced in Refs. 4, 6, and 7.

To summarize, the apparent differences between the latter three presentations[4,6,7] are that the localization length increases even as the frequency is rather large and there is a minimum of the localization length with respect to the frequency. The last is not present in Fig. 8 of Ref. 1. The possible reason, as we explained above, could be traced from both the perturbation and numerical methods Bahraminasab *et al.* adopted in Ref. 1, which are all limited to the low frequency regime.

The author is grateful for the hospitality of the Chern Shiing-Shen Institute of Mathematics, Nankai University.

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