Resonance oscillations of magnetoresistance in double quantum wells

N. C. Mamani, G. M. Gusev, T. E. Lamas, and A. K. Bakarov[*]

Instituto de Física da Universidade de São Paulo, Caixa Postale 66318, CEP 05315-970 São Paulo, SP, Brazil

O. E. Raichev

Institute of Semiconductor Physics, National Academy of Sciences of Ukraine, Prospekt Nauki 45, Kiev 03028, Ukraine
(Received 22 April 2008; published 23 May 2008)

We present the experimental and theoretical studies of the magnetoresistance oscillations induced by the resonance transitions of electrons between the tunnel-coupled states in double quantum wells. The suppression of these oscillations with increasing temperature is irrelevant to the thermal broadening of the Fermi distribution and reflects the temperature dependence of the quantum lifetime of electrons. The gate control of the period and amplitude of the oscillations is demonstrated.

I. INTRODUCTION

The Landau quantization[1] of electron states manifests itself in oscillations of various physical quantities as functions of the applied magnetic field. The magnetoresistance oscillations in solids belong to the most fundamental quantum phenomena of this kind.[2] In particular, the Shubnikov–de Haas oscillations[2] (SdHOs) originating from the sequential passage of Landau levels through the Fermi level are always present in three-dimensional (3D) and two-dimensional (2D) electron systems with a metallic type of conductivity. The investigation of these oscillations is a powerful source of information about band structure, Fermi surface, and electron interaction mechanisms.

The SdHOs are strongly damped with increasing temperature as the thermal broadening of the Fermi distribution exceeds the cyclotron energy. In quasi-2D electron systems, which are realized in quantum wells with two (or more) occupied subbands, the possibility of intersubband transitions of the electrons leads to another kind of magnetoresistance oscillations,[3] which are not considerably damped when the temperature rises. Such oscillations, known as the magneto-intersubband oscillations (MISOs), have been studied in single quantum wells with two populated 2D subbands[3–8] and, theoretically, in 2D layers where the electron states are split owing to the spin-orbit interaction.[9] The physical mechanism of the MISOs is the periodic modulation of probability of the intersubband transitions by the magnetic field as the different Landau levels of the two subbands sequentially come in alignment. Oscillations of similar origin are present in 3D systems such as layered organic conductors with weak coupling between the layers.[10–12]

As compared to the SdHOs, the MISOs show a stronger suppression by the disorder, so their observation requires clean (high-mobility) samples. A decrease in the MISO amplitude with the increasing temperature reflects the temperature dependence of the quantum lifetime of electrons. The experimental studies of the MISOs allow one to determine the quantum lifetimes in the region of temperatures where the SdHOs completely disappear. Another advantage of the MISOs is the possibility of a direct and precise determination of the intersubband energy gap, which otherwise can be found from an analysis of the double periodicity of the SdHOs observed at low temperatures in various quasi-2D elec-

tron systems such as single quantum wells with two populated 2D subbands,[4,6] double quantum wells (DQWs),[13,14] and 2D systems with spin-orbit splitting.[15,16]

The DQWs, which consist of two quantum wells separated by a barrier and where the electrons occupy two 2D subbands coupled by tunneling, should be recognized as the most convenient system for studying the MISO phenomenon, owing to the unique possibility to control both the intersubband energy gap and the probabilities of the intersubband transitions in a wide range by using barriers of different widths and biasing the structure by using external gates. Nevertheless, no studies of the MISOs have been reported so far for these particular systems. In this paper, we report on the first observation and systematic investigation of the MISOs in symmetric high-mobility ($\mu \approx 10^6$ cm^2/V s) GaAs DQWs with different barrier widths. Along with the experimental results, we present a theoretical description of the magnetotransport in DQWs, which explains the results of our measurements.

The paper is organized as follows. The experimental results are described in Sec. II. The theoretical description and the discussion of the results are presented in Sec. III. The concluding remarks are given in the last section.

II. EXPERIMENT

The samples are symmetrically doped GaAs DQWs with equal widths, $d_W = 14$ nm, separated by Al$_x$Ga$_{1-x}$As barriers of different thickness, d_b, varied from 1.4 to 5 nm. The samples have a high total sheet electron density $n_s \approx 9 \times 10^{11}$ cm^{-2} (4.5×10^{11} cm^{-2} per one layer). Both layers are shunted by Ohmic contacts. The densities in the wells are variable by a gold top gate. The voltage of the gate, V_g, changes the density of the well closest to the sample surface with the carrier density in the other well being almost constant. The system is balanced (has equal densities in the wells) at $V_g = 0$. The sample parameters are shown in Table I. Over a dozen specimens of both the Hall bar and the van der Pauw geometries from the four wafers have been studied. We measure both the longitudinal and the Hall resistances at temperatures T from 0.3 to 50 K and at the magnetic fields B, which is perpendicularly directed to the well plane, up to 12 T by using conventional ac-locking techniques with a bias current of 0.1–1 μA.

TABLE I. The sample parameters: d_W is the well width, d_b the barrier thickness, n_s the electron density, and μ the zero-field mobility. Δ_{SAS}^{theor} is the symmetric–antisymmetric splitting (SAS) energy determined from self-consistent calculations and Δ_{SAS}^{exp} is determined from the periodicity of the oscillations observed.

d_b (Å)	d_W (Å)	n_s (10^{11} cm^{-2})	μ (10^3 cm^2/V s)	Δ_{SAS}^{theor} (meV)	Δ_{SAS}^{exp} (meV)
14	140	9.32	970	3.87	4.22
20	140	9.8	900	2.59	2.07
31	140	9.19	870	1.24	1.38
50	140	8.43	830	0.325	0.92

Figure 1 shows the low-field part of the magnetoresistance $R_{xx}(B)$ for the sample with $d_b=1.4$ nm and the balanced densities at different temperatures. At $T=0.3$ K, we see a double-periodic SdHO in the form of a beating pattern[13,14] since the energy splitting of 2D subbands in the DQWs is small in comparison to the Fermi energy. With the increase in T, the small-period SdHOs are strongly suppressed and the remaining long-period oscillations can be identified as the MISOs by analyzing their $1/B$ periodicity and temperature dependence, as described below. First, assuming that the magnetoresistance maxima correspond to the Landau-level alignment condition[5] $\Delta_T=k\hbar\omega_c$, where Δ_T is the intersubband splitting energy ($\Delta_T=\Delta_{SAS}$ for balanced DQWs), $\omega_c=|e|B/mc$ is the cyclotron frequency (e is the electron charge, m the effective mass, and c the velocity of light), and k is an integer, we find $\Delta_T=4.22$ meV, which is close enough to the calculated value of Δ_{SAS} (see Table I). Next, the amplitudes of the long-period oscillations, in contrast to the SdHO amplitudes, are saturated at low temperatures. The dependence of $R_{xx}(B)$ in the temperature interval $4.2<T<25$ K [Fig. 1(b)] shows that the oscillations are present at high temperatures. The amplitudes of the oscillations are up to 30% of the total resistance, so these oscillations cannot be attributed to the magnetophonon resonances whose relative amplitudes are much smaller.[17]

We have checked the $1/B$ periodicity of the oscillations for the structures with different barriers. Figure 2 shows the magnetoresistance traces versus $1/B$ while the results of the fast Fourier transform (FFT) of the magnetoresistance data are shown in the inset. The peak corresponding to the main period shifts toward the lower frequencies with an increasing d_b and the frequency of the FFT peaks has a nearly exponential dependence on d_b. Since the splitting energy in the balanced DQWs is known to exponentially decrease with an increasing barrier width, this observation is another confirmation of the intersubband origin of the oscillations.

Figure 3 shows the low-field part of the magnetoresistance for the sample with $d_b=1.4$ nm at various fixed gate voltages away from the symmetry point at $V_g=0$ V. As the absolute value of V_g increases, the oscillations are suppressed and their frequency becomes higher. The positive magnetoresistance is clearly seen in these plots.

III. THEORY AND DISCUSSION

The mechanism responsible for the observed oscillations relies on the intersubband scattering, which is resonantly modulated by the Landau quantization. A quantitative description of the magnetotransport in DQWs, taking into ac-
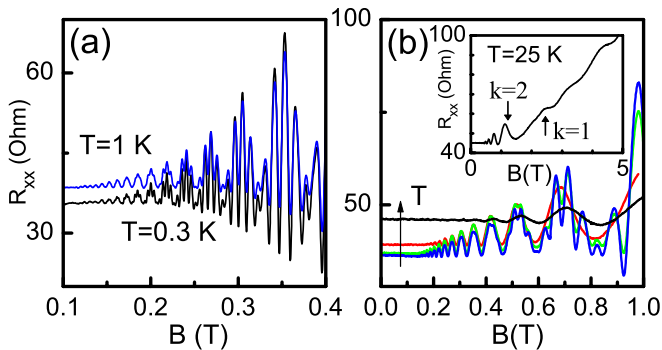

FIG. 1. (Color online) (a) Magnetoresistance oscillations in double wells with a barrier thickness, $d_b=14$ Å, at $T=0.3$ K and $T=1$ K. The curves are shifted for clarity. (b) Magnetoresistance oscillations at various temperatures T (K): 18, 10, 8, and 4.2. The inset shows a high-temperature magnetoresistance in a wide magnetic field interval, the arrows indicate the calculated positions of the two last peaks, $k=\Delta_T/\hbar\omega_c$.
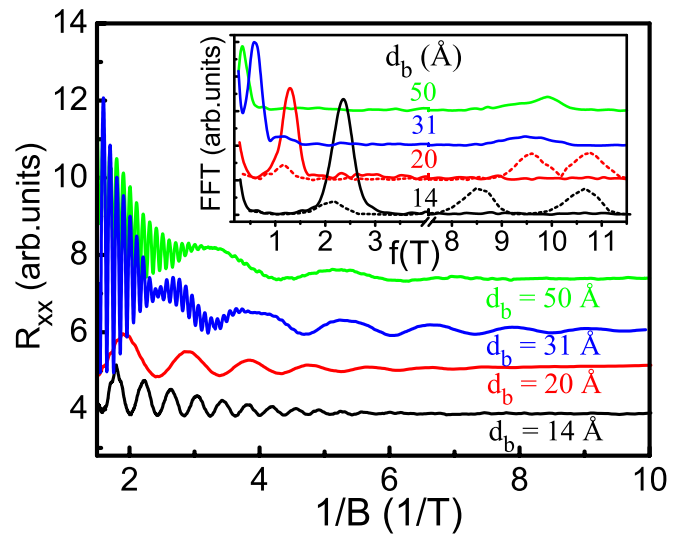

FIG. 2. (Color online) Magnetoresistance of double wells with various barrier widths vs the inverse magnetic field, measured at $T=4.2$ K for the samples with $d_b=14$ and 20 Å, and at $T=1.6$ K for the samples with $d_b=50$ and 31 Å. The curves are shifted for clarity. The inset shows the Fourier power spectra of R_{xx}, and the additional dashed curves correspond to the measurements at $T=50$ mK and demonstrate two SdHO frequencies for two subbands.
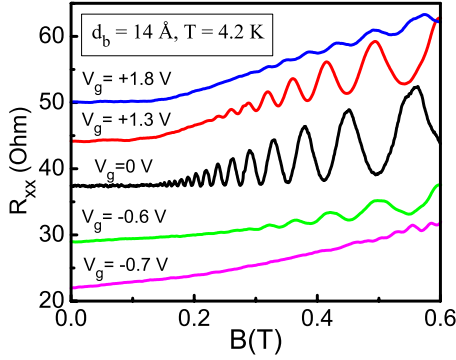
FIG. 3. (Color online) Low-field magnetoresistance oscillations for the double wells with a barrier width $d_b = 14$ Å as a function of the magnetic field for different gate voltages. The curves are shifted for clarity. The corresponding areal densities n_s are 14, 12, 9.3, 8, and 7×10^{11} cm^{-2} (from up to down).

count the elastic scattering of electrons, is based on the Hamiltonian written as the following 2×2 matrix in the single-well eigenstate basis:[18]

$$\hat{H} = \begin{vmatrix} \boldsymbol{\pi}^2/2m + \Delta/2 + V_{\mathbf{r}}^u & -\Delta_{SAS}/2 \\ -\Delta_{SAS}/2 & \boldsymbol{\pi}^2/2m - \Delta/2 + V_{\mathbf{r}}^l \end{vmatrix}, \quad (1)$$

where $\boldsymbol{\pi} = -i\hbar\nabla_{\mathbf{r}} - (e/c)\mathbf{A_r}$ is the kinematic momentum operator, $\mathbf{A_r}$ is the vector potential describing the magnetic field, $V_{\mathbf{r}}^i$ are the random potentials of the impurities and other inhomogeneities in the upper $(i=u)$ and lower $(i=l)$ wells, and $\mathbf{r} = (x, y)$ is the in-plane coordinate vector. Next, Δ_{SAS} denotes the subband splitting owing to the tunnel coupling and Δ is the splitting in the absence of the tunneling. In symmetric (balanced) DQWs, $\Delta = 0$. According to Eq. (1), the free-electron energy spectrum is given by two sets of Landau levels of the upper $(+)$ and lower $(-)$ subbands: $\varepsilon_n^{\pm} = \pm\Delta_T/2 + \varepsilon_n$, where $\varepsilon_n = \hbar\omega_c(n+1/2)$ and $\Delta_T = \sqrt{\Delta_{SAS}^2 + \Delta^2}$ is the total intersubband gap. In symmetric DQWs, $\Delta_T = \Delta_{SAS}$ is the energy gap between the states with the symmetric and antisymmetric wave functions $\psi_z^{(-)}$ and $\psi_z^{(+)}$. Since we consider using weak enough magnetic fields, the Zeeman splitting is neglected.

The calculation of the diagonal (σ_d) and nondiagonal $(\sigma_\perp)$ components of the conductivity tensor uses the Kubo formalism and Green's function technique in the Landau-level representation.[19] In the model of the short-range symmetric scattering potential characterized by the correlator $\langle\langle V_{\mathbf{r}}^i V_{\mathbf{r}'}^{i'} \rangle\rangle \simeq w\delta_{ii'}\delta(\mathbf{r} - \mathbf{r}')$, and in the limit of the weak disorder when the scattering rate $1/\tau = mw/\hbar^3$ is small in comparison to $\Delta_T/\hbar$, one has

$$\sigma_d = \frac{e^2\omega_c}{\pi^2}\int d\varepsilon \left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right)[\Phi_\varepsilon^+ + \Phi_\varepsilon^-], \quad (2)$$

and $\sigma_\perp = |e|cn_s/B + \delta\sigma_\perp$ with

$$\delta\sigma_\perp = -\frac{2e^2}{\pi^2\hbar}\int d\varepsilon \left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right)[\Sigma_\varepsilon^{(+)''}\Phi_\varepsilon^+ + \Sigma_\varepsilon^{(-)''}\Phi_\varepsilon^-], \quad (3)$$

where f_ε is the equilibrium Fermi distribution and

$$\Phi_\varepsilon^{\pm} = \frac{2\Sigma_\varepsilon^{(\pm)''}S_\varepsilon^{(\pm)''}[\varepsilon - \Sigma_\varepsilon^{(\pm)'}]}{(\hbar\omega_c)^2 + (2\Sigma_\varepsilon^{(\pm)''})^2}, \quad S_\varepsilon^{(\pm)} = \sum_{n=0}^{\infty} \frac{1}{\varepsilon - \varepsilon_n - \Sigma_\varepsilon^{(\pm)}}. \quad (4)$$

The quantities $\Sigma_\varepsilon^{(\pm)}$ are the self-energies for the $+$ and $-$ states. Their real $(\Sigma_\varepsilon^{(\pm)'})$ and imaginary $(\Sigma_\varepsilon^{(\pm)''})$ parts can be found using the self-consistent Born approximation (SCBA)[20] generalized for DQWs,

$$\Sigma_\varepsilon^{(\pm)} = \pm\frac{\Delta_T}{2} + \frac{\hbar^2\omega_c}{4\pi\tau}[(1 \pm \delta^2)S_\varepsilon^{(+)} + (1 \mp \delta^2)S_\varepsilon^{(-)}], \quad (5)$$

where $\delta^2 = (\Delta/\Delta_T)^2$. The energies $\pm\Delta_T/2$ are included in the self-energies for notational convenience. Equations (2)–(5) give the full description of the magnetotransport in the DQWs with a symmetric short-range scattering within the SCBA.

In the weak magnetic fields, both $\Sigma_\varepsilon^{(\pm)}$ and $S_\varepsilon^{(\pm)}$ can be represented as serial expansions in small Dingle factors $e^{-\alpha}$, where $\alpha = \pi/\omega_c\tau$ (see Ref. 20). This procedure gives

$$S_\varepsilon^{(\pm)} = \frac{i\pi}{\hbar\omega_c}\left\{1 - 2e^{-\alpha}\exp\left[-i\frac{2\pi(\varepsilon \mp \Delta_T/2)}{\hbar\omega_c}\right] + \dots\right\},$$

and leads to the following expression for the resistivity $\rho_{xx} = \sigma_d/(\sigma_d^2 + \sigma_\perp^2)$:

$$\rho_{xx} \simeq \rho_0\left\{1 - 4e^{-\alpha}\mathcal{T}\cos\left(\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right)\cos\left(\frac{\pi\Delta_T}{\hbar\omega_c}\right) + e^{-2\alpha}\left[a_+ + a_-\cos\left(\frac{2\pi\Delta_T}{\hbar\omega_c}\right)\right]\right\}, \quad (6)$$

where $\rho_0 = m/e^2n_s\tau$ is the zero-field Drude resistivity, and $a_\pm = 1 \pm \delta^2 - (1 \pm \delta^4)/[1 + (\omega_c\tau)^2]$. The Fermi energy ε_F is counted from the middle point between the subbands and the function $\mathcal{T} = (2\pi^2 T/\hbar\omega_c)/\sinh(2\pi^2 T/\hbar\omega_c)$ describes the thermal suppression of the resistivity oscillations. The second $(\propto e^{-\alpha})$ term in Eq. (6) describes usual SdHOs with beatings while the last $(\propto e^{-2\alpha})$ term contains the MISO contribution, which does not have the thermal suppression (the temperature-dependent corrections to this term are neglected because they are small in comparison to the second term). The expression for the coefficients $a_\pm$ can be rewritten in terms of the intersubband and the intrasubband scattering times in DQWs, which are given by $\tau_{\text{inter}} = 2\tau/(1 - \delta^2)$ and $\tau_{\text{intra}} = 2\tau/(1 + \delta^2)$, respectively, so that $\tau^{-1} = \tau_{\text{intra}}^{-1} + \tau_{\text{inter}}^{-1}$. This representation shows that the MISO contribution (at the coefficient a_-) is proportional to the intersubband scattering rate τ_{inter}^{-1}.

As the temperature rises, the second term in the right-hand side of Eq. (6) vanishes and only the MISOs remain. These oscillations are periodic in $1/B$ and have a maxima under the condition $\Delta_T = k\hbar\omega_c$. In balanced DQWs ($\delta=0$ and $a_+ = a_-$), when the electron density distributions $|\psi_z^{(-)}|^2$ and $|\psi_z^{(+)}|^2$ for the $-$ and $+$ states are equal, the probability of the intersubband scattering is high. As the system is driven out of balance by the gates (δ^2 increases), the wave functions for the $-$ and $+$ subbands are localized in the different wells, and the intersubband scattering is suppressed, so the MISO am-
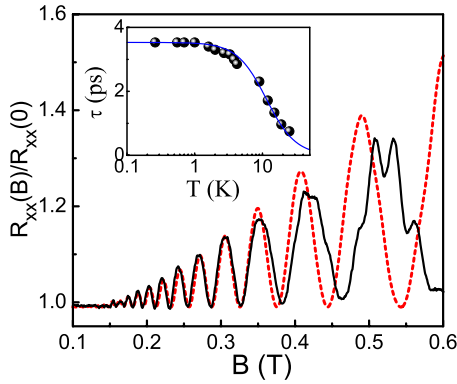
FIG. 4. (Color online) Comparison between the experimental (solid line) and theoretical (dashed line) magnetoresistances for balanced DQWs with $d_b = 14$ Å at 4.2 K. The theoretical curve is slightly shifted down to account for a weak negative magnetoresistance at low fields. The inset shows the temperature dependence of the quantum lifetime of electrons extracted from the amplitude fit of the MISOs (circles) and the theoretical dependence assuming T^2 scaling of $1/\tau$ (line).

plitude (coefficient a_-) is reduced, while the nonoscillating positive magnetoresistance (coefficient a_+) becomes larger. This behavior is seen in the experiment (Fig. 3), along with the enhanced frequency of the oscillations owing to a larger intersubband separation. Equation (6) shows that the resistivity in the minima, when $\Delta_T = (k+1/2)\hbar\omega_c$, is equal to ρ_0 for the balanced DQWs in the low magnetic fields. This feature, also seen in our experiment, is described by the fact that the total density of states, $(m\omega_c/\pi^2\hbar)[S_\varepsilon^{(+)''} + S_\varepsilon^{(-)''}]$, given by a superposition of the densities of states in the + and − subbands, does not oscillate with the energy under the condition $\Delta_T = (k+1/2)\hbar\omega_c$, so the magnetic field has no effect on the scattering and the resistivity under this condition.

For a more detailed comparison between the theory and the experiment, we have carried out a fit of the magnetoresistance [Eq. (6)] to the data obtained for the balanced DQWs with $d_b = 1.4$ nm at $T = 4.2$ K. In order to fit the MISO frequency at low B, we have taken $\Delta_T = 4.22$ meV, while the amplitude fit has been done assuming that the ratio of the transport time $\tau_{tr} = 37$ ps, determined from the mobility to the quantum lifetime τ standing in the Dingle factor, is equal to 12 (large ratios of τ_{tr} to τ are normally met in modulation-doped heterostructures). With these parameters used, we obtain an excellent agreement between the theory and the experiment in the region of the magnetic fields up to 0.4 T (see Fig. 4). A good agreement is also obtained for lower temperatures when the SdHOs are present. Above 0.4 T, the observed MISO frequency becomes slightly smaller than expected from the theory. We do not know the exact origin of this discrepancy and assume that it is possibly related to the modification of the tunnel coupling by the magnetic field.[21]

The amplitude fit of the MISOs in a wide region of temperatures allows us to plot the temperature dependence of the quantum lifetime of electrons. We have found that τ is constant at $T < 1$ K and decreases at higher temperatures. This decrease cannot be attributed to the quasielastic scattering of

electrons by acoustic phonons because the acoustic-phonon scattering rates in the GaAs quantum wells are more than one order smaller in the described temperature region. On the other hand, the consideration of the electron–electron scattering in DQWs[18,22] gives a reasonable explanation for the observed temperature behavior. To prove this, we have compared the experimental dependence shown in the inset to Fig. 4 with the theoretically predicted dependence[18,22] $\tau(T) = \tau(0)/[1 + \lambda(T/T_0)^2]$, where $T_0 = \sqrt{\varepsilon_F \hbar/\tau(0)}$ and λ is a numerical coefficient of the order of unity, which is considered here as a single fitting parameter. Using the experimentally determined values $\varepsilon_F \simeq 17$ meV and $\hbar/\tau(0) \simeq 0.2$ meV, we have found good agreement between the experiment and the theory in the whole temperature region for $\lambda = 3.5$.

IV. CONCLUSIONS

In conclusion, we have observed and investigated the magnetoresistance oscillations caused by the resonant modulation of the elastic intersubband scattering in DQWs by the Landau quantization. These oscillations survive at high temperatures and represent a prominent and well-reproducible feature of the magnetotransport in high-mobility DQWs.

The oscillations we observe are caused by the same physical mechanism as the MISOs previously studied in single quantum wells with two populated subbands. However, there are qualitatively different features in the manifestation of the MISO phenomenon in the DQWs. First, since the Fermi energy in the DQWs is typically much larger than the subband separation, the MISOs have a large period, as compared to the SdHO period, and they are clearly identifiable without an additional Fourier analysis. For the same reason, one can observe all the intersubband resonances up to the last one, when the cyclotron energy equals the subband separation, in relatively weak magnetic fields [see Fig. 1(b)]. Next, since the intersubband scattering in DQWs is easily controllable by modifying the wave functions of the electrons via the gate, the MISOs appear to be very sensitive to the gate voltage. We also point out that the observation of the large-amplitude MISOs requires large quantum lifetimes and a high probability of intersubband scattering. These two requirements are difficult to simultaneously satisfy in single quantum wells with two populated subbands because large quantum lifetimes are usually attainable in the modulation-doped systems where the scattering probability strongly decreases with the increasing momentum transfer, and the intersubband scattering, which requires a large momentum transfer, is suppressed.[5] In contrast, the intersubband scattering in DQWs does not require a large momentum transfer and its probability is comparable to the probability of intrasubband scattering.

The theoretical consideration of the magnetotransport in DQWs leads to an analytical expression for the oscillating resistivity, which survives at high temperatures [the last term in Eq. (6)]. Its simple form is a consequence of the symmetric scattering when the subbands are characterized by a single quantum lifetime τ, which is common for both subbands. The quantum lifetime can be extracted from the SdHO amplitudes at low temperatures while the measure-

ments of the MISO amplitudes allow us to find its temperature dependence in a wide region of temperatures. This dependence becomes an independent confirmation of the influence of the electron–electron scattering on the quantum lifetime of 2D electrons.

*Present address: Institute of Semiconductor Physics, Novosibirsk 630090, Russia.

[1] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Nonrelativistic Theory* (Pergamon, New York, 1997).

[2] D. Shoenberg, *Magnetic Oscillations in Metals* (Cambridge University Press, Cambridge, 1984).

[3] V. Polyanovsky, Fiz. Tekh. Poluprovodn. (S.-Peterburg) **22**, 2230 (1988).

[4] D. R. Leadley, R. Fletcher, R. J. Nicholas, F. Tao, C. T. Foxon, and J. J. Harris, Phys. Rev. B **46**, 12439 (1992).

[5] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **49**, 5531 (1994).

[6] T. H. Sander, S. N. Holmes, J. J. Harris, D. K. Maude, and J. C. Portal, Phys. Rev. B **58**, 13856 (1998).

[7] N. S. Averkiev, L. E. Golub, S. A. Tarasenko, and M. Willander, J. Phys.: Condens. Matter **13**, 2517 (2001).

[8] A. C. H. Rowe, J. Nehls, R. A. Stradling, and R. S. Ferguson, Phys. Rev. B **63**, 201307(R) (2001).

[9] M. Langenbuch, M. Suhrke, and U. Rossler, Phys. Rev. B **69**, 125303 (2004).

[10] V. Polyanovsky, Phys. Rev. B **47**, 1985 (1993).

[11] M. V. Kartsovnik, P. D. Grigoriev, W. Biberacher, N. D. Kushch, and P. Wyder, Phys. Rev. Lett. **89**, 126802 (2002).

[12] P. D. Grigoriev, Phys. Rev. B **67**, 144401 (2003).

[13] G. S. Boebinger, A. Passner, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **43**, 12673 (1991).

[14] R. Fletcher, M. Tsaousidou, T. Smith, P. T. Coleridge, Z. R. Wasilewski, and Y. Feng, Phys. Rev. B **71**, 155310 (2005).

[15] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).

[16] J. P. Heida, B. J. van Wees, J. J. Kuipers, T. M. Klapwijk, and G. Borghs, Phys. Rev. B **57**, 11911 (1998).

[17] M. A. Zudov, I. V. Ponomarev, A. L. Efros, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **86**, 3614 (2001).

[18] Y. Berk, A. Kamenev, A. Palevski, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **51**, 2604 (1995).

[19] See, for example, F. T. Vasko and O. E. Raichev, *Quantum Kinetic Theory and Applications* (Springer, New York, 2005).

[20] T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).

[21] I. L. Aleiner and L. I. Glazman, Phys. Rev. B **52**, 11296 (1995).

[22] M. Slutzky, O. Entin-Wohlman, Y. Berk, A. Palevski, and H. Shtrikman, Phys. Rev. B **53**, 4065 (1996).