Transmittance and transparency of subwavelength-perforated conducting films in the presence of a magnetic field

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We theoretically and numerically studied the transmission of light through a subwavelength-perforated metal film, as well as through a homogeneous metal film with a Drude ac conductivity tensor, in the presence of a static magnetic field. Both the perforated and the homogeneous metal films are found to exhibit a magnetoinduced light transparency and a decreasing reflectivity due to cyclotron resonance. In particular, the cyclotron resonance and the surface plasmon resonance of the perforated metal film move to higher frequencies with increasing magnetic field, bringing about large changes in the extraordinary light transmission peaks predicted to occur in such a film. The practical possibility of changing the sample transparency by application of a static magnetic field is discussed.

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I. INTRODUCTION

We discuss some phenomena that can be used for manipulating the transmission of light through a thin metal film by applying an external magnetic field. Although this has been extensively discussed before in the context of a uniform film,[1] we now apply this idea in the context of a film with an ordered array of subwavelength perforations. Such films have been shown to exhibit "extraordinary light transmission" (ELT)[2,3] as a result of surface plasmons, which can propagate through holes and along the film surfaces.

In this paper, we show how the ELT can be manipulated by the application of a strong dc magnetic field. This possibility arises whenever the Hall resistivity of the film exceeds its ohmic resistivity. It allows one to switch the optical transmissivity of the film from totally opaque to highly transparent by changing either the magnitude of the magnetic field or its direction. It also allows the perforated film to be put in a state wherein the transmissivity strongly depends upon the polarization of the incident light.

The ELT through a metal film that is perforated by a periodic array of subwavelength holes[2] is widely believed to result from the coupling of light to the plasmons on the surface of the patterned metal film. Continuing this idea, the effect of an applied static magnetic field on the ELT has been discussed.[4–9] It was shown that an applied static magnetic field shifts the surface plasmon resonance and the ELT to higher frequencies. Now, the influence of a magnetic field on the ELT is under intensive study.[10–21] In Refs. 4–9, it was also mentioned that, in addition to surface plasmon resonance, there also exists a so-called cyclotron resonance, which can play, in principle, some role in light transmission.

The phenomenon of cyclotron resonance was first seriously discussed by Azbel[22–24] in 1956, and the effect of a magnetic field on surface plasmon resonance was first discussed at approximately the same time (1955) by Dresselhaus *et al.*[25,26] The enhancement effect of a magnetic field on optical transmission was also discussed long ago.[1,27] In this paper, we discuss the joint effect of cyclotron resonance and the magnetic-field dependence of surface plasmon resonance on the optical transmissivity. For simplicity, we use the quasistatic approximation and the Drude model for the permittivity tensors.

The remainder of this paper is organized as follows: In Sec. II, we describe our method of calculating the effective permittivity tensor $\hat{\varepsilon}_e$ for both random and ordered arrangements of elongated holes in a metallic film. We also present numerical results for both cases. A brief discussion of the results and of some possible practical applications appear in Sec. III.

II. FORMALISM AND NUMERICAL RESULTS

In the quasistatic regime, the electric permittivity tensor $\hat{\varepsilon}_M$ has the form[4–6,28]

$$\hat{\varepsilon}_M = \varepsilon_0 \cdot \hat{I} + i\frac{4\pi}{\omega}\hat{\sigma}$$
$$= \varepsilon_0 \cdot \hat{I} + \frac{i\omega_p^2\tau}{\omega}\begin{pmatrix} \frac{1-i\omega\tau}{(1-i\omega\tau)^2+H^2} & \frac{-H}{(1-i\omega\tau)^2+H^2} & 0 \\ \frac{H}{(1-i\omega\tau)^2+H^2} & \frac{1-i\omega\tau}{(1-i\omega\tau)^2+H^2} & 0 \\ 0 & 0 & \frac{1}{1-i\omega\tau} \end{pmatrix}, \quad (1)$$

where the conductivity tensor $\hat{\sigma}$ is taken in the free-electron Drude approximation (with $\mathbf{B}_0 \| z$), ε_0 is the scalar dielectric constant of the background ionic lattice, which we take to be 1 in this paper, and $\hat{I}$ is the unit tensor. The magnetic field enters only through the Hall-to-ohmic resistivity ratio $H \equiv \rho_H/\rho = \sigma_{yx}/\sigma_{xx} = \mu|\mathbf{B}_0| = \omega_c\tau$, where $\omega_c = eB/mc$ is the cyclotron frequency, τ is the conductivity relaxation time, $\omega_p = (4\pi e^2 N_0/m)^{1/2}$ is the plasma frequency, N_0 is the charge

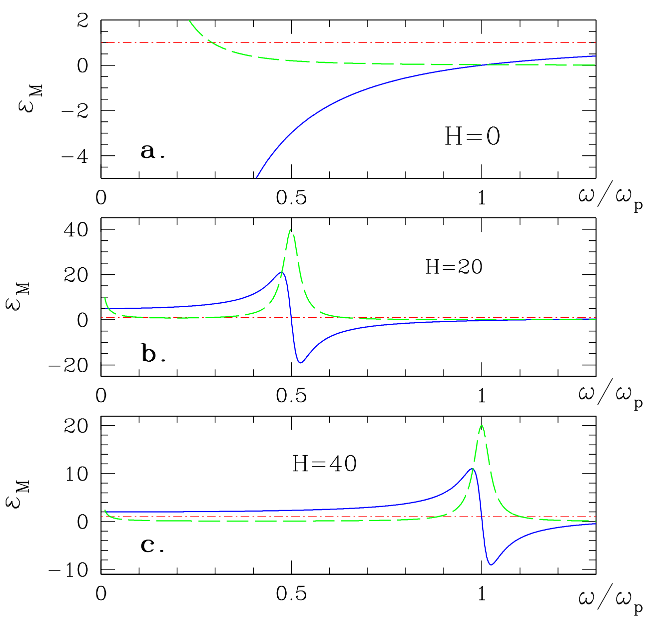

FIG. 1. (Color online) Real (solid lines) and imaginary (dashed lines) parts of $\varepsilon_M \equiv \varepsilon_{xx}^{(M)}$ [given by Eq. (1)] vs ω/ω_p for different values of the dimensionless magnetic field $H=\omega_c\tau=0$, 20, and 40 (when $\omega_p\tau=40$, these correspond to $\omega_c/\omega_p=0$, 0.5, and 1). The horizontal dashed-dotted lines show the value $\varepsilon_M=1$.

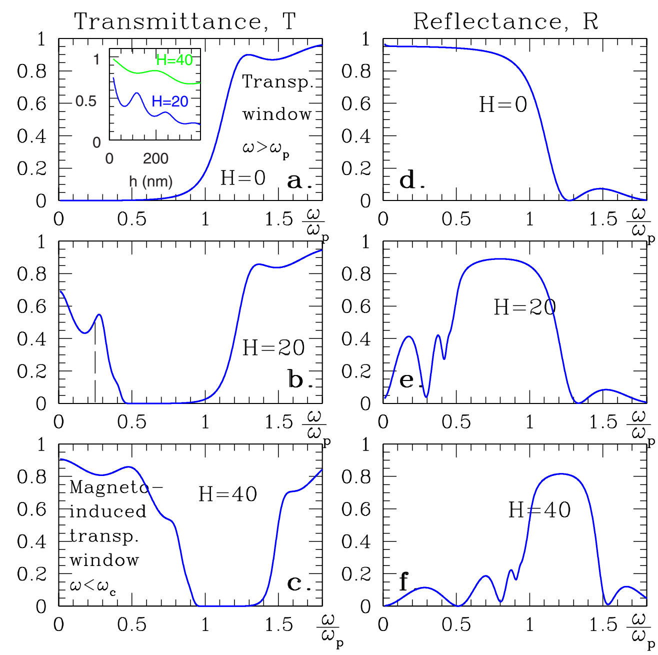

FIG. 2. (Color online) Transmittance T and reflectance R of a uniform metal film vs frequency ω/ω_p for different strengths $H=\omega_c\tau=0$, 20, and 40 of an applied perpendicular magnetic field, which are computed from the values of $\varepsilon_M(\omega,H)$ shown in Fig. 1. The film thickness is $h=100$ nm. $\hat{\varepsilon}_I=\hat{I}$, $\varepsilon_0=1$, $\omega_p\tau=40$, and $\omega_p=3\times10^{15}$ rad/s. Inset: Transmittance T [at $\omega=0.25\omega_p$, shown in Fig. 2(b) by a dashed line] vs film thickness for different magnetic fields ($H=20$ and 40).

carrier concentration, m is the effective mass of the charge carriers, and μ is the Hall mobility.[29–37]

When $\omega\tau \gg 1$, the transverse (i.e., the xx and yy) components of permittivity tensor (1) simplify to

$$\varepsilon_{xx}^{(M)} \simeq \varepsilon_0 + \omega_p^2/(\omega_c^2-\omega^2). \tag{2}$$

From this, it is clear that for $H=\omega_c\tau\neq0$, these components have a resonance at $\omega=\omega_c$, which is called the cyclotron resonance.[22,24–27] In Fig. 1, we show the real and imaginary parts of $\varepsilon_{xx}^{(M)}$ (corresponding to polarization $\mathbf{E_0}\|x$) vs ω/ω_p (these results are for films without any holes). Even without invoking an explicit expression for the light transmittance T, it is clear that, the closer is the value of the film permittivity to that of air, which is 1, the greater will be T. For $H=0$ [see Fig. 1(a)], ε_M approaches 1 only when $\omega>\omega_p$ (ε_M is a shorthand for the transverse diagonal elements of $\hat{\varepsilon}_M$). At frequencies below ω_p, ε_M is negative. However, when $H>0$, the situation changes and ε_M can be within the vicinity of 1 for values of ω that are even less than ω_c and are well below ω_p [see Figs. 1(b) and 1(c)]. The closer ε_M is to 1 (i.e., the larger H is), the greater T will be. At $\omega\sim\omega_c/2$, ε_M tends to the approximate expression $1+\frac{4}{3}(\tau\omega_p)^2/H^2$. Thus, for $\tau\omega_p=40$ at $H=20$, $\varepsilon_M\sim6$, while for $H=40$, $\varepsilon_M\sim2.3$.

When a sufficiently strong external magnetic field is applied perpendicular to the film plane, a dramatic increase is observed to occur in the transmittance T of a uniform metal film for frequencies that satisfy $\omega<\omega_c$. Along with this, a decrease in the reflectance R is also observed for those same frequencies—see Fig. 2. The dependence of T and R on frequency ω, for different film thicknesses, can be obtained from the value of ε_M by using the known expressions[8,38,39] for $T=|d|^2$ and $R=|r|^2$, where

$$d=(1-r_{12}^2)/D, \tag{3}$$

$$r=-2ir_{12}\sin\chi/D, \tag{4}$$

$$D=\exp(-i\chi)-r_{12}^2\exp(i\chi), \tag{5}$$

$$r_{12}=(1-n)/(1+n), \tag{6}$$

$$n=\sqrt{\varepsilon_{xx}^{(M)}(\omega)}, \tag{7}$$

$$\chi=(\omega/c)hn=(\omega/\omega_p)\xi n, \tag{8}$$

$$\xi=\omega_p h/c, \tag{9}$$

and h is the film thickness.[8,9,38,39] In Fig. 2, we plot the transmittance $T(\omega)$, which exhibits the magnetic-field-induced transparency and was obtained by using the dielectric functions in Fig. 1. At $H=0$ [see Fig. 2(a)], light of frequencies below the plasma frequency is reflected because the electrons in the metal screen the ac electric field. Light of frequencies above the plasma frequency is transmitted because the electrons cannot respond fast enough to screen that field. In most metals, the plasma frequency is in the ultraviolet range, making them shiny (reflective) in the visible range. In doped semiconductors, the plasma frequency is usually in the infrared range.[4–6] When $H>0$ [see Figs. 2(b) and 2(c)], the nontransparency interval shifts to higher frequencies and a new transparency window appears at frequencies $\omega<\omega_c$,

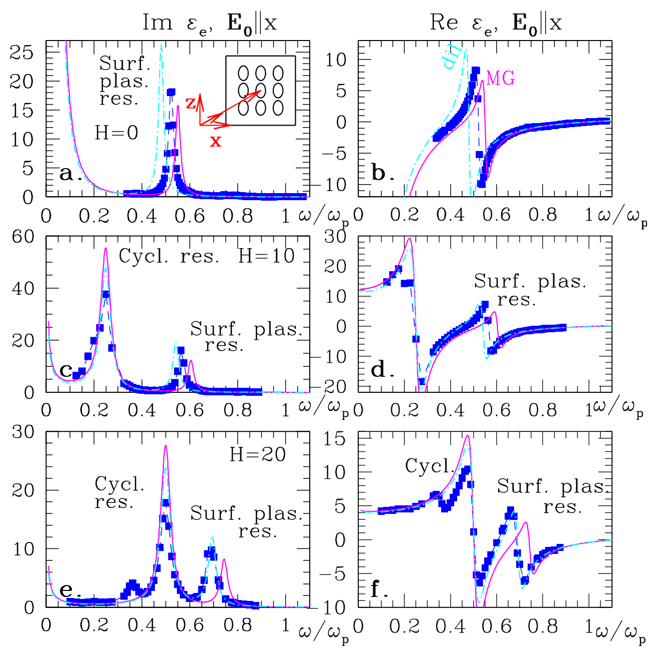

FIG. 3. (Color online) [(a)–(f)] Real and imaginary parts of the macroscopic electric permittivity ε_e of a random dilute array of elliptical holes (analytical results obtained in dilute and Maxwell Garnett approximations for a volume fraction of holes $p_{\text{holes}}=0.094$ are shown by the dashed-dotted lines and the solid lines, respectively) and simple-square periodic array of rectangular holes [$p_{\text{holes}}=0.12$; numerical results (Refs. 29–33) are shown by the filled symbols connected by a thin dashed line) for $\mathbf{E_0}\|x$ light polarization [see the inset of (a)] for different values of the dimensionless magnetic field: $H=0$, 10, and 20. [(a), (c), and (e)] Im $\varepsilon_{xx}^{(e)}$ vs ω/ω_p. [(b), (d), and (f)] Re $\varepsilon_{xx}^{(e)}$ vs ω/ω_p. Inset of (a): A metal film with a periodic, simple-square array of elliptical holes. The coordinate axes x and z lie along the principal axes of the array and the applied static magnetic field lies in the film plane. The incident light beam is normal to the film surface. The rectangular hole sizes ($l_x=0.2d$ and $l_z=0.6d$, where d is the unit cell size) used in our numerical calculations were such that their aspect ratio is identical to that used in the analytical calculations on elliptical holes ($l_x/l_z=a/c=0.33$, where $a=0.1d$ and $c=0.3d$ are the semiaxes of those holes). The film thickness is $h=100$ nm. $\hat{\varepsilon}_I=\hat{I}$, $\varepsilon_0=1$, $\omega_p\tau=40$, and $\omega_p=3\times10^{15}$ rad/s.

as soon as a strong static magnetic field is applied.

It is interesting to compare the effect of the cyclotron resonance to that of the surface plasmon resonance in the case of a metal film with an array of subwavelength elliptical holes. A similar calculation can also be carried out for circular holes but we preferred to consider elliptical holes in order to get a handle on the light polarization.[7,40,41] In Fig. 3, we show the macroscopic electric permittivity $\varepsilon_{xx}^{(e)}$ of such an array vs ω/ω_p. This is done by using the dilute approximation and the Maxwell Garnett approximation (closed form results, as shown by the thin dotted lines and the thin solid lines, respectively), and also by using numerical computations[4,5,7,29–35] on a simple-square array of rectangular holes (filled symbols and dashed lines that connect them). The results are obtained for $\mathbf{E_0}\|x$ light polarization and for different strengths of the dimensionless magnetic field ($H=0$, 10, and 20) by using Eqs. (3)–(9), where the permittivity tensor component $\varepsilon_{xx}^{(M)}$ in Eq. (7) should be replaced by the macroscopic or bulk effective electric permittivity $\varepsilon_{xx}^{(e)}$. A cyclotron resonance peak and a surface plasmon (SP) resonance peak appear at $\omega=\omega_c$ and $\omega=\omega_{sp}$, respectively, and move to higher frequencies with increasing H (see the curves for $H=0$, 10, and 20). In Fig. 4, we show similar plots of the transmittance T vs ω/ω_p and also vs the actual wavelength λ of the incident light for magnetic field strengths $H=0$ and 10. We have marked as "Surf. plas." those transmission peaks whose frequency or wavelength coincides with that of the surface plasmon resonance, as it appears in Fig. 3. Similarly, we have marked with an arrow a small transmission peak whose frequency coincides with that of the cyclotron resonance, which also appears in Fig. 3.

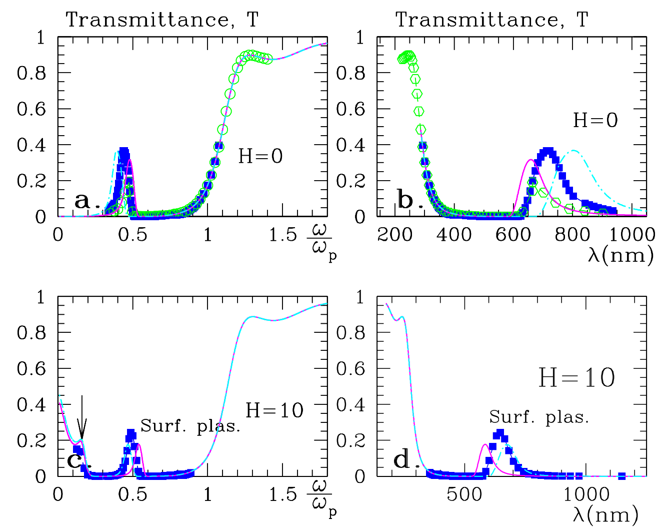

FIG. 4. (Color online) [(a) and (c)] Transmittance T of the perforated film vs frequency ω/ω_p for different values of magnetic field $H=0$ and 10. T is calculated by using the values of ε_e from Fig. 3 and similar conventions for the lines and symbols. The circles in (a) and (b) correspond to the case of elliptical holes of volume fraction $p_{\text{holes}}=0.12$. The arrow in (c) marks the peak related to cyclotron resonance, which corresponds to the peak of ε_e shown in Fig. 3(c) (note that the frequencies of these two peaks are slightly different). [(b) and (d)] The same as (a) and (c) but plotted vs wavelength λ.

III. DISCUSSION AND CONCLUSIONS

In summary, we have shown that an applied static magnetic field can be used to manipulate the transmission of light through nonmagnetic metal films both with and without an array of subwavelength perforations. This could form the basis for a new type of a magneto-optical switch and other magneto-optical devices. As a material that may be suitable for this purpose, bismuth can be considered, wherein the low free-charge density ($\sim3\times10^{17}$ cm^{-3}) can make the carrier cyclotron frequency ω_c equal to or greater than the plasma frequency ω_p.[42–45]

Another possibility is to use highly doped semiconductors such as GaAs and InAs:[4–6,46] In that case, it is possible to obtain large values of the Hall mobility μ and, therefore, also large values of the dimensionless magnetic field $H=\mu|\mathbf{B}_0|$ by using lower values of $\mathbf{B}_0$. The value of $\omega_p\tau$ in this case can be of the same order of magnitude as that of conventional metals (in our calculations, we assumed $\omega_p\tau=40$, while for a typical free-electron metal such as Al, we have $\omega_p\tau\approx 100$). Thus, such magnetic-field-dependent extraordinary optical transmission in the infrared range of frequencies can be sought in heavily doped semiconductor films with an array of holes with a submicron periodicity. Detailed estimations of the above mentioned parameters, which are typical of GaAs and InAs, can be found in Refs. 4–6. Also to be found there is a discussion of how small the size scale of the microstructure should be in order for the quasistatic limit to be applicable.

In Fig. 5, we show the curves for transmittance T vs dimensionless magnetic field H, which were calculated for a number of frequencies corresponding to the cyclotron [see Fig. 5(a)] and surface plasmon [see Fig. 5(b)] resonance regions. The sensitivity $\partial T/\partial H$ at $\omega/\omega_p=0.2$ [i.e, at the cyclotron resonance—see the vertical arrow in Fig. 4(c)] is on the order of 0.2, while at $\omega/\omega_p=0.5$ [this is the frequency of the 2D surface plasmon resonance of an isolated ellipse of the shape that we considered when $\mathbf{E}_0\|x$—see Fig. 3(a) and its inset], $\partial T/\partial H\sim -0.06$. The approximate dependences of the dimensionless magnetic field $H=\mu B$ on the magnetic field, which is measured in tesla, are shown in the inset of Fig. 5(a) for Bi [estimated values for polycrystalline film samples at temperature of 5 K (Ref. 45)] and for GaAs [estimated values for n-doped samples at temperature of 9 K (Ref. 46)]. From this, we find that the sensitivity $\partial T/\partial B$ (measured in inverse tesla) at $\omega/\omega_p=0.5$ can reach the order of $\sim 4.4\ \mathrm{T}^{-1}$ for Bi and of $\sim 0.05\ \mathrm{T}^{-1}$ for GaAs.

In transition metal films made of Ag or Au, wherein surface plasmons are easily observed in visible light, stronger magnetic fields would be needed in order to observe the behavior described here. This is due to the lower values of the electronic Hall mobility μ in those metals. A possible way to overcome this handicap would be to use layered ferromagnet-Ag (or Au) sandwiches.[15–19] A better treatment of the effects studied here would require going beyond the quasistatic approximation. Experimental tests of our predictions would be very desirable.

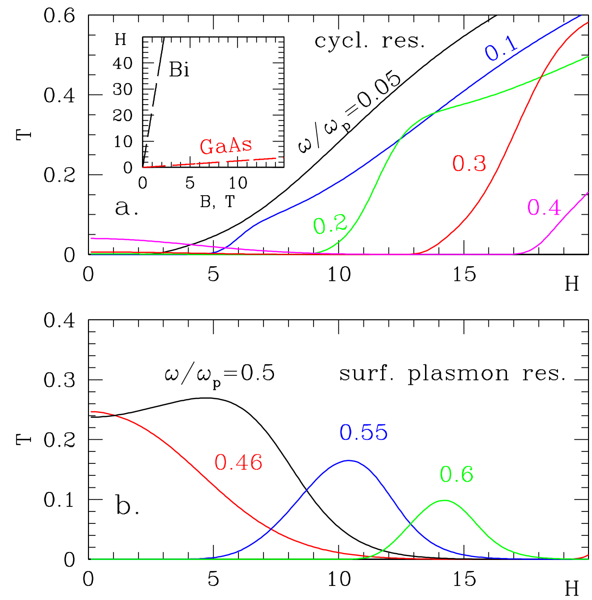

FIG. 5. (Color online) Transmittance T of a perforated film vs dimensionless magnetic field H for different frequencies ω/ω_p calculated by using the dilute approximation. (a) Magnetoinduced transparency in the vicinity of the cyclotron resonance ($\omega/\omega_p=0.05$, 0.1, 0.2, 0.3, and 0.4). (b) Similar to (a) but in the vicinity of the surface plasmon resonance ($\omega/\omega_p=0.46$, 0.5, 0.55, and 0.6). Inset of (a): Approximate dependencies of the dimensionless magnetic field $H=\mu B$ vs magnetic field B measured in tesla for Bi (see Ref. 45) and GaAs (see Ref. 46).

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