Electronic properties in a two-dimensional disordered electron liquid: Spin-valley interplay

I. S. Burmistrov and N. M. Chtchelkatchev
L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 117940 Moscow, Russia
and Department of Theoretical Physics, Moscow Institute of Physics and Technology, 141700 Moscow, Russia
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We report a detailed study of the influence of the spin and valley splittings on such physical observables of the two-dimensional disordered electron liquid as resistivity and spin and valley susceptibilities. We explain qualitatively the nonmonotonic dependence of the resistivity on temperature in the presence of a parallel magnetic field. In the presence of either spin or valley splitting we predict a temperature dependence of the resistivity with two maximum points.

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I. INTRODUCTION

Disordered two-dimensional (2D) electron systems have been in the focus of experimental and theoretical research for several decades.[1] Recently, the interest in 2D electron systems has been renewed because of the experimental discovery of a metal-insulator transition (MIT) in a high-mobility silicon metal-oxide-semiconductor field-effect transistor (Si-MOSFET).[2,3] Although during the last decade behavior of the resistivity similar to that of Refs. 2 and 3 has been found experimentally in a wide variety of two-dimensional electron systems,[4] the MIT in 2D calls still for a theoretical explanation.

Very likely, the most promising framework is provided by the microscopic theory, initially developed by Finkelstein, that combines disorder and strong electron-electron interaction.[5] Punnoose and Finkelstein[6] have shown the possibility for the MIT existence in a special model of a 2D electron system with an infinite number of spin and valley degrees of freedom. The current theoretical results[7,8] do not support the MIT existence for electrons without spin and valley degrees of freedom. Therefore, it is natural to assume that the spin and valley degrees of freedom play a crucial role for the MIT in 2D disordered electron systems.[9]

Usually, in the vicinity of the MIT, from the metallic side, i.e., for an electron density higher than the critical one, and at low temperatures $T \ll \tau_{\mathrm{tr}}^{-1}$, the initial increase of the resistivity (ρ) with lower temperature is replaced by a decrease of ρ as T becomes lower than some sample-specific temperature.[4] Here, τ_{tr} denotes the elastic scattering time. This nonmonotonic behavior of the resistivity was predicted from a renormalization group (RG) analysis of the interplay between disorder and electron-electron interaction in 2D disordered electron systems.[5,10] As a weak magnetic field B is applied parallel to the 2D plane, the decrease of the resistivity is stopped at some temperature and ρ increases again.[11] A further increase of B leads to monotonic growth of the resistivity as temperature is lowered, i.e., to an insulating behavior, in the whole T range. These experimental results suggest the significance of the electron spin for the existence of the metallic phase in 2D disordered electron systems.

As is well known, in both Si-MOSFETs (Ref. 1) and n-type AlAs quantum wells,[12] 2D electrons can populate two valleys. Therefore, these systems offer the unique opportunity for an experimental investigation of the interplay between the spin and valley degrees of freedom. In Si-MOSFETs the spin-valley interplay has been probed by varying the parallel magnetic field in the metallic region for relatively large electron concentrations.[13,14] Recently, using a symmetry-breaking strain to tune the valley occupation of the 2D electron system in an n-type AlAs quantum well, as well as a parallel magnetic field to adjust the spin polarization, the spin-valley interplay has been experimentally studied.[15,16] However, the electron concentrations in the experiment were at least three times larger than the critical one.[12] Therefore, the spin-valley interplay has been studied in the region of a good metal very far from the metal-insulator transition.

In the present paper we report detailed theoretical results for the T behavior of a 2D electron system with two valleys in the vicinity of the MIT. In particular, we study the effect of a parallel magnetic field and/or valley splitting (Δ_v) on the transport, and the spin and valley susceptibilities. We find that in the presence of either a magnetic field or valley splitting the metallic behavior of the resistivity survives down to zero temperature.[17] For example, this result implies that at $B=0$ the metallic $\rho(T)$ dependence can be observed experimentally at temperatures $T \ll \Delta_v$. Only if both the magnetic field and the valley splitting are present does the metallic behavior of the resistivity cross over to the insulating one. Next, we predict T behavior of the resistivity *with two maximum points*, in the presence of a magnetic field and/or valley splitting. Finally, we find that as T vanishes the ratio of the valley (χ_v) to the spin susceptibility (χ_s) becomes sensitive to the ratio of the valley to the spin splitting. At high temperatures the ratio χ_v/χ_s is temperature independent and can be chosen equal to unity. If the spin splitting is larger (smaller) than the valley splitting, then at low temperatures the ratio $\chi_v/\chi_s < \ (>)\ 1$. If the spin and valley splittings are equal to each other, then the ratio $\chi_v/\chi_s=1$ as the temperature vanishes.

The presence of the parallel magnetic field and the symmetry-breaking strain introduces new energy scales $\Delta_s = g_L \mu_B B$ and Δ_v in the problem. Here, g_L and μ_B stand for the g factor and the Bohr magneton, respectively. Let us assume that the following conditions hold: $\tau_{so}^{-1}, \tau_v^{-1} \ll \Delta_v \ll \Delta_s \ll \tau_{\mathrm{tr}}^{-1}$, where τ_{so} and τ_v are spin-relaxation and intervalley scattering times, respectively. These assumptions are relevant for experiments on Si-MOSFETs as discussed below in Sec. VI.

As is well known,[18–20] the spin and valley splittings do not change the weak-localization ("cooperon") contribution to the resistivity in the absence of spin-relaxation processes and intervalley transitions. However, the interaction (Altshuler-Aronov) correction[21] to the resistivity is affected by the presence of Δ_s and Δ_v. Due to the symmetry breaking, the spin and valley splittings set the cutoff for a pole in the diffusion modes ("diffusons") with opposite spin and valley isospin projections. In the temperature range $\Delta_s \ll T \ll \tau_{\rm tr}^{-1}$, this cutoff is irrelevant and the 2D electron system behaves as if no symmetry-breaking terms are applied. The temperature behavior of the resistivity is governed by one singlet and 15 triplet diffusive modes.[10] At low temperatures $\Delta_v \ll T \ll \Delta_s$, eight diffusive modes with opposite spin projections do not contribute. Then, the $\rho(T)$ dependence is determined by the remaining one singlet and seven triplet modes. As we shall demonstrate below, the behavior of the resistivity can be either metallic or insulating. Surprisingly, we found that the seven triplet diffusive modes are not equivalent. They have to split into two groups of six and one modes for the spin susceptibility be T independent. For temperatures $\tau_{so}^{-1}, \tau_v^{-1} \ll T \ll \Delta_v$, the next four diffusive modes with opposite isospin projections become ineffective. In this case, the temperature dependence of the resistivity is determined by one singlet and three triplet diffusive modes. Although the number of remaining diffusive modes corresponds formally to single-valley electrons with spin, the $\rho(T)$ behavior is insulating.

The paper is organized as follows. In Sec. II we introduce the nonlinear σ model that describes the disordered interacting electron system. Then, we consider the short length scales at which the system has SU(4) symmetry in the combined spin and valley space (Sec. III). The behavior of the system at intermediate and long length scales is studied in Secs. IV and V, respectively. We end the paper with discussion of our results and with conclusions (Sec. VI).

II. FORMALISM

A. Microscopic Hamiltonian

To start out, we consider 2D interacting electrons with two valleys in the presence of quenched disorder and a parallel magnetic field at low temperatures $T \ll \tau_{\rm tr}^{-1}$. For the case of a Si(001)-MOSFET, which is of main interest for us, we write an electron annihilation operator with the help of one-electron orbital functions as[22,23]

$$\psi_\sigma(\mathbf{R}) = \sum_{\tau=\pm} \psi_\tau^\sigma(\mathbf{r})\varphi(z)(e^{izQ/2} + \tau e^{-izQ/2})/\sqrt{2}, \qquad (1)$$

where z denotes the coordinate perpendicular to the 2D plane, $\mathbf{r}$ the in-plane coordinate vector, and $\mathbf{R}=\mathbf{r}+z\mathbf{e}_z$. The subscript τ enumerates the two valleys and ψ_τ^σ is the annihilation operator of an electron with the spin and isospin projections equal to $\sigma/2$ and $\tau/2$, respectively. Let us choose the envelope function $\varphi(z)$ to be normalized, and assume negligible overlap $\int dz\, \varphi^2(z)\sin(Qz)$. The vector $\mathbf{Q}=(0,0,Q)$ corresponds to the shortest distance between the valley minima in the reciprocal space: $Q \sim a_{\rm lat}^{-1}$, with $a_{\rm lat}$ being the lattice constant.[1]

In the path-integral formulation 2D interacting electrons in the presence of the random potential $V(\mathbf{r})$ are described by the following grand partition function:

$$Z = \int \mathcal{D}[\bar{\psi},\psi] e^{S[\bar{\psi},\psi]}, \qquad (2)$$

with the imaginary time action

$$S = \int_0^{1/T} dt[-\bar{\psi}_\tau^\sigma(\mathbf{r},t)\partial_t \psi_\tau^\sigma(\mathbf{r},t) - \mathcal{H}_0 - \mathcal{H}_{\rm dis} - \mathcal{H}_{\rm int}]. \qquad (3)$$

The one-particle Hamiltonian

$$\mathcal{H}_0 = \int d\mathbf{r}\, \bar{\psi}_\tau^\sigma(\mathbf{r})\left(-\frac{\nabla^2}{2m_e} - \mu + \frac{\Delta_s}{2}\sigma + \frac{\Delta_v}{2}\tau\right)\psi_\tau^\sigma(\mathbf{r}) \qquad (4)$$

describes a 2D quasiparticle with mass m_e in the presence of the parallel magnetic field and the valley splitting. Here, μ denotes the chemical potential. Next,

$$\mathcal{H}_{\rm dis} = \int d\mathbf{r}\, \bar{\psi}_{\tau_1}^\sigma(\mathbf{r}) V_{\tau_1\tau_2}(\mathbf{r})\psi_{\tau_2}^\sigma(\mathbf{r}) \qquad (5)$$

involves matrix elements of the random potential

$$V_{\tau_1\tau_2}(\mathbf{r}) = \frac{1}{2}\int dz\, V(\mathbf{R})\varphi^2(z)(1 + \tau_1\tau_2 + \tau_1 e^{izQ} + \tau_2 e^{-izQ}). \qquad (6)$$

In general, the matrix elements $V_{\tau_1\tau_2}(\mathbf{r})$ induce both the intravalley and intervalley scattering. We suppose that $V(\mathbf{R})$ has a Gaussian distribution, and

$$\langle V(\mathbf{R})\rangle = 0, \quad \langle V(\mathbf{R}_1)V(\mathbf{R}_2)\rangle = W(|\mathbf{r}_1-\mathbf{r}_2|,|z_1-z_2|), \qquad (7)$$

where the function W decays at a typical distance d. If d is larger than the effective width of the 2D electron system, i.e., $d \gg [\int dz\, \varphi^4(z)]^{-1}$, then one can neglect the z dependence of $V(\mathbf{R})$ under the integral sign in Eq. (6). In this case, only the intravalley scattering survives:

$$\langle V_{\tau_1\tau_2}(\mathbf{r})V_{\tau_3\tau_4}(0)\rangle = W(|\mathbf{r}|,0)\delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4}. \qquad (8)$$

In the opposite case, $d \ll [\int dz\, \varphi^4(z)]^{-1}$, one finds[24]

$$\begin{aligned}\langle V_{\tau_1\tau_2}(\mathbf{r})V_{\tau_3\tau_4}(0)\rangle = \int dz\, \varphi^4(z)\{&\delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4}[\tilde{W}(|\mathbf{r}|,0) \\ &+ \tau_1\tau_3\tilde{W}(|\mathbf{r}|,Q)/2] + (\delta_{\tau_1\tau_4}\delta_{\tau_2\tau_3} \\ &- \delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4})\tilde{W}(|\mathbf{r}|,Q)/2\},\end{aligned} \qquad (9)$$

where $\tilde{W}(|\mathbf{r}|,Q) = \int dz\, W(|\mathbf{r}|,|z|)\exp(iQz)$. The other correlation functions vanish due to integration over the $(z_1+z_2)/2$ coordinate. It is the last term in Eq. (9) that contributes to the intervalley scattering rate $1/\tau_v$. Assuming $Q^{-1} \ll d$, one can neglect the intervalley scattering rate in comparison with the intravalley scattering rate $1/\tau_i \sim \tilde{W}(r,0)$. Finally, allowing for a low electron concentration n_e in 2D electron systems, we consider the case when the inequality $n_e d^2 \ll 1$ holds. Then, both Eqs. (8) and (9) read

$$\langle V_{\tau_1\tau_2}(\mathbf{r}_1) V_{\tau_3\tau_4}(\mathbf{r}_2)\rangle = \frac{1}{2\pi\nu\tau_i}\delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4}\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (10)$$

$$\frac{1}{\tau_i} = 2\pi\nu\int d^2\mathbf{r}\, dz_1 dz_2 W(|\mathbf{r}|,|z_1-z_2|)\varphi^2(z_1)\varphi^2(z_2),$$

$$Q^{-1} \ll d, \quad \left(\int \varphi^4(z)dz\right)^{-1} \ll n_e^{-1/2}. \quad (11)$$

Here, ν is the thermodynamic density of states. Under the conditions (11), the interaction part of the Hamiltonian is invariant under global SU(4) rotations of the electron operator ψ_τ^σ in the combined spin-valley space:

$$\mathcal{H}_{\text{int}} = \frac{e^2}{2\epsilon}\int d\mathbf{r}_1 d\mathbf{r}_2 \frac{\bar{\psi}^{\sigma_1}_{\tau_1}(\mathbf{r}_1)\psi^{\sigma_1}_{\tau_1}(\mathbf{r}_1)\bar{\psi}^{\sigma_2}_{\tau_2}(\mathbf{r}_2)\psi^{\sigma_2}_{\tau_2}(\mathbf{r}_2)}{|\mathbf{r}_1-\mathbf{r}_2|}. \quad (12)$$

The dielectric constant of the substrate is denoted as ϵ. The low-energy part of $\mathcal{H}_{\text{int}}$ can be written as[5,25–27]

$$\mathcal{H}_{\text{int}} = \frac{1}{2}\int d\mathbf{r}_1 d\mathbf{r}_2[\rho(\mathbf{r}_1)\Gamma_s(\mathbf{r}_1-\mathbf{r}_2)\rho(\mathbf{r}_2) + m^a(\mathbf{r}_1)\Gamma_t(\mathbf{r}_1-\mathbf{r}_2)m^a(\mathbf{r}_2)], \quad (13)$$

where

$$\rho(\mathbf{r}) = \sum_{\sigma\tau}\bar{\psi}^\sigma_\tau(\mathbf{r})\psi^\sigma_\tau(\mathbf{r}), \quad (14)$$

$$m^a(\mathbf{r}) = \sum_{\sigma_1\sigma_2;\tau_1\tau_2}\bar{\psi}^{\sigma_1}_{\tau_1}(\mathbf{r})(t^a)^{\sigma_1\sigma_2}_{\tau_1\tau_2}\psi^{\sigma_2}_{\tau_2}(\mathbf{r}). \quad (15)$$

Here, $\Gamma_s(\mathbf{q}) = U(q) + F_0^\rho/(4\nu)$ involves the long-range part of the Coulomb interaction $U(q)=2\pi e^2/(q\epsilon)$ and $\Gamma_t(\mathbf{q}) = F_0^\sigma/(4\nu)$. The quantities F_0^σ and F_0^ρ are the standard Fermi liquid interaction parameters in the singlet and triplet channels, respectively. The matrices t^a with σ'_{xx} are the nontrivial generators of the SU(4) group.

B. Nonlinear σ model

At low temperatures, $T\tau_{\text{tr}} \ll 1$, the effective quantum theory of 2D disordered interacting electrons described by the Hamiltonian (3) is given in terms of the nonlinear σ model. As we have mentioned above, the weak-localization (cooperon) contribution to the resistivity is independent of the presence of Δ_s and Δ_v.[18–20] Therefore, we shall ignore the Cooper channel in the intermediate calculations for the sake of simplicity and shall discuss its role in Sec. IV. Then, the effective theory involves unitary matrix field variables $Q^{\alpha_1\alpha_2;\sigma_1\sigma_2}_{mn;\tau_1\tau_2}(\mathbf{r})$ which obey the nonlinear constraint $Q^2(\mathbf{r})=1$. The integers $\alpha_j = 1,2,\ldots,N_r$ denote the replica indices. The integers m,n correspond to the discrete set of Matsubara frequencies $\omega_n = \pi T(2n+1)$. The integers $\sigma_j = \pm 1$ and $\tau_j = \pm 1$ are spin and valley indices, respectively. The effective action is

$$\mathcal{S} = \mathcal{S}_\sigma + \mathcal{S}_F + \mathcal{S}_{sb} + \mathcal{S}_{vb} + \mathcal{S}_0, \quad (16)$$

where $\mathcal{S}_\sigma$ represents the free electron part[28]

$$\mathcal{S}_\sigma = -\frac{\sigma_{xx}}{32}\text{Tr}(\nabla Q)^2. \quad (17)$$

Here, σ_{xx} denotes the mean-field conductivity in units e^2/h. The symbol Tr stands for the trace over replicas, Matsubara frequencies, and spin and valley indices as well as integration over space coordinates.

The term[5]

$$\mathcal{S}_F = 4\pi Tz\ \text{Tr}\ \eta(Q-\Lambda) + \pi T\Gamma\int d^2\mathbf{r}\sum_{\alpha n}\text{tr}\ I_n^\alpha Q\ \text{tr}\ I_{-n}^\alpha Q - \pi T\Gamma_2\int d^2\mathbf{r}\sum_{\alpha n}(\text{tr}\ I_n^\alpha Q)\otimes(\text{tr}\ I_{-n}^\alpha Q) \quad (18)$$

involves the electron-electron interaction amplitudes which describe the scattering on small (Γ) and large (Γ_2) angles and the quantity z originally introduced by Finkelstein,[5] which is responsible for the specific heat renormalization.[29] The interaction amplitudes are related to the standard Fermi liquid parameters as[5,25,27] $\Gamma_2 = -zF_0^\sigma/(1+F_0^\sigma)$, $4\Gamma = \Gamma_2 + zF_0^\rho/(1+F_0^\rho)$, and $z=\pi\nu^\star/2$ where $\nu^\star = m^\star/(2\pi)$ with $m^\star$ being the effective mass. The case of the Coulomb interaction corresponds to the so-called "unitary" limit[20] $F_0^\rho \to \infty$.

The symbol tr involves the same operations as in Tr except for the integration over space coordinates, and $\text{tr}\ A\otimes\text{tr}\ B = A^{\alpha\alpha;\sigma_1\sigma_2}_{nn;\tau_1\tau_2}B^{\beta\beta;\sigma_2\sigma_1}_{mm;\tau_2\tau_1}$. The matrices Λ, η, and I_k^γ are given as

$$\Lambda^{\alpha\beta;\zeta_1\zeta_2}_{nm} = \text{sgn}(\omega_n)\delta_{nm}\delta^{\alpha\beta}\delta^{\zeta_1\zeta_2},$$

$$\eta^{\alpha\beta;\zeta_1\zeta_2}_{nm} = n\delta_{nm}\delta^{\alpha\beta}\delta^{\zeta_1\zeta_2},$$

$$(I_k^\gamma)^{\alpha\beta;\zeta_1\zeta_2}_{nm} = \delta_{n-m,k}\delta^{\alpha\gamma}\delta^{\beta\gamma}\delta^{\zeta_1\zeta_2}. \quad (19)$$

In the absence of Δ_s and Δ_v, the action $\mathcal{S}_\sigma + \mathcal{S}_F$ is invariant under the global rotations $Q^{\alpha\beta;\sigma_1\sigma_2}_{nm;\tau_1\tau_2}(\mathbf{r}) \to u^{\tau_1\tau_3}_{\sigma_1\sigma_3}Q^{\alpha\beta;\sigma_3\sigma_4}_{nm;\tau_3\tau_4}(\mathbf{r})[u^{-1}]^{\tau_4\tau_2}_{\sigma_4\sigma_2}$ in the combined spin-valley space for $u\in\text{SU}(4)$. The presence of the parallel magnetic field and the valley splitting generates the symmetry-breaking terms

$$\mathcal{S}_{sb} = iz_s\Delta_s\ \text{Tr}\ \sigma_z Q, \quad \mathcal{S}_{vb} = iz_v\Delta_v\ \text{Tr}\ \tau_z Q, \quad (20)$$

where σ_z and τ_z are Pauli matrices in the spin and valley spaces, respectively. The Q-independent part of the action reads[5,30]

$$\mathcal{S}_0 = -2\pi Tz\ \text{Tr}\ \eta\Lambda + \frac{N_r}{2T}\int d^2\mathbf{r}(\chi_s^0\Delta_s^2 + \chi_v^0\Delta_v^2), \quad (21)$$

with $\chi^0_{s,v} = 2z_{s,v}/\pi$ being the bare value of the spin (valley) susceptibility.

C. $\mathcal{F}$ algebra

The action (16) involves matrices which are formally defined in the infinite Matsubara frequency space. In order to operate with them we have to introduce a cutoff for the Matsubara frequencies. Then the set of rules which is called $\mathcal{F}$ algebra can be established.[30] At the end of all the calculations the cutoff should tend to infinity.

The global rotations of Q with the matrix $\exp(i\hat{\chi})$ where $\hat{\chi}=\Sigma_{\alpha,n}\chi_n^\alpha I_n^\alpha$ play the important role.[30,31] For example, $\mathcal{F}$ algebra allows us to establish the following relations:

$$\text{sp } I_n^\alpha e^{i\hat{\chi}} Q e^{-i\hat{\chi}} = \text{sp } I_n^\alpha Q + 2in\chi_{-n}^\alpha,$$

$$\text{tr } \eta e^{i\hat{\chi}} Q e^{-i\hat{\chi}} = \text{tr } \eta Q + \sum_{\alpha n} in(\chi_n^\alpha)_{\tau_1\tau_2}^{\sigma_1\sigma_2} \text{sp } I_n^\alpha Q_{\tau_2\tau_1}^{\sigma_2\sigma_1} - \sum_{\alpha n} n^2 (\chi_n^\alpha)_{\tau_1\tau_2}^{\sigma_1\sigma_2} (\chi_{-n}^\alpha)_{\tau_2\tau_1}^{\sigma_2\sigma_1}, \tag{22}$$

where sp stands for the trace over replicas and the Matsubara frequencies.

D. Physical observables

The most significant physical quantities in the theory containing information on its low-energy dynamics are the physical observables σ'_{xx}, z', and $z_{s,v}$ associated with the mean-field parameters σ_{xx}, z, and $z_{s,v}$ of the action (16). The observable σ'_{xx} is the dc conductivity, as one can obtain from the linear response to an electromagnetic field. The observable z' is related to the specific heat.[29] The observables z'_s and z'_v determine the static spin (χ'_s) and valley (χ'_v) susceptibilities of the 2D electron system[5,32] as $\chi'_{s,v}=2z'_{s,v}/\pi$. It is extremely important to recall that the observable parameters σ'_{xx}, $z_{s,v}$, and z' are precisely the same as those determined by the background field procedure.[33]

The conductivity σ'_{xx} is obtained from

$$\sigma'_{xx}(i\omega_n) = -\frac{\sigma_{xx}}{16n}\langle \text{tr}[I_n^\alpha, Q][I_{-n}^\alpha, Q]\rangle + \frac{\sigma_{xx}^2}{64\mathbb{D}n}\int d\mathbf{r}' \times \langle\langle \text{tr } I_n^\alpha Q(\mathbf{r}) \nabla Q(\mathbf{r}) \text{tr } I_{-n}^\alpha Q(\mathbf{r}') \nabla Q(\mathbf{r}')\rangle\rangle \tag{23}$$

after the analytic continuation to the real frequencies, $i\omega_n \to \omega + i0^+$ at $\omega\to 0$. Here, $\mathbb{D}=2$ stands for the space dimension, and the expectations are defined with respect to the theory (16).

A natural definition of z' is obtained[30] through the derivative of the thermodynamic potential Ω per unit volume with respect to T,

$$z' = \frac{1}{2\pi \text{ tr } \eta\Lambda}\frac{\partial}{\partial T}\frac{\Omega}{T}. \tag{24}$$

The observables $z_{s,v}$ are given by

$$z'_{s,v} = \frac{\pi}{2N_r}\frac{\partial^2\Omega}{\partial\Delta_{s,v}^2}. \tag{25}$$

III. SU(4)-SYMMETRIC CASE

A. $\mathcal{F}$ invariance

At short length scales $L \ll L_s, L_v$ where $L_{s,v} = \sqrt{\sigma_{xx}/(16 z_{s,v}\Delta_{s,v})}$, the symmetry-breaking terms $\mathcal{S}_{sb}$ and $\mathcal{S}_{vb}$ can be omitted and the effective theory becomes SU(4) invariant in the combined spin-valley space. Then, Eqs. (17) and (18) should be supplemented by the important constraint that the combination $z+\Gamma_2-4\Gamma$ remains constant in the course of the RG flow. Physically, this corresponds to the conservation of the particle number in the system.[5] In the special case of the Coulomb or other long-ranged interactions which are of main interest for us in this paper the relation

$$z + \Gamma_2 - 4\Gamma = 0 \tag{26}$$

holds. With the help of Eqs. (22), one can check that Eq. (26) guarantees the so-called $\mathcal{F}$ invariance[30] of the action $\mathcal{S}_\sigma + \mathcal{S}_F$ under the global rotation of the matrix Q:

$$Q(\mathbf{r}) \to e^{i\hat{\chi}} Q(\mathbf{r}) e^{-i\hat{\chi}}, \quad \hat{\chi} = \sum_{\alpha n} \chi_n^\alpha I_n^\alpha. \tag{27}$$

Here, χ_n^α is the unit matrix in the spin-valley space. By virtue of Eq. (26), it is convenient to introduce the triplet interaction parameter $\gamma=\Gamma_2/z$ such that $\Gamma=(1+\gamma)z/4$. We notice that the triplet interaction parameter is related to F_0^σ as $\gamma=-F_0^\sigma/(1+F_0^\sigma)$.

B. Perturbative expansions

To define the theory for the perturbative expansions we use the "square-root" parametrization

$$Q = W + \Lambda\sqrt{1-W^2}, \quad W = \begin{pmatrix} 0 & w \\ w^\dagger & 0 \end{pmatrix}. \tag{28}$$

The action (16) can be written as an infinite series in the independent fields $w_{n_1 n_2;\tau_1\tau_2}^{\alpha_1\alpha_2,\sigma_1\sigma_2}$ and $w_{n_4 n_3;\tau_1\tau_2}^{\dagger\alpha_1\alpha_2,\sigma_1\sigma_2}$. We use the convention that the Matsubara frequency indices with odd subscripts $n_1, n_3, \ldots$ run over nonnegative integers whereas those with even subscripts $n_2, n_4, \ldots$ run over negative integers. The propagators can be written in the following form:

$$\langle w_{n_1 n_2;\tau_1\tau_2}^{\alpha_1\alpha_2;\sigma_1,\sigma_2}(\mathbf{p}) w_{n_4 n_3;\tau_4\tau_3}^{\dagger\alpha_4\alpha_3;\sigma_4\sigma_3}(-\mathbf{p})\rangle = \frac{16}{\sigma_{xx}}\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\delta_{n_{12},n_{34}}\Bigg[\delta^{\sigma_1\sigma_3}\delta^{\sigma_2\sigma_4}\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4}\left(\delta_{n_1,n_3}D_p(\omega_{12}) - \frac{32\pi T z\gamma}{\sigma_{xx}}\delta^{\alpha_1\alpha_2}D_p(\omega_{12})D_p^t(\omega_{12})\right) + \frac{8\pi T z(1+\gamma)}{\sigma_{xx}}\delta^{\alpha_1\alpha_2}\delta^{\sigma_1\sigma_3}\delta^{\sigma_2\sigma_4}\delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4}D_p^s(\omega_{12})D_p^t(\omega_{12})\Bigg], \tag{29}$$

where $\omega_{12}=\omega_{n_1}-\omega_{n_2}$ and

$$D_p^{-1}(\omega_n)=p^2+\frac{16z\omega_n}{\sigma_{xx}},\quad [D_p^s(\omega_n)]^{-1}=p^2,$$

$$[D_p^t(\omega_n)]^{-1}=p^2+\frac{16(z+\Gamma_2)\omega_n}{\sigma_{xx}}. \quad (30)$$

C. Relation of $z_{s,v}$ with z and γ

The dynamical spin susceptibility $\chi_s(\omega,\mathbf{p})$ can be obtained from[5]

$$\chi_s(i\omega_n,\mathbf{p})=\chi_s^0-Tz_s^2\langle \mathrm{tr}\, I_n^\alpha\sigma_z Q(\mathbf{p})\mathrm{tr}\, I_{-n}^\alpha\sigma_z Q(-\mathbf{p})\rangle \quad (31)$$

by the analytic continuation to the real frequencies, $i\omega_n\to\omega+i0^+$. A similar expression is valid for the valley susceptibility. Evaluating Eq. (31) in the tree level approximation with the help of Eqs. (29), we obtain

$$\chi_s(i\omega_n,\mathbf{p})=\frac{2z_s}{\pi}\left(1-\frac{16z_s\omega_n}{\sigma_{xx}}D_p^t(\omega_n)\right). \quad (32)$$

In the case $\Delta_s=\Delta_v=0$ the total spin is conserved, i.e., $\chi(\omega,\mathbf{p}=\mathbf{0})=0$. In order to be consistent with this physical requirement, the relation

$$z_s=z+\Gamma_2\equiv z(1+\gamma) \quad (33)$$

should hold. Similarly, the total valley isospin conservation guarantees that

$$z_v=z+\Gamma_2\equiv z(1+\gamma). \quad (34)$$

Being related to the conservation laws, Eqs. (33) and (34) are valid also for the observables

$$z_s'=z_v'=z'(1+\gamma'). \quad (35)$$

Therefore, the three physical observables σ_{xx}', z', and γ' completely determine the renormalization of the theory (16) at short length scales $L\ll L_s,L_v$.

D. One-loop renormalization group equations

As is shown in Ref. 10, the standard one-loop analysis for the action $\mathcal{S}_\sigma+\mathcal{S}_F$ yields the following renormalization group functions that determine the zero-temperature behavior of the observable parameters when the length scale L changes:

$$\frac{d\sigma_{xx}}{d\xi}=\beta_\sigma=-\frac{2}{\pi}[1+15f(\gamma)], \quad (36)$$

$$\frac{d\gamma}{d\xi}=\beta_\gamma=\frac{(1+\gamma)^2}{\pi\sigma_{xx}}, \quad (37)$$

$$\frac{d\ln z}{d\xi}=\gamma_z=\frac{15\gamma-1}{\pi\sigma_{xx}}. \quad (38)$$

Here, $f(\gamma)=1-(1+\gamma^{-1})\ln(1+\gamma)$, $\xi=\ln L/l$, and we omit primes for brevity. Physically, the microscopic length l is the mean free path length. It is the length at which the bare parameters of the action (16) are defined. The renormalization group equations (36)–(38) are valid at short length scales $L\ll L_s,L_v$.

As is well known,[10] the solution of the RG equations (36) and (37) yields the dependence of the resistivity $\rho=1/\pi\sigma_{xx}$ on ξ, which has a maximum point and $\gamma(\xi)$ dependence that monotonically increases with ξ.

IV. SU(2)×SU(2) SYMMETRY CASE

A. Effective action

In this and the next sections we assume that the spin splitting is much larger than the valley splitting, $\Delta_s\gg\Delta_v$. Then, at intermediate length scales $L_s\ll L\ll L_v$ the symmetry-breaking term $\mathcal{S}_{\mathrm{sb}}$ becomes important. In the quadratic approximation it reads

$$\mathcal{S}_{\mathrm{sb}}=\frac{iz_s\Delta_s}{2}\int d\mathbf{r}\sum_{n_j,\tau_j}^{\alpha_j,\sigma_j}(\sigma_2-\sigma_1)w_{n_1n_2;\tau_1\tau_2}^{\alpha_1\alpha_2;\sigma_1\sigma_2}\bar{w}_{n_2n_1;\tau_2\tau_1}^{\alpha_2\alpha_1;\sigma_2\sigma_1}. \quad (39)$$

The symmetry-breaking term modifies the propagators in Eq. (29), e.g., $D_q^t(\omega_n)$ becomes

$$[D_q^t(\omega_n,\sigma_1,\sigma_2)]^{-1}=[D_q^t(\omega_n)]^{-1}+i\frac{8z_s\Delta_s}{\sigma_{xx}}(\sigma_1-\sigma_2). \quad (40)$$

Hence, the diffusive pole for the modes in $Q_{nm;\tau_1\tau_2}^{\alpha\beta;\sigma_1\sigma_2}$ with $\sigma_1\neq\sigma_2$ is cut off at $q\sim L_s^{-1}$. Therefore, these modes are negligible at length scales $L\gg L_s$. As a result, Q becomes a diagonal matrix in the spin space. Then, according to Eqs. (31) and (32), the spin susceptibility has no renormalization on these length scales, i.e.,

$$\frac{d\chi_s}{d\xi}=\frac{dz_s}{d\xi}=0,\quad L_s\ll L\ll L_v. \quad (41)$$

The absence of the spin susceptibility renormalization at length scales $L\gg L_s$ can be explained as follows. On the one hand, χ_s is renormalized only due to the diffusion modes with the total spin projection $S_z=\pm1$ in the particle-hole channel. On the other hand, these modes cannot typically diffuse over distances larger than L_s and, therefore, they cannot lead to infrared singularities in the spin susceptibility.

Let us denote $Q_{nm;\tau_1\tau_2}^{\alpha\beta;\pm1\pm1}=[Q_{nm;\tau_1\tau_2}^{\alpha\beta}]_\pm$. Then, the action (16) becomes $\mathcal{S}=\mathcal{S}_\sigma+\mathcal{S}_F+\mathcal{S}_{\mathrm{vb}}$ where

$$\mathcal{S}_\sigma=-\frac{\sigma_{xx}}{32}\sum_{\sigma=\pm}\int d^2\mathbf{r}\,\mathrm{tr}(\nabla Q_\sigma)^2 \quad (42)$$

and

$$\mathcal{S}_F=4\pi Tz\sum_\sigma\int d^2\mathbf{r}\,\mathrm{tr}\,\eta(Q_\sigma-\Lambda)$$
$$+\pi T\int d^2\mathbf{r}\sum_{\alpha n}\sum_{\sigma_1,\sigma_2=\pm}\Gamma_{\sigma_1\sigma_2}\,\mathrm{tr}\,I_n^\alpha Q_{\sigma_1}\,\mathrm{tr}\,I_{-n}^\alpha Q_{\sigma_2}$$
$$-\pi T\Gamma_2\int d^2\mathbf{r}\sum_{\alpha n}\sum_{\sigma=\pm}(\mathrm{tr}\,I_n^\alpha Q_\sigma)\otimes(\mathrm{tr}\,I_{-n}^\alpha Q_\sigma). \quad (43)$$

Now, the symbol tr stands for the trace over the replicas, the

Matsubara frequencies, and the valley indices, whereas Tr $=\int d^2\mathbf{r}$ tr. The action (43) corresponds to the following low-energy part of the Hamiltonian describing electron-electron interactions:

$$\mathcal{H}_{\text{int}} = \frac{1}{2}\int d\mathbf{r}\left(\sum_{\sigma_1,\sigma_2} \rho^{\sigma_1}\Gamma_s^{\sigma_1\sigma_2}\rho^{\sigma_2} + m^a\Gamma_t m^a\right),$$

$$\rho^{\sigma} = \sum_{\tau} \bar{\psi}^{\sigma}_{\tau}\psi^{\sigma}_{\tau}, \quad m^a = \sum_{\sigma\tau\tau'} \bar{\psi}^{\sigma}_{\tau}(t^a)^{\sigma\sigma}_{\tau\tau'}\psi^{\sigma}_{\tau'}. \tag{44}$$

It is worth mentioning that Eq. (44) is in agreement with the ideas of Refs. 34 and 35.

The symmetry-breaking part reads

$$\mathcal{S}_{\text{sb}} = iz_v\Delta_v \sum_{\sigma=\pm} \int d^2\mathbf{r} \text{ tr } \tau_z Q_{\sigma}. \tag{45}$$

At length scales $L \sim L_s$, the couplings $\Gamma_{\sigma_1\sigma_2}$ are all equal to each other, $\Gamma_{\sigma_1\sigma_2}(L\sim L_s)=\Gamma=(z+\Gamma_2)/4$. However, the symmetry allows the following matrix structure of $\hat{\Gamma}$:

$$\hat{\Gamma} = \begin{pmatrix} \Gamma_{++} & \Gamma_{+-} \\ \Gamma_{+-} & \Gamma_{++} \end{pmatrix}. \tag{46}$$

As we shall see below, this matrix structure is consistent with the renormalization group. Physically, Γ_{++} and Γ_{+-} describe interactions between electrons with the same and opposite spins, respectively.

The action (42) and (43) is invariant under the global rotations $[Q^{\alpha\beta}_{nm;\tau_1\tau_2}]_{\sigma}(\mathbf{r}) \to u_{\sigma}^{\tau_1\tau_3}[Q^{\alpha\beta;}_{nm;\tau_3\tau_4}]_{\sigma}(\mathbf{r})[u^{-1}]_{\sigma}^{\tau_4\tau_2}$ in the valley space for $u_{\sigma}\in \text{SU}(2)$. In order to preserve the invariance under the global rotations

$$Q_{\pm}(\mathbf{r}) \to e^{i\hat{\chi}}Q_{\pm}(\mathbf{r})e^{-i\hat{\chi}}, \quad \hat{\chi} = \sum_{\alpha n}\chi_n^{\alpha}I_n^{\alpha}, \tag{47}$$

where χ_n^{α} is the unit matrix in the valley space, the following relation has to be satisfied:

$$z + \Gamma_2 - 2\Gamma_{++} = 2\Gamma_{+-}. \tag{48}$$

Physically, this equation corresponds to the particle number conservation and is completely analogous to Eq. (26).

B. Perturbative expansions

In order to resolve the constraint $Q_{\pm}^2=1$ we use the square-root parametrization

$$Q_{\pm} = W_{\pm} + \Lambda\sqrt{1-W_{\pm}^2}. \tag{49}$$

Then, the action (42) and (43) determines the propagators as follows:

$$\langle [w^{\alpha_1\alpha_2;\tau_1,\tau_2}_{n_1n_2}(q)]_{\sigma}[w^{\dagger\alpha_4\alpha_3;\tau_4\tau_3}_{n_4n_3}(-q)]_{\sigma'}\rangle = \frac{32}{\sigma_{xx}}\hat{\mathcal{D}}_{\sigma\sigma'}, \tag{50}$$

where

$$\begin{aligned}\hat{\mathcal{D}} = \delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\delta_{n_{12},n_{34}}\Big(&\delta_{n_1,n_3}\delta^{\tau_1\tau_3}\delta^{\tau_2\tau_4}D_q(\omega_{12},\tau_1,\tau_2) \\ &- \frac{32\pi T}{\sigma_{xx}}\Gamma_2\delta^{\alpha_1\alpha_2}\delta^{\tau_1\tau_3}\delta^{\tau_2\tau_4}D_q(\omega_{12},\tau_1,\tau_2)D_q^t(\omega_{12},\tau_1,\tau_2) \\ &+ \frac{32\pi T}{\sigma_{xx}}\hat{\Gamma}\delta^{\alpha_1\alpha_2}\delta^{\tau_1\tau_2}\delta^{\tau_3\tau_4}\hat{D}_q^s(\omega_{12})D_q^t(\omega_{12})\Big)\end{aligned} \tag{51}$$

with

$$[\hat{D}_q^s(\omega_n)]^{-1} = q^2 + \frac{16}{\sigma_{xx}}(z+\Gamma_2-2\hat{\Gamma})\omega_n, \tag{52}$$

$$D_q^{-1}(\omega_n,\tau_1,\tau_2) = D_q^{-1}(\omega_n) + i\frac{8z_v\Delta_v}{\sigma_{xx}}(\tau_1-\tau_2), \tag{53}$$

$$[D_q^t(\omega_n,\tau_1,\tau_2)]^{-1} = [D_q^t(\omega_n)]^{-1} + i\frac{8z_v\Delta_v}{\sigma_{xx}}(\tau_1-\tau_2). \tag{54}$$

In the same way as in Sec. III C, the conservation of the total valley isospin guarantees the relation $z_v=z+\Gamma_2$. The conservation of the z component of the total spin, $\rho^+-\rho^-$, implies that $z_s=4\Gamma_{+-}$ [see Eq. (31)]. Therefore,

$$\frac{d\ln\Gamma_{+-}}{d\xi} = 0 \tag{55}$$

for the length scales $L_s \ll L \ll L_v$. Equation (55) originates from the same mechanism as the absence of the renormalization for the spin susceptibility [Eq. (41)]. As one can check, the renormalization of Γ_{+-} involves the diffusion modes with the total spin projection $S_z=\pm 1$ in the particle-hole channel. However, the diffusion pole in these modes is cut off at Δ_s. Therefore, they cannot lead to infrared singularities for the Γ_{+-} coupling.

C. One-loop approximation

Evaluation of the conductivity according to Eq. (23) in the one-loop approximation yields

$$\begin{aligned}\sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^8\pi}{\mathbb{D}\sigma_{xx}}\int_p p^2 T\sum_{\omega_m>0}\min\left\{\frac{\omega_m}{\omega_n},1\right\} \\ \times D_p^t(\omega_m)D_p(\omega_m+\omega_n) \\ \times\left(\sum_{\sigma=\pm}[\hat{\Gamma}\hat{D}_p^s(\omega_m)]_{\sigma\sigma} - 4\Gamma_2 D_p(\omega_m)\right).\end{aligned} \tag{56}$$

Hence, we find

$$\begin{aligned}\sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^7\pi}{\mathbb{D}\sigma_{xx}}\int_p p^2 T\sum_{\omega_m>0}\min\left\{\frac{\omega_m}{\omega_n},1\right\} \\ \times D_p(\omega_m)D_p(\omega_m+\omega_n)[zD_p^s(\omega_m) - 6\Gamma_2 D_p^t(\omega_m) \\ -(z+2\Gamma_2-4\Gamma_{++})\tilde{D}_p^t(\omega_m)],\end{aligned} \tag{57}$$

where

$$[\tilde{D}_q^t(n)]^{-1} = q^2 + \frac{64}{\sigma_{xx}}\omega_n\Gamma_{+-}. \tag{58}$$

Performing the analytic continuation to the real frequencies, $i\omega_n \to \omega + i0^+$, in Eq. (57), one obtains the dc conductivity in the one-loop approximation:

$$\sigma'_{xx} = \sigma_{xx} - \frac{2^8\pi}{\mathbb{D}\sigma_{xx}}\int_p p^2 \int_0^\infty d\omega\, D_p^2(\omega)[zD_p^s(\omega) - 6\Gamma_2 D_p^t(\omega) - (z + 2\Gamma_2 - 4\Gamma_{++})\tilde{D}_p^t(\omega)]. \tag{59}$$

In order to compute z' and z'_v we have to evaluate the thermodynamic potential Ω in the presence of the finite valley splitting Δ_v. In the one-loop approximation we find

$$T^2\frac{\partial \Omega/T}{\partial T} = 8N_r T\sum_{\omega_n>0}\omega_n\Bigg[z + \frac{4}{\sigma_{xx}}\int_p (2\Gamma_{+-}\tilde{D}_p^t(\omega_n) - (z+\Gamma_2)D_p^t(\omega_n) + (z+\Gamma_2)\sum_{\tau_1,\tau_2} D_p^t(\omega_n,\tau_1,\tau_2) - z\sum_{\tau_1,\tau_2} D_p(\omega_n,\tau_1,\tau_2))\Bigg]. \tag{60}$$

Following the definitions (24) and (25) of the physical observables, we obtain from Eq. (60)

$$z' = z + \frac{8}{\sigma_{xx}}(2\Gamma_2 - \Gamma_{++})\int_p D_p(0) \tag{61}$$

and

$$z'_v = z_v\left[1 + 4\pi\left(\frac{16}{\sigma_{xx}}\right)^3 (z+\Gamma_2)T\sum_{\omega_n>0}\omega_n \int_p \times[zD_p^3(\omega_n) - (z+\Gamma_2)D_p^{t3}(\omega_n)]\right]. \tag{62}$$

We mention that the results (59), (61), and (62) can be obtained with the help of the background field procedure[36] applied to the action (42) and (43).

D. One-loop RG equations

Using the standard method,[37] we derive from Eqs. (59), (61), and (62) one-loop results for the RG equations which determine the $T=0$ behavior of the physical observables when the length scale L changes. It is convenient to define $\gamma_v = \Gamma_2/z$ and $\gamma_s = -1 + 4\Gamma_{+-}/z$. Then, for $\mathbb{D}=2$ we obtain

$$\frac{d\sigma_{xx}}{d\xi} = -\frac{2}{\pi}[1 + 6f(\gamma_v) + f(\gamma_s)], \tag{63}$$

$$\frac{d\gamma_v}{d\xi} = \frac{1+\gamma_v}{\pi\sigma_{xx}}(1 + 2\gamma_v - \gamma_s), \tag{64}$$

$$\frac{d\gamma_s}{d\xi} = \frac{1+\gamma_s}{\pi\sigma_{xx}}(1 - 6\gamma_v - \gamma_s), \tag{65}$$

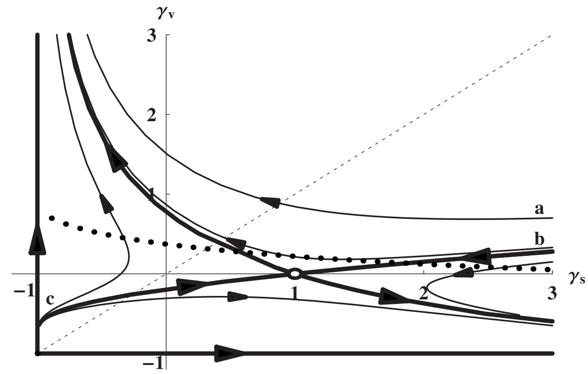

FIG. 1. Projection of the RG flow in the three-dimensional parameter space $(\sigma_{xx}, \gamma_v, \gamma_s)$ onto the (γ_v, γ_s) plane for the SU(2)×SU(2) symmetry case [Eqs. (63)–(65)]. Dots denote the line at which $1+6f(\gamma_v)+f(\gamma_s)=0$. The dashed line indicates the line $\gamma_v=\gamma_s$ (see text).

$$\frac{d\ln z}{d\xi} = -\frac{1}{\pi\sigma_{xx}}(1 - 6\gamma_v - \gamma_s). \tag{66}$$

Equations (63)–(66) constitute one of the main results of the present paper and describe the system at the intermediate length scales $L_s \ll L \ll L_v$. We mention that the length scale l involved in $\xi = \ln L/l$ is now of the order of L_s.

In Fig. 1 we present the projection of the RG flow in the three-dimensional parameter space $(\sigma_{xx}, \gamma_v, \gamma_s)$ onto the (γ_v, γ_s) plane. There is an unstable fixed point at $\gamma_v=0$ and $\gamma_s=1$. However, for the physical system considered the fixed point is inaccessible since an initial point of the RG flow is always situated near the line $\gamma_v=\gamma_s$. As shown in Fig. 2, there are possible three distinct types of $\rho(\xi)$ behavior for such initial points. Along the RG flow line a (Fig. 1) that crosses the curve d described by the equation $1+6f(\gamma_v)+f(\gamma_s)=0$ the resistance demonstrates metallic behavior: ρ decreases as ξ grows. If we move along the RG flow line b which intersects the curve d twice, then the resistance develops a minimum and a maximum. Finally, the resistance on the RG flow line c which has a single crossing with the curve d has a maximum. Remarkably, in all three cases, the behavior of the resistance is of the metallic type

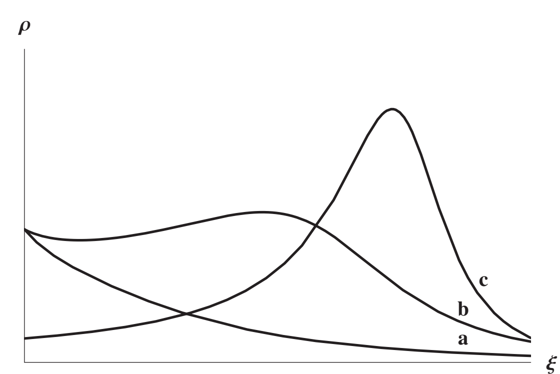

FIG. 2. Schematic dependence of the resistance $\rho=1/(\pi\sigma_{xx})$ on ξ. Curves a, b, and c correspond to the flow lines a, b, and c in Fig. 1 (see text).

for relatively large L. The reason for this metallic behavior can be understood from the following arguments. At large ξ, the coupling γ_v flows to large positive values whereas $\gamma_s \to -1$. Then, $\Gamma_{+-}/\Gamma_{++} \sim 1/\gamma_v \ll 1$ and the RG equations (63)–(66) transform into equations for a single-valley system with conductance equal $\sigma_{xx}/2$. The metallic behavior of this system is well known.[5]

V. COMPLETELY SYMMETRY-BROKEN CASE

A. Effective action

At the long length scales $L \gg L_v$ the symmetry-breaking term $\mathcal{S}_{\rm vb}$ becomes important. In the quadratic approximation it reads

$$\mathcal{S}_{\rm vb} = \frac{iz_v\Delta_v}{2}\int d^2\mathbf{r}\sum_{n_j,\tau_j}^{\alpha_j,\sigma}(\tau_2-\tau_1)[w^{\alpha_1\alpha_2}_{n_1n_2;\tau_1\tau_2}]_\sigma[\bar{w}^{\alpha_2\alpha_1}_{n_2n_1;\tau_2\tau_1}]_\sigma. \tag{67}$$

Hence, the modes in $[Q^{\alpha\beta}_{nm;\tau_1\tau_2}]_\sigma$ with $\tau_1 \neq \tau_2$ acquire a finite mass of the order of $z_v\Delta_v$. Therefore, they are negligible at long length scales $L \gg L_v$. As a result, the matrix Q becomes diagonal in the valley isospin space. The valley susceptibility remains constant under the action of the renormalization group on these length scales:

$$\frac{dz_v}{d\xi} = 0, \quad L \gg L_v. \tag{68}$$

The physical origin of Eq. (68) is analogous to the absence of the renormalization of the spin susceptibility for $L \gg L_s$ [cf. Eq. (41)]. Let us define

$$Q^{\alpha\beta}_j = \{[Q^{\alpha\beta}_{11}]_+,[Q^{\alpha\beta}_{-1-1}]_+,[Q^{\alpha\beta}_{11}]_-,[Q^{\alpha\beta}_{-1-1}]_-\}. \tag{69}$$

Then the action $\mathcal{S}=\mathcal{S}_\sigma+\mathcal{S}_F$ reads

$$\mathcal{S}_\sigma = -\frac{\sigma_{xx}}{32}\sum_j\int d^2\mathbf{r}\,\mathrm{tr}(\nabla Q_j)^2 \tag{70}$$

and

$$\mathcal{S}_F = \pi T\int d^2\mathbf{r}\sum_{j,k}\sum_{\alpha n}\mathrm{tr}\, I^\alpha_n Q_j\hat{\Gamma}_{jk}\,\mathrm{tr}\, I^\alpha_{-n}Q_k + 4\pi Tz\sum_j\int d^2\mathbf{r}\,\mathrm{tr}\,\eta Q_j. \tag{71}$$

The couplings $\hat{\Gamma}_{jk}$ are elements of the following 4×4 matrix:

$$\hat{\Gamma} = \begin{pmatrix} \Gamma_{++}-\Gamma_2 & \tilde{\Gamma}_{++} & \tilde{\Gamma}_{+-} & \Gamma_{+-} \\ \tilde{\Gamma}_{++} & \Gamma_{++}-\Gamma_2 & \Gamma_{+-} & \tilde{\Gamma}_{+-} \\ \tilde{\Gamma}_{+-} & \Gamma_{+-} & \Gamma_{++}-\Gamma_2 & \tilde{\Gamma}_{++} \\ \Gamma_{+-} & \tilde{\Gamma}_{+-} & \tilde{\Gamma}_{++} & \Gamma_{++}-\Gamma_2 \end{pmatrix}. \tag{72}$$

Initially, at the length scale of the order of L_v, the coupling $\tilde{\Gamma}_{++}=\Gamma_{++}$ and $\tilde{\Gamma}_{+-}=\Gamma_{+-}$. However, the more general structure (72) is consistent with the renormalization group. It is worthwhile to mention that if the matrix $\hat{\Gamma}$ is diagonal then the theory (70) and (71) would include four copies of the singlet U(1) theory studied in Refs. 7 and 8. The action (71) corresponds to the following low-energy part of the electron-electron interaction Hamiltonian:

$$\mathcal{H}_{\rm int} = \frac{1}{2}\int d\mathbf{r}\sum_{\sigma\sigma';\tau\tau'}\rho^\sigma_\tau[(\Gamma_s)^{\sigma\sigma'}_{\tau\tau'} + \Gamma_t(t^a)^{\sigma\sigma}_{\tau\tau}(t^a)^{\sigma'\sigma'}_{\tau'\tau'}]\rho^\sigma_\tau,$$

$$\rho^\sigma_\tau = \bar{\psi}^\sigma_\tau\psi^\sigma_\tau. \tag{73}$$

In order to have invariance under the global rotations

$$Q_j \to e^{i\hat{\chi}}Q_je^{-i\hat{\chi}}, \quad \hat{\chi} = \sum_{\alpha n}\chi^\alpha_n I^\alpha_n, \tag{74}$$

the following relation has to be satisfied:

$$z + \Gamma_2 - \Gamma_{++} - \tilde{\Gamma}_{++} = \Gamma_{+-} + \tilde{\Gamma}_{+-}. \tag{75}$$

B. Perturbative expansions

As above, in order to resolve the constraints $Q_j^2=1$, we shall use the square-root parametrization for each Q_j: $Q_j = W_j+\Lambda\sqrt{1-W_j^2}$. Then, the propagators are defined by the theory (70) and (71) as

$$\langle[w^{\alpha_1\alpha_2}_{n_1n_2}(q)]_j[w^{\dagger\alpha_4\alpha_3;\tau_4\tau_3}_{n_4n_3}(-q)]_k\rangle = \frac{32}{\sigma_{xx}}\hat{\mathcal{D}}_{jk}, \tag{76}$$

$$\hat{\mathcal{D}} = \delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\delta_{n_{12},n_{34}}\Bigg(\delta_{n_1,n_3}D_q(\omega_{12}) + \frac{32\pi T}{\sigma_{xx}}\hat{\Gamma}\delta^{\alpha_1\alpha_2}D_q(\omega_{12})\hat{D}^c_q(\omega_{12})\Bigg), \tag{77}$$

where

$$[\hat{D}^c_q(\omega_n)]^{-1} = q^2 + \frac{16}{\sigma_{xx}}(z-\hat{\Gamma})\omega_n. \tag{78}$$

The conservation of the z components of the total spin, $\Sigma_{\sigma\tau}\sigma\rho^\sigma_\tau$, and the total valley isospin, $\Sigma_{\sigma\tau}\tau\rho^\sigma_\tau$, implies [see Eq. (31)] that $z_s=2\Gamma_{+-}+2\tilde{\Gamma}_{+-}$ and $z_v=2\tilde{\Gamma}_{++}+2\tilde{\Gamma}_{+-}$. Since, for $L \gg L_v$ both z_s and z_v are not renormalized, we obtain

$$\frac{d\tilde{\Gamma}_{+-}}{d\xi} = \frac{d\Gamma_{+-}}{d\xi} = \frac{d\tilde{\Gamma}_{++}}{d\xi} = 0. \tag{79}$$

Since both $\tilde{\Gamma}_{+-}$ and Γ_{+-} coincide at the length scales $L \sim L_v$ and they are not renormalized, we shall not distinguish $\tilde{\Gamma}_{+-}$ and Γ_{+-} from here onward. If we introduce γ_s and γ_v such that $\Gamma_{+-}=\tilde{\Gamma}_{+-}=z(1+\gamma_s)/4$ and $\tilde{\Gamma}_{++}=z(1+2\gamma_v-\gamma_s)/4$ then both γ_s and γ_v coincide with the corresponding couplings of the previous sections at the length scales $L \sim L_v$.

C. One-loop approximation

Evaluating the conductivity with the help of Eq. (23) in the one-loop approximation, we find

$$\sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^8\pi}{\mathbb{D}\sigma_{xx}}\int_p p^2 T \sum_{\omega_m>0} \min\left\{\frac{\omega_m}{\omega_n},1\right\} \times D_p(\omega_m+\omega_n)D_p(\omega_m)\sum_j [\hat{\Gamma}\hat{D}^c_p(\omega_m)]_{jj}. \quad (80)$$

Hence,

$$\sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^8\pi z}{\mathbb{D}\sigma_{xx}}\int_p p^2 T \sum_{\omega_m>0} \min\left\{\frac{\omega_m}{\omega_n},1\right\} \times D_p(\omega_m+\omega_n)D_p(\omega_m)[D^s_p(\omega_m) - 2\gamma_v \bar{D}^t_p(\omega_m) - \gamma_s \tilde{D}^t_p(\omega_m)], \quad (81)$$

where

$$[\bar{D}^t_q(\omega_n)]^{-1} = q^2 + \frac{32}{\sigma_{xx}}(\Gamma_{++}+\tilde{\Gamma}_{++})\omega_n. \quad (82)$$

Performing the analytic continuation to the real frequencies in Eq. (82), we find

$$\sigma'_{xx} = \sigma_{xx} - \frac{2^8\pi z}{\mathbb{D}\sigma_{xx}}\int_p p^2 \int_0^\infty d\omega D^2_p(\omega)[D^s_p(\omega) - 2\gamma_v \bar{D}^t_p(\omega) - \gamma_s \tilde{D}^t_p(\omega)]. \quad (83)$$

As in the previous section, in order to compute z' we evaluate the thermodynamic potential in the one-loop approximation. The result is

$$T^2\frac{\partial\Omega/T}{\partial T} = 8TN_r z \sum_{\omega_n>0}\omega_n\left[1 + \frac{8}{\sigma_{xx}}\int_p\left(\frac{(1+\gamma_s)}{2}\tilde{D}^t_p(\omega_n) + (1+\gamma_v)\bar{D}^t_p(\omega_n) - 2D_p(\omega_n)\right)\right]. \quad (84)$$

Hence, we obtain

$$z' = z + \frac{16}{\sigma_{xx}}(\Gamma_2 - \Gamma_{++})\int_p D_p(0). \quad (85)$$

We mention[36] that the results (79), (83), and (85) can be obtained with the help of the background field procedure applied to the action (70) and (71).

D. One-loop RG equations

Equations (79), (83), and (85) allow us to derive the following one-loop results for the renormalization group functions which determine the $T=0$ behavior of the physical observables when the length scale L changes ($\mathbb{D}=2$):

$$\frac{d\sigma_{xx}}{d\xi} = -\frac{2}{\pi}[1+2f(\gamma_v)+f(\gamma_s)], \quad (86)$$

$$\frac{d\gamma_v}{d\xi} = \frac{1+\gamma_v}{\pi\sigma_{xx}}(1-2\gamma_v-\gamma_s), \quad (87)$$

$$\frac{d\gamma_s}{d\xi} = \frac{1+\gamma_s}{\pi\sigma_{xx}}(1-2\gamma_v-\gamma_s), \quad (88)$$

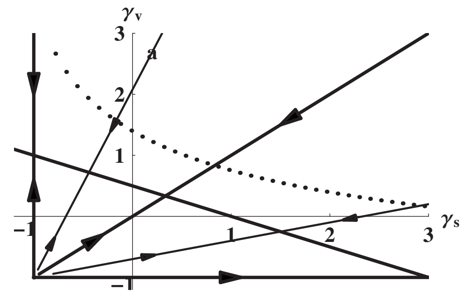

FIG. 3. Projection of the RG flow in the three-dimensional parameter space $(\sigma_{xx},\gamma_v,\gamma_s)$ onto the (γ_v,γ_s) plane for the completely symmetry-broken case [Eqs. (86)–(88)]. Dots denote the line at which $1+2f(\gamma_v)+f(\gamma_s)=0$ (see text).

$$\frac{d\ln z}{d\xi} = -\frac{1}{\pi\sigma_{xx}}(1-2\gamma_v-\gamma_s). \quad (89)$$

The renormalization group equations (86)–(89) constitute one of the main results of the present paper. We mention that the length scale l involved in $\xi=\ln L/l$ is now of the order of L_v and Eqs. (86)–(89) describe the system at the long length scales $L \gg L_v$.

The projection of the RG flow for Eqs. (86)–(88) on the γ_v-γ_s plane is shown in Fig. 3. There exits a line of the fixed points that is described by the equation $2\gamma_v+\gamma_s=1$. If the initial point has large γ_v or γ_s then the RG flow line crosses the curve that is determined by the condition $1+2f(\gamma_v)+f(\gamma_s)=0$. Therefore, the $\rho(\xi)$ dependence along the RG flow line develops a minimum and will be of the insulating type shown in Fig. 4.

VI. DISCUSSION AND CONCLUSIONS

The renormalization group equations discussed above describe the $T=0$ behavior of the observable parameters with changing length scale L. At finite temperatures $T \gg \sigma_{xx}/(zL^2_{\text{sample}})$, where L_{sample} is the sample size, the temperature behavior of the physical observables can be found from the RG equations stopped at the inelastic length L_{in} rather than at the sample size. Formally, it means that one should substitute $\xi_T=\frac{1}{2}\ln\sigma_{xx}/(zTl^2)$ for ξ in the RG equations with ξ_T obeying the following equation:[38]

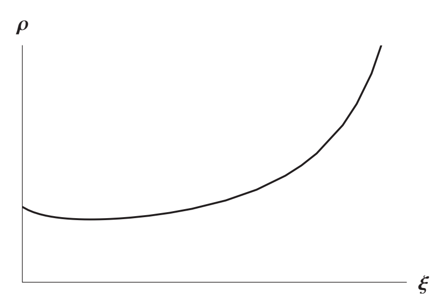

FIG. 4. Schematic dependence of the resistance $\rho=1/(\pi\sigma_{xx})$ on ξ along the flow line a in Fig. 3 (see text).

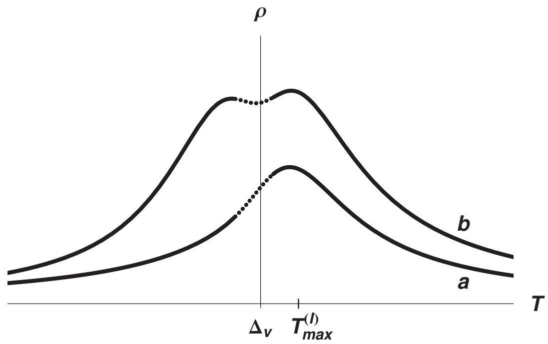

FIG. 5. Schematic $\rho(T)$ dependence in the case of zero parallel magnetic field (see text).

$$\frac{d\xi_T}{d\xi} = 1 - \frac{1}{2}\frac{d \ln z}{d\xi}. \tag{90}$$

Having in mind Eq. (90), we find that the T behavior of the resistivity at $B=0$ is described by Eqs. (36) and (37) for $T \gg \Delta_v$ and Eqs. (63)–(65) with interchanged γ_v and γ_s for $\tau_{so}^{-1}, \tau_v^{-1} \ll T \ll \Delta_v$. The experimental data in Si-MOSFETs suggest that the valley splitting $\Delta_v \lesssim 1$ K and it depends weakly on the electron concentration.[13,14,39] The intervalley scattering rate in Si-MOSFETs has been recently extracted from the data on the weak-localization magnetoresistance.[24] It was found that $1/\tau_v$ is of the order of hundreds of millikelvins. However, the experiments of Ref. 24 were limited to the case of electron concentrations at least four times larger than the critical one. The estimate for the intervalley scattering rate for electron concentrations near the metal-insulator transition is not known at present. The experiments in Si-MOSFETs on the magnetoresistance in parallel magnetic field demonstrate no significant anisotropy for different bias current orientations with respect to the direction of the magnetic field.[40] This indicates a negligible role of the spin-orbit interaction and, therefore, spin-relaxation processes for the two-dimensional electron system in Si-MOSFETs.

In what follows, we assume that $\Delta_v < T^{(I)}_{\max}$ where $T^{(I)}_{\max}$ denotes the temperature of the maximum point that appears in $\rho(T)$ according to the RG equations (36) and (37). For example, in Si-MOSFETs the value of $T^{(I)}_{\max}$ is about several kelvins.[41] Then, depending on the initial conditions at $T \sim 1/\tau$ two types of $\rho(T)$ behavior are possible as is shown in Fig. 5. The curve a represents the typical $\rho(T)$ dependence that was observed in transport experiments on two-valley 2D electron systems in Si-MOS samples[2] and n-type AlAs quantum wells.[42,43] Surprisingly, other behavior with two maximum points is possible, as illustrated by curve b in Fig. 5. So far, this interesting nonmonotonic $\rho(T)$ dependence has been neither observed experimentally nor predicted theoretically. At very low temperatures $T \ll \Delta_v$, the metallic behavior of $\rho(T)$ dominates even in the presence of the valley splitting.

In the presence of a sufficiently low parallel magnetic field $\Delta_s < T^{(I)}_{\max}$, $\rho(T)$ behavior of three distinct types is possible as plotted in Fig. 6. In all three cases, the $\rho(T)$ dependence has a maximum point at temperature $T = T^{(\mathrm{I})}_{\max}$ and is of the insulating type as $T \to 0$. As follows from Fig. 2, in the intermediate temperature range, when T is between Δ_s and Δ_v, the metallic (curve a), insulating (curve b), and nonmonotonic (curve c) types of $\rho(T)$ behavior emerge. As a result, the $\rho(T)$ dependence with two maximum points has to exist in the presence of B.

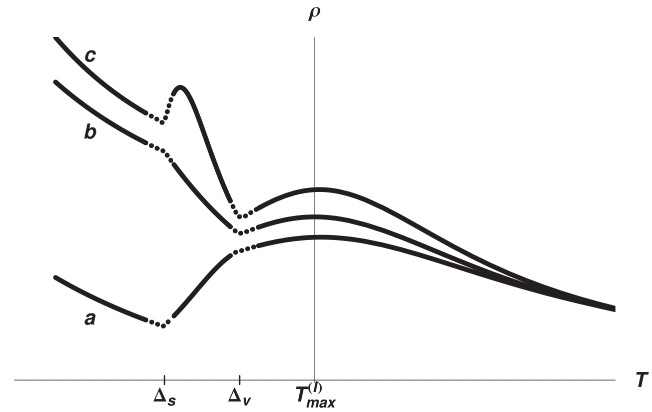

FIG. 6. Schematic $\rho(T)$ dependence in the presence of both spin and valley splitting in the case $\Delta_s < \Delta_v$. For the opposite case, the behavior will be similar (see text).

For high magnetic fields such that $\Delta_s > T^{(\mathrm{I})}_{\max}$, the maximum point at $T = T^{(\mathrm{I})}_{\max}$ is absent, and two types of $\rho(T)$ behavior are possible as is shown in Fig. 7. If $T^{(\mathrm{II})}_{\max} < \Delta_v$, then the dependence of the resistivity is monotonic and insulating; see the curve a in Fig. 7. Here, $T^{(\mathrm{II})}_{\max}$ denotes the temperature of the maximum point that appears in the resistivity in accordance with the RG equations (63) and (64). In the opposite case $T^{(\mathrm{II})}_{\max} > \Delta_v$, a typical $\rho(T)$ dependence is illustrated by the curve b in Fig. 7. Therefore, if the valley splitting is sufficiently large, i.e., $\Delta_v > T^{(\mathrm{II})}_{\max}$, then monotonic insulating behavior of the resistivity appears in a parallel magnetic field which corresponds to $\Delta_s \sim T^{(\mathrm{I})}_{\max}$. This is the case for experiments on the magnetotransport in Si-MOSFETs.[11,41] However, if the valley splitting is small, $\Delta_v < T^{(\mathrm{II})}_{\max}$, then the maximum point of the $\rho(T)$ dependence survives even in high magnetic fields but shifts down to lower temperatures.

In addition to the interesting T dependences of the resistivity, the theory predicts strong renormalization of the electron-electron interaction with temperature. In order to characterize this renormalization, we consider the ratio χ_v/χ_s

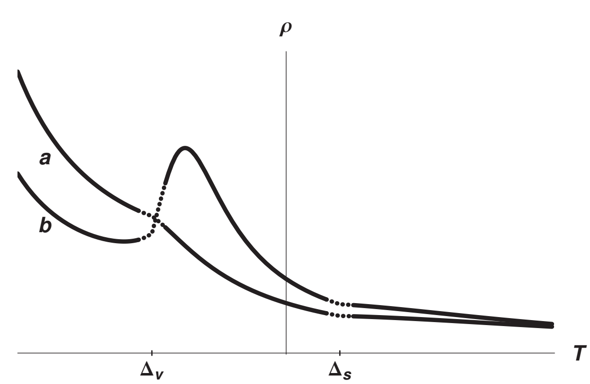

FIG. 7. Schematic $\rho(T)$ dependence in the presence of a strong parallel magnetic field: $\Delta_v, T^{(I)}_{\max} < \Delta_s$ (see text).

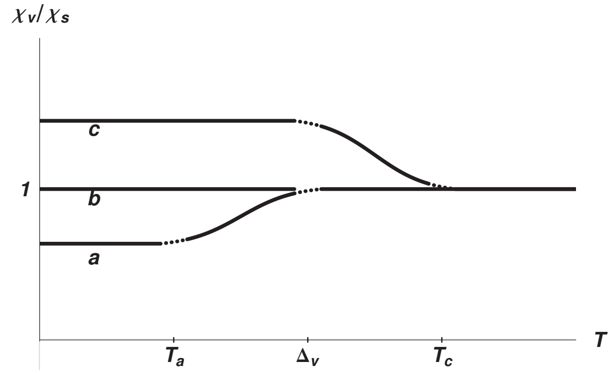

FIG. 8. Schematic dependence of the ration χ_v/χ_s on temperature: (a) for $\Delta_s<\Delta_v$; (b) for $\Delta_s=\Delta_v$; (c) for $\Delta_s>\Delta_v$. The temperature scales $T_{a,c}\equiv\Delta_s$.

of valley and spin susceptibilities. In Fig. 8, we present the schematic dependence of χ_v/χ_s on T for a fixed valley splitting but with varying spin splitting. At high temperatures $T\gg\Delta_v,\Delta_s$, the ratio of the susceptibilities equals unity, $\chi_v/\chi_s=1$. At low temperatures $T\ll\Delta_v,\Delta_s$, we find

$$\frac{\chi_v}{\chi_s}\begin{cases}<1, & \Delta_s<\Delta_v,\\ =1, & \Delta_s=\Delta_v,\\ >1, & \Delta_s>\Delta_v.\end{cases} \tag{91}$$

Therefore, the ratio χ_v/χ_s at $T\to 0$ is sensitive to the ratio Δ_v/Δ_s. This can be used for the experimental determination of the valley splitting in a 2D electron system.

Finally, we recall that we do not consider above the contribution to the one-loop RG equations from the particle-particle (Cooper) channel. As is well known,[18–20] neither the spin splitting nor the valley splitting changes the weak-localization (cooperon) contribution to the RG equations in the one-loop approximation. Therefore, the weak-localization contribution to the RG equations discussed above can be taken into account by the substitution of $1+2$ for 1 in the square brackets of Eqs. (36), (63), and (86). It does not change the qualitative behavior of the resistivity, or the valley and spin susceptibilities discussed above.

To summarize, we have obtained the temperature behavior of such physical observables as the resistivity and spin and valley susceptibilities in 2D electron liquids with two valleys in the vicinity of the MIT and in the presence of both a parallel magnetic field and the valley splitting. First, we found that the metallic behavior of the resistivity at low temperatures survives in the presence of only a parallel magnetic field or the valley splitting. If both spin and valley splitting exist, then the metallic ρ dependence crosses over to insulating behavior at low temperatures. Second, we have predicted the existence of a nonmonotonic dependence of the resistivity at zero and finite magnetic field in which $\rho(T)$ has two maximum points. It would be an experimental challenge to identify this regime.

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