



Deconfined quantum criticality driven by Dirac fermions in SU(2) antiferromagnets

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Quantum electrodynamics in 2+1 dimensions is an effective gauge theory for the so-called algebraic quantum liquids. A new type of such a liquid, the algebraic charge liquid, has been proposed recently in the context of deconfined quantum critical points [R. K. Kaul *et al.*, Nat. Phys. **4**, 28 (2008)]. In this context, by using the renormalization group in $d=4-\varepsilon$ space-time dimensions, we show that a deconfined quantum critical point occurs in a SU(2) system provided the number of Dirac fermion species $N_f \geq 4$. The calculations are done in a representation where the Dirac fermions are given by four-component spinors. The critical exponents are calculated for several values of N_f . In particular, for $N_f=4$ and $\varepsilon=1$ ($d=2+1$), the anomalous dimension of the Néel field is given by $\eta_N=1/3$, with a correlation length exponent $\nu=1/2$. These values considerably change for $N_f > 4$. For instance, for $N_f=6$, we find $\eta_N \approx 0.75191$ and $\nu \approx 0.66009$. We also investigate the effect of chiral symmetry breaking and analyze the scaling behavior of the chiral holon susceptibility, $G_\chi(x) \equiv \langle \bar{\psi}(x)\psi(x)\bar{\psi}(0)\psi(0) \rangle$.

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I. INTRODUCTION

In very simple terms, we can define spinons as elementary excitations carrying spin but no charge. They emerge in two-dimensional Mott insulators as a consequence of electron fractionalization.¹ Similar to quarks in quantum chromodynamics, spinons do not emerge so easily out of a confined state. As a consequence, *explicit* observation of spinons is very difficult. Quarks, although confined, are essential to explain the properties and structure of matter. In a similar way, spinons, in spite of their confinement, seem to be essential in explaining all the remarkable properties of Mott insulators, including the doped ones, which open the way for the understanding of high- T_c superconductors.²

In several systems, spinons may deconfine at a quantum critical point.¹ One prominent example is the critical point separating different types of order in quantum antiferromagnets, for example, in the phase transition between a Néel and a valence-bond solid (VBS) state,³ which is a paramagnetic state breaking lattice symmetries. Spinon deconfinement also occurs in some paramagnetic Mott insulating states where no symmetries are broken, for example, the spin liquid state in pyrochlore antiferromagnets,^{4,5} and in some theories for the underdoped state of high- T_c superconductors.⁶ A remarkable property of deconfined spinons is that they are interacting at low energies (i.e., they are asymptotically interacting in the infrared), in contrast to deconfined quarks, which are free at high energies (asymptotic freedom).⁷ Therefore, the field theory of a deconfined quantum critical point is a highly nontrivial conformal field theory.⁸ This deconfined quantum criticality (DQC) arises due to a destructive interference between the Berry phases of a quantum antiferromagnet and the instantons. This mechanism makes instanton events irrelevant at large distances, which in turn allows the spinons to deconfine. Furthermore, since deconfined spinons are strongly interacting, the critical exponents are nontrivial. Interestingly, the critical exponents following from such a theory have unusual values with respect to those arising from the Landau–Ginzburg–Wilson (LGW) paradigm of phase

transitions.⁸ The difference can already be seen in the mean-field theory. It is easy to see that mean-field theory based on the LGW paradigm gives an anomalous dimension $\eta_N=0$ for the Néel field $\mathbf{S}_i=(-1)^i \mathbf{S} \mathbf{n}_i$, where i is the lattice site, S is the spin, and $\mathbf{n}_i^2=1$. However, this is not the case for a mean-field theory in the DQC scenario. There, the direction field is written in terms of the spinon fields using a CP¹ representation, i.e., $n_{i,a}=\mathbf{z}_i^\dagger \sigma_a \mathbf{z}_i$, $a=1,2,3$, where $\mathbf{z}_i=(z_{i1}, z_{i2})$, and σ_a are the Pauli matrices, and the constraint $\mathbf{n}_i^2=1$ implies $\mathbf{z}_i^\dagger \mathbf{z}_i=1$. In mean-field theory, the dimension of fields is simply given by the dimensional analysis of the action functional. In such a case, this simply gives $\dim[z_{i\alpha}]=(d-2)/2$, where d is the dimension of the space-time. Therefore, the mean-field theory leads to $\dim[n_{i,a}]=(d-2+\eta_N)/2=2 \dim[z_{i\alpha}]$, which implies $\eta_N=d-2$, or $\eta_N=1$ in 2+1 dimensions. Thus, we see that the field theory of deconfined spinons leads to a large anomalous dimension of the Néel field already at the mean-field level. Quantum fluctuations typically reduce this value, but it still remains considerably larger than the prediction of the LGW scenario,^{9,10} whose fluctuation corrections lead to an anomalous dimension $\eta_N \approx 0.03$.¹¹

A deconfined quantum critical point governs a second-order phase transition between different quantum states of matter at zero temperature, the paradigmatic situation in this case being the transition from a Néel state to a VBS. Thus, it is important to check whether the proposed models for DQC actually undergo a second-order phase transition with a large anomalous dimension. In recent years, there has been a lot of activity in this direction. Indeed, some of the proposed models have been extensively checked in large scale Monte Carlo (MC) simulations.^{9,10,12–16} Particularly important are the MC simulations for the well studied $S=1/2$ quantum antiferromagnet with easy-plane anisotropy. Due to its global U(1) symmetry and the additional *local* U(1) symmetry, the model is self-dual.⁹ This model was predicted to exhibit a deconfined quantum critical point.^{8,9} However, recent MC simulations^{12,13} have clearly shown that in this case actually a first-order phase transition takes place. This result is confirmed by a recent renormalization group (RG) analysis.¹⁷

Remarkably, despite the absence of quantum criticality, the destructive interference mechanism between the Berry phases and instantons was shown to work,¹³ so that we can still talk about spinon deconfinement. The situation here is reminiscent of the Z_2 -Higgs theory in three space-time dimensions, although in this case we have a discrete gauge symmetry rather than a continuous one. This model is also self-dual,¹⁸ but is known to undergo a first-order phase transition along the self-dual line.¹⁹ However, the Z_2 -Higgs theory has the peculiarity of having a deconfined phase bounded by two lines of second-order phase transitions that become first order when they are very near to each other and to the self-dual line. At present, it is not known whether a similar behavior for the easy-plane frustrated quantum antiferromagnet is possible.

For the SU(2) case, a MC simulation on a Heisenberg model with hedgehog suppression⁹ seems to support the DQC scenario. Another interesting model is the $S=1$ Heisenberg antiferromagnet with a biquadratic interaction between the spins,²⁰ where numerical evidence for DQC has also been found.¹⁴ Strong evidence for DQC has been reported in Ref. 10, where a model featuring a four-spin interaction was simulated. There, the anomalous dimension of the Néel field was found to be $\eta_N \approx 0.26$. Further MC studies¹⁵ confirm the analysis of Ref. 10, in which a transition from a Néel state to a VBS occurs. There, the obtained value of the anomalous dimension is $\eta_N \approx 0.35$. However, there is a recent paper¹⁶ on the same model where MC simulations are reported to lead to a first-order phase transition.

Due to the inherently nonperturbative character of deconfined quantum critical points, purely analytical and well controlled studies are not easy to perform. For instance, the perturbative RG applied to the SU(N) case can only access a quantum critical point if $N > 182.9$.¹⁷ The smallest value of N , which produces a critical point, $N=183$, leads to an anomalous dimension $\eta_N = 609/671 \approx 0.9076$. In principle, the result of this RG analysis would mean that the phase transition for the SU(2) case is a first-order one. However, it was argued in Ref. 17 that, similarly to the Ginzburg–Landau (GL) superconductor with N complex order parameter fields,²¹ the actual critical behavior at low N and in the strong-coupling limit may be compatible with a second-order phase transition. For a GL model with a single complex order field, this expectation is confirmed both by duality and MC analysis of the model.^{22–28} There are RG studies where low- N critical points were found,^{29–32} but they all have some kind of problem, being either not well controlled or involving *ad hoc* assumptions. The two-loop perturbation in terms of the ε expansion should be, in principle, a well controlled approach.³³ The resummation of the two-loop result³⁰ led to a critical point for $N=1$. However, the unresummed result, although correct, contains unacceptable pathologies, for example, a critical exponent ν for $182.9 < N \leq 200$ larger than the $N \rightarrow \infty$ result at fixed dimension, and the absence of a fixed point for the gauge coupling if $N < 18\varepsilon$. In view of these pathologies, the resummed two-loop result cannot be completely trusted. The ideal approach would be to obtain a resummed three-loop result. Unfortunately, to this order, the only available RG function is the β function for the gauge coupling.³⁴ Anyway, the important argument from Ref. 17 to

be retained here is the following. The existence of a critical value of N in the weak-coupling analysis in $d=4-\varepsilon$ space-time dimensions, above which a critical point exists, is thought of as an indication that in the strong-coupling limit a quantum critical point at lower values of N may exist. This is, of course, a conjecture that must be tested further. Note, however, that the easy-plane case, also studied in Ref. 17, when generalized to a theory with a global $O(N) \times O(N)$ symmetry, does not have any fixed points at nonzero gauge coupling for all values of N , i.e., this theory does not have a critical value of N .

The behavior of the ε expansion for the GL model with a single complex order field considerably improves if the theory is coupled via the gauge field to N_f Dirac fermion species.³⁵ Interestingly, it is not necessary to have a large value of N_f to obtain an infrared stable fixed point. In the context of the present paper, it should be noted that the SU(2) model for deconfined spinons is exactly the same as a GL model with two complex order fields.

In this paper, we will consider the Lagrangian:

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_f, \tag{1}$$

where

$$\begin{aligned} \mathcal{L}_b = & \frac{1}{2e_0^2} (\varepsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 + \sum_{\alpha=1}^{N_b} |(\partial_\mu - iA_\mu)z_\alpha|^2 + r_0 \sum_{\alpha=1}^{N_b} |z_\alpha|^2 \\ & + \frac{u_0}{2} \left(\sum_{\alpha=1}^{N_b} |z_\alpha|^2 \right)^2 \end{aligned} \tag{2}$$

and

$$\mathcal{L}_f = \sum_{\alpha=1}^{N_f} \bar{\psi}_\alpha (\not{\partial} + i\not{A}) \psi_\alpha. \tag{3}$$

The Lagrangian \mathcal{L}_b is precisely the model for deconfined spinons proposed in Ref. 8. In this context, we are primarily interested in the case with $N_b=2$. The Lagrangian \mathcal{L}_b is supposed to govern the universality class of the phase transition between a Néel state and a VBS.

The Lagrangian \mathcal{L}_f contains N_f species of four-component Dirac fermions in $2+1$ Euclidean dimensions.³⁶ The representation of γ matrices in this case is given by

$$\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \tag{4}$$

where σ_1 , σ_2 , and σ_3 are the Pauli matrices.

The Lagrangian \mathcal{L}_f is typical of a so-called algebraic quantum liquid. There are three such quantum liquids: the algebraic spin liquid (ASL),³⁷ the algebraic Fermi liquid (AFL),³⁸ and the recently introduced algebraic charge liquid (ACL).³⁹ Let us briefly mention the differences between these algebraic quantum liquids.

The ASL emerges out of the so-called staggered flux phase⁴⁰ in the large N_f limit of a SU(N_f) Heisenberg antiferromagnet. In this case, the spin operators are written in terms of fermion bilinears such that the Heisenberg model reads

$H = -(J/N_f) \sum_{\langle i,j \rangle} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta}$, with the constraint $f_{i\alpha}^\dagger f_{i\alpha} = N_f/2$. A lattice gauge field arises from the phase fluctuations of the Hubbard–Stratonovich link field $\chi_{ij} = \langle f_{i\alpha}^\dagger f_{j\alpha} \rangle$. This gives rise to a compact U(1) gauge theory. Linearizing around the nodes $\pm(\pi/2, \pi/2)$ of the quasiparticle spectrum $E_{\mathbf{k}} = 2|\chi_0| \sqrt{\cos^2 k_x + \cos^2 k_y}$ and taking the continuum limit lead to the Lagrangian (3). For large enough N_f , the fermionic spinons deconfine^{41–43} and we obtain a Mott insulating state having no broken symmetries, i.e., a spin liquid. For low enough N_f , the spin liquid is no longer stable⁴³ and the chiral symmetry is probably broken due to spinon confinement. However, in this theory, chiral symmetry breaking (CSB) takes place even in the noncompact case.^{36,44} This CSB is associated with the development of an insulating antiferromagnetic state.⁴⁵

The AFL, although having essentially the same Lagrangian \mathcal{L}_f , is physically very different from the ASL. Here, the gauge field is not compact and has a completely different origin.^{38,46} While the ASL is associated with a Mott insulator, the AFL originates from a d -wave superconductor. The Dirac fermions are obtained by identifying the low-energy quasiparticles on the nodes of the d -wave gap. The gauge field follows from the coupling of these quasiparticles to vortices. The way this is done is very subtle and involves the manipulation of singular gauge transformations.³⁸ The fermions in the AFL, just like the ones of the ASL, carry no charge. Interestingly, the number N_f in the AFL is related to the number of layers of the system. For example, a single-layer system has $N_f=2$, while a bilayer system has $N_f=4$. Note that, in this case, there is no need for other gauge fields in the corresponding layers, since the gauge field arises from a vortex-antivortex excitation. Thus, at large distances, a vortex in one layer is always connected with a vortex in the second layer, the same happening for the antivortices. The end result is that Dirac fermions in different layers couple to the same gauge field.⁴⁷ Of course, similarly to the ASL, CSB also occurs here for small enough N_f ,^{46,48} and is once more associated with antiferromagnetism.

In the ACL, a fractional fermionic particle with charge e and no spin, a *holon*, couples to bosonic spinons via a gauge field.³⁹ Therefore, it is necessarily related to the concept of DQC and in this case the whole Lagrangian (1) has to be considered. Note that in contrast to the ASL and the AFL, the Dirac fermions in the ACL carry charge. This state of matter bears some resemblances with earlier ideas on spin-charge separation in the cuprates.⁶ An important difference is that in earlier theories superconductivity is obtained by doping a spin liquid, while in the ACL superconductivity arises by doping an antiferromagnetic state near the deconfined quantum critical point.⁴⁹ Thus, we can either dope a Néel or a VBS state. Like in the two other algebraic quantum liquids, here the chiral symmetry can also be broken. However, since the Dirac fermions now carry charge, we have a situation where charge density waves (CDWs) develop as a result of CSB. It is important to note that the coupling to bosons leads to a reduction in the value of N_f below which the chiral symmetry breaks.⁴⁵

From the above discussion, we see that among the three algebraic quantum liquids, only the ACL cannot exist without the coupling to bosons. The results of this paper will

mainly concern this case. Quantum criticality in U(1) gauge theories with both bosonic and fermionic matter has been also considered in a recent paper.⁵⁰ There, the authors considered a CP^{N_b-1} model coupled to N_f Dirac fermion species and analyzed the model for large N_b and N_f , while keeping N_f/N_b arbitrary. Here, we have followed Ref. 8 and softened the CP^{N_b-1} constraint. This leads to a theory that is tractable in a RG framework in $d=4-\varepsilon$ space-time dimensions. The main advantage of this approach is that we will be able to work with a fixed $N_b=2$ and N_f does not need to be large to obtain a quantum critical point. Part of our results follow directly from Ref. 35, the main difference being that here we are interested in $N_b=2$ rather than $N_b=1$.⁵¹ However, in this paper, we will substantially improve the previous analysis and present many results, which are relevant for the quantum critical behavior of the ACL. One of the main results of this paper will be the calculation of η_N for $N_b=2$ and $N_f \geq 4$ in terms of the crossover exponent ϕ . As shown in Ref. 17, the critical exponent η_N is related to the crossover exponent by the following formula:

$$\eta_N = d + 2(1 - \phi/\nu), \quad (5)$$

where ν is the correlation length exponent. The crossover exponent is related to a mass anisotropy of the Lagrangian. Although the Lagrangian \mathcal{L}_b does not have any mass anisotropy, the correlation function $\langle \mathbf{z}^*(x) \cdot \mathbf{z}(0) \mathbf{z}(x) \cdot \mathbf{z}^*(0) \rangle$ is calculated through the insertion of the operator $z_{\alpha}^* z_{\beta}$. For $\alpha \neq \beta$, this insertion allows us to calculate ϕ .⁵²

The plan of this paper is as follows. In order to explain why a second-order phase transition is easier to obtain when Dirac fermions are included, in Sec. II we will calculate the effective potential associated with the Lagrangian (1). There, we compare the cases with and without fermions. In the absence of fermions, the one-loop effective potential typically describes a first-order phase transition. When fermions are included, the situation changes and we can show that for small N_f a first-order transition takes place, while for larger values of N_f the effective potential features a second-order phase transition. In Sec. III, we proceed with the RG analysis and the calculation of the critical exponents. In Sec. IV, we briefly discuss the effect of CSB and show that it does not affect the quantum critical regime analyzed in Sec. III. We also compute the anomalous dimension of the chiral holon susceptibility. Section V concludes this paper. The Appendix sketches the details of the calculations of the anomalous dimension η_N .

II. EFFECTIVE POTENTIALS AND THE ORDER OF THE PHASE TRANSITION

A. Simple example

In order to illustrate the subtlety involving the determination of the order of the phase transition in models for deconfined spinons, we will consider the following simple Lagrangian for an interacting scalar theory in $d=2+1$ Euclidean dimensions:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{m_0^2}{2}\varphi^2 + \frac{u_0}{4!}\varphi^4. \quad (6)$$

At the mean-field level, the above model obviously exhibits a second-order phase transition with a critical point at $m_0^2=0$. Let us calculate the effective potential for the above model at one-loop order. This is more easily done by writing $\varphi = \bar{\varphi} + \delta\varphi$, where $\bar{\varphi}$ is a constant background field, and integrating out the quadratic fluctuations in $\delta\varphi$ while disregarding the higher order ones. This calculation is very simple, since it just involves a Gaussian integration in $\delta\varphi$. We obtain

$$U_{\text{eff}}(\bar{\varphi}) = \frac{m_0^2}{2}\bar{\varphi}^2 + \frac{u_0}{4!}\bar{\varphi}^4 + \frac{1}{2V} \left[\text{Tr} \ln \left(-\partial^2 + m_0^2 + \frac{u_0}{2}\bar{\varphi}^2 \right) - \text{Tr} \ln(-\partial^2) \right], \quad (7)$$

where V is the (infinite) volume. Explicit evaluation of the trace-log term yields

$$U_{\text{eff}}(\bar{\varphi}) = \frac{1}{2} \left(m_0^2 + \frac{\Lambda u_0}{2\pi^2} \right) \bar{\varphi}^2 + \frac{u_0}{4!} \bar{\varphi}^4 - \frac{1}{12\pi} \left(m_0^2 + \frac{u_0}{2} \bar{\varphi}^2 \right)^{3/2}, \quad (8)$$

where Λ is an ultraviolet cutoff and we have neglected terms, which are independent of $\bar{\varphi}$. The above effective potential can be written in terms of renormalized quantities m^2 and u defined by the normalization conditions $U''_{\text{eff}}(0) = m^2$ and $U'''_{\text{eff}}(0) = u$, where the primes denote derivatives with respect to $\bar{\varphi}$. This leads to

$$m^2 = m_0^2 + \frac{\Lambda u_0}{2\pi^2} \quad (9)$$

and

$$u = u_0 - \frac{3u_0^2}{16\pi m}, \quad (10)$$

where in Eq. (10) we have replaced in the second term m_0^2 with m^2 , since the error involved in this replacement contributes to an order higher than the one being calculated here. Thus, the effective potential becomes

$$U_{\text{eff}}(\bar{\varphi}) = \frac{m^2}{2}\bar{\varphi}^2 + \frac{u_0}{4!}\bar{\varphi}^4 - \frac{1}{12\pi} \left(m^2 + \frac{u_0}{2}\bar{\varphi}^2 \right)^{3/2}. \quad (11)$$

The above potential is typical of a system exhibiting a second-order phase transition. The order parameter is obtained by extremizing the effective potential. It is given by

$$\bar{\varphi}_\pm = \pm \frac{\sqrt{3}}{8\pi u_0} (3u_0^3 - 128\pi^2 m^2 u_0 - \sqrt{9u_0^6 - 512\pi^2 m^2 u_0^4})^{1/2}, \quad (12)$$

and we see that the critical point is given by $m^2=0$. Looking at Eq. (10), we may think that u will get large and negative as the critical point is approached, such that the effective potential would become unstable against a φ^6 interaction. However, this is not the case, since up to one-loop accuracy,

we can write the following RG equation for the dimensionless coupling $\hat{u} = u/(16\pi m)$:

$$m \frac{\partial \hat{u}}{\partial m} = -\hat{u} + 3\hat{u}^2. \quad (13)$$

We see that as $m \rightarrow 0$, the above β function vanishes at the infrared stable fixed point $\hat{u}_* = 1/3$. Thus, u actually does not diverge as the critical point is approached.

B. Effective potential for the SU(2) antiferromagnet

Let us consider now the one-loop effective potential for the Lagrangian (2). Like in the example of a single scalar field theory, all we have to do is to integrate out the Gaussian fluctuations. Note that the Lagrangian is already quadratic in the gauge field, so that the latter can be immediately integrated out exactly. We will consider a nonzero background for the fields z_1 and z_2 , leaving a vanishing background for the remaining $N_b - 2$ fields. Thus, we will have $z_1 = \bar{z}_1 + \delta z_1$, $z_2 = \bar{z}_2 + \delta z_2$, and $z_\alpha = \delta z_\alpha$ for $\alpha \geq 3$. Thus, we have the background spin orientation field $\bar{\mathbf{n}} = \bar{z}_\alpha \boldsymbol{\sigma}_{\alpha\beta} \bar{z}_\beta$ for $N_b = 2$. The result is

$$U_{\text{eff}}^{\text{AF}}(\bar{z}_1, \bar{z}_2) = m^2(|\bar{z}_1|^2 + |\bar{z}_2|^2) + \frac{u_0}{2}(|\bar{z}_1|^2 + |\bar{z}_2|^2)^2 - \frac{\sqrt{2}e_0^3}{3\pi}(|\bar{z}_1|^2 + |\bar{z}_2|^2)^{3/2} - \frac{1}{12\pi} \{ [m^2 + u_0(3|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} + [m^2 + u_0(|\bar{z}_1|^2 + 3|\bar{z}_2|^2)]^{3/2} + 2(N_b - 1)[m^2 + u_0(|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} \}, \quad (14)$$

where m^2 is given by

$$m^2 = r_0 + \left(\frac{N_b + 1}{2} u_0 + \frac{4e_0^2}{3} \right) \frac{\Lambda}{\pi^2}. \quad (15)$$

The third term in Eq. (14) arises from the integration over the gauge field and is reminiscent from the fluctuation-corrected mean-field theory in the GL superconductor.^{21,53} Due to this term, the symmetry will be broken for a $m^2 > 0$, leading in this way to a weak first-order phase transition.²¹ Note that, for $e_0 = 0$, we have a second-order phase transition in the universality class of a $O(2N_b)$ symmetric classical magnetic system in three dimensions.

C. Effective potential for the algebraic charge liquid

Finally, let us consider the one-loop effective potential for the Lagrangian (1). First, we note that the one-loop contribution following from integrating out the Dirac fermions is given by

$$\mathcal{L}_f^{\text{eff}} = \frac{N_f e_0^2}{32} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F_{\mu\nu}, \quad (16)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (note that the above expression is multiplied by a factor of 2 in the case of two-component

Dirac fermions). The above expression simply follows by calculating the one-loop vacuum polarization in 2+1 dimensions. The background fields are chosen in the same way as in Sec. II B. After integrating out the quadratic gauge fluctuations for a fixed spinon background, we obtain the following correction:

$$\delta\mathcal{L}_f^{\text{eff}} = \frac{1}{V} \left\{ \text{Tr} \ln \left[-\partial^2 + \frac{N_f e_0^2}{8} \sqrt{-\partial^2} + 2e_0^2(|\bar{z}_1|^2 + |\bar{z}_2|^2) \right] - \text{Tr} \ln(-\partial^2) \right\}. \quad (17)$$

After evaluating the above trace-log and integrating out the spinon Gaussian fluctuations, we obtain

$$\begin{aligned} U_{\text{eff}}^{\text{ACL}}(\bar{z}_1, \bar{z}_2) = & m^2(|\bar{z}_1|^2 + |\bar{z}_2|^2) + \frac{u_0}{2}(|\bar{z}_1|^2 + |\bar{z}_2|^2)^2 + \frac{N_f e_0^4}{16\pi^2} \left(|\bar{z}_1|^2 + |\bar{z}_2|^2 - \frac{N_f^2 e_0^2}{384} \right) \ln \left[\frac{2e_0^2(|\bar{z}_1|^2 + |\bar{z}_2|^2)}{\Lambda^2} \right] \\ & + \frac{(N_f e_0^2/8)^3 - (5N_f e_0^4/4)(|\bar{z}_1|^2 + |\bar{z}_2|^2) + (128e_0^2/N_f)(|\bar{z}_1|^2 + |\bar{z}_2|^2)^2}{12\pi^2 \sqrt{1 - \frac{512}{N_f^2 e_0^2}(|\bar{z}_1|^2 + |\bar{z}_2|^2)}} \ln \left[\frac{1 - \sqrt{1 - \frac{512}{N_f^2 e_0^2}(|\bar{z}_1|^2 + |\bar{z}_2|^2)}}{1 + \sqrt{1 - \frac{512}{N_f^2 e_0^2}(|\bar{z}_1|^2 + |\bar{z}_2|^2)}} \right] \\ & - \frac{1}{12\pi} \{ [m^2 + u_0(3|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} + [m^2 + u_0(|\bar{z}_1|^2 + 3|\bar{z}_2|^2)]^{3/2} + 2(N_b - 1)[m^2 + u_0(|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} \}, \quad (18) \end{aligned}$$

where m^2 is still given by Eq. (15). Note that the annoying third term of Eq. (14) disappeared once Dirac fermions were included. For $N_f \rightarrow 0$, the above equation reduces to Eq. (14), as it should. Thus, we can expect that for sufficiently small N_f a first-order phase transition occurs. On the other hand, for N_f above some critical value, a second-order phase transition should take place. The leading small N_f correction shifts the mass term and adds a logarithmic correction, which result in

$$\begin{aligned} U_{\text{eff}}^{\text{ACL}}(\bar{z}_1, \bar{z}_2) = & U_{\text{eff}}^{\text{AF}} + \frac{e_0^4 N_f}{48\pi^2} (|\bar{z}_1|^2 + |\bar{z}_2|^2) \\ & \times \left\{ 2 + 3 \ln \left[\frac{2e_0^2(|\bar{z}_1|^2 + |\bar{z}_2|^2)}{\Lambda^2} \right] \right\} + \mathcal{O}(N_f^2). \quad (19) \end{aligned}$$

The phase transition described by the above effective potential is certainly a first-order one.

To see how the second-order phase transition takes place, we perform a large N_f expansion with the limit $N_f \rightarrow \infty$ being taken with $N_f e_0^2$ kept fixed. Up to order $1/N_f$, we have

$$\begin{aligned} U_{\text{eff}}^{\text{ACL}}(\bar{z}_1, \bar{z}_2) = & (m^2 - m_c^2)(|\bar{z}_1|^2 + |\bar{z}_2|^2) + \left(\frac{u_0}{2} - \frac{8e_0^2}{N_f \pi^2} \right) \\ & \times (|\bar{z}_1|^2 + |\bar{z}_2|^2)^2 \\ & - \frac{1}{12\pi} \{ [m^2 + u_0(3|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} \\ & + [m^2 + u_0(|\bar{z}_1|^2 + 3|\bar{z}_2|^2)]^{3/2} + 2(N_b - 1) \\ & \times [m^2 + u_0(|\bar{z}_1|^2 + |\bar{z}_2|^2)]^{3/2} \} + \mathcal{O}\left(\frac{1}{N_f^2}\right), \quad (20) \end{aligned}$$

where

$$m_c^2 = \frac{e_0^4 N_f}{24\pi^2} \left[3 \ln \left(\frac{8\Lambda}{N_f e_0^2} \right) - 1 \right]. \quad (21)$$

The $(|\bar{z}_1|^2 + |\bar{z}_2|^2)^2$ term is reminiscent of the Landau expansion of the dual formulation of the GL model of a superconductor,^{23,54} where a tricritical point was shown to exist. By defining a ‘‘Ginzburg parameter’’ $\kappa^2 = u_0/(2e_0^2)$, we see that the effective potential (20) is stable only for $\kappa^2 > 8/(N_f \pi^2)$, in which case a second-order phase transition occurs. The precise characterization of the change of behavior as a function of N_f will be studied in Sec. III by means of the RG.

III. QUANTUM CRITICAL BEHAVIOR OF THE SU(2) ALGEBRAIC CHARGE LIQUID

The β functions at one-loop order in $d=4-\varepsilon$ dimensions are easily calculated using standard methods.^{11,55} The dimensionless couplings are given as a function of the momentum scale μ by $f = Z_A \mu^{-\varepsilon} e_0^2$ and $g = Z_z \mu^{-\varepsilon} u_0/Z_u$, where Z_A and Z_z are the wave function renormalizations for the gauge field and spinon field, respectively. Z_u is obtained from the vertex function associated with spinon self-interaction. The renormalization factor Z_A is directly obtained from the one-loop vacuum polarization. The β functions $\beta_f \equiv \mu \partial f / \partial \mu$ and $\beta_g \equiv \mu \partial g / \partial \mu$ are given by

$$\beta_f = -\varepsilon f + \frac{4N_f + N_b}{3} f^2 \quad (22)$$

and

$$\beta_g = -\varepsilon g - 6fg + (N_b + 4)g^2 + 6f^2, \quad (23)$$

where f and g were rescaled as $f \rightarrow 8\pi^2 f$ and $g \rightarrow 8\pi^2 g$ in order to remove unnecessary geometrical factors. Note that, at one loop, the number of fermion components N_f appears

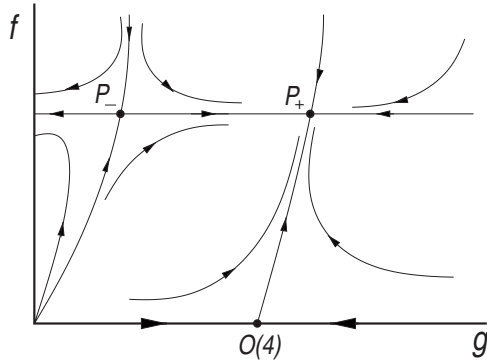


FIG. 1. Schematic flow diagram associated with the β functions (22) and (23) for $N_b=2$ and $N_f \geq 4$. In the figure, $P_{\pm} = (g_{\pm}, f_*)$. Note that only P_+ is infrared stable, thus corresponding to a quantum critical point. For a vanishing gauge coupling, we have a second-order phase transition governed by a $O(4)$ Wilson–Fisher fixed point. This fixed point is unstable for $f \neq 0$.

only in Eq. (22). Of course, for $N_f \rightarrow 0$, the above result coincides with the one for the Ginzburg–Landau superconductor.^{21,30,33}

For $N_b=2$, and provided $N_f \geq 4$, there are two fixed points (g_{\pm}, f_*) at nonzero gauge coupling whose coordinates are given by

$$f_* = \frac{3\varepsilon}{2(2N_f + 1)} \quad (24)$$

and

$$g_{\pm} = \frac{5 + N_f \pm \sqrt{N_f^2 + 10N_f - 56}}{6(2N_f + 1)} \varepsilon. \quad (25)$$

From the above fixed points, only (g_+, f_*) is infrared stable, thus corresponding to the quantum critical point where the critical exponents will be calculated. A schematic flow diagram is shown in Fig. 1. We see that the effect of Dirac fermions on the phase structure is quite remarkable, since neither N_b nor N_f need to be large in order to attain criticality. This behavior is in strong contrast to the one obtained in the limit $N_f \rightarrow 0$, where quantum criticality occurs only for $N_b > 182.9$.¹⁷

The case $N_f=4$ exhibits a peculiar critical behavior, as the calculations of the critical exponents will show. In fact, for $N_f=4$, the fixed points (g_-, f_*) and (g_+, f_*) coincide.

The critical exponent ν is obtained through the calculation of the insertion of the operator $\sum_{\alpha} |z_{\alpha}|^2$ in the correlation function $\langle z_{\alpha}(x) z_{\alpha}^*(y) \rangle$.^{11,55} This leads to singularities, so that we have to introduce a renormalization constant, Z_2 , associated with this operator insertion. In pure scalar theories, Z_2 is very simply calculated by differentiating the two-point function at zero momentum with respect to the mass. However, in general, it is better to perform the renormalization at nonzero momenta in order to avoid certain infrared divergences, especially when the scalar fields are coupled to a gauge field, like the case studied here. The standard way to calculate ν is to compute the following RG function at the infrared stable fixed point:

$$\gamma_2 \equiv \mu \frac{\partial \ln(Z_2/Z_2)}{\partial \mu}, \quad (26)$$

which leads to

$$\frac{1}{\nu} = 2 + \eta_2, \quad (27)$$

where η_2 is the value of γ_2 at the infrared stable fixed point. At one-loop order, we have

$$\gamma_2 = 3f - (N_b + 1)g. \quad (28)$$

By inserting f_* and g_+ for $N_b=2$ in the above equations and expanding up to first order in ε , we obtain

$$\nu = \frac{1}{2} + \frac{N_f - 4 + \sqrt{N_f^2 + 10N_f - 56}}{4(1 + 2N_f)} \varepsilon + \mathcal{O}(\varepsilon^2). \quad (29)$$

The critical exponent η_N is also derived through a quadratic operator insertion. However, in this case, there is a mass anisotropy involved. For a general N_b , the correlation function $\mathcal{G}(x) \equiv \langle \mathbf{n}(x) \cdot \mathbf{n}(0) \rangle$ is given in terms of spinon fields as

$$\mathcal{G}(x) = 2 \langle \mathbf{z}^*(x) \cdot \mathbf{z}(0) \mathbf{z}(x) \cdot \mathbf{z}^*(0) \rangle - \frac{2}{N_b} \langle |\mathbf{z}(x)|^2 |\mathbf{z}(0)|^2 \rangle. \quad (30)$$

For $N_b=2$, we simply have

$$\begin{aligned} \mathcal{G}(x) &= 2 \langle z_1^*(x) z_1(0) z_2(x) z_2^*(0) \rangle + 2 \langle z_1(x) z_1^*(0) z_2^*(x) z_2(0) \rangle \\ &\quad - 2 \langle |z_1(x)|^2 |z_2(0)|^2 \rangle + \langle |z_1(x)|^2 |z_1(0)|^2 \rangle \\ &\quad + \langle |z_2(x)|^2 |z_2(0)|^2 \rangle. \end{aligned} \quad (31)$$

The critical exponent η_N is defined by the behavior of $\mathcal{G}(x)$ at the quantum critical point:

$$\mathcal{G}(x) \sim \frac{1}{|x|^{d-2+\eta_N}}, \quad (32)$$

where η_N is given in terms of the crossover exponent ϕ in Eq. (5).

We also note the following scaling behavior at the quantum critical point:

$$\left\langle \sum_{\alpha} |z_{\alpha}(x)|^2 \sum_{\beta} |z_{\beta}(0)|^2 \right\rangle \sim \frac{1}{|x|^{d-2+\eta_4}}, \quad (33)$$

where

$$\eta_4 = d + 2(1 - 1/\nu). \quad (34)$$

Thus, the scaling dimension of the operator $\sum_{\alpha} |z_{\alpha}(x)|^2$ is given by

$$\dim \left[\sum_{\alpha} |z_{\alpha}(x)|^2 \right] = d - 1/\nu. \quad (35)$$

That η_4 depends only on ν is very easy to see, since it follows from the correlation function between operators $\sum_{\alpha} |z_{\alpha}|^2$, whose singular behavior leads to the renormalization constant Z_2 and consequently to ν . From the expression (29) for the critical exponent ν , we obtain

$$\eta_4 = 2 - \frac{9 + 2\sqrt{N_f^2 + 10N_f - 56}}{1 + 2N_f} \varepsilon + \mathcal{O}(\varepsilon^2). \quad (36)$$

The calculation of η_N depends on the calculation of the crossover exponent ϕ . The calculation is facilitated by observing that the scaling dimension of each component of \mathbf{n} is given by $d - \phi/\nu$. In particular, we have

$$\dim[|z_1|^2 - |z_2|^2] = d - \phi/\nu. \quad (37)$$

Thus, ϕ can be calculated from the insertions of $|z_1|^2$ and $|z_2|^2$ in the correlation function $G_{11}(x) \equiv \langle z_1(x)z_1^*(0) \rangle$.⁵² This calculation is sketched in the Appendix. By denoting those insertions by $G_{11,\alpha}$, where $\alpha=1,2$, we can obtain the crossover exponent from the renormalization of $G_{11,1}(x) - G_{11,2}(x)$. This procedure leads to a renormalization constant Z'_2 and, analogously to Eq. (26), we can define the following RG function:

$$\gamma'_2 \equiv \mu \frac{\partial \ln(Z'_2/Z_z)}{\partial \mu}, \quad (38)$$

which, at the infrared stable fixed point, yields

$$\eta'_2 = \frac{\phi}{\nu} - 2. \quad (39)$$

By comparing the above equation to Eq. (5), we see that $\eta_N = d - 2 - 2\eta'_2$. At one-loop order, we have

$$\gamma'_2 = 3f - g, \quad (40)$$

and therefore

$$\eta_N = 2 + \frac{\sqrt{N_f^2 + 10N_f - 56} - 5N_f - 16}{3(1 + 2N_f)} \varepsilon + \mathcal{O}(\varepsilon^2), \quad (41)$$

while the crossover exponent is given by

$$\phi = 1 + \frac{5(N_f + \sqrt{N_f^2 + 10N_f - 56}) - 11}{12(1 + 2N_f)} \varepsilon + \mathcal{O}(\varepsilon^2). \quad (42)$$

The critical exponents for three space-time dimensions ($\varepsilon=1$) are shown for several values of N_f in Table I. As already mentioned, the case $N_f=4$ is peculiar. Indeed, for $N_f=4$, the critical exponents are rational and, moreover, it corresponds to the only case where $\eta_N < \eta_4$. Note that for $N_f=10$, the exponents are also rational, but in this case $\eta_N > \eta_4$, just like for all of the other values of N_f . Interestingly, we have that $\eta_N > 1$ for $N_f > 10$. This result is consistent with the analysis at large N_f and N_b with N_b/N_f arbitrary.⁵⁰ Indeed, in that case, we find that $\eta_N > 1$ if $N_f/N_b > 3$ (if four-component Dirac fermions are used, the latter inequality becomes $N_f/N_b > 3/2$).

IV. CHIRAL SYMMETRY BREAKING AND CHIRAL SUSCEPTIBILITY

We have already mentioned in the Introduction that CSB constitutes an important physical aspect of both algebraic Fermi and spin liquids. In this section, we want to investigate the effect of CSB on the ACL.⁵⁶ This is typically a phenom-

TABLE I. Values of the critical exponents for $N_b=2$ and different values of $N_f \geq 4$. We have set $\varepsilon=1$.

N_f	ν	η_N	η_4	ϕ
4	1/2	1/3	1	13/12
5	0.62179	0.61694	0.38929	1.27117
6	0.66009	0.75191	0.33468	1.3245
7	0.68229	0.84305	0.3417	1.35381
8	0.69678	0.90943	0.36696	1.37208
9	0.70689	0.96007	0.39749	1.38429
10	5/7	1	3/7	39/28
11	0.71988	1.0323	0.45837	1.39907
20	0.73978	1.17336	0.64274	1.41792
50	0.74817	1.27148	0.83646	1.42103

enon that takes place at low energies. In the case of AFL and ASL, it occurs for $|p| \ll N_f e_0^2$.^{36,44} For the ACL, this roughly corresponds to $|p| \ll \alpha \equiv (N_f + N_b/2)e_0^2$. In this regime, the gauge field propagator is dominated by the one-loop vacuum polarization and is given by

$$D_{\mu\nu}(p) \approx \frac{16}{(2N_f + N_b)e_0^2|p|} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \quad (43)$$

where we have used the Landau gauge and assumed a three-dimensional Euclidean space-time. Following the standard literature,^{36,44} we consider the one-loop fermion Green function with a dressed fermionic propagator and where the above gauge field propagator has been inserted. The inverse propagator is given by $G_\psi^{-1}(p) = i\not{p}Z(p) + \Sigma(p)$ and the corresponding self-consistent equation reads

$$G_\psi^{-1}(p) = i\not{p} + \frac{16}{2N_f + N_b} \times \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{\gamma_\mu [\Sigma(k-p) + i(\mathbf{k} - \mathbf{p})Z(k-p)] \gamma_\mu}{[Z^2(k-p)(k-p)^2 + \Sigma^2(k-p)]|k|} - \int \frac{d^3k}{(2\pi)^3} \frac{\mathbf{k}[\Sigma(k-p) + i(\mathbf{k} - \mathbf{p})Z(k-p)]\mathbf{k}}{[Z^2(k-p)(k-p)^2 + \Sigma^2(k-p)]|k|^3} \right\}. \quad (44)$$

Note that the only difference with respect to the standard analysis is the coefficient in front of the curly brackets, which gets modified due to the spinon loop contribution to the vacuum polarization. We will set $N_b=2$, which is the case of physical interest to us. Furthermore, in order to simplify the analysis, we will assume that $Z(p) \approx 1$.^{44,57} After standard manipulations involving the algebra of the gamma matrices, we obtain the self-consistent equation for the self-energy:

$$\Sigma(p) = \frac{32}{2N_f + 1} \int \frac{d^3k}{(2\pi)^3} \frac{\Sigma(k)}{[k^2 + \Sigma^2(k)]|k+p|}. \quad (45)$$

Integrating over the angles, we obtain

$$\begin{aligned}
 \Sigma(p) &= \frac{8}{(2N_f+1)\pi^2|p|} \int_0^\alpha d|k| \frac{|k|\Sigma(k)(|k|+|p|-|k-p|)}{k^2+\Sigma^2(k)} \\
 &= \frac{16}{(2N_f+1)\pi^2|p|} \left[\int_0^{|p|} d|k| \frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)} \right. \\
 &\quad \left. + |p| \int_{|p|}^\alpha d|k| \frac{|k|\Sigma(k)}{k^2+\Sigma^2(k)} \right], \quad (46)
 \end{aligned}$$

which can easily be converted to the differential following equation:

$$\frac{d}{dp} \left[p^2 \frac{d\Sigma(p)}{dp} \right] = - \frac{16}{\pi^2(2N_f+1)} \frac{p^2 \Sigma(p)}{p^2 + \Sigma^2(p)}. \quad (47)$$

The above equation can be linearized in the regime $|p| \gg \Sigma(p)$ where it can be solved. We will omit the details here, since the analysis is well known.⁴⁴ The solution here is only slightly changed due to the spinon degrees of freedom. The result is a dynamically generated mass gap $\Sigma(0) \sim \langle \bar{\psi}\psi \rangle$ of the following form:

$$\Sigma(0) = \alpha \exp \left[- \frac{2\pi}{\sqrt{\frac{64}{\pi^2(2N_f+1)} - 1}} \right]. \quad (48)$$

This mass generation is associated with the development of a chiral condensate, which signals the appearance of holon charge density waves in the system. The above mass gap vanishes for $N_f > N_{\text{ch}}$, where

$$N_{\text{ch}} = \frac{1}{2} \left(\frac{64}{\pi^2} - 1 \right) \approx 2.74, \quad (49)$$

which is smaller than the corresponding value in the absence of spinons.⁵⁸ Thus, CSB seems to occur only in the regime where no quantum critical point is found. Therefore, by accepting the calculations and underlying approximations employed so far in this paper, it seems that CSB does not spoil the quantum critical behavior.

Another correlation function of interest is the chiral susceptibility,

$$G_\chi(x) = \langle \bar{\psi}(x)\psi(x)\bar{\psi}(0)\psi(0) \rangle, \quad (50)$$

whose scaling behavior for a large number of fermion components and 2+1 dimensions has been calculated in Refs. 59 and 60. At the quantum critical point, the chiral susceptibility behaves like

$$G_\chi(x) \sim \frac{1}{|x|^{d-2+\tilde{\eta}_4}}, \quad (51)$$

which defines the critical exponent $\tilde{\eta}_4$. If fluctuations are ignored, we can simply write

$$G_\chi(x) \sim \frac{1}{|x|^{2(d-1)}}, \quad (52)$$

which leads to $\tilde{\eta}_4 = d$.

We can obtain the anomalous dimension $\tilde{\eta}_4$ in 2+1 dimensions for large N_f and N_b , with N_f/N_b arbitrary, directly

from the result of Ref. 60 by the simple replacement $8/N \rightarrow 8/(N_f+N_b/2)$. The result agrees, of course, with the one obtained in Ref. 50, up to a trivial difference related to the fact that the latter reference works with two-component Dirac fermions.

Let us compute now $\tilde{\eta}_4$ for the SU(2) ACL in first order in ε . Similarly to the case of four-spinon correlation functions, the calculation can be done by computing the insertion of the operator $\bar{\psi}\psi$ in the correlation function $G_\psi(x)$. This amounts in computing the scaling dimension of the operator $\bar{\psi}\psi$, i.e.,

$$\dim[\bar{\psi}\psi] = d - 1 - \tilde{\eta}_2, \quad (53)$$

where $\tilde{\eta}_2$ is the fixed point value of the RG function related to mass renormalization. Thus, the easiest way to compute the above scaling dimension is by adding a bare mass term in the Lagrangian \mathcal{L}_f . The calculation of the needed renormalization constant is standard and can be found in quantum field theory textbooks.⁵⁵ We obtain the result $\tilde{\eta}_2 = 3f_*$ at one loop. From Eqs. (51) and (53), we obtain

$$\tilde{\eta}_4 = d - 2\tilde{\eta}_2, \quad (54)$$

and, therefore,

$$\tilde{\eta}_4 = 2 \left[2 - \frac{N_f+5}{2N_f+1} \varepsilon + \mathcal{O}(\varepsilon^2) \right]. \quad (55)$$

In particular, for $N_f=4$ and $\varepsilon=1$, we obtain $\tilde{\eta}_4=2$. In this case, we have that the chiral susceptibility behaves at the quantum critical point as

$$G_\chi(x) \sim \frac{1}{|x|^3}, \quad (56)$$

which leads to a logarithmic behavior in momentum space.

V. CONCLUSION AND DISCUSSION

In this paper, we have studied the quantum critical behavior of a field theory associated with the algebraic charge liquid (abbreviated as ACL throughout this paper), which is a new type of quantum liquid proposed recently in Ref. 39. Our study was made in the framework of the ε expansion. To lowest order, it leads to a quantum critical point provided the number of fermion components $N_f \geq 4$. Since we are considering a representation using four-component spinors, the physically relevant number of fermions is $N_f=2$.⁶¹ Thus, in the framework of the present study, we are unable to find a deconfined quantum critical point for $N_f=2$. However, it is likely that an improved approximation will show that the theory is also critical for $N_f=2$. The point is that the ε expansion for the Lagrangian (2) should, in a nonperturbative framework, exhibit an infrared stable fixed point for all values of $N_b \geq 1$. The ε expansion cannot capture such a strong-coupling regime. At the moment, there is no reliable nonperturbative scheme for this problem. As discussed in the Introduction, a resummed three-loop ε expansion would provide the desirable controlled approximation. The inclusion of Dirac fermions makes the ε expansion more reliable, since they essentially make the gauge coupling weaker. It is re-

markable that in such a case it is not any longer necessary to have a large number of spinons to obtain a quantum critical point. Interestingly, the number of Dirac fermions does not need to be large either. Thus, we were able to explicitly exhibit a deconfined quantum critical point for an SU(2) theory of bosonic spinons.

In order to convincingly show how the Dirac fermions make the full theory more well behaved, let us have a look at the two-loop β function for the gauge coupling. This result can be easily derived by using the earlier two-loop result for the Lagrangian (2),³³ combined with the higher order result for the Lagrangian (3).^{62,63} The result is

$$\beta_f = -\varepsilon f + \frac{N_b + 4N_f}{3} f^2 + 2(N_b + 2N_f) f^3, \quad (57)$$

which has a nontrivial fixed point at

$$f_* = \frac{3\varepsilon}{N_b + 4N_f} \left[1 - \frac{18}{N_b + 4N_f} \varepsilon + \mathcal{O}(\varepsilon^2) \right]. \quad (58)$$

For $N_f=0$, we obtain that $f_* > 0$ only for $N_b > 18\varepsilon$. Furthermore, in this limit, the two-loop result for β_g shows that we still have an infrared stable fixed point only for $N_b > 182.9$. Thus, besides the two-loop result for $N_f=0$ being unable to produce a quantum critical point for low values of N_b , it introduces a difficulty, since a nontrivial fixed point for f no longer exists for all $N_b \geq 1$. However, the situation considerably improves if $N_f \neq 0$. Indeed, in this case, we have that $f_* > 0$ for $N_b > 18\varepsilon - 4N_f$. Thus, for $N_b=2$, we should have $N_f > (9\varepsilon - 1)/2$, which after setting $\varepsilon=1$ becomes $N_f > 4$. While this clearly shows that the introduction of Dirac fermions improves the behavior of the SU(2) antiferromagnet, it is a little bit frustrating to see that at two loop, no quantum critical point for $N_f=4$ is found anymore. However, we should keep in mind that the theory in the absence of fermions is not being considered in a strong-coupling regime. Once this is done in some way, a critical regime even for values as low as $N_f=2$ should be expected to occur. Note that even if such a result is achieved, we may have to face another problem, namely, CSB. The calculation in Sec. IV indicates that CSB leads to gapped fermions for $N_f=2$. If this number is really correct, then there will be no deconfined quantum critical point anyway. Physically, this means that CDW will screen the interspinon interaction in such a way as to make the spinon correlation length finite, and the only possible phase transition, if any, would be a first-order one. Adopting a more optimistic point of view, let us mention that there is strong evidence that the values of N_{ch} calculated using methods like the one we have used in Sec. IV tend to be overestimated.^{64–66} Since the spinons contribute further to reduce the value of N_{ch} , we may hope that the quantum critical point for $N_f=2$ will survive.

Larger values of N_f are physically relevant for multilayer systems. As discussed in the Introduction, this is precisely the case with the AFL.³⁸ However, we must be aware of the fact that in the case of the ACL, additional gauge fields may play a role, so that the analysis made here would have to be accordingly modified.

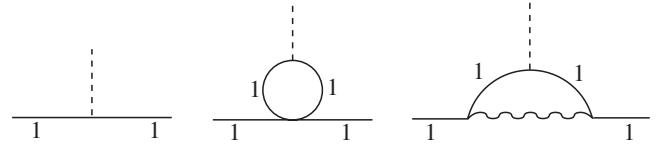


FIG. 2. One-loop Feynman diagrams representing the insertion of the operator $|z_1|^2$ (dashed line) in the correlation function $\langle z_1(x)z_1^*(0) \rangle$. Here, the lines labeled 1 or 2 are associated with the fields z_1 and z_2 , respectively. The wavy line represents the gauge field propagator.

The main achievement of this paper was the computation of the anomalous dimensions of composite operators associated with the quantum critical behavior of the Lagrangian (1). Particularly important was the calculation of the anomalous dimension η_N of the Néel field. The results presented here constitute an explicit and well controlled example of deconfined quantum critical point for a system having SU(2) invariance. The computed values of η_N are substantially larger than the ones that would follow from a LGW analysis, which reflect a typical feature of the DQC scenario.

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APPENDIX: CALCULATION OF η_N

In this appendix, we sketch the calculation of η_N based on the insertion of the operators $|z_1|^2$ and $|z_2|^2$ in the correlation function $G_{11}(x) = \langle z_1(x)z_1^*(0) \rangle$. The calculation follows the method of Ref. 52, i.e., it is based on dimensional regularization in the minimal subtraction scheme.

In Fig. 2, the one-loop Feynman diagrams associated with the insertion of $|z_1|^2$ are shown. For the insertion of $|z_2|^2$, on the other hand, only one diagram is produced at one loop (Fig. 3).

The renormalization constant Z'_2 is determined by the normalization condition in momentum space as follows:

$$Z'_2 [G_{11,1}^{-1}(p_1, p_2; p_3) - G_{11,2}^{-1}(p_1, p_2; p_3)]_{\text{SP}} = 1, \quad (\text{A1})$$

where $p_3 = -(p_1 + p_2)$ and SP denotes the symmetry point defined by

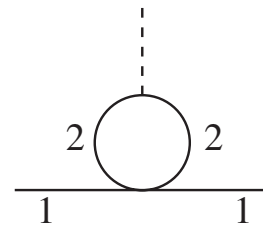


FIG. 3. For the insertion of the operator $|z_2|^2$ (dashed line) in the correlation function $\langle z_1(x)z_1^*(0) \rangle$, only one diagram contributes.

$$p_i \cdot p_j = \frac{\mu^2}{4} (4\delta_{ij} - 1). \quad (\text{A2})$$

We will also need to compute Z_z , which can be determined from the following normalization condition:

$$Z_z \frac{\partial G_{11}^{-1}(p)}{\partial p^2} \Big|_{p^2=\mu^2} = 1. \quad (\text{A3})$$

A straightforward way to understand the insertion of the operators $|z_1|^2$ and $|z_2|^2$ is by adding a source term of the following form:

$$\mathcal{L}_{\text{source}} = r_1(x)|z_1(x)|^2 + r_2(x) \leq f t |z_2(x)|^2 \quad (\text{A4})$$

to the Lagrangian. Thus,

$$G_{11,\alpha}(x;x') = \frac{\delta}{\delta r_\alpha(x')} G_{11}(x). \quad (\text{A5})$$

Note that both Z_z and Z'_2 are gauge dependent. However, this gauge dependence can be exactly calculated from the Ward identities.⁵⁵ It can be shown that Z_z and Z'_2 have both the same gauge dependence to all orders in perturbation theory. Thus, the ratio Z'_2/Z_z is gauge independent, which should be physically expected, since it leads to the anomalous dimension of the gauge invariant operator $|z_1|^2 - |z_2|^2$. A similar argument holds for Z_2/Z_z , which leads to the critical exponent ν . Since Z'_2/Z_z is gauge independent, any gauge can be fixed to calculate Z_z and Z'_2 . Here, we are going to use the Feynman gauge, so that the gauge field propagator is given in imaginary time by

$$D_{\mu\nu}(p) = \frac{\delta_{\mu\nu}}{p^2}. \quad (\text{A6})$$

At the quantum critical point and at one-loop order, we have

$$\begin{aligned} G_{11}^{-1}(p) &= p^2 - e_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{(2p_\mu - k_\mu)(2p_\nu - k_\nu)}{(p-k)^2} D_{\mu\nu}(k) \\ &= p^2 \left(1 - \frac{e_0^2 |p|^{-\varepsilon}}{4\pi^2 \varepsilon} \right) + \text{finite}, \end{aligned} \quad (\text{A7})$$

where we have made use of the following integral:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(p-k)^2 k^2} = \frac{\Gamma(2-d/2)\Gamma^2(d/2-1)}{(4\pi)^{d/2}\Gamma(d-2)} |p|^{d-4} \quad (\text{A8})$$

and the usual properties of dimensional regularization.^{11,55} Next, we use Eq. (A3) to obtain

$$Z_z = 1 + \frac{e_0^2 \mu^{-\varepsilon}}{4\pi^2 \varepsilon}, \quad (\text{A9})$$

which can be rewritten in terms of the renormalized dimensionless coupling up to one-loop accuracy as

$$Z_z = 1 + \frac{2f}{\varepsilon}. \quad (\text{A10})$$

The calculation of Z'_2 is similarly made. In this case, standard manipulations in Eq. (A1) lead once more to integrals of the form (A8). An integral of the following form:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(p_1-k)^2 (p_2+k)^2 k^2}, \quad (\text{A11})$$

also appears in Eq. (A1). However, there is no need to worry with the above integral, since it does not have poles at $\varepsilon=0$. At the end, we obtain the following result:

$$Z'_2 = 1 + \frac{g}{\varepsilon} - \frac{f}{\varepsilon}. \quad (\text{A12})$$

Therefore,

$$\frac{Z'_2}{Z_z} = 1 + \frac{g}{\varepsilon} - \frac{3f}{\varepsilon}, \quad (\text{A13})$$

which leads to Eq. (40).

The renormalization constant Z'_2 can also be obtained by computing $G_{11,\alpha}$, assuming that the sources in Eq. (A4) are independent. In this case, by setting $r_0=0$, we have

$$\begin{aligned} G_{11}^{-1}(p) &= p^2 + r_1 + (2u_0 + e_0^2) \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + r_1} \\ &\quad + u_0 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + r_2} + 2e_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{r_1 - p^2}{[(p-k)^2 + r_1]k^2} \\ &\quad - 2e_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2}. \end{aligned} \quad (\text{A14})$$

Thus,

$$\begin{aligned} G_{11,1}^{-1}(0,p;-p) &= \frac{\partial}{\partial r_1} G_{11}^{-1}(p) \\ &= 1 - (2u_0 + e_0^2) \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + r_1)^2} \\ &\quad + 2e_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(p-k)^2 + r_1]k^2} \\ &\quad - 2e_0^2 p^2 \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(p-k)^2 + r_1]^2 k^2} \end{aligned} \quad (\text{A15})$$

and

$$G_{11,2}^{-1}(0,p;-p) = \frac{\partial}{\partial r_2} G_{11}^{-1}(p) = -u_0 \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + r_2)^2}. \quad (\text{A16})$$

The renormalization constant Z'_2 follows from

$$\lim_{p \rightarrow 0} [G_{11,1}^{-1}(0,p;-p) - G_{11,2}^{-1}(0,p;-p)]|_{r_1=r_2=\mu^2} = Z'_2{}^{-1}, \quad (\text{A17})$$

and the result (A12) is obtained once more.

In order to obtain η_N , we first note that the renormalized

direction field, which is a composite operator, is given by

$$\mathbf{n}_R = \frac{Z'_2}{Z_z} \mathbf{n}. \quad (\text{A18})$$

Since \mathbf{n}_R is finite, its dimension is just given by dimensional analysis, i.e., $\dim[\mathbf{n}_R] = d - 2$. Near the quantum critical point, we have that $Z'_2/Z_z \sim \mu^{\eta'_2}$, where η'_2 is the value of γ'_2 [recall Eq. (38)] at the infrared stable fixed point. Therefore, the scaling dimension of the Néel field is given by

$$\dim[\mathbf{n}] = d - 2 - \eta'_2. \quad (\text{A19})$$

The anomalous dimension η_N is defined by the scaling behavior (32). Thus, we also have that

$$\dim[\mathbf{n}] = \frac{d - 2 + \eta_N}{2}, \quad (\text{A20})$$

and therefore we obtain

$$\eta_N = d - 2 - 2\eta'_2. \quad (\text{A21})$$

Using Eq. (39), we obtain the form given in Eq. (5).

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