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Mobility and its temperature dependence in underdoped $La_{2-x}Sr_xCuO_4$ interpreted as viscous motion of charges

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We argue that charges in underdoped $La_{2-x}Sr_xCuO_4$ move in a dissipative environment of strong spatial and temporal fluctuations. The unusual temperature dependence of the Hall angle known as "the separation of lifetimes" is reinterpreted and attributed to the appearance of the thermally activated component in the effective number of carriers with the temperature increase. We consider the temperature interval above T_c where localization effects can be neglected.

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The normal phase properties of high-temperature superconducting (HT_c) cuprates differ strongly from those of ordinary superconductors. This fact has been correctly attributed to the proximity of new materials to the Mott metal-insulator (MI) transition, where metallic features come about due to doping of external carriers into the CuO_2 plane. Mott's physics has been best traced in the evolution of two parent materials, La_2CuO_4 (LCO, or 214) and $YBa_2Cu_3O_6$ ($YBCO_6$, or 123), from the antiferromagnetic (AFM) insulator state into the HT_c material at doping. La_2CuO_4 doped by the divalent Sr transforms into the single-plane HT_c superconductor, $La_{2-x}Sr_xCuO_4$ (LSCO). $YBCO_{6+y}$ is a classic example of the two-plane HT_c material doped by excessive oxygen.

We concentrate on the transport characteristics of LSCO. Better single crystals have been available for these materials, which have been examined extensively. Early resistivity and Hall effect studies were conducted usually below room temperatures (for a brief review of old results, see, e.g., Ref. 1). Recent data[2] for resistivity and the Hall effect in LSCO now cover the broad range of concentration up to 1000 K. Below, we apply our analysis mostly to those findings.[2]

In this Rapid Communication, we interpret the T dependence of resistivity taking into account the increased number of carriers with temperature.

Transport data for cuprates could not be easy explained in terms of Fermi liquid (FL) theory. Thus, resistivity in optimally doped LSCO has been found to increase linearly with temperature up to 1000 K without any tendency toward saturation.[3] This dependence was interpreted in the framework of the phenomenological marginal Fermi liquid (MFL) theory.[4] According to Ref. 4, all measured quantities are expected to scale with the only possible dimensional parameter, namely the temperature; hence the linear in T dependence of the relaxation rate is $1/\tau \propto T$. A challenge to MFL arose with the observation of an almost quadratic temperature dependence for the Hall angle [more precisely of $\cot(\theta_H) = \rho_{xx}/\rho_{yx}$, where ρ_{xx} and ρ_{yx} are the longitudinal and the transverse resistivity components, respectively].[5] Quadratic T dependence for the Hall angle, as opposed to the T-linear resistivity, seemed to be evidence in favor of two scales in the relaxation processes for carriers.[6] (This controversy is sometimes referred to as "the separation of lifetimes"; e.g., see Ref. 7).

In Ref. 8, the puzzle was attributed to "spin-charge separation," a concept borrowed from the physics of one-dimensional conductors and merely postulated for HT_c cuprates. The quadratic T dependence for $\cot(\theta_H)$ was ascribed in Ref. 9 to small-angle scattering of carriers on dopants, e.g., on the Sr^{2+} ions located far enough from the conducting CuO_2 planes.

Treatment of transport phenomena in metals and semiconductors is based on the fundamental concept of quasiparicles and on the subsequent use of the Boltzmann-like equation. In the MFL theory,[4] the energy spectrum of electronic liquid bears a singular character and the well-defined quasiparticles are absent. MFL scaling[4] does not immediately include small-angle scattering, and the Boltzmann equation approach had to be generalized in Ref. 9.

The electron energy spectrum in cuprates was directly addressed in the angle-resolved photoemission spectroscopy (ARPES) experiments. Consensus is that within the current resolution, well-defined quasiparticle excitations exist only at crossing of the "Fermi surface (FS) locus" along the nodal directions. These regions of the FS are termed "the Fermi arcs," with the arcs' lengths increasing with the increase of temperature. Broad features are seen instead for all other directions (see a summary of recent ARPES findings in Ref. 10 together with a discussion concerning possible implications of the "arcs" to transport properties).

The textbook expression for conductivity is

$$\sigma = ne^2\tau_{\rm tr}/m^*, \tag{1}$$

where m^* is the effective mass and $\tau_{\rm tr}$ is the transport scattering time [Eq. (1) can be equally expressed through the mobility $\mu \equiv \tau/m^*$].

The Hall coefficient R_H in metals and semiconductors must also be derived using the Boltzmann equation. For the parabolic energy spectrum, the well-known result is

$$R_H = 1/nec. \quad (2)$$

R_H preserves its form, Eq. (2), for interacting electrons with the isotropic energy spectrum.[11] In a more general case, however, the expression for R_H would depend on the model. Recall that even for semiconductors with small elliptic pockets, the expression (2) should be multiplied by a factor that depends on the anisotropy of masses. In metals, even the sign of R_H may depend on the FS topology.[12]

Equation (2) becomes exact in the limit of strong magnetic fields.[13] At weaker fields, expression (2) is nothing more than an *estimate for effective number of carriers*. To the best of the authors' knowledge, the only example when R_H in its form (2) measures the exact number of carriers is given by the motion of charged particles in electric and magnetic fields in a viscous media. All the more interesting is that in the case of $La_{2-x}Sr_xCuO_4$, experimentally the number of carriers n calculated as in Eq. (2) coincides exactly with x at small x.[2,14,15]

It is common in the literature to consider peculiarities in transport properties of cuprates above T_c as being due to the nonexistence of quasiparticles in a system of strongly interacting electrons. Strong interactions are of course important in a system near the Mott MI transition, but the view itself does not lead to theoretical understanding. Below we suggest that anomalies in the cuprates' transport may actually stem from some qualitatively different physics. It concerns, first of all, homogeneity of the electronic liquid in cuprates.

Interpretations of the electronic spectra as obtained from the ARPES data, for instance, always implicitly infer that studied samples are homogeneous both in space and time. This is definitely not so. Abstracting from nonhomogeneity caused by external doping, it is now a well-established experimental fact that spatial and temporal fluctuations between nonmagnetic regions and incommensurate antiferromagnetic (ICAFM) regions (known also as "stripes") constitute the ubiquitous feature of the so-called pseudogap (PG) phase on the (T,x) plane for LSCO. At lower temperatures, the two fluctuating phases realize themselves as static SC regions that coexist spatially with ICAFM areas. *Static* phase coexistence is established in the elastic neutron experiments both with[16,17] and without magnetic fields,[18] from the NMR data[19] and from the μSR experiments.[20] The "granular" character of LSCO samples manifests itself in the anomalous "ln T resistivity" at low temperatures for SC suppressed by magnetic fields.[21]

Temporal phases' fluctuations at higher temperatures were seen by the inelastic neutron scattering[22] and in the NMR experiments.[23] Slowing down of the fluctuations at cooling, for instance, was directly traced as "wipe out" of the ^{63}Cu signal at the temperature decrease.[24]

In the complex dynamic regime of fluctuating subphases, kinetic properties cannot be obtained from a Boltzmann-like approach. Note the important role of nonelastic events in such a regime.

The clue to the following analysis is this. As was already emphasized above, for LSCO at small x the number of externally doped holes is known *a priori* and is equal to the number of dopants, Sr^{2+}. The remarkable fact is then that

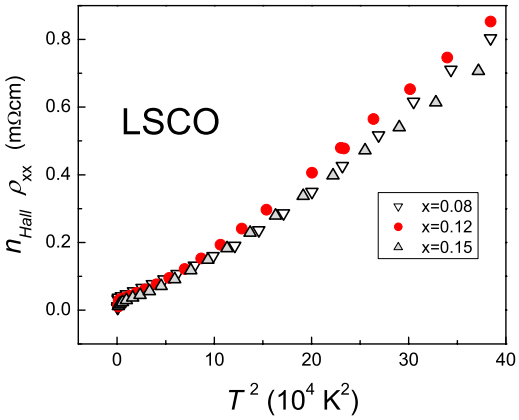

FIG. 1. (Color online) The resistivity multiplied by $n_{\mathrm{Hall}}(T,x)$ from Eq. (3) plotted against T^2 for LSCO at selected x deduced from Ref. 2.

from Eq. (2) and the experimental R_H,[2] one indeed obtains exactly x carriers per Cu site (at small x and temperatures around 100 K).[2,14] It encourages us to add more significance to measurements of the Hall coefficient in LSCO. More specifically, *we assume that from expression* (2) *for* R_H *one obtains the true number of carriers*, n_{eff} (T,x), *at all given* x *and temperature*, T.[14] In accordance with the introductory remarks above, we conclude that *at low* x *and finite* T, *single charges move in a dissipative media*.

Analysis of the Hall data from Ref. 2 performed in Refs. 14 and 15 has shown that the number of holes per Cu atom, $n_{\mathrm{Hall}}(T,x)=nV_{\mathrm{Cu}}$, in LSCO changes with temperature,

$$n_{\mathrm{Hall}}(T,x) = n_0(x) + n_1(x)\exp[-\Delta(x)/T], \quad (3)$$

where $n_0(x)=x$ at low x, n_1 is a constant ($\sim$2.8) but decreases rapidly above $x\sim 0.2$, and V_{Cu} is the unit volume per Cu. Note that the activation character of the T-dependent term in Eq. (3) is the thermodynamic feature.

In the ARPES experiments, one also measures the energy position of the "van Hove flat band" with respect to the chemical potential. The gap, $\Delta(x)$, in Eq. (3) and this energy do coincide, and therefore in Refs. 14 and 15 the activation gap has been interpreted as the ionization energy of coupled electron-hole pairs on Cu-O clusters.

With Eq. (3) in mind, it becomes tempting to extract the proper relaxation rate, $1/\tau(T)$, for a single moving charge by making use of Eq. (1). (At least, at small enough x this quantity should not depend on the holes concentration.) In Fig. 1, we have plotted (for a few concentrations) resistivity, $\rho(T,x)$, multiplied by $n_{\mathrm{Hall}}(T,x)$.[2,14] One sees that three curves at $T<300-400$ K superimpose on each other, the result consistent with the notion of the single charge–carrier mobility.

The T dependence in Fig. 1 is very close to the quadratic law, T^2. One may try to interpret this dependence as the FL behavior of the carriers forming small Fermi pockets. We show that such an interpretation is not correct. Indeed, assuming a Fermi energy, T_F, for such a hypothetical pocket, one may attempt to rewrite $\hbar/\tau$ as

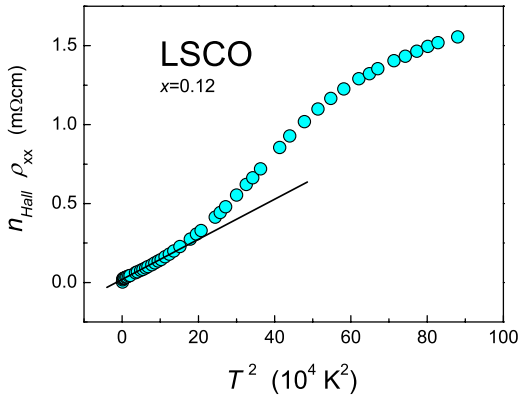

FIG. 2. (Color online) The resistivity multiplied by $n_{\mathrm{Hall}}(T,x)$ plotted vs T^2 for LSCO ($x=0.12$) (deduced from Ref. 2).

$$\hbar/\tau = \mathrm{const} T(m/m^*)(T/T_F) \tag{4}$$

and estimate T_F from Fig. 1. After trivial calculations, one arrives at the value $T_F \sim 120-160$ K [for the effective mass, m^*, we use its optical value $\sim(3-4)m_0$ from Ref. 25]. It is obvious that the FL concept is not applicable: the quadratic dependence on temperature in the FL frameworks is justified only at $T \ll T_F$.

There is no immediate explanation for the T-squared dependence that would follow from the picture of a charge moving along fluctuating subphases. Most probably, the form of Eq. (4) is nothing but a good numerical fit to the data. Note that the quadratic dependence in Fig. 1 is actually the same T^2 dependence that was first observed in old measurements for the Hall angle and comes about according to Eq. (3) after multiplying resistivity by $n_{\mathrm{Hall}}(T,x) \propto 1/R_H$.[26] More recently, the very question of whether the linear and the quadratic T behavior in resistivity and the Hall angle correspondingly, are ubiquitous for cuprates became the subject of debate (see, e.g., Ref. 7).

In Fig. 2, for completeness, we plotted the same value for $x=0.12$ for the whole temperature interval available in Ref. 2. There are two regions of a seemingly quadratic dependence in T, separated by an intermediate temperature range. The transition between the two regimes occurs near the pseudogap temperature, $T^*(x)$, for this concentration. [According to Ref. 14, $T^*(x)$ is defined as a temperature at which the number of holes, $n_0(x)$, introduced through the external doping and the number of the thermally activated holes in Eq. (3) become approximately equal.] Although at higher temperatures the carriers' concentration also increases rapidly and charges may be not independent anymore, qualitatively the result would mean that the dissipation rate grows enormously above $\sim 100-150$ K (the mobility decreases).

It is only natural to wonder whether the concept elaborated on above for LSCO applies to other HT$_c$ materials. Unlike LSCO, in other cuprates there is no easy way to know the amount of holes, p, introduced by the external (chemical) doping, especially close to the onset of superconductivity. Judgments about the actual hole concentration are then often based on the shape of the so-called "superconducting dome,"

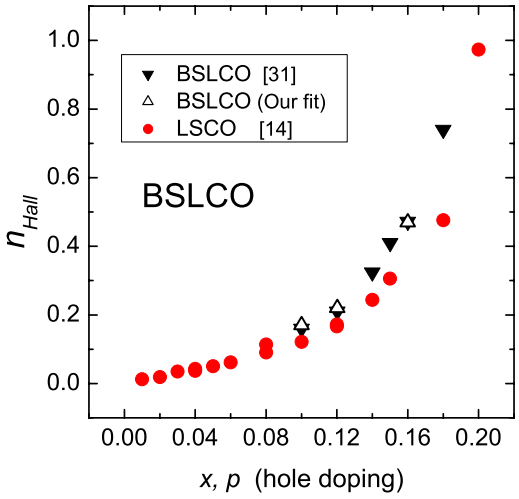

FIG. 3. (Color online) The dependence of the Hall carrier number on the concentration of the dopants in the CuO_2 plane for LSCO (Ref. 14) (circles) and BSLCO found in Ref. 31 from R_H at 100 K (filled triangles) and from the fitting of Eq. (3) to the $R_H(T)$ dependencies (open triangles).

its dependence on the dopants' concentration, and the subsequent comparison with those in LSCO as a template (or estimated otherwise from the thermopower experiments above T_c). As to the shape of the "superconducting dome," it is worth remembering that for a d-wave SC, T_c is sensitive to defects introduced by the doping process.

Consider briefly two examples. The low-temperature Hall effect in the normal state was measured in Ref. 31 for $Bi_2Sr_{2-x}La_xCuO_{6+y}$ (BSLCO, or La-doped Bi-2201) where the hole number, p, was defined according to such a procedure in Ref. 32. Superconductivity occurs only above $p \sim 0.10$. We have calculated the number of holes in La-doped Bi-2201 from the $R_H(T,x)$ (Ref. 31) according to Eq. (3). The results indeed turned out to be very close to our results for LSCO (see Fig. 3). Together with the analysis[14] for LSCO, one may conclude that the Hall coefficient in the form of Eq. (2) does indeed serve as the measure of the actual number of carriers. Another argument in favor of the applicability of the above physics to other materials is that rough estimations gave us (for 0.12–0.15 doping) close values of $T_F \sim 50(m^*/m)$ K for LSCO, YBCO,[33] and BiSrLaCuO. Correspondingly, the Hall angle (recalculated for the equal field values) is practically the same for these compounds. [Writing down $\cot(\theta_H) \approx AT$,[2] it is straightforward to obtain the expression for characteristic temperature as $T_F = k_B m^* c/(Ae\hbar B_z)$. Here parameter A is inversely proportional to B_z, the magnetic field normal to the CuO_2 plane. The material-dependent quantities here are A and m^*.]

To summarize, according to Refs. 2 and 14, the number of carriers in SC cuprates (above T_c) changes with temperature and deviates from simple proportionality to the amount of holes given by concentration of chemical dopants. Therefore, the effective mobility [or $\hbar/\tau(T)$] must be calculated from Eq. (1) after taking $n_{\mathrm{Hall}}(T,x)$ from Eq. (3) into account. No traditional approach is able to explain the quadratic T dependence, which we consider to be a purely numerical artifact.

Mobility shows a dramatic decrease above ~100 K. We ascribe this behavior to the motion of charges in a viscous media.[29] For cuprates, spatial and temporal competition between the two phases is ubiquitous at these temperatures.

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