Entanglement between charge qubits induced by a common dissipative environment

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We study entanglement generation between two charge qubits due to the strong coupling with a common bosonic environment (Ohmic bath). The coupling to the boson bath is a source of both quantum noise (leading to decoherence) and an indirect interaction between qubits. As a result, two effects compete as a function of the coupling strength with the bath: entanglement generation and charge localization induced by the bath. These two competing effects lead to a nonmonotonic behavior of the concurrence as a function of the coupling strength with the bath and, importantly, to steady-state entanglement. As an application, we present results for charge qubits based on double quantum dots.

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I. INTRODUCTION

Solid state nanostructures have become promising candidates for quantum information processing,[1] with basic operations such as single-qubit manipulation and readout having been demonstrated during the past few years. However, to go beyond single-qubit manipulations, and study effects such as entanglement generation and quantum gate operations, one needs some kind of interaction between the qubits. Although this interaction usually comes from a direct coupling between qubits (like the Coulomb interaction for charge qubits or exchange coupling for spin qubits), entanglement can also be generated by coupling two qubits (which do not interact with each other) to a common third system.[2–11] In most of these studies, the indirect interaction comes from the coupling to one or a few external degrees of freedom. Examples include the coupling to electromagnetic modes in a cavity (see, for example, Ref. 2, where the authors study entanglement of atoms within a single-mode cavity field) or to a harmonic oscillator representing a mode in a thermal environment.[10,11] Importantly, entanglement can also be induced when the environment is made by an infinitely large number of degrees of freedom, namely, a bath, as demonstrated by Braun in Ref. 12. This is an important case because entanglement is generated exclusively by *incoherent means*. In this context, different works have studied the coupling of two noninteracting qubits to fermionic[13–16] or bosonic[11,12,17–19] baths.

Indirect qubit interactions have recently attracted attention because the information distribution among distant entangled particles is the base of quantum cryptography,[20] quantum teleportation,[21–23] quantum dense code,[23–25] different processes proposed for testing Bell inequalities,[26–29] and even certain steps within quantum computation algorithms. The possibility of entangling two quantum systems that do not directly interact is therefore highly desirable, with various aspects of current interest such as "entanglement swapping"[26,30,31] and "entanglement transfer."[32–35]

In this paper, we study entanglement generation between two charge qubits due to the strong coupling with a common bosonic environment (Ohmic bath). For concreteness, we focus on charge qubits based on double quantum dots (DQDs) but we point out that our results can also be applied to Cooper pair boxes in a resistive environment. In a DQD, the electron charge degree of freedom is used to construct a qubit,[36–41] with logical states $|0\rangle$ and $|1\rangle$ corresponding to the localization of one excess electron on each one of the quantum dots (QDs). One of the advantages of these charge qubits is their controllability through external voltage handling, as demonstrated in recent experiments[38] where the charge has been coherently manipulated. We model the two DQD systems as two independent two-level systems strongly coupled to the same Ohmic bath [two spin-boson (SB) models[42,43]]. In addition, we consider that one of the DQDs is coupled to electron reservoirs[44] (Fig. 1) in order to allow electronic transport. The coupling to electronic reservoirs is treated using a Markovian approach,[41,44–46] which is valid in the sequential tunneling limit and large bias voltages. The non-Markovian character of the strong coupling with the boson bath is, on the other hand, taken into account by using a polaron approach.[41,44,45,47,48] As a result of this strong cou-

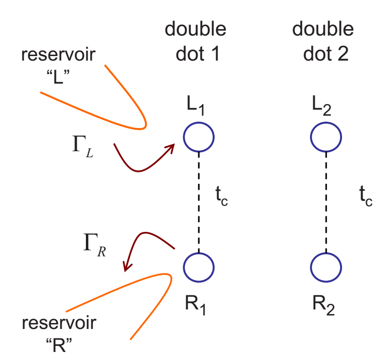

FIG. 1. (Color online) Schematic representation of two charge qubits based on double quantum dots. Interdot hopping, t_c, is allowed only in each double dot. The first double quantum dot is open to electron reservoirs, with probabilities for tunneling in and out given by Γ_L and Γ_R, respectively. The coupling to a common dissipative environment (not shown) generates a coupling between both qubits that are otherwise uncoupled.

pling, an indirect Ising-type interaction between qubits is induced by the bath.

By combining both Markovian and non-Markovian approximations, we derive a master equation for the reduced density matrix (RDM) of the system, including boson correlation functions in Laplace space. The resulting density matrix is used to calculate the degree of entanglement (quantified by Wootters' concurrence[49]) as well as the probability for each one of the Bell states as a function of the coupling strength by the bath.

Our results complement the previous work in Ref. 19 of Vorrath and Brandes, who studied a similar problem within a Markovian approach. We also mention some recent works[11,17,18] in which related models are treated.

The paper is organized as follows: in Sec. II, the model describing the DQDs coupled to both electronic reservoirs and the bosonic environment is discussed. We also present in Sec. II the general solution scheme for the density matrix equations. The coupling with the leads is treated by using a standard Born–Markov approximation, whereas the strong coupling with the bath is treated within a polaron approach. Section III shows the main results obtained, and, finally, we conclude in Sec. IV.

II. MODEL

An array of two parallel DQDs in the strong Coulomb blockade regime, and coupled to the same bosonic environment, is considered. Interdot tunneling is allowed only in each double dot, which defines an array of two charge qubits (Fig. 1). The first DQD is weakly coupled to two electron reservoirs in such a way that electronic transport through this double dot is possible (the excess charge in this DQD fluctuates between 1 and zero). The second DQD is closed and, therefore, has always one excess electron. Note that such configuration is close to the one realized in very recent experiments.[50] The Hilbert space includes two-particle states $|1\rangle=|L_1L_2\rangle$, $|2\rangle=|L_1R_2\rangle$, $|3\rangle=|R_1L_2\rangle$, and $|4\rangle=|R_1R_2\rangle$ [where $L_i(R_i)$ represents the charge localized in the upper (lower) QD of the ith DQD], as well as one particle states $|5\rangle=|0_1L_2\rangle$ and $|6\rangle=|0_1R_2\rangle$ (where 0 means no extra electron in the first DQD]. The completeness of the system is therefore given by $\Sigma_{k=1}^{6}|k\rangle\langle k|$.

The total Hamiltonian describing this system reads

$$H = H_{sys} + H_{res} + H_B + H_{SR} + H_{SB}. \quad (1)$$

The free part of the Hamiltonian, i.e., without couplings, contains three terms. The first term corresponds to the Hamiltonian of two independent DQDs, which in pseudospin language can be written as

$$H_{sys} = \sum_{i}^{2} \frac{1}{2}\Delta\varepsilon_i\sigma_z^i + t_c\sigma_x^i, \quad (2)$$

where $\Delta\varepsilon_i$ is the energy difference between quantum dots of each pair being $\Delta\varepsilon_1=\varepsilon_{L_1}-\varepsilon_{R_1}$ and $\Delta\varepsilon_2=\varepsilon_{L_2}-\varepsilon_{R_2}$ (ε_{i_j} is the on-site energy of the ith QD of the pair j), σ_j^i is the jth Pauli matrix acting on each DQD, and t_c is the electron tunneling amplitude which is considered identical for both DQDs.[44,45]

The Hamiltonian of the reservoirs, referred to as L and R, reads[41,44,45]

$$H_{res} = \sum_k \{\epsilon_k^L c_{k,L}^\dagger c_{k,L} + \epsilon_k^R c_{k,R}^\dagger c_{k,R}\}, \quad (3)$$

where $c_{k,\beta}^\dagger$ and $c_{k,\beta}$ are fermion creation and annihilation operators in lead β with corresponding energy ϵ_k^β. Finally, the third term corresponds to the boson bath, which is described as a set of harmonic oscillators with frequency ω_q:

$$H_B = \sum_q \hbar\omega_q a_q^\dagger a_q, \quad (4)$$

where $a_q^\dagger(a_q)$ is the creation (annihilation) boson operator. The coupling to the electron reservoir is given by

$$H_{SR} = \sum_{k,i\in 1,2} \{V_k^L(c_{k,L}^\dagger s_{L,i} + \text{c.c.}) + V_k^R(c_{k,R}^\dagger s_{R,i} + \text{c.c.})\}, \quad (5)$$

with V_k^β being the coupling with the lead β. The Lindblad-type operators $s_{L,i}$ ($s_{R,i}$) describe tunneling into (out of) the first DQD by taking into account the two possible configurations ($\Sigma_{i\in 1,2}$) in the second DQD, namely, $s_{L,1}=|5\rangle\langle 1|$, $s_{L,2}=|6\rangle\langle 2|$, $s_{R,1}=|5\rangle\langle 3|$, and $s_{R,2}=|6\rangle\langle 4|$. Although we consider the coupling of only one of the DQDs to reservoirs, the generalization for both double dots is straightforward.

The electron-boson interaction is described with a spin-boson Hamiltonian, where the bath "force" operator $\xi^i=\Sigma_q\gamma_q^j(a_q^\dagger+a_q)$ linearly couples to each qubit's σ_z^i (Refs. 42 and 43) (here, we consider that such interaction is identical for both DQDs,[51] $\gamma_q^j=\gamma_q$):

$$H_{SB} = \sum_q \sum_i \frac{1}{2}\gamma_q\sigma_z^i(a_q^\dagger + a_q). \quad (6)$$

Note that this coupling, which is longitudinal in the local basis of each qubit, contains both longitudinal and transversal components in the basis that diagonalizes the qubit Hamiltonian.[52] The bath effects can be encapsulated in the spectral density $J(\omega)=\Sigma_q\gamma_q^2\delta(\omega-\omega_q)$. In the following, we use a generic Ohmic bath: $J(\omega)=2\alpha\omega e^{-\omega/\omega_c}$, where ω_c is a cutoff frequency and α is a dimensionless parameter that reflects the dissipation strength.[42,43,53,54] As we shall see in the next section, the coupling of both qubits to the same quantum heat bath leads to decoherence *and* to an effective interaction between qubits.

A. Polaron transformation

Due to the strong coupling of the qubits with the boson bath, a proper description of the system must take into account non-Markovian effects. Among the different approaches available to deal with this,[55–58] we use a "polaron transformation,"[48] which for an arbitrary operator O is given by

$$\bar{O} = e^S O e^{-S},$$

$$S=\sum_q \sum_i \frac{1}{2}\sigma_z^i \frac{\gamma_q}{\omega_q}(a_q^\dagger - a_q). \tag{7}$$

This approach, which is well known for treating problems in which bosonic modes couple to localized electronic states, has been successfully used for studying single[59,60] and double quantum dots[41,44,45,47] strongly coupled to a bath of phonons.

By using this transformation (see Appendix A), we obtain an effective Hamiltonian:

$$\bar{H}=\bar{H}_0+\bar{H}_T+\bar{H}_{SR}, \tag{8}$$

$$\bar{H}_0=\sum_i \frac{1}{2}\Delta\varepsilon_i \sigma_z^i - \frac{1}{4}\kappa\sum_{i,j}\sigma_z^i\sigma_z^j + H_B + H_{res}, \tag{9}$$

$$\bar{H}_T=\sum_i t_c(\sigma_+^i X + \sigma_-^i X^\dagger), \tag{10}$$

where $\sigma_\pm^i$ are the ladder spin operators on each DQD, whereas X and $X^\dagger$ correspond to polaronic phases[48] given by

$$X=e^A, \tag{11}$$

with $A=\Sigma_q \gamma_q(a_q^\dagger - a_q)$.

The effect of the canonical transformation is threefold:

(i) The electron-boson interaction H_{SB} has been transformed away.

(ii) The state of the bosonic system is strongly modified every time an electron tunnels between dots (boson "shake-up").[61] As a result, the interdot tunneling amplitude [Eq. (10)] becomes renormalized with environment-dependent phases through the operators $X=e^A$. These time-dependent exponential phases, which appear as a result of the nonperturbative treatment of the electron-boson interaction, lead to nontrivial effects. In particular, this implies that non-Markovian effects become relevant and need to be considered in the dynamics of the reduced density matrix. Note also that, in principle, this renormalization of tunneling has to be taken into account also in $\bar{H}_{SR}$ through the operators $\bar{s}_{L,i}=s_{L,i}e^{-A/2}$ and $\bar{s}_{R,i}=s_{R,i}e^{A/2}$. However, this is no longer true in the limit of large bias voltages, where the coupling to the reservoirs becomes Markovian (see the next section).

(iii) The transformed Hamiltonian contains an *effective interaction* between qubits $H_{eff}=-\frac{\kappa}{4}\Sigma_{i,j}\sigma_z^i\sigma_z^j$ due to the coupling with the common bath; this interaction has an Ising form and depends on the parameter $\kappa=\Sigma_q\gamma_q^2/\omega_q$ (which for Ohmic dissipation used here reads $\kappa=2\alpha\omega_c$), favoring states with the same charge distribution in both DQDs.

B. Master equation

To obtain the master equation, we start from the transformed frame. Defining the RDM of the two DQDs plus the boson bath in the interaction picture as $\tilde{\rho}(t)$ and applying the second order Born–Markov approximation on the electronic leads, we obtain the following equation of motion for $\tilde{\rho}(t)$:

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}(t) = &-i[\tilde{H}_T(t),\tilde{\rho}(t)] - \frac{\Gamma_L}{2}\sum_{i\in 1,2}\{\tilde{s}_{L,i}(t')\tilde{s}_{L,i}^\dagger(t')\tilde{\rho}(t') \\ &- 2\tilde{s}_{L,i}^\dagger(t')\tilde{\rho}(t')\tilde{s}_{L,i}(t') + \tilde{\rho}(t')\tilde{s}_{L,i}(t')\tilde{s}_{L,i}^\dagger(t')\} \\ &- \frac{\Gamma_R}{2}\sum_{i\in 1,2}\{\tilde{s}_{R,i}^\dagger(t')\tilde{s}_{R,i}(t')\tilde{\rho}(t') - 2\tilde{s}_{R,i}(t')\tilde{\rho}(t')\tilde{s}_{R,i}^\dagger(t') \\ &+ \tilde{\rho}(t')\tilde{s}_{R,i}^\dagger(t')\tilde{s}_{R,i}(t')\},\end{aligned} \tag{12}$$

in which the transformed operators are in the interaction picture, as explained in Appendix B, and where Γ_L and Γ_R are the electron tunneling rates in and out of the first DQD, respectively.

As we mentioned already, the fact that the coupling with the reservoirs becomes Markovian in this limit implies, in particular, that the renormalization of tunneling due to the bosonic bath becomes ineffective [for example, $\bar{s}_{L,1}(t')\bar{s}_{L,1}^\dagger(t')=s_{L,1}(t')s_{L,1}^\dagger(t')$].

By denoting the projector operators over the system states as $Y_{nm}=|n\rangle\langle m|$, the corresponding expectation values can be written as $\langle Y_{nm}\rangle=\mathrm{Tr}_{dot}\{\rho^S Y_{nm}\}=\langle m|\rho^S|n\rangle=\mathrm{Tr}_{dot}\{\tilde{\rho}^S\tilde{Y}_{nm}\}$, where we have performed the partial trace over the DQD array, Tr_{dot}, and the RDM of the DQD array (system) is defined as the trace over the boson bath states: $\rho^S=\mathrm{Tr}_B\,\rho$. It is therefore possible to obtain matrix elements of the reduced density operator by just calculating the expectation value for the suitable $\tilde{Y}_{nm}(t)$ operators *directly* from the master equation (12). Using the notation $\rho_{mn}^S\equiv\langle m|\rho^S|n\rangle$, we obtain the following set of exact equations:

$$\begin{aligned}\rho_{nm}^S(t) = {}& \rho_{nm}^S(0) - i\int_0^t \mathrm{Tr}_{dot,B}\{\tilde{\rho}(t')[\tilde{Y}_{nm}(t),\tilde{H}_T(t')]\}dt' \\ &- \frac{\Gamma_L}{2}\int_0^t \mathrm{Tr}_{dot,B}\{[\tilde{s}_L(t')\tilde{s}_L^\dagger(t')\tilde{\rho}(t') - 2\tilde{s}_L^\dagger(t')\tilde{\rho}(t')\tilde{s}_L(t') \\ &+ \tilde{\rho}(t')\tilde{s}_L(t')\tilde{s}_L^\dagger(t')]\tilde{Y}_{mn}(t)\}dt' \\ &- \frac{\Gamma_R}{2}\int_0^t \mathrm{Tr}_{dot,B}\{[\tilde{s}_R^\dagger(t')\tilde{s}_R(t')\tilde{\rho}(t') - 2\tilde{s}_R(t')\tilde{\rho}(t')\tilde{s}_R^\dagger(t') \\ &+ \tilde{\rho}(t')\tilde{s}_R^\dagger(t')\tilde{s}_R(t')]\tilde{Y}_{mn}(t)\}dt'.\end{aligned} \tag{13}$$

The complete expressions for the density matrix elements are, as expected, quite intricate. As explained in Appendix B, one needs to use some physical assumptions in order to handle the master equation. However, the resulting expressions are not exact anymore. We then obtained a set of coupled equations for $\rho^S(t)$, which in matrix form can be written as

$$\rho^S(t)=\rho^S(0)+\int_0^t\{\mathbf{M}(t-t')\rho^S(t')+\mathbf{\Gamma}\}dt', \tag{14}$$

where the vector $\mathbf{\Gamma}$ contains the terms related to the coupling of the first DQD to the reservoirs, and $\mathbf{M}(t-t')$ is a non-Markovian time-dependent kernel that contains the bath correlation functions $C(t-t')$ as defined in Appendix B.[44] Equation (14) can be solved in the Laplace space[41,44] as

$$\rho^S(z) = [z - z\mathbf{M}(z)]^{-1}[\rho^S(0) + (\boldsymbol{\Gamma}/z)]. \tag{15}$$

The kernel $\mathbf{M}(z)$ contains the Laplace transform of the bath correlation functions $C_\varepsilon^{(*)} = \int_0^\infty e^{-z\tau} e^{(-)i\varepsilon\tau} C^{(*)}(\tau) d\tau$ evaluated at different energies corresponding to the involved transitions.[41,44,45]

III. ENTANGLEMENT

The full time-dependent density matrix can be obtained by algebraically solving Eq. (15) and performing an inverse Laplace transformation, which is a formidable task. Fortunately, the entanglement generated by the bath is finite at long times, namely, in the *stationary state*, as we will show. The stationary solution of Eq. (15), ρ_∞, is obtained by extracting the $1/z$ coefficient in a Laurent series of $\rho^S(z)$ for $z \to 0$.[41,44,45] For entanglement quantification, we use Wootters' concurrence[49] for a general state of two qubits,

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{16}$$

where the λ's are the eigenvalues in decreasing order of the non-Hermitian matrix $\rho^S(\sigma_y \otimes \sigma_y)(\rho^S)^*(\sigma_y \otimes \sigma_y)$. The concurrence ranges from $C=0$ for nonentangled states to $C=1$ for the maximum degree of entanglement. That maximum entanglement is shown by the Bell states.[49] In the basis of triplet and singlet states, $|S_0\rangle = \frac{1}{\sqrt{2}}(|L_1R_2\rangle - |R_1L_2\rangle)$, $|T_0\rangle = \frac{1}{\sqrt{2}}(|L_1R_2\rangle + |R_1L_2\rangle)$, $|T_+\rangle = |L_1L_2\rangle$, and $|T_-\rangle = |R_1R_2\rangle$, the Bell states read $|\Psi^+\rangle = |T_0\rangle$, $|\Psi^-\rangle = |S_0\rangle$, $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|T_-\rangle + |T_+\rangle)$, and $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|T_-\rangle - |T_+\rangle)$.

Importantly, the stationary density matrix in our problem corresponds to a transport situation and, therefore, a proper generalization of concurrence to *nonequilibrium* is needed. Following Ref. 16, we quantify nonequilibrium entanglement via the concurrence C of the stationary state $\hat{P}\rho_\infty$, where $\hat{P}$ is the projection onto doubly occupied states including proper normalization. The projection $\hat{P}$ corresponds to taking the limit $\Gamma_L \to \infty$, where both qubits are always occupied with one single electron. For concreteness, we focus on the zero-temperature case.

The concurrence as a function of the coupling α always shows the same qualitative behavior: for very small α, there is a switching behavior, indicating that below a minimum interaction strength κ the concurrence vanishes [cf. Fig. 2(a) for identical QDs ($\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$)]. As α increases, two effects compete: entanglement generation and charge localization induced by the bath. At small α, the two electronic delocalized states $|S_0\rangle$ and $|T_0\rangle$ have a finite weight, which depends in a nontrivial way on the ratio $\frac{t_c}{\alpha}$. On the other hand, for strong coupling, the bath completely freezes the charges on the left dots and the triplet $|T_+\rangle = |L_1, L_2\rangle$ becomes fully occupied [cf. Fig. 2(b)]. These two competing effects lead to the nonmonotonic behavior of the concurrence vs α, with an optimal value, $\alpha = \alpha^*$, at which the concurrence presents a maximum. We note, however, that the concurrence is maximum for values of $\alpha^* \ll \alpha_c$, where α_c is the value of the coupling at which the spin-boson model undergoes a quantum phase transition to a localized phase.[42,43] Qualitatively,

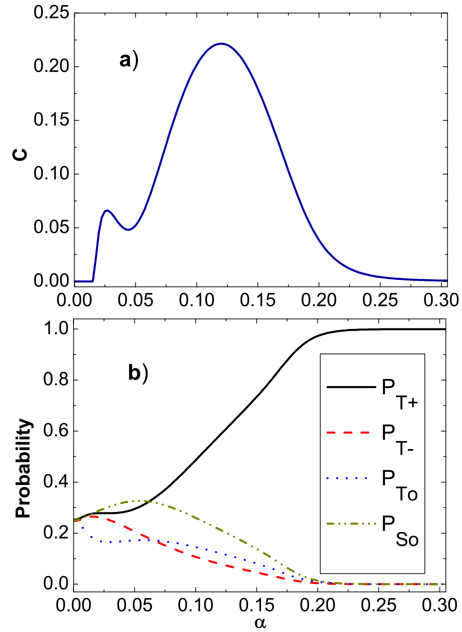

FIG. 2. (Color online) (a) Concurrence as a function of the strength of dissipation α. (b) Population of triplet and singlet states. Parameters: $\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$, $t_c = 3.5$, $\Gamma_L = 10$, and $\omega_c = 500$ (in units of $\Gamma_R = 1$ μeV). These parameters correspond to typical experimental values in AlGaAs-GaAs lateral DQDs (Refs. 36, 50, and 62).

we can understand this from the point of view of a single qubit, say, qubit 1. The presence of qubit 2 induces an effective level detuning, namely, $\Delta\tilde{\varepsilon}_1 = \Delta\varepsilon_1 - \frac{1}{4}\kappa\sigma_z^2$, which, in general, is nonzero even for $\Delta\varepsilon_1 = 0$. For $\Delta\tilde{\varepsilon}_1 \neq 0$, the von Neumann entropy (the figure of merit that quantifies qubit-bath entanglement) of the spin-boson model is a nonmonotonic function of α, with a maximum at $\alpha^{SB} \ll \alpha_c$ at which the qubit becomes maximally entangled with the bath.[63,64] We argue that, although the maximum in the von Neumann entropy of the spin-boson model cannot be directly related to the maximum of concurrence in our case, the nonmonotonic behavior of both quantities and the fact that both present maxima at values of α below α_c probably have the same physical origin.[65]

The population of each Bell state is shown in Fig. 3. The

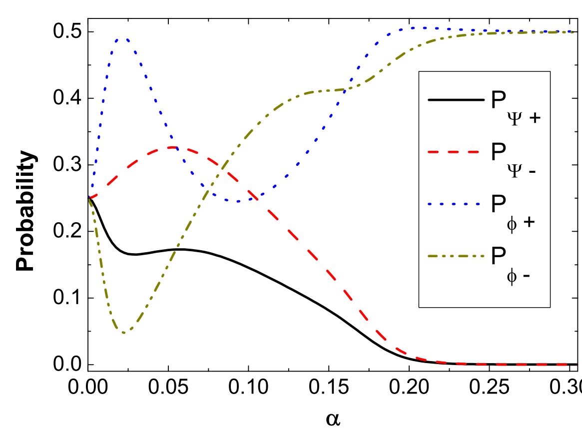

FIG. 3. (Color online) Population of the Bell states as a function of α. Same parameters as in Fig. 2.

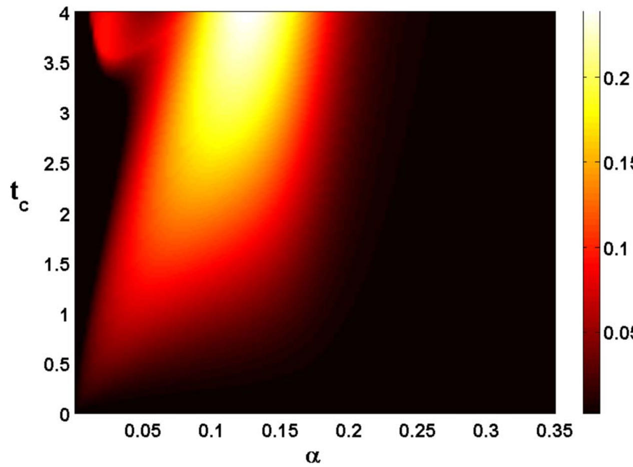

FIG. 4. (Color online) Color map of concurrence vs interdot tunneling t_c and α. The rest of the parameters are the same as in Fig. 2.

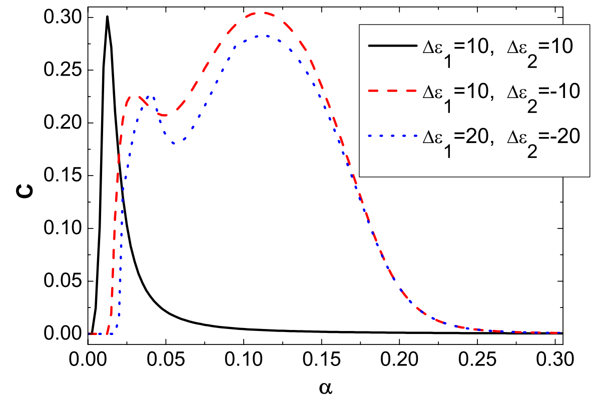

FIG. 6. (Color online) Concurrence as a function of α for different level detunings. The rest of the parameters are the same as in Fig. 2.

system does not originate a preferred Bell state and therefore both maxima in the concurrence contain contributions from all states. The first concurrence peak is formed by a combination of the four bell states with a symmetric contribution of $|\Psi^-\rangle$ and $|\phi^+\rangle$, whereas on the second peak, $|\phi^-\rangle$ probability is slightly dominant. Electrons localization in "parallel" charge states is reflected in the large probability for both $|\phi^+\rangle$ and $|\phi^-\rangle$ states for $\alpha \gtrsim 0.2$.

The concurrence as a function of both t_c and α is shown in Fig. 4. For $2t_c < \Gamma_R$, the dephasing induced by the leads suppresses interdot coherence, and the contribution of the delocalized states $|S_0\rangle$ and $|T_0\rangle$ is negligible. Thus, the concurrence is almost zero for all α. For $2t_c > \Gamma_R$, entanglement is finite in a region $\alpha_{min} < \alpha < \alpha_{max}$; both α_{min} and α_{max} increase with t_c. For $2t_c \gg \Gamma_R$, the system presents a maximum in the concurrence at $\alpha \approx 0.15$ with values $C \approx 0.3$.

The effect of Γ_R on concurrence is shown in Fig. 5. Here, we also find the switching behavior described above: starting from $\alpha = 0$, the state of the system is strongly mixed for small Γ_R. Therefore, the concurrence is zero below a minimal value α_{min}. This threshold value decreases as Γ_R increases. Interestingly, this threshold behavior has been found in other open quantum systems exhibiting steady-state entanglement: Hartmann *et al.* discussed in Ref. 66 the same effect in an open system at finite temperature and far from equilibrium. Moreover, Huelga and Plenio recently discussed in Ref. 67 a driven coupled qubit system (in contact with a Markov bath) where, again, a value of the environmental noise above a given threshold is needed in order to obtain steady-state entanglement. Finally, Lambert *et al.* find in Ref. 16 the same effect in capacitively coupled charge qubits open to reservoirs. We speculate that this behavior, which seems to be ubiquitous, may be general and, therefore, imply that a finite amount of environmental decoherence is needed in order to have steady-state entanglement in open quantum systems.

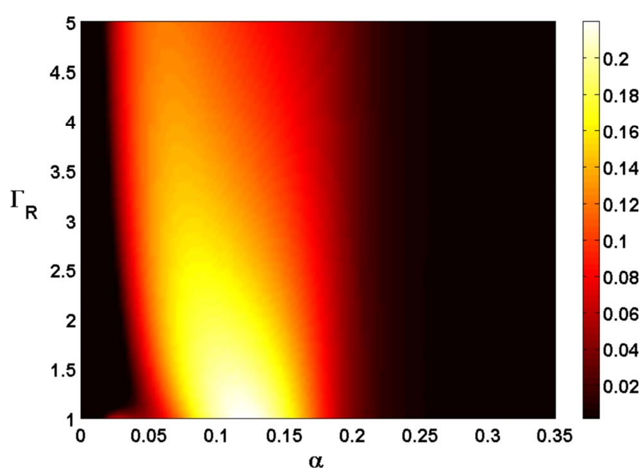

FIG. 5. (Color online) Color map of concurrence vs tunneling rate to the right lead Γ_R and α. The rest of the parameters are the same as in Fig. 2.

At fixed α, the entanglement decreases as one increases Γ_R. For very large Γ_R, the pure localized triplet $|T_+\rangle = |L_1, L_2\rangle$ is reached and thus the entanglement is zero. This effect, which is a transport version of the quantum Zeno effect, is also found in Ref. 16.

A finite detuning $\Delta\varepsilon_i > 0$ ($\Delta\varepsilon_i < 0$) localizes the charge on the lower (upper) QD of each pair and, therefore, the entanglement should depend on whether $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$ or $\Delta\varepsilon_1 = -\Delta\varepsilon_2 > 0$. The concurrence of the latter case is very similar to the one for $\Delta\varepsilon_1 = \Delta\varepsilon_2 = 0$ and, therefore, the population of singlet and triplet states show also the same kind of behavior [Fig. 7(a)]. On the contrary, the concurrence for $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$ is different with a narrow resonance at small α (cf. Fig. 6). This resonance corresponds to a maximum in the population of the triplet T_0 [cf. Fig. 7(b)], followed by a fast decay of both T_0 and S_0 and an enhanced population of T_+ (and, hence, zero concurrence). The overall qualitative behavior is in agreement with Ref. 19, where the current through two DQDs coupled to the same phonon bath is analyzed in the Markovian limit. The indirect interaction due to a bath leads to an enhancement of the inelastic current at $\Delta\varepsilon_1 = \Delta\varepsilon_2 > 0$ and a maximum population of the triplet T_0, which is a transport version of the Dicke effect. In close analogy to the Dicke effect in quantum optics, this superradiance should turn into subradiance as the probability of finding the system in the singlet S_0, rather than in the triplet T_0, increases.[19] We do not find, however, the subradiance counterpart in our analysis of concurrence.

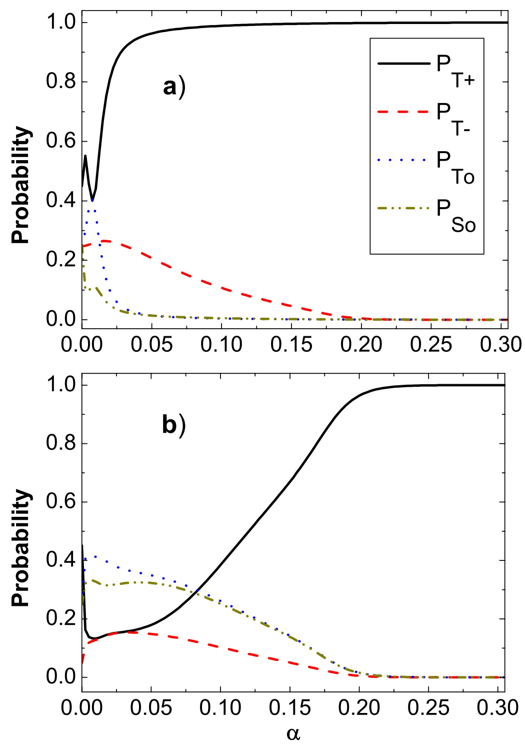

FIG. 7. (Color online) (a) Population of triplet and singlet states for $\Delta\varepsilon_1=\Delta\varepsilon_2=10$. (b) The same for $\Delta\varepsilon_1=\Delta\varepsilon_2=-10$.

IV. CONCLUSION

We have shown that two noninteracting charge qubits can get entangled, in a stationary limit, by the strong coupling with a common bosonic bath. The Ising-type interaction between qubits induced by the bath yields up to two different effects: entanglement generation and further charge localization. These effects lead to a nonmonotonic behavior of the concurrence as a function of the coupling strength with the bath.

In the specific realization based on double quantum dots, we have also analyzed the effects of different parameters, which can be experimentally controlled, on the degree of entanglement obtained. We find that the concurrence strongly depends on tunneling and on the energy difference on each DQD. In particular, the charge distribution promoted by such parameters compete with the bath effects.

In general, small concurrence values are obtained here ($C<0.5$) and a preferred Bell state is not formed. Therefore, the use of this setup may not be an optimal choice for entanglement studies in the solid state realm. Note, however, that the model considered here is the minimal implementation of a fully tunable two charge-qubit system coupled to a bosonic bath. The bath describes environmental noise, namely, unavoidable voltage fluctuations that couple to the qubit. Experimentally, these voltage fluctuations correspond to, e.g., phonons[62] or to fluctuations of the electromagnetic environment.[68] Interestingly, in this latter case, the coupling to the environment α depends on the external impedance of the circuit[69,70] (namely, on how environmental voltage fluctuations couple to the gate voltages used to tune the qubit parameters) and is, therefore, *tunable*. From this point of view, this realization is an attractive benchmark in which to study the interplay between quantum coherence, entanglement, and decoherence. Another interesting extension of our work would be to study shot noise cross correlations[16,71–73] between the electrical currents through each double dot and their relation to charge entanglement as induced by the bath.

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APPENDIX A: POLARON TRANSFORMATION

By using the Baker–Hausdorff lemma, the canonical transformation given in Eq. (7) is performed over the relevant operators in our model. Thus, these take the form

$$\bar{\sigma}_z^i = \sigma_z^i,$$

$$\bar{\sigma}_x^i = \sigma_+^i X + \sigma_-^i X^\dagger,$$

$$\bar{a}_q = a_q - \frac{1}{2}\frac{\lambda_q}{\omega_q}\sum_i \sigma_z^i,$$

$$\bar{s}_{L,i} = s_{L,i} e^{-A/2},$$

$$\bar{s}_{R,i} = s_{R,i} e^{A/2}, \tag{A1}$$

where σ_+^i and σ_-^i are the ladder spin operators acting on the ith qubit, and the explicit form of the polaron operators X and $X^\dagger$ are given in Eq. (11).

Substituting these *transformed* operators into Eqs. (2)–(6), the effective Hamiltonians presented in Eqs. (8)–(10) are obtained.

APPENDIX B: MASTER EQUATION

We define the total density operator of the open system as $\chi(t)=e^{-iHt}\chi(0)e^{iHt}$, which, after the polaron transformation (7), can be written in the interaction picture as[41,45] $\tilde{\chi} = e^{i\bar{H}_0 t}\bar{\chi}(t)e^{-i\bar{H}_0 t}$, with $\bar{\chi}(t)=e^{-i\bar{H}t}\bar{\chi}(0)e^{i\bar{H}t}$. By taking the partial trace over the reservoir degrees of freedom, the RDM of the two DQDs plus the boson bath is obtained as $\tilde{\rho}(t) = \mathrm{Tr}_{res}\,\tilde{\chi}(t)$. Considering a weak coupling of the system with the electronic leads, we arrive at the following equation of motion for $\tilde{\rho}(t)$:

$$\frac{d}{dt}\tilde{\rho}(t) = -i[\tilde{H}_T(t),\tilde{\rho}(t)] - \sum_{k,i\in 1,2,j\in L,R}\int_0^t dt'|V_k^j|^2 f^j(\epsilon_k^j)e^{i\epsilon_k^j(t-t')} \times\{\tilde{s}_{j,i}(t)\tilde{s}_{j,i}^\dagger(t')\tilde{\rho}(t') - \tilde{s}_{j,i}^\dagger(t')\tilde{\rho}(t')\tilde{s}_{j,i}(t)\} - \sum_{k,i\in 1,2,j\in L,R}\int_0^t dt'|V_k^j|^2[1-f^j(\epsilon_k^j)]e^{-i\epsilon_k^j(t-t')}$$

$$\times\{\tilde{s}^{\dagger}_{j,i}(t)\tilde{s}_{j,i}(t')\tilde{\rho}(t') - \tilde{s}^{\dagger}_{j,i}(t')\tilde{\rho}(t')\tilde{s}_{j,i}(t)\}$$

$$- \sum_{k,i\in 1,2,j\in L,R}\int_0^t dt'|V_k^j|^2 f^j(\epsilon_k^j)e^{i\epsilon_k^j(t-t')}$$

$$\times\{\tilde{\rho}(t')\tilde{s}_{j,i}(t')\tilde{s}^{\dagger}_{j,i}(t) - \tilde{s}^{\dagger}_{j,i}(t')\tilde{\rho}(t')\tilde{s}_{j,i}(t')\}$$

$$- \sum_{k,i\in 1,2,j\in L,R}\int_0^t dt'|V_k^j|^2[1-f^j(\epsilon_k^j)]e^{-i\epsilon_k^j(t-t')}$$

$$\times\{\tilde{\rho}(t')\tilde{s}^{\dagger}_{j,i}(t')\tilde{s}_{j,i}(t) - \tilde{s}_{j,i}(t)\tilde{\rho}(t')\tilde{s}^{\dagger}_{j,i}(t')\}, \qquad \text{(B1)}$$

where $f^j(\epsilon_k^j)=\mathrm{Tr}_{res}\{R_0 c^\dagger_{k,j}c_{k,j}\}$ are the Fermi distributions of each contact (R_0 is the density matrix of the electron reservoirs, considered in thermal equilibrium).[41,45,47] Equation (B1) can be simplified by rewriting the sums over k as integrals $\Sigma_k|V_k^j|^2 f^j(\epsilon_k^j)e^{i\epsilon_k^j(t-t')}=\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}\Gamma_j(\epsilon)f^j(\epsilon)e^{i\epsilon(t-t')}$, where $\Gamma_j(\epsilon)\equiv 2\pi\Sigma_k|V_k^j|^2\delta(\epsilon-\epsilon_k^j)$ are the tunneling rates in and out of the DQD. By working in an "infinite bias regime" between the reservoirs (such that $f^L\rightarrow 1$ and $f^R\rightarrow 0$) and assuming a constant density of states in the leads, the coupling with the reservoirs becomes Markovian: $\Sigma_k|V_k^L|^2 f^L(\epsilon_k^L)e^{i\epsilon_k^L(t-t')}=\Gamma_L\delta(t-t')$ and $\Sigma_k|V_k^R|^2[1-f^R(\epsilon_k^R)]\times e^{i\epsilon_k^R(t-t')}=\Gamma_R\delta(t-t')$ and, therefore, Eq. (B1) reads as Eq. (12).

Invariance under unitary operations implies that the expected value of any dot operator can be written as $\langle O(t)\rangle=\mathrm{Tr}_{dot}\{\mathrm{Tr}_B\{\tilde{\rho}(t)\}\mathrm{Tr}_B\{\tilde{O}(t)\}\}=\mathrm{Tr}_{dot,B}\{\tilde{\rho}(t)\tilde{O}(t)\}$, where Tr_B is the trace over the bath states. In particular, as we have pointed out previously, the expected value of the projector operators over the system states Y_{nm} yield to the elements $\rho^S_{mn}\equiv\langle m|\rho^S|n\rangle$ as shown in Eq. (13).

The full expression of the density matrix is too large to give it here (its total dimension is 6×6) and we show just two examples for $\rho^S_{14}(t)$ and $\rho^S_{13}(t)$ elements:

$$\dot{\rho}^S_{13}(t) = -i\{t_c e^{-i(\bar{\varepsilon}_1-\bar{\varepsilon}_3)(t-t')}(\langle\tilde{Y}_{33}(t')X_t^\dagger X_{t'}\rangle - \langle\tilde{Y}_{11}(t')X_{t'}X_t^\dagger\rangle$$

$$+ \langle\tilde{Y}_{32}(t')^\dagger X_t^\dagger X_{t'}\rangle - \langle\tilde{Y}^\dagger_{41}(t')X_{t'}X_t^\dagger\rangle) \ , \qquad \text{(B2a)}$$

$$\dot{\rho}^S_{14}(t) = -i\{t_c e^{-i(\bar{\varepsilon}_1-\bar{\varepsilon}_4)(t-t')}(\langle\tilde{Y}^\dagger_{43}(t')X_{t'}X_t^\dagger X_t^\dagger X_{t'}\rangle$$

$$+ \langle\tilde{Y}^\dagger_{21}(t')X_{t'}X_{t'}X_t^\dagger X_{t^e}\rangle + \langle\tilde{Y}^\dagger_{42}(t')X_{t'}X_t^\dagger X_t^\dagger X_{t'}\rangle$$

$$- \langle\tilde{Y}^\dagger_{31}(t')X_{t'}X_{t'}X_t^\dagger X_t^\dagger\rangle) \ , \qquad \text{(B2b)}$$

with $\bar{\varepsilon}_n|n\rangle=[\Sigma_i(1/2)\Delta\varepsilon_i\sigma_z^i-(1/4)\kappa\Sigma_{i,j}\sigma_z^j\sigma_z^j]|n\rangle$.

Note that Eqs. (B2a) and (B2b) are not closed. They contain expectation values involving products of dot and boson operators, as, for example, $\langle\tilde{Y}_{33}(t')X_t^\dagger X_{t'}\rangle=\mathrm{Tr}_{dot,B}\{\tilde{\rho}(t')\tilde{Y}_{33}(t')X_t^\dagger X_{t'}\}$. Since the boson operators correspond to a continuum of modes, we can assume that the bath remains at thermal equilibrium at all times[41] with a density matrix ρ_B and inverse temperature β, such that the reduced density operator can be approximated as $\tilde{\rho}(t')\approx\rho_B(0)\otimes\mathrm{Tr}_B\,\tilde{\rho}(t')$. By using this Born approximation, we can decouple higher order correlation functions as $\langle\tilde{Y}_{33}(t')X_t^\dagger X_{t'}\rangle\approx\langle\tilde{Y}_{33}(t')\rangle\langle X_t^\dagger X_{t'}\rangle$, etc. Note that this decoupling, which corresponds to the so-called noninteracting blip approximation in the spin-boson problem,[42] neglects back action of the system onto the bath. This neglect of back action is, of course, invalid in interesting cases such as few-mode environments, detectors, or nonequilibrium baths, in which a full quantum treatment is needed.[74]

For such equilibrium boson bath, one can write the *correlation functions* as[41] $C(t-t')=\langle X_t X^\dagger_{t'}\rangle=e^{-\Phi(t-t')}$ with $\Phi(\tau)=\int_0^\infty\frac{J(\omega)}{\omega^2}\{[1-\cos(\omega\tau)]\coth(\beta\omega/2)+i\sin(\omega\tau)\}$. Note that in our problem we also need $C_2(t-t')=\langle X_t X_t X^\dagger_{t'} X^\dagger_{t'}\rangle=e^{-2\Phi(t-t')}$, which appear from coherences involving interdot processes such as $|1\rangle\leftrightarrow|4\rangle$ [namely, $|L_1,L_2\rangle\leftrightarrow|R_1,R_2\rangle$, see Eqs. (B2a) and (B2b)]. In principle, terms involving half phases $e^{-(1/2)\Phi(t-t')}$ also appear in tunneling processes to the reservoirs (like, for example, $|R_1,L_2\rangle\overset{\Gamma_R}{\rightarrow}|0_1,L_2\rangle$) but, again, they do not contribute in the Markovian limit [see discussion after Eq. (12)].

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