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Isotope effect on superconductivity in Josephson coupled stripes in underdoped cuprates

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Inelastic neutron scattering data for YBaCuO as well as for LaSrCuO indicate incommensurate neutron scattering peaks with an incommensuration $\delta(x)$ away from the (π,π) point. $T_c(x)$ can be replotted as a linear function of the incommensuration for these materials. This linear relation implies that the constant that relates these two quantities, where one is the incommensuration (momentum) and the other is $T_c(x)$ (energy), has the dimension of velocity, which we denote by v^*: $k_B T_c(x) = \hbar v^* \delta(x)$. We argue that this experimentally determined relation can be obtained in a simple model of Josephson coupled stripes. Within this framework, we address the role of the $O^{16} \rightarrow O^{18}$ isotope effect on $T_c(x)$. We assume that the incommensuration is set by the *doping* of the sample and is not sensitive to the oxygen isotope given the fixed doping. We find therefore that the only parameter that can change with the O isotope substitution in the relation $T_c(x) \sim \delta(x)$ is the velocity v^*. We predict an oxygen isotope effect on v^* and expect it to be $\simeq 5\%$.

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I. INTRODUCTION

The isotope effect has played an important role in understanding the underlying pairing mechanism in superconductors. Historically, it was used to identify the role of the electron-lattice interaction in superconductivity. Experimental evidence as to the nature of the interaction that causes superconductivity was first provided in 1950 by Maxwell[1] and Reynolds *et al.*[2] They showed that $T_c \propto M^{-\alpha}$, where $\alpha = 0.5 \pm 0.05$ and M is the mean mass of the different isotopes of a superconductor. These findings indicate that ion mass and, therefore, lattice vibrations and phonons are important to the mechanism of superconductivity.

In the same year, Fröhlich,[3] who was unaware of the experiments on the isotope effect, and Bardeen[4] provided theories of the phonon-electron interaction, which, in turn, led to models of superconductivity that are dependent on phonon energies.

For high-T_c superconductors, the study of the isotope effect does not paint a simple and straightforward picture. The role of the electron-lattice interactions in the mechanism of superconductivity was initially ruled out, and the pairing mechanism was ascribed to antiferromagnetic exchange and fluctuations.[5,6] As time goes by, we witness the growing acknowledgment that interactions of the lattice with the carriers in high T_c might be important. In current discussions, the role of lattices and phonons is again gaining importance.[7–14] The isotope effect on T_c is believed to be small at optimal doping but increases to the BCS value in the underdoped regime.[15] What complicates the discussion on the isotope effect is the fact that underdoped LaSrCuO (LSCO) (Ref. 16) is electronically inhomogeneous. Inhomogeneity is also well established in a Bi2212 superconductor.[9] The situation is different for $YBa_2Cu_3O_{6+x}$ (YBCO) compounds, which are believed to be more homogeneous. In both the LSCO and the YBCO cases, the incommensuration, which is possibly related to stripes, is certain.

For the purposes of this discussion, we would like to point out a distinction between the isotope effect we consider here and the notion of an isotope effect in inhomogeneous systems. In the case of conventional homogeneous superconductors, a discussion of the isotope effect is centered on an exponent that describes the effect of ionic isotope substitution on superconducting T_c. In the case of spatially inhomogeneous systems and materials with more than one energy scale, e.g., superconducting gap vs pseudogap energy scale, the notion of an isotope effect has to be expanded to address the difference between changes that could be caused by isotope substitution on different energy scales.[17–24] Similar arguments can be made about the effect isotope substitution can have on pairing gap vs superfluid stiffness. The very notion of a *single exponent* for the isotope effect in the presence of an electronic inhomogeneity, which characterizes the whole sample by a single exponent, has to be viewed at best as a very crude average description of what is really happening in these materials.

Related to the isotope effect is a question on the role of inhomogeneity as a mechanism to modify superconducting properties. This question has not been addressed in detail. One can make arguments that inhomogeneity, in fact, is beneficial for superconductivity in correlated systems. One possible route to increase T_c in inhomogeneous systems has recently been discussed by Abrikosov.[25]

We will take the view that there are stripes in the underdoped cuprates and address how they modify the isotope effect. The discussion on the precise real space shape of the stripes in the presence of disorder has revealed a variety of complicated patterns.[26,27] We are not concerned here with the specific form of stripe order since we assume some typical stripe-stripe distance.

Recent scanning tunneling microscopy data on two lightly hole-doped cuprates, $Ca_{1.88}Na_{0.12}CuO_2Cl_2$ and $Bi_2Sr_2Dy_{0.2}Ca_{0.8}Cu_2O_{8+x}$, by Kohsaka *et al.*[28] reported the presence of a cluster glass with a large pairing amplitude of the localized pairs on the oxygen sites that form a real space glasslike pattern. We therefore assume that the superconductivity in the underdoped regime develops through the onset of a phase coherence between the superconducting regions

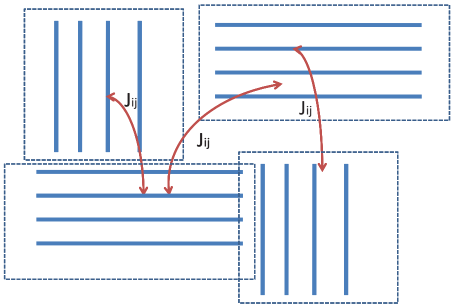

FIG. 1. (Color online) Schematic of the Josephson coupling J_{ij} between grains that contain stripe pieces. The average J determined by averaging over an ensemble of J_{ij} determines T_c.

that communicate with each other via Josephson coupling.[29] The precise real space arrangement of these regions is not crucial for our analysis except for the fact that the superconducting regions look like quasi-one-dimensional clusters with a broken orientational symmetry.

In this paper, we discuss the effect of isotope substitution on Josephson coupled stripes. In doing so, our starting point will be the linear relation between the incommensurate peak splitting and T_c observed in YBCO and LSCO.[30,31] In the case of YBCO, this relation does not extend as far as for LSCO as a function of doping.

Inelastic neutron scattering data for YBCO as well as for LSCO indicate incommensurate neutron scattering peaks, with an incommensuration $\delta(x)$ away from the (π,π) point. It is also known that $T_c(x)$ taken as a function of the doping x can be replotted as a linear function of the incommensuration for these materials, a so-called Yamada plot. This proportionality implies that the constant that relates these two quantities, one being the incommensuration (momentum) and the other being $T_c(x)$, or energy, has the dimension of velocity and is denoted by v^* as follows:

$$k_B T_c(x) = \hbar v^* \delta(x). \tag{1}$$

This experimentally derived relation can be obtained in a simple model of Josephson coupled stripes (Fig. 1). We address the role of the $O^{16} \rightarrow O^{18}$ isotope effect on $T_c(x)$ within this framework. We argue that the incommensuration is set by the doping of the sample and is not sensitive to the oxygen isotope given the fixed doping. We find therefore that the only parameter that can change in the relation $T_c(x) \sim \delta(x)$ is the velocity v^*. We estimate that the effect of the isotope substitution on v^* is on the order of 5% for both LSCO and YBCO materials.

II. DISCUSSION

The progress in neutron scattering has allowed for a multitude of inelastic neutron scattering data to be gathered for the high-T_c superconductor $YBa_2Cu_3O_{6+x}$. If one follows the off-resonance spectrum to lower energies, one finds incommensurate peaks with an incommensuration δ that is doping dependent. From the neutron data for YBCO (Fig. 2) for an oxygen concentration x, where $0.45 \leq x \leq 0.95$, with max $T_c(x) = 93$ K, a simple linear relation between T_c and δ for the doping range $x \leq 0.6$ was found to follow Eq. (1) where $\hbar v^* = 37$ meV Å (see Ref. 31).

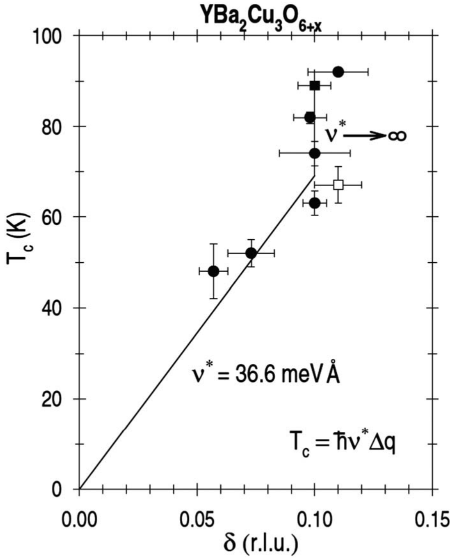

FIG. 2. Linear relation between the incommensurate peak splitting δ and T_c for the YBCO superconductor. The linear slope of the curve means that $k_B T_c(x) = \hbar v^* \delta$. We find $\hbar v^* = 37$ meV Å. This figure is taken from Fig. 25 of Ref. 31.

Another well-studied system is LaSrCuO. Inelastic neutron scattering on La_{214} compounds show incommensurate peaks at $(\pi \pm \delta, \pi)$ and $(\pi, \pi \pm \delta)$ (see Ref. 30). It was experimentally determined that T_c is a linear function of δ up to the optimal Sr doping value,[30] and Eq. (1) holds as well. Here, the constant of proportionality for LSCO is $\hbar v^* = 20$ meV Å. For both materials, the velocity $\hbar v^*$ is 2 orders of magnitude smaller than the Fermi velocity of nodal quasiparticles, $\hbar v_F \simeq 1$ eV Å (see Ref. 32). Furthermore, the velocity $\hbar v^*$ is 1 order of magnitude smaller than the spin-wave velocity $\hbar v_{\mathrm{sw}} \simeq 0.65$ eV Å of the parent compound.[30] Similarly, for La_{214}, the inferred velocity is much smaller than the spin-wave velocity $\hbar v_{\mathrm{sw}} \simeq 0.85$ eV Å (see Ref. 33).

Does the relation in Eq. (1) imply the existence of an excitation with such a velocity? An interpretation of this relation is to connect the superconductivity mechanism to the existence of fluctuating stripes. The simple relation above gives an inverse proportionality between $T_c(x)$ and the doping dependent length $\ell(x)$ determined from neutron scattering, $\ell(x) = 1/\delta(x)$. Josephson tunneling of pairs between stripe segments can produce such a relation.

A model Hamiltonian of random stripe separation and inter- and intrastripe random Josephson coupling is

$$\mathcal{H} = \sum_{ij} J_{ij} \exp[\mathbf{i}(\phi_i - \phi_j)], \tag{2}$$

where the summation is taken over coarse-grained regions i with well-defined phases and where $J_{ij} = J(r_{ij}) = t_0 / r_{ij}^{\beta}$. Here, it is assumed that $J(r)$ has an exponential cutoff at lengths much larger than the stripe-stripe distance in order to have a well-defined thermodynamic limit. The stripe-stripe distance r is given by a probability distribution $P(r,\theta)$. For simplicity, we will, as was done in Ref. [29], assume a uniform distribution in two dimensions with $P(r,\theta)=C$ for $\ell-a\leq r\leq\ell+a$ and $0\leq\theta<2\pi$; otherwise, $P(r,\theta)=0$. Here, $a=\nu\ell$, where ν is a bounded parameter. This gives

$$\int_0^{2\pi}\int_0^{\infty}P(r,\theta)rdrd\theta=2\pi C\int_{\ell-a}^{\ell+a}rdr=4\pi C\ell a=1$$

so that $C=(4\pi\ell a)^{-1}$. This gives the expected $\langle r\rangle$,

$$\langle r\rangle=\int_0^{2\pi}\int_0^{\infty}P(r,\theta)r^2drd\theta=2\pi C\int_{\ell-a}^{\ell+a}r^2dr=\ell+\frac{a^2}{3\ell}\propto\ell,$$

and the expected $\langle J(r)\rangle$,

$$\langle J(r)\rangle=\langle t_0/r^{\beta}\rangle=\int_0^{2\pi}\int_0^{\infty}P(r,\theta)t_0r^{1-\beta}drd\theta$$

$$=2\pi Ct_0\int_{\ell-a}^{\ell+a}r^{1-\beta}dr=\frac{2\pi Ct_0}{2-\beta}[(\ell+a)^{2-\beta}-(\ell-a)^{2-\beta}],$$

which for $\beta=1$ gives $\langle J(r)\rangle=t_0/\ell$ so that one recovers the experimentally observed relation[29]

$$T_c(x)\simeq\langle J(r)\rangle\propto\langle r\rangle^{-1}\propto\delta(x). \quad (3)$$

The velocity v^* cannot be determined for this simple model without any further assumptions. We suggest that v^* is related to the phase dynamics of the superconducting regions (stripes).

From the simple relation in Eq. (1), we can now investigate the effect of isotope substitution on T_c. Because the hole concentration does not change and since $\delta(x)$ does not change with isotope substitution, the only parameter left that can be isotope dependent is $\hbar v^*$. Since v^* is related to the phase dynamics of the stripes, it is natural to expect that v^* will not be changed much by isotope substitution because of its slight effect on the band structure. From the measured oxygen isotope effect on T_c of YBCO,[34] we predict v^*_{18}/v^*_{16} to be at least 0.95, where the velocity v^* is indexed by the isotope mass, which is in agreement with our expectation.

The prediction on the change in v^* with isotope substitution is made as follows: The isotope effect parameter α is calculated as

$$\alpha=\frac{\ln[1-(T_c^{16}-T_c^{18})/T_c^{16}]}{\ln(m_{16}/m_{18})},$$

where m_{16} and m_{18} are the oxygen isotope masses. Furthermore, due to Eq. (1), $T_c^{18}/T_c^{16}=v^*_{18}/v^*_{16}$. This leads to

$$\frac{v^*_{18}}{v^*_{16}}=\left(\frac{16}{18}\right)^{\alpha}.$$

For YBCO, $\alpha=0.27$ ($T_c^{16}=60$ K),[34] so we get $v^*_{18}/v^*_{16}=0.969$; for LSCO, $\alpha=0.38$ ($T_c^{16}=38.3$ K),[34] so $v^*_{18}/v^*_{16}=0.956$.

We find that if the doping level is kept the same in isotope substitution, then the typical stripe-stripe distance, which is controlled by the doping x, does not change with x. Therefore, the only parameter that can change with isotope substitution is the coefficient that relates T_c to $\delta(x)$: $k_BT_c(x)=\hbar v^*\delta(x)$. This coefficient v^* has the dimension of velocity and describes the phase dynamics in Josephson coupled superconductors. Since v^* is related to an electronic degree of freedom, it is hardly surprising that it is only weakly dependent on O isotope substitution. We estimated the isotope effect on v^*, or equivalently on T_c, to be less than 5%. We can therefore predict the change in v^* to be on the order of a few percent within a wide doping range. This estimate is consistent with the isotope effect observed for the superfluid stiffness ρ_s for underdoped LCSO.[35,36]

III. CONCLUSION

In conclusion, we have considered the role of the $O^{16}\rightarrow O^{18}$ isotope effect on the T_c of Josephson coupled stripes and its implications for v^*. We find that the effect is small and is on the order of 5% at most. We argue that the effect on v^* is small because the underdoped materials enter into a superconducting state due to phase fluctuations and, therefore, the main effect that controls T_c is a Josephson coupling between superconducting regions. If these phase fluctuations are due to electronic degrees of freedom, lattice dynamics has a small but observable effect on v^*, which we estimate to be on the order of $\simeq 5\%$. This estimate would imply a similar scale effect on superfluid stiffness ρ_s, an estimate that is consistent with other experiments.[35]

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