

Magnetic field of Josephson vortices outside superconductors

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We consider the structure of Josephson vortices approaching the junction boundary with vacuum in large-area Josephson junctions with the Josephson length λ_J large relative to the London penetration depth λ_L . Using the stability argument for one-dimensional solitons with respect to two-dimensional perturbations, it is shown that on the scale λ_J , the Josephson vortices do not spread near the boundary in the direction of the junction. The field distribution in vacuum due to the Josephson vortex is evaluated, the information needed for the scanning superconducting quantum interference device microscopy.

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An experimental technique, scanning SQUID microscopy (SSM), has recently been developed for measuring magnetic field distributions due to vortices exiting superconducting samples.¹ Knowing the distributions, one can, in principle, extract the London penetration depth λ_L (either isotropic or anisotropic) and its temperature dependence.²

To extract the superconducting parameters from the measured field above the sample, one has to solve the problem of the field created by a vortex terminating at the sample surface inside and outside the sample. For isotropic bulk materials, the solution has been given by Pearl, who utilized the cylindrical symmetry of a vortex normal to the sample surface.³ For the anisotropic case, a more general approach should be employed to match solutions of London equations inside and of Maxwell equations outside.⁴ The most likely situation of a vortex perpendicular to the sample surface was described and implemented in the analysis of SSM data in Ref. 5. These calculations showed that vortices spread out in the superconductor while approaching the surface. For the vortex axis along z , the transverse components $h_{x,y}$ appear, describing field lines bending out of the vortex axis in the subsurface layer of a depth on the order of λ_L .

Originally, the SSM technique has been designed for studies of Josephson boundaries between misoriented high- T_c superconducting crystallites. Some of the most convincing demonstrations of the d -wave symmetry of these materials were obtained by using SSM to detect spontaneous half-flux-quantum vortices at the intersection of three Josephson boundaries between crystallines of a proper misorientation.^{6,7} Other configurations involving Josephson boundaries have been shown to contain spontaneous magnetic flux (e.g., closed triangular and hexagonal boundaries⁸ and faceted boundaries with alternating critical current⁹). However, the absence of theoretical description of Josephson vortices terminating on the sample surface prevented a reliable interpretation of a large body of SSM data.

The Josephson length λ_J for the boundaries in question (the scale of the field distribution in the Josephson vortex that plays a role of λ_L for the bulk vortices) might be on the order of microns. This suggests that a considerable spreading effect may take place at the distance of order of λ_J near the surface. Such an effect, if it exists, should be taken into account when relating the outside field distributions to the internal structure of Josephson vortices (J vortices).

We argue in this Brief Report that Josephson vortices do

not spread in the direction of the junction in the limit

$$\lambda_L \ll \lambda_J. \quad (1)$$

This conclusion makes the SSM data analysis not only possible but quite straightforward.

Let us start with an infinite two-dimensional (2D) Josephson junction in the plane (x, z) between two superconducting banks occupying the half-spaces $y > 0$ and $y < 0$. The stationary J vortex directed along z in such a junction is described by the sine-Gordon equation

$$\lambda_J^2 \varphi'' = \sin \varphi, \quad (2)$$

where $\varphi(x)$ is the gauge-invariant phase difference.¹⁰⁻¹² The Josephson length $\lambda_J \propto j_c^{-1/2}$ may vary due to differences in crystal misalignment and in quality of the boundary (j_c is the critical Josephson current density). The primes denote differentiation with respect to x . One follows the standard procedure: multiply Eq. (2) by φ' to get the first integral $\lambda_J^2 \varphi'^2 / 2 = C - \cos \varphi$. The Josephson current and the field ($\propto \varphi'$) at the junction should vanish as $|x| \rightarrow \infty$; this yields $C = 1$ for $\varphi(\infty) = 2\pi$. We then obtain

$$\varphi = 4 \tan^{-1}(e^{x/\lambda_J}). \quad (3)$$

The field at the junction plane,

$$h_z(x) = \frac{\phi_0}{4\pi\lambda_L} \frac{d\varphi}{dx} = \frac{\phi_0}{2\pi\lambda_L\lambda_J} \operatorname{sech} \frac{x}{\lambda_J}, \quad (4)$$

where $\phi_0 = \pi\hbar c / |e|$ is the flux quantum (the thickness of the insulating layer is assumed small relative to λ_L). The vortex described by Eqs. (3) and (4) is infinite in the z direction; the problem and the solution are, in fact, one dimensional.

Let us now remove the superconductors from the half-space $z > 0$; thus, the plane $z = 0$ is a free boundary with vacuum. The question arises what changes the J vortex (occupying now only $z < 0$) should undergo? The problem is no longer uniform in the z direction, and the z independence of the solution of Eq. (2) cannot be assumed. In other words, we now have $\varphi = \varphi(x, z)$ and the sine-Gordon equation in two spatial dimensions,

$$\lambda_J^2 \nabla^2 \varphi = \sin \varphi, \quad (5)$$

where ∇^2 is the 2D Laplacian. We look for solutions at the half-plane $-\infty < z < 0$ describing a J vortex somehow spread-

ing as $z \rightarrow -0$. The solution should satisfy certain boundary conditions: at $z \rightarrow -\infty$, φ must be of the form of Eq. (5). Besides, at any $z < 0$, the vortex field should vanish far from the vortex “core,”

$$\left. \frac{\partial \varphi}{\partial x} \right|_{x \rightarrow \pm \infty} = 0. \quad (6)$$

This formidable, at first sight, problem does have an exact and unique solution due to the “nonexistence” theorem by Derrick¹³ (see also Eilenberger¹⁴) that states that there are no *stable static* solutions of the sine-Gordon equation in more than one spatial dimension.

We reproduce this argument for the 2D case of interest here. The energy functional $E[\varphi(x, z)]$, which generates Eq. (5), can be written in the form

$$E = \int \int dx dz \left[(\nabla \varphi)^2 + \frac{4}{\lambda_J^2} \sin^2 \frac{\varphi}{2} \right] = K + U, \quad (7)$$

where we have dropped a constant prefactor irrelevant for the following. Let us assume that a function $\varphi(x, z)$ exists such that $\delta E / \delta \varphi = 0$; in other words, that the sine-Gordon equation [Eq. (5)] has a 2D bounded solution. We now evaluate the energy for the phase $\varphi(\eta x, \eta z)$ with an arbitrary scaling factor η . Substitute this in the integral [Eq. (7)] and change integration variables to $\eta x, \eta z$ to obtain

$$E(\eta) = K + U / \eta^2, \quad (8)$$

where the kinetic and potential parts, K and U , are defined for the original phase $\varphi(x, z)$. Since $\varphi(\eta x, \eta z)$ is a solution of $\delta E = 0$ for $\eta = 1$, we must have $dE(\eta) / d\eta|_{\eta=1} = 0$. Equation (8), however, gives $dE(\eta) / d\eta|_{\eta=1} = -2U < 0$. Thus, there are no 2D static solutions—either stable or unstable—of Eq. (5).

Note that for the one-dimensional (1D) case, $E(\eta) = \eta K + U / \eta$, $dE(\eta) / d\eta|_{\eta=1} = K - U$, and 1D solutions with $K = U$ do exist and are stable.

It should be noted that the above argument implies an infinite plane x, z ; both integrals [Eq. (7)] are taken on the interval $(-\infty, \infty)$. For the junction of our interest, $-\infty < z < 0$. Clearly, the scaling employed above can still be used for the

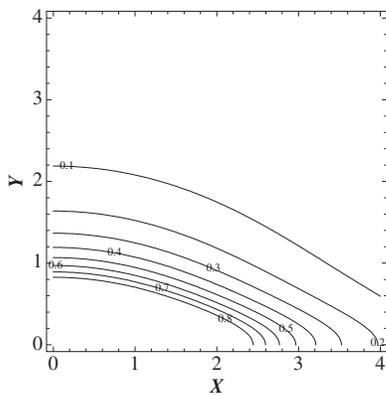


FIG. 1. Contours of $h_z(x, y) = \text{const}$ for $\lambda_J = 1$ and $z_0 = 0.5$. The horizontal axis is x along the junction. The field in units of $\phi_0 / 4\pi^2 \lambda_J^2$ reaches maximum at the origin $h_z(0, 0) = 3.507$. Values of h_z are indicated on the contours.

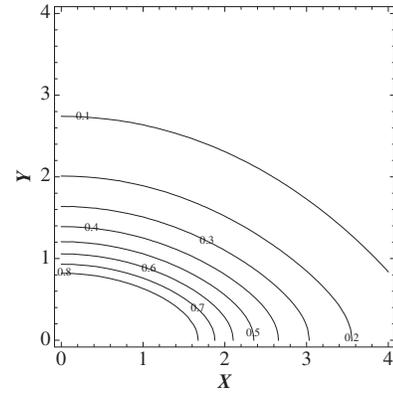


FIG. 2. Same as Fig. 1 for $z_0 = 1$. The maximum field is $h_z(0, 0) = 1.474$.

half-plane, provided that the coordinate z is counted from the junction edge so that the edge is left in place under the scaling transform.

The variational minimization of the energy [Eq. (7)] defined on the half-plane involves the integration by parts, which yields the boundary condition

$$\left. \frac{\partial \varphi}{\partial z} \right|_{z \rightarrow -0} = 0, \quad (9)$$

which is invariant with respect to the scaling employed above. This condition translates to $h_x(x, 0, 0) = 0$ along with $g_z(x, +0, -0) = g_y(x, +0, -0) = 0$.

We thus conclude that Eq. (3) with z independent $\varphi(x)$ is the only solution possible. In other words, the static Josephson vortex does not spread approaching the sample surface. It is worth mentioning that dynamic 2D solutions of the time dependent sine-Gordon equation do exist. A few examples are given in Ref. 15 where the initial essentially two-dimensional solitonlike structure has been shown (numerically) to evolve in time toward a straight 1D soliton.

We stress that the above conclusion holds when effects on the scale of London penetration depth are neglected. Of course, at distances of the order λ_L from the surface, the field deviates from the z direction and some spreading takes place. This effect, however, is hardly relevant for the SSM method

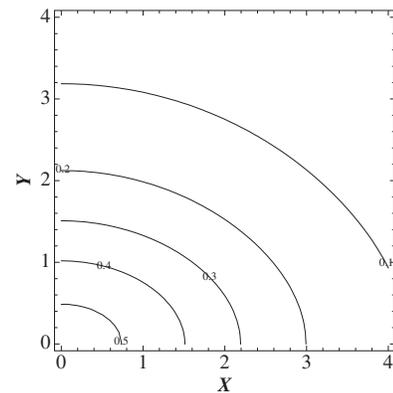


FIG. 3. Same as Fig. 1 for $z_0 = 2$. The maximum field is $h_z(0, 0) = 0.5388$.

because the low bound of the spatial resolution of the superconducting quantum interference device (SQUID) loops is set by the loop size, which exceeds by much the London depth.¹

Also, one has to keep in mind that near the free surface, the stray fields outside the superconductor may affect the very equation governing the behavior of the phase difference φ . This question has been discussed for thin-film junctions,^{16–18} where it was shown that the governing equation replacing the “bulk” equation [Eq. (5)] becomes integral; in other words, the stray fields make the problem non-local. This perturbation, however, penetrates the system only down to depths on the order of λ_L and can be disregarded in our approximation.

Since the field \mathbf{h} in vacuum satisfies $\text{curl } \mathbf{h} = \text{div } \mathbf{h} = 0$, one can introduce in the upper half-space $z > 0$ a scalar potential,

$$\mathbf{h} = \nabla \psi, \quad \nabla^2 \psi = 0. \quad (10)$$

The boundary conditions for the Laplace equation are provided by the Josephson field “sticking out” of the plane $z = 0$ from a belt of the width $2\lambda_L$ along the junction. With the help of Eq. (4), we obtain

$$\left. \frac{\partial \psi(x, y, z)}{\partial z} \right|_{z=0} = 2\lambda_L \delta(y) \frac{\phi_0}{2\pi\lambda_L\lambda_J} \text{sech} \frac{x}{\lambda_J}. \quad (11)$$

In magnetostatic terms, this corresponds to a linear “charge” with the density,

$$-\frac{\phi_0}{4\pi^2\lambda_J} \text{sech} \frac{x}{\lambda_J}. \quad (12)$$

Therefore, the potential ψ reads

$$\psi(x_0, y_0, z_0) = -\frac{\phi_0}{4\pi^2\lambda_J} \int_{-\infty}^{\infty} \frac{dx}{R \cosh(x/\lambda_J)}, \quad (13)$$

where

$$R^2 = (x - x_0)^2 + y_0^2 + z_0^2. \quad (14)$$

The field component h_z , which is actually measured by SSM, is readily obtained,

$$h_z(x_0, y_0, z_0) = \frac{\phi_0 z_0}{4\pi^2\lambda_J} \int_{-\infty}^{\infty} \frac{dx}{R^3 \cosh(x/\lambda_J)}. \quad (15)$$

This integral is well convergent at any finite z and is easy for numerical evaluation. It diverges at the junction line at $z=0$; the divergence, however, should be truncated at distances of the order λ_L , which are disregarded in our model.

Contours of $h_z(x, y) = \text{const}$ are shown in Figs. 1–3. For this calculation, we set $\lambda_J = 1$ and $z_0 = 0.5, 1, \text{ and } 2$. In actual SSM, the field $h_z(x, y; z)$ at a fixed height z should be integrated over the SQUID area. In some implementations, it is more convenient to have the 2D Fourier transform $h_z(k_x, k_y; z)$ instead of the real space distribution. After straightforward algebra, we obtain

$$h_z(k_x, k_y; z) = \frac{\phi_0 e^{-kz}}{\cosh(\pi k_x \lambda_J / 2)}, \quad k = \sqrt{k_x^2 + k_y^2}. \quad (16)$$

In particular, this shows that the total flux through any plane $z = z_0$ is $h_z(\mathbf{k} = 0; z_0) = \phi_0$.

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