Spin transport in proximity-induced ferromagnetic graphene

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Ferromagnetic insulators deposited on graphene can induce ferromagnetic correlations in graphene. We estimate that induced exchange splittings $\Delta \sim 5$ meV can be achieved by, e.g., using the magnetic insulator EuO. We study the effect of the induced spin splittings on the graphene transport properties. The exchange splittings in proximity-induced ferromagnetic graphene can be determined from the transmission resonances in the linear response conductance or, independently, by magnetoresistance measurements in a spin-valve device. The spin polarization of the current near the Dirac point increases with the length of the barrier, so that long systems are required to determine Δ experimentally.

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I. INTRODUCTION

The two-dimensional honeycomb lattice of graphene is a conceptual basis for describing carbon structures such as fullerenes, carbon nanotubes, and individual layers of graphite. The fabrication of free and stable monolayers of graphene a few years ago transformed this concept into an experimental reality that has attracted a tremendous interest from the research community.^{1–3} The low energy excitations of charge carriers in graphene are similar to massless relativistic Dirac (or rather Weyl) particles. The Hamiltonian is^{4,5}

$$H = -i\hbar v \,\boldsymbol{\sigma} \cdot \nabla + U(\boldsymbol{r}),\tag{1}$$

where the velocity $v \approx 10^6$ m/s is the analog of the Dirac electron speed of light (in the sense of limiting velocity) in graphene and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ is a two-dimensional vector of Pauli matrices acting on two-spinor states related to the two triangular sublattices constituting graphene's honeycomb lattice. The approximate Hamiltonian (1) is valid near the Dirac points *K* and *K'* in the reciprocal lattice. The two inequivalent Dirac points introduce a twofold valley degeneracy.⁶

The carrier concentrations are typically in the range $10^{11}-10^{12}$ cm⁻², corresponding to a Fermi wavelength of $\lambda_F \approx 50-100$ nm.^{3,7} The mean free path (mfp) has been reported to be of the order $l_{mfp} \approx 0.4 \ \mu m.^1$ With improved control over the fabrication process of graphene, we expect to see the realization of even cleaner samples with longer mean free paths.

Spintronics aim to inject, manipulate, and detect spins in electronic devices. Electrical spin injection in normal metals is routinely achieved by contacting ferromagnets like Fe, Ni, and Co with normal metals such as Cu and Al, and driving a current through the system. In semiconductors, electrical spin injection is more challenging because of the resistance mismatch between the semiconductor and possible ferromagnetic metal contacts.⁸ Nevertheless, spin injection into a semiconductor is feasible from a conventional ferromagnet when the interface resistance between the semiconductor and the ferromagnet is sufficiently large, as recently demonstrated by using Fe Schottky contacts in Ref. 9. Spin injection tubes contacted to ferromagnets has also been reported.¹⁰

Graphene is clearly an interesting candidate for spintronics applications since the carrier concentration can be controlled by gate voltages. Also, it has a very weak spin-orbit interaction, leading to the possibility of relatively long spin flip lengths.^{11,12} In a recent experiment on spin injection in single layer graphene, the spin flip (sf) length is found to be $l_{sf} \approx 1 \ \mu m$ at room temperature in dirty samples.¹³ Cleaner samples are expected to have even longer spin flip lengths.

We explore another possibility of spin dependent transport by envisioning that graphene is put in close proximity to a magnetic insulator. Via the magnetic proximity effect, exchange splittings will be induced in graphene. Strong proximity induced exchange splittings due to ferromagnetic insulators have been observed at EuO/Al interfaces.^{14–16} The effect was attributed to the nonvanishing overlap between the wave functions of the localized moments in the magnetic insulator and the itinerant electrons in the metal.¹⁷ The electronic wave functions can be described by atomiclike wave functions at the surface of thin Al films.¹⁸ The spatial range is similar for the atomic wave functions in Al and graphene, so we expect the overlap between the localized moments and itinerant electrons in graphene at EuO/graphene interfaces to induce exchange interactions comparable to those observed for EuO/Al. Based on the results reported in Refs. 15-17 and 19, we roughly estimate that exchange splittings in graphene due to the ferromagnetic insulator EuO could be of the order of 5 meV (see Appendix A for details).

In this paper, we show that proximity-induced splittings can be observed in the tunneling conductance associated with a tunable barrier created by the ferromagnetic insulator gate on top of graphene. First, for highly doped barriers, we demonstrate that the splitting Δ can be directly observed from the transmission resonances in the conductance.^{7,20,21} Moreover, for low doping of the barrier, we show that the spin polarization of the tunneling current, directly related to the spin splitting Δ , increases with increasing length of the barrier. The spin polarization can be studied by magnetoresistance (MR) measurements in a spin-valve device where two magnetic gates are deposited in series. Such MR measurements could also allow us to independently determine the induced spin splitting Δ .

This paper is organized as follows: We present a model of a magnetic gate in Sec. II. Section III reminds the reader of the results obtained in Refs. 7 and 22 for the conductance of



FIG. 1. A ferromagnetic insulator on top of graphene induces an exchange splitting in graphene. A metallic gate on top of the insulator controls the electrostatic potential.

a square barrier in graphene. Then we discuss how to obtain analytical expressions for the conductance both far from and close to the Dirac point. We extend the spinless situation to a spin dependent barrier with an exchange splitting Δ between the two spin channels in Sec. IV. First, we discuss the possibilities for extracting the splitting Δ directly from the conductance of a single highly doped barrier. Second, we study the dependence of the current spin polarization on the barrier height and length. Section V discusses the MR effect in a double barrier spin-valve device and discusses how it can be used to extract Δ . Finally, our conclusions are in Sec. VI.

II. MODEL

A possible way to construct a ferromagnetic gate is to deposit a magnetic insulator, such as EuO, on top of a graphene sample with a metallic gate above it (see Fig. 1). So far, experimental efforts have focused on depositing non-magnetic gates on graphene.^{23,24} The presence of a magnetic insulator will induce an exchange splitting in graphene. The normal metal gate allows us to control the Fermi level locally, i.e., to create a tunable barrier in graphene. In this way, both control of the charge and spin carrier concentrations can be achieved.

We assume in this paper that the normal metal gate induces a sharp potential barrier below it. This is a reasonable assumption provided the distance *d* between the gate and the graphene layer is much shorter than the Fermi wavelength λ_F , which is relatively long in graphene, $\lambda_F \approx 50-100$ nm.⁷ Recently, a method for manufacturing top gates where the distance from the gate to the graphene layer is of the order of λ_F has been demonstrated.²³ Observation of resonance effects due to sharp potential steps, therefore, seems feasible in graphene.

The exchange interaction between the localized magnetic moments in the ferromagnetic insulator and the spins of the electrons creates an additional spin dependent offset of the barrier potential, leading to the possibility of spin dependent tunneling. We estimate in Appendix A that the exchange splitting due to the magnetic insulator EuO can be around 5 meV. Here, we assume that the exchange interaction is not affected by the gate voltage of the top metallic gate.

III. TUNNELING PROBABILITY

For completeness, we first review the results for tunneling through a square barrier in graphene, and follow the deriva-



FIG. 2. Square barrier of length L.

tion in Refs. 7 and 22. We will later extend this discussion to a spin dependent barrier. The charge carriers we consider are Dirac quasiparticles, described by the Hamiltonian (1). These quasiparticles originate from reservoirs to the left and to the right of the ballistic graphene sample. E_F is the Fermi energy measured with respect to the Dirac point of the undoped graphene layer. At zero temperature, the transport properties are governed by quasiparticles that approach a square barrier of height U and length L (see Fig. 2) at energy E_F . We assume ballistic transport across the barrier, and also that the spin flip length l_{sf} is much longer than the other length scales of the problem. Given the values for l_{mfp} and l_{sf} reported for graphene,^{1,11,13} this regime should be realistic.

The Hamiltonian (1) has the following plane wave solutions in regions *I*, *II*, and *III* of Fig. 2, respectively:^{7,22}

$$\psi_{(I)} = \left[\begin{pmatrix} 1 \\ \alpha e^{i\theta} \end{pmatrix} e^{ik_x x} + r \begin{pmatrix} 1 \\ -\alpha e^{-i\theta} \end{pmatrix} e^{-ik_x x} \right] e^{ik_y y}, \qquad (2)$$

$$\psi_{(II)} = \left[a \begin{pmatrix} 1 \\ \beta e^{i\phi} \end{pmatrix} e^{iq_x x} + b \begin{pmatrix} 1 \\ -\beta e^{-i\phi} \end{pmatrix} e^{-iq_x x} \right] e^{iq_y y}, \quad (3)$$

$$\psi_{(III)} = \begin{pmatrix} 1\\ \alpha e^{i\theta} \end{pmatrix} e^{ik_x(x-L)} e^{ik_y y}.$$
 (4)

The momentum of the incident particle makes an angle θ = arctan(k_y/k_x) with the *x* axis. The angle of refraction, i.e., the corresponding angle inside the barrier, is ϕ = arctan(q_y/q_x). We consider only elastic scattering at the interfaces and define

$$k_F \equiv (k_x^2 + k_y^2)^{1/2} = (\hbar v)^{-1} |E_F|$$
(5)

and
$$q_F \equiv (q_x^2 + q_y^2)^{1/2} = (\hbar v)^{-1} |E_F - U|.$$
 (6)

The parameters $\alpha = \text{sign}(E_F)$ and $\beta = \text{sign}(E_F - U)$ determine the wave function in the corresponding regions as either electronlike (positive sign) or holelike (negative sign). Translational invariance in the transverse (y) direction implies conservation of transverse momentum:

$$k_{v} = q_{v} \Longrightarrow k_{F} \sin \theta = q_{F} \sin \phi. \tag{7}$$

It is convenient to introduce the dimensionless variable

$$\xi = \frac{E_F - U}{E_F} \tag{8}$$

as a measure of the gate voltage U. $\xi=1$ corresponds to the case of no barrier. Throughout the rest of the paper, we will

make the substitution $u = \sin \theta$, and we recall that by definition $\alpha k_F = E_F / \hbar v$ and $\beta q_F = (E_F - U) / \hbar v$.

Matching the wave functions at the interfaces, $\psi_{(I)}(x=0) = \psi_{(II)}(x=0)$ and $\psi_{(II)}(x=L) = \psi_{(III)}(x=L)$, and solving for *t* give the transmission probability $T \equiv |t|^2$ for a given incoming angle^{7,22} θ :

$$T(u) = \frac{(\xi^2 - u^2)(1 - u^2)}{(\xi^2 - u^2)(1 - u^2) + u^2(1 - \xi)^2 \sin^2(q_x L)},$$
 (9)

where

$$q_x L = k_F L \sqrt{\xi^2 - \sin^2 \theta}.$$
 (10)

Both *t* and *T* are invariant under the transformation $k_y \rightarrow -k_y$ as a consequence of the continuity condition (7).

In a real device, the sample has a finite width *W*. The allowed incoming angles θ are, therefore, determined by the channel index *n*, due to the quantization of the transverse modes. This quantization condition is, for the infinite mass boundary condition, $k_y \rightarrow k_n = (n + \frac{1}{2})/W$, where *n* are integers in the range $0-N_{\text{max}} = [k_F W/ \pi - 1/2]$, and the transverse states are superpositions of states with positive and negative k_y .^{22,25} Provided that the transverse momentum is conserved across the barrier interfaces, Eq. (9) is valid for systems of both finite and infinite width.²²

The conductance through the barrier for each spin independent channel is given in the Landauer-Büttiker formalism as

$$G = g_v \frac{e^2}{h} \sum_{n=0}^{N_{\text{max}}} T_n,$$
 (11)

where $g_v=2$ is the valley degeneracy and T_n is the transmission probability [Eq. (9)] for a given transverse channel k_n . When the number of channels N becomes large, i.e., $k_FW \ge 1$, we can replace the summation over channels with an integration over transverse momenta, such that the conductance becomes

$$G = \mathcal{G}_0 \int_0^1 du T(u) = \mathcal{G}_0 g, \qquad (12)$$

with \mathcal{G}_0 defined as

$$\mathcal{G}_0 = \frac{2e^2}{h} \frac{k_F W}{\pi}.$$
(13)

The dimensionless conductance g in Eq. (12) is plotted in Fig. 3 as a function of the dimensionless gate voltage ξ .

From Eq. (10), we see that the longitudinal momenta in the barrier region, q_x , can be either purely real $(\xi^2 > u^2)$ or purely imaginary $(\xi^2 < u^2)$, corresponding to propagating and evanescent modes, respectively.²² The contribution to the conductance from the evanescent modes becomes dominant around $\xi=0$, and the scaling of the conductance with length at this point resembles that of a diffusive system.^{22,26} For $|\xi| < 1$, the conductance (12) can be split into the contributions from propagating and evanescent modes:



FIG. 3. (Color online) Conductance $g=G/\mathcal{G}_0$ as a function of $\xi=(E_F-U)/E_F$ normalized to one spin channel when $k_FL=14$. The solid (blue) line shows the numerical result using Eq. (12), while the dashed (red) line is computed using the approximation (17). $\mathcal{G}_0=2e^2k_FW/h\pi$.

$$g = \int_{0}^{|\xi|} du T(u) + \int_{|\xi|}^{1} du T(u) = g_{\text{prop}} + g_{\text{evan}}, \qquad (14)$$

from which it is readily seen that the evanescent modes dominate in the region near $\xi=0$ as long as T(u) > 0 for at least some $u > |\xi|$ (see Appendix B for details). For $k_F L \ge 1$ and setting $\xi=0$ in Eq. (9):

$$T(u) \approx \frac{1}{\cosh^2(k_F L u)}.$$
(15)

This corresponds to the limit $N_{\text{max}} \gg W/L$ in Ref. 22. Upon insertion of Eq. (15) into the integral (12), we find that the conductance at the Dirac point is inversely proportional to the system length:

$$g \approx \frac{1}{k_F L}.$$
 (16)

This corresponds to the so-called minimal conductivity $g_s G \times L/W = g_s g_v e^2/h\pi$,²² $g_s = 2$ being the spin degeneracy.

For $|\xi| < 1$ and $k_F L \ge 1$, we can approximate the conductance by the expression

$$g \approx (a_1 + a_2 \xi) |\xi| + \frac{1}{k_F L} \exp(-k_F L |\xi|),$$
 (17)

with $a_1=0.79$ and $a_2=0.21$ (see Appendix C for details and Fig. 3 for a comparison with the exact solution). Equation (17) reduces to Eq. (16) when $\xi \rightarrow 0$.

For $|\xi| > 1$, corresponding to a well or a large barrier, only propagating modes contribute, and we would expect to see resonances in the conductance due to quasibound²⁷ states in the barrier region. In the limit $|\xi| \ge 1$, using that $u^2 \le 1$, the tunneling probability (9) becomes

$$T(u) \approx \frac{1 - u^2}{1 - u^2 \cos^2(k_F L\xi)},$$
 (18)

resulting in the expression



FIG. 4. Ferromagnetic proximity effect splits the barrier according to spin such that $U^{\pm} = U \mp \Delta$.

$$g \approx \frac{|\cos(k_F L\xi)| - \sin^2(k_F L\xi) \operatorname{arctanh}[|\cos(k_F L\xi)|]}{|\cos(k_F L\xi)|^3}$$
(19)

for the dimensionless conductance (see Appendix B for details). The period of g as a function of ξ is π/k_FL . Also g oscillates between 2/3 and 1. The transmission probability analogous to Eq. (9) for a square barrier in a nonchiral twodimensional system,⁷

$$T_{\text{nonchiral}} = \frac{4(\xi^2 - u^2)(1 - u^2)}{4(\xi^2 - u^2)(1 - u^2) + (1 - \xi^2)^2 \sin^2(q_x L)}, \quad (20)$$

also gives oscillation with the same periodicity, but, in this case, the conductance oscillates between 0 and 1. The fact that the conductance given by Eq. (19) oscillates between 2/3 and 1 is due to the perfect tunneling of carriers near normal incidence in graphene. Another difference between graphene and a nonchiral system is that the transmission probability of the latter [Eq. (20)] is symmetric around $\xi=0$, while the transmission probability for graphene [Eq. (9)] depends also on the sign of ξ through the $(1-\xi)$ factor in the denominator. The asymmetry for the case of graphene can readily be seen in Fig. 3.

IV. SPIN DEPENDENT BARRIER

We now turn to a situation where the two spin channels see barriers of different heights, i.e., the bottom of the conduction band at the barrier is shifted differently according to spin. Such a shift can arise through a Zeeman interaction due to an in-plane magnetic field or exchange field.

The exchange term Δ splits the system into two separate subsystems according to spin. For an external magnetic field *B*, the splitting is given by $\Delta \approx 2\mu_B B$. We introduce the spin dependent variables

$$\xi^{\pm} = \xi \pm \delta = \frac{E_F - U}{E_F} \pm \frac{\Delta}{E_F},\tag{21}$$

where \pm denotes spins parallel (+) or anti parallel (-) to the exchange field (see Fig. 4). In the following, we will let $g^{+(-)}$ denote the spin resolved conductance for spins parallel (antiparallel) to the exchange field. Assuming no spin flip, $l_{sf} \ge L$, the total conductance g_T across the barrier is given by the sum:

$$g_T = g^+ + g^- = g(\xi^+) + g(\xi^-). \tag{22}$$



FIG. 5. (Color online) Spin resolved conductance through a square barrier for $k_F L=14$ and $\delta = \Delta/E_F = 0.05$. The normalization of conductance is chosen as in Fig. 3 to correspond to g(1)=1 for each spin channel.

Because $\Delta/B \approx 5.8 \times 10^{-2}$ meV/T, a direct interaction of the spins with an external magnetic field gives only a very weak effect (about 1 meV per 20 T), and one will have to rely on more indirect effects to observe such spin splittings.

We propose depositing a ferromagnetic insulator, e.g., EuO, on top of the graphene sample to induce an exchange splitting in graphene. A normal gate on top of the insulator allows us to control the Fermi level in the same region. The resulting potential profile is sketched in Fig. 4. A rough estimate suggests that the splitting energy can be of order $\Delta \approx 5$ meV at EuO/graphene interfaces (see Appendix A).

As can be seen from Fig. 5, the effect of the splitting is simply to shift the conductance of each spin channel with respect to the other, leading to a broadening of the dip in the total conductance g_T near the Dirac point $\xi=0$. To be able to observe the splitting directly in the g_T near the Dirac point, the magnitude of the splitting must be larger than the width of the dip of each spin resolved conductance, $g^{+(-)}$. A measure $w=(k_FL)^{-1}$ of the width of the dip is discussed in Appendix C, leading to the condition

$$L > \frac{\hbar v}{|\Delta|} \tag{23}$$

for observation of the splitting directly in g_T at the Dirac point. However, the broadening of the dip due to a spin splitting would be difficult to distinguish from a broadening due to other effects.

From Fig. 5, it is apparent that the spin splitting has a more dramatic effect on the total conductance g_T at large barrier doping, since, due to the transmission resonances, g^+ and g^- can differ substantially at a given ξ . The asymptotic expression (19) for $|\xi| \ge 1$ implies that g_T has periodicity $\pi/k_F L$ in ξ for $\delta=0$, as shown at the bottom of Fig. 6. With increasing δ , each peak of g_T gradually splits into two spin resolved peaks. The splitting measured from the conductance 2δ equals $2\Delta/E_F$ (see Fig. 6), so, in principle, Δ can be determined directly from the total conductance across the barrier in this way.



FIG. 6. (Color online) Total conductance $g_T = g^+ + g^-$ when $|\xi| \ge 1$ for a range of different splittings $\delta = \Delta/E_F$. For clarity the curves are shifted upward in steps of 0.5 with increasing δ .

On the other hand, it is also possible to study the splitting by examining the spin polarization across the barrier.

We define a normalized spin polarization p along the direction of the exchange field as

$$p = \frac{g^+ - g^-}{g^+ + g^-}.$$
 (24)

Inserting the approximate expression for the conductance from Eq. (17) and comparing to exact numerical calculations, we find good agreement in the whole region $|\xi| < 1$ (see Fig. 7).

Equation (17) implies that the polarization becomes more pronounced with increasing barrier length L (see Fig. 8), owing to the fact that the evanescent modes are increasingly suppressed as L increases.

V. MAGNETORESISTANCE

Placing two magnetic gates a distance *D* apart in the graphene sample is a possible way to probe the polarization *p* in Eq. (24). We assume either that *D* is much larger than the mean free path $l_{\rm mfp}$ (but still much shorter than $l_{\rm sf}$), or that the experimental setup is realized as a three-terminal experiment, where the middle terminal completely randomizes the momenta between the two barriers (see Fig. 9).



FIG. 7. (Color online) The polarization p from the approximation (17) compared to the exact numerical result obtained directly from Eq. (12). Both plots are for $k_F L$ =14 and δ =0.05.



FIG. 8. (Color online) Polarization p as a function of $\xi = (E_F - U)/E_F$ for different barrier lengths L with $\delta = 0.05$.

Assuming that no spin flip processes take place in the sample, the conductance for each spin channel is found by treating the two barriers as resistors connected in series. Arranging the magnetizations of the ferromagnetic barriers parallel or antiparallel to each other gives different conductances $g_{\uparrow\uparrow\uparrow}$ and $g_{\uparrow\downarrow\downarrow}$, corresponding to the two situations in Fig. 9, respectively. We study the polarization using the "pessimistic" definition of the magnetoresistance: $MR = (g_{\uparrow\uparrow\uparrow} - g_{\uparrow\downarrow\downarrow})/g_{\uparrow\uparrow\uparrow}$. For the general case of different left (*L*) and right (*R*) barriers, we obtain

$$MR = \frac{4p_L g_L p_R g_R}{(g_L + g_R)^2 - (p_L g_L - p_R g_R)^2},$$
 (25)

assuming that the resistance of the region *D* between the barriers is negligible compared to the typical resistances of the barriers. For clarity, we have suppressed the subscript *T* denoting total conductance of the left (right) barrier: $g_{L(R)} \equiv g_{L(R)}^+ + g_{L(R)}^-$.

For identical barriers, MR reduces to the simple expression

$$MR = p^2.$$
(26)

The combination of Eqs. (17), (24), and (25) allows us to experimentally determine Δ from magnetoresistance measurements. The change of sign in the polarization, shown in



FIG. 9. Measuring tunneling magnetoresistance by placing two short barriers a distance $D \gg l_{mfp}$ apart.



FIG. 10. (Color online) Magnetoresistance for two barriers of equal height. The curves are shifted upward in steps of 0.25 for clarity.

Fig. 8, is directly related to the relative shift of the conductances corresponding to each spin channel. The coefficient MR is proportional to p^2 , which produces the double peak structure seen in Figs. 10 and 11. The condition for observing MR effects is also given by Eq. (23), $L > \hbar v / |\Delta|$. However, since the MR signal is only sensitive to the spin degree of freedom, we expect MR experiments to be a more direct probe of a spin induced splitting. Any broadening of the dip introduced by sources other than Δ will also be less important, since the polarization p changes sign around $\xi=0$.

For a splitting of Δ =5 meV, the condition in Eq. (23) gives L>110 nm (or, equivalently, $k_FL>20$). As can be seen in Fig. 8, the features in the polarization becomes sharper when increasing the length above this value. This also translates into a clearer signal in the magnetoresistance, which is plotted in Figs. 10 and 11 for barriers of equal and unequal heights, respectively.

Finally, even if the top gate creates a smooth tunable barrier, far from the perfectly square potential discussed here, magnetoresistance measurements should still provide an experimental demonstration of proximity-induced ferromagnetism in graphene, as the magnetic insulator still creates a sharp splitting of the spin up and spin down states in the region underneath the magnetic insulator. The exact dependence of the polarization p on the splitting Δ may be different in this case than the one presented here.

VI. CONCLUSIONS

We suggest using magnetic insulators deposited on top of graphene to create ferromagnetic graphene. The exchange



FIG. 11. (Color online) Same as in Fig. 10, with one barrier being lower than the other $(|\xi_L - \xi_R| = 0.1)$.

interaction between electrons in graphene and the localized magnetic moments in the insulator will give rise to a proximity-induced exchange splitting Δ . We have estimated that the graphene exchange splitting due to the magnetic insulator EuO in close proximity can be around Δ =5 meV.

We have studied how the conductance of a square barrier in graphene is modified by the presence of a ferromagnetic insulator. We show that for large barriers or deep wells, $|\xi| \ge 1$, the splitting Δ can be determined directly from the total conductance across the barrier, provided that the barrier is sharp enough for transmission resonances to appear. For a barrier of length $L > \hbar v / |\Delta|$, where v is the Fermi velocity of the charge carriers in graphene, Δ should be observable in the polarization of the tunneling current near the Dirac point of the barrier, irrespective of whether the barrier is smooth or sharp.

Demonstration of proximity-induced ferromagnetism in graphene should be possible through magnetoresistance measurements both for smooth and sharp barriers.

Note added. Recently, we became aware of a related work by Semenov *et al.*,²⁸ where a similar system with a magnetic gate is considered. Their work discusses the possibility of a spin field effect transistor, the feasibility of which relies on variations of the spin splitting across the sample of the same order of magnitude as our estimate for the splitting itself.

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APPENDIX A: ESTIMATION OF EXCHANGE SPLITTING AT EuO/ GRAPHENE INTERFACES

Experiments on depairing at EuO/Al interfaces suggest that the superconducting quasiparticles of Al experience an exchange field due to the Eu²⁺ moments.^{14,19} This interaction is short ranged; essentially, only the nearest layer of Eu²⁺ ions contributes. It has be shown that the exchange interaction between Eu²⁺ ions and charge carriers can be described as a Zeeman splitting^{14–17,19}

$$\Delta \approx c J \langle S_z \rangle, \tag{A1}$$

where *c* is the fractional density of Eu^{2+} ions to that of itinerant electrons in Al at the interface, *J* is the spatial average of the exchange integral, and $\langle S_z \rangle$ is the average spin of Eu^{2+} ions at a given temperature.

Perpendicular to the surface of thin Al films, the electronic wave functions can be well approximated by atomiclike wave functions.¹⁸ The spatial range of an atomic wave function is determined by the ratio Z/n,²⁹ where Z is the atomic number and *n* is the energy level. Since this ratio is approximately the same for the 3*s* and 3*p* orbitals in Al $(Z/n=13/3\approx4.3)$ and the 2*p* orbitals in graphene (Z/n=6/2=3), we expect the overlap between the wave functions of localized moments and itinerant electrons at EuO/ graphene interfaces to be comparable to those for EuO/Al. Accordingly, we assume that the exchange interaction between Eu²⁺ ions and itinerant electrons are the same at EuO/Al and EuO/graphene interfaces. Reference 17 reports the value J=15 meV for Eu/Al interfaces, which also agrees with the exchange energy $h_{ex}=0.1$ meV estimated in Ref. 19.

Using a nearest neighbor distance in graphene of 1.42 Å,³⁰ we obtain for the areal density of itinerant electrons $n_{\rm C} \approx 40$ nm⁻¹. Similarly, the areal density of Eu²⁺ ions at the surface of EuO is $n_{\rm Eu^{2+}} \approx 4$ nm⁻¹. Together this gives $c = n_{\rm Eu^{2+}}/n_{\rm C} \approx 10^{-1}$.

The temperature dependence of the average spin of Eu²⁺ ions in EuO is calculated in Ref. 31, showing that 3.5 $\geq \langle S_z \rangle \geq 3$ for 0 < T < 30 K.

Collecting all together, we arrive at the estimate

$$\Delta \approx 5 \text{ meV}$$
 (A2)

for the exchange splitting in graphene due to EuO. We stress that this is a very rough estimate which needs to be tested experimentally.

APPENDIX B: LIMITING CASES FOR THE CONDUCTANCE

1. Large potential

For large barriers or deep wells, $|E_F - U| \ge |E_F|$, $\xi^{-1} \ll 1$. The transmission probability (9) then becomes

$$T(u) \approx \frac{1 - u^2}{1 - u^2 \cos^2(k_F L\xi)}.$$
 (B1)

The conductance in this case is

$$g \approx \int_{0}^{1} du \frac{1 - u^{2}}{1 - u^{2} \cos^{2}(k_{F}L\xi)}$$
$$= \frac{|\cos(k_{F}L\xi)| - \sin^{2}(k_{F}L\xi) \operatorname{arctanh}[|\cos(k_{F}L\xi)|]}{|\cos(k_{F}L\xi)|^{3}},$$
(B2)

which oscillates between the values 2/3 and 1 with period $\pi/k_F L$ as a function of ξ .

2. Evanescent modes

When $|E_F - U| \ll |E_F|$, the evanescent modes dominate the transport so we neglect the contribution from the propagating modes. Using that $|\xi| \le u$ for evanescent modes, and that $|\xi| \ll 1$, we find

$$T(u) \approx \frac{1}{\cosh^2(k_F L u)},$$
 (B3)

which is valid for $k_F L \ge 1$. The conductance then becomes

$$g \approx \int_0^1 \frac{1}{\cosh^2(k_F L u)} \approx \frac{1}{k_F L}.$$
 (B4)

APPENDIX C: DIP OF THE TUNNELING CONDUCTANCE AROUND THE DIRAC POINT

We study how the width of the dip in the conductance around the Dirac point scales with barrier length L. The contributions from both evanescent and propagating modes must be considered when $|\xi| < 1$.

The conductance due to propagating modes can be written as

$$g_{\rm prop} = f(\xi) |\xi|, \qquad (C1)$$

where

$$f(\xi) = \int_0^1 dv \frac{1}{1 + \frac{v^2(1-\xi)^2}{(1-v^2)(1-\xi^2 v^2)} \sin^2(k_F L\xi \sqrt{1-v^2})}.$$
(C2)

When $k_F L \ge 1$, $f(\xi)$ is well approximated by a linear curve $a_1 + a_2 \xi$ for all $|\xi| < 1$. The function $f(\xi)$ deviates from linearity in an oscillatory fashion in a small region around $|\xi|=0$, but $f(\xi)$ is allways of order unity. For $k_F L \ge 1$, the conductance due to propagating waves in the region $|\xi| < 1$ can, therefore, be approximated by

$$g_{\text{prop}} \approx (a_1 + a_2 \xi) |\xi|, \qquad (C3)$$

where the value of the constants $a_1=0.79$ and $a_2=0.21$ depend weakly on $k_F L$ when $k_F L \ge 1$, and are found by fitting Eq. (C3) to numerical calculations.

We have not been able to obtain an analytical expression for the contribution due to evanescent modes. However, we note that the contribution from T(u) in Eq. (9) to evanescent modes can be well approximated by a decaying exponential function. We have fitted our numerical calculations of g_{evan} = $\int_{|\mathcal{E}|}^{1} du T(u)$ to an exponentially decaying function of $|\xi|$:

$$g_{\rm evan} \sim A e^{-B|\xi|}.$$
 (C4)

The constant A is found to be $1/k_FL$ by letting $\xi \rightarrow 0$ and comparing with Eq. (16). Numerical evidence suggest that $B = Ck_FL$, with C of order unity.

We define the width w of the dip in the conductance at the Dirac point as $w=2|\xi_c|$, where $|\xi_c|$ is the value of $|\xi|$ for which $g_{\text{prop}}(\xi_c)=g_{\text{evan}}(\xi_c)$. Taking advantage of the fact that g_{evan} decays rapidly away from $|\xi|=0$, we ignore the second order term in the expression for g_{prop} for the purpose of estimating the width w. We find that

$$w = 2|\xi_c| \approx \frac{1}{k_F L},\tag{C5}$$

using $2\mathcal{W}(1/a_1) \approx 1$, where \mathcal{W} is the Lambert W function.

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