

Rectified dc voltage versus magnetic field in a superconducting asymmetric figure-of-eight-shaped microstructure

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We have measured periodic oscillations of rectified dc voltage versus magnetic field $V_{dc}(B)$ in a superconducting aluminum thin-film circular-asymmetric figure-of-eight microstructure threaded by a magnetic flux and biased with a sinusoidal alternating current (without a dc component) near the critical temperature. The Fourier spectra of these $V_{dc}(B)$ functions contain fundamental frequencies representing periodic responses of the larger and smaller asymmetric circular loops, composing the microstructure, to the magnetic field. The higher harmonics of the obtained fundamental frequencies result from the nonsinusoidal character of loop circulating currents. The presence of the difference and summation frequencies in these spectra points to the interaction between the quantum states of both loops. Magnitudes of the loop responses to the bias ac and magnetic field vary with temperature and the bias current amplitude, both in absolute values and with respect to each other. The strongest loop response appears when the average resistive state of the loop corresponds to the midpoint of the superconducting-normal phase transition.

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I. INTRODUCTION

Superconducting loops interrupted by tunnel junctions are used in superconducting quantum interference devices¹ and typical superconducting flux qubits.² This work deals with double superconducting circular-asymmetric loops without tunnel contacts. For the first time, it has been found³ that a single superconducting asymmetric circular loop is a simple and very efficient rectifier of ac voltage.

Nonzero time-averaged rectified dc voltage $V_{dc}(B)$ was experimentally observed³ in a single superconducting aluminum asymmetric circular loop threaded by a magnetic flux Φ and biased with a sinusoidal alternating current (without a dc component) with an amplitude close to critical and frequencies up to 1 MHz at temperatures slightly below the superconducting critical temperature T_c . This $V_{dc}(B)$ voltage as a function of magnetic field B oscillates with the period $\Delta B = \Phi_0/S$, where Φ_0 is the superconducting magnetic flux quantum and S is the effective loop area.^{4,5} It was also shown³ that the magnitude of rectified voltage could easily be increased by a serial connection of such loops.

The $V_{dc}(B)$ oscillations as well as the Little–Parks (LP) ones⁶ are determined by the requirement of superconducting fluxoid quantization,⁷ or, set it another way, quantization of the circulation of a total quantum angular momentum along a closed contour. Unlike $R(B)$ oscillations in the LP effect,⁶ $V_{dc}(B)$ voltage is the odd function of magnetic field.³

The work by S. Dubonos *et al.*³ supplies indirect experimental arguments that $V_{dc}(B)$ voltage in a single asymmetric circular loop is directly proportional to a magnetic-field-dependent circulating current $I_R(B)$ of the loop. If this is the case, the measurement of $V_{dc}(B)$ (as we think) permits a complete determination of the quantum state of the loop, viz., both the magnitude and direction of the circulating current at different values of the magnetic field.

It would be of interest to investigate the quantum behavior of more complicated asymmetric multiply connected

structures by measuring rectified dc voltage $V_{dc}(B)$ as a function of the magnetic field. The $V_{dc}(B)$ measurements could be expected to allow the quantum state determination of each loop and possible interaction between the loops in a system of serial circular-asymmetrical loops with different areas. Therefore, we measured the $V_{dc}(B)$ voltage both in a system of two serial loops coupled by a wire (results obtained will be presented elsewhere) and in a system of two directly coupled loops forming a figure-of-eight structure (Fig. 1).

This work was inspired by a supposition made in Ref. 8. The subject discussed was the possibility to use an asymmetric circular loop like the one considered in Ref. 3, although with extremely thin walls, as an element for a flux qubit without tunnel contacts. At present, we do not know of any experimental work dealing with a superconducting flux qubit without tunnel contacts.

Quantum phase-slip centers (QPSCs)^{9–11} can be formed in nanostructures with cross-sectional dimensions of less than 10 nm and at temperatures below $0.5T_c$. Another superconducting flux qubit with QPSCs instead of tunnel contacts has recently been theoretically considered.¹² We believe that an asymmetric figure-of-eight structure, like the one considered here (Fig. 1), but with extremely thin loop walls, can be a prototype of two directly coupled different flux qubits. Need-

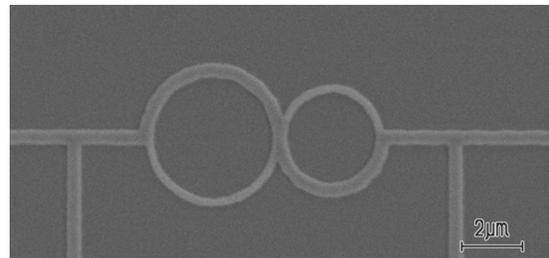


FIG. 1. Scanning electron microscopy image of the structure. The scale bar: 2 μm .

less to say that quantum tunneling between two distinct macroscopic quantum states cannot be realized in a structure with the geometry and external parameters used here.

Quantum behavior of systems of two superconducting loops as a function of the magnetic field was studied earlier. A double superconducting loop composed of two equal squares having a common side was used to study the features of the $T_c(B)$ function.^{13,14} Magnetic coupling between two superconducting coaxial square loops was experimentally studied.¹⁵ This work essentially differs from those reported in Refs. 13–15 by a circular-asymmetric geometry of the structure (Fig. 1). Unlike symmetric structures,^{13–15} the geometry of the structure provides a chance to use $V_{dc}(B)$ measurements to detect quantum states of both a figure-of-eight double loop taken as a whole and each circular loop individually.

The goal of the work is to experimentally study the quantum behavior of rectified dc voltage $V_{dc}(B)$ versus perpendicular magnetic field and bias sinusoidal low-frequency current (without a dc component) at temperatures slightly below T_c in a superconducting aluminum figure-of-eight structure (Fig. 1). Moreover, we hope to evaluate relative contributions of both circular loops of the structure into the total dc voltage and to detect a presupposed interaction in the structure.

II. SAMPLES AND EXPERIMENTAL PROCEDURE

Structures were fabricated by thermal aluminum deposition onto Si substrates using the lift-off process of electron-beam lithography. The NANOMAKER program package with correction for the proximity effect was employed. The central region of the structure (Fig. 1) is figure-of-eight shaped and circular asymmetric, with the widths of wide wires $w_w = 0.47 \mu\text{m}$ and narrow wires $w_n = 0.24 \mu\text{m}$ and the film thickness $d = 70 \text{ nm}$. The structure consists of two circularly asymmetric loops of different areas having a common area. The average area of the larger loop determined as a sum of average areas of the upper and lower semiloops is equal to $S_L^g = 14.51 \mu\text{m}^2$. The area of the smaller loop is $S_S^g = 8.51 \mu\text{m}^2$. The effective mean radii of the larger and smaller loops calculated from S_L^g and S_S^g are equal to $r_L = 2.15 \mu\text{m}$ and $r_S = 1.65 \mu\text{m}$, respectively.

The total normal-state resistance at $T = 4.2 \text{ K}$ is $R_{4.2} = 8.39 \Omega$. The ratio of room-temperature resistance to the helium one is $R_{300}/R_{4.2} = 2.22$, and the sheet resistance is $R_{\square} = 0.33 \Omega$, hence, the resistivity is $\rho = 2.37 \times 10^{-8} \Omega \text{ m}$. From the known mean value of the product¹⁶ $\rho l = 6 \times 10^{-16} \Omega \text{ m}^2$, the electron mean free path in the structure is $l = 25 \text{ nm}$. The superconducting coherence length of pure aluminum at zero temperature is $\xi_0 = 1.6 \mu\text{m}$. Because $l \ll \xi_0$ in our structure, it can be regarded as a “dirty” superconductor.¹⁷ The critical superconducting temperature $T_c = 1.324 \pm 0.001 \text{ K}$ was determined in the midpoint of normal-superconducting transition $R(T)$ at very small currents in a zero field.

At temperatures slightly below T_c , the temperature-dependent coherence length of the dirty superconductor and the field penetration depth are determined by the

expressions:^{17,18} $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$ and $\lambda_d(T) = \lambda_d(0)(1 - T/T_c)^{-1/2}$, respectively, where $\xi(0) = 0.85(\xi_0/l)^{1/2}$ and $\lambda_d(0) = 0.615\lambda_L(0)(\xi_0/l)^{1/2}$. Here, $\lambda_L(0)$ is the London penetration depth of a pure superconductor at zero temperature. In pure aluminum superconductors,¹⁸ $\lambda_L(0) = 0.05 \mu\text{m}$. Hence, $\xi(0) = 0.17 \mu\text{m}$ and $\lambda_d(0) = 0.25 \mu\text{m}$ in our structure.

We present four-probe measurements of the $V_{dc}(B)$ oscillations in the structure (Fig. 1). Magnetic field B was perpendicularly applied to the structure surface. The structure was periodically switched to the resistive state by a sinusoidal current (without a dc component) $I_{bias}(t) = I_{bias} \sin(2\pi\nu t)$, with the amplitude I_{bias} close to critical at frequencies ν of up to 1 MHz at temperatures slightly below T_c . The $V_{dc}(B)$ voltage was measured at slowly varying B with a sweep period Δt_B . The $V_{dc}(B)$ was equal to the time-averaged momentary pulsating voltage $V(t)$ over a large time interval Δt_L . Moreover, the condition $\Delta t_B > 20\Delta t_L > 400\Delta t_I$ was valid in all the experimental cases. Here, Δt_I is the period of bias ac. So, the measured voltage $V_{dc}(B)$ was practically equal to $\frac{1}{\Delta t_L} \int_0^{\Delta t_L} V(t) dt$. The experimental $V_{dc}(B)$ function is probably the result of multiple time-averaged measurements of the structure quantum state. Similar $V_{dc}(B)$ functions were obtained for six structures of similar geometries and similar external parameters.

III. RESULTS AND DISCUSSION

Figure 2 shows the $V_{dc}(B)$ curves at different temperatures slightly below T_c in the structure biased with a sinusoidal current of an amplitude I_{bias} , close to critical, and a frequency of 1.5 kHz (without a dc component). The magnitudes of critical bias current $I_c(T, B=0)$ are shown in Fig. 2 at several temperatures T in the zero field. In higher fields, the $V_{dc}(B)$ oscillations were damped due to the suppression of the superconducting order parameter in a wire of finite width. For a detailed analysis, fast Fourier transforms (FFTs) of the $V_{dc}(B)$ functions were calculated. The left insets of Fig. 2 show the FFT spectra of the $V_{dc}(B)$ functions. The right inset of Fig. 2(a) presents the FFT spectrum of $V_{dc}(B)$ curve (the curve is not shown) with external parameters given in the inset. FFT spectra were obtained using two¹² uniformly distributed points in magnetic fields ranging from -50 to $+50 \text{ G}$. In the insets, numerous peaks are observed at certain frequencies which are the inverse magnitudes of different oscillation periods of the $V_{dc}(B)$ functions. The values of fundamental frequencies (first harmonics) f_{S1} and f_{L1} are equal to the inverse magnitudes of the oscillation periods corresponding to the effective areas of the smaller S_S and the larger S_L loops of the figure-of-eight structure and are

$$f_{S1} = 1/\Delta B_S = S_S/\Phi_0, \quad f_{L1} = 1/\Delta B_L = S_L/\Phi_0, \quad (1)$$

respectively.

Using average geometric values of areas for the smaller S_S^g and the larger S_L^g circular loops instead of the values of effective areas S_S and S_L , we obtain “geometric” values of the fundamental frequencies $f_{S1}^g = 0.41 \text{ G}^{-1}$ and $f_{L1}^g = 0.70 \text{ G}^{-1}$. Indeed, the FFT spectra exhibit corresponding peaks at frequencies of 0.37 and 0.63 G^{-1} close to these geometric

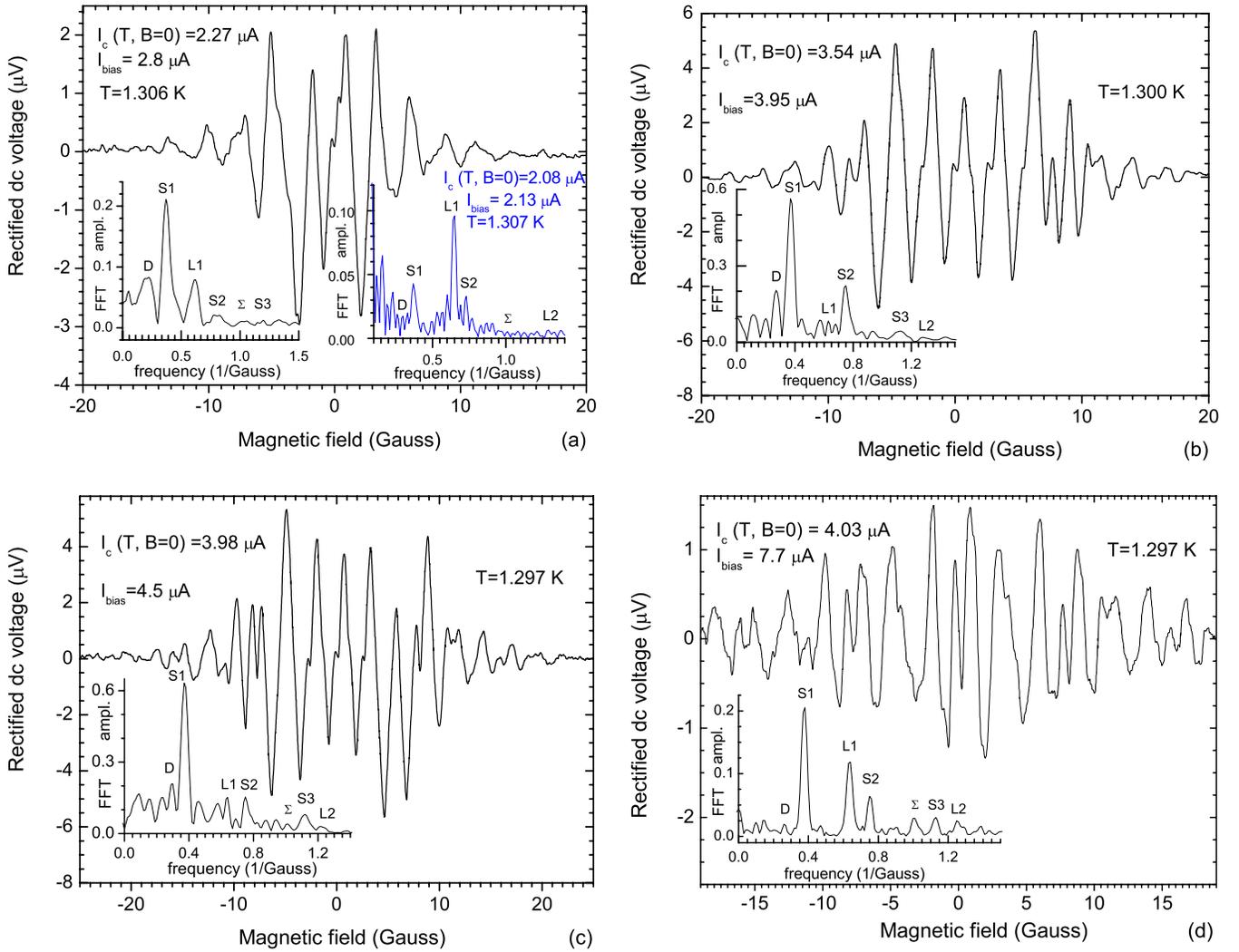


FIG. 2. (Color online) [(a)–(d)] Experimental $V_{dc}(B)$ plots. The left insets show the FFT spectra of the $V_{dc}(B)$ curves. The right inset of (a) presents the FFT spectrum of an experimental $V_{dc}(B)$ curve measured at parameters given in the inset [this $V_{dc}(B)$ curve is not shown here]. The symbols $S1$, $S2$, $S3$, and $L1$, $L2$ in the insets refer to the spectral peaks corresponding to the fundamental frequencies and their higher harmonics for the smaller and larger loops, respectively. The symbols D and Σ refer to the spectral peaks corresponding to the difference and summation frequencies, respectively.

values. The measured values of the fundamental Fourier frequencies, f_{S1} and f_{L1} , are smaller than their geometric values. The difference is probably due to the fact that the effective area of the loop is smaller than the averaged geometric one.

We found that quantum resistive contributions of both loops into the total rectified voltage can vary with temperature and bias current amplitude, both in absolute values and with respect to each other (Fig. 2). Unexpectedly, the amplitude of the peak corresponding to the fundamental frequency of the larger loop turned out to considerably exceed the amplitude of the peak corresponding to the fundamental frequency of the smaller loop [the right inset of Fig. 2(a)]. In addition to the fundamental frequencies f_{S1} and f_{L1} , the Fourier spectra contain higher harmonics of these frequencies, $f_{Sm} = mf_{S1}$ and $f_{Lm} = mf_{L1}$, where $m = 2, 3, 4, \dots$, and difference and summation frequencies, $f_D = f_{L1} - f_{S1}$ and $f_\Sigma = f_{L1} + f_{S1}$, respectively.

Let us now discuss the results obtained. It can be expected that at temperatures slightly below T_c , the most efficient rectification of alternating voltage is realized when a joint effect of bias ac $I_{bias}(t)$ and loop circulating current $I_R(B)$ periodically switch the structure from a superconducting S state to that with finite resistance (close to normal N state) and back. Moreover, a time-averaged resistive state of the structure would correspond to the midpoint of the S – N transition.

We guess that there are two reasons as to why the difference between the magnitudes of circulating currents in the loops forming the figure-of-eight-shaped structure can result in different relative resistive contributions of the loops into the total rectified dc voltage in the structure. On the one hand, if $V_{dc}(B) \propto I_R(B)$ in each circular loop, then the higher magnitude of the loop circulating current should result in a higher contribution into the total dc voltage. On the other hand, the relative contribution of each loop to the total dc

voltage should be a nonmonotonic function of the bias ac amplitude³ I_{bias} . Moreover, the contributions of the loops should reach their maximum at different values of the I_{bias} amplitude close to the critical current. A larger contribution of a loop should be expected when the average resistive state of the loop is closer to the midpoint of the $S-N$ transition.

To check the assumption that the difference between the magnitudes of circulating currents results in different relative contributions of the loops, we calculated circulating currents of each loop composing the structure at four values of T shown in Fig. 2 and in the right inset of Fig. 2(a). We used the Ginzburg-Landau theory for finite-wall-thickness asymmetric circular loops, neglecting radial variations of the order parameter and self-field generated by superconducting currents, which is reasonable because $\lambda_d(T) > d = 70$ nm, $\xi(T) \approx 1 \mu\text{m} > w_n = 0.24 \mu\text{m}$, and $w_w = 0.47 \mu\text{m}$. The calculated circulating currents of the smaller $I_{RS}(B)$ and the larger $I_{RL}(B)$ loops are not strongly harmonic functions. In higher fields, $I_{RS}(B)$ and $I_{RL}(B)$ oscillations as well as the $V_{dc}(B)$ ones were damped.

Provided that $V_{dc}(B) \propto I_R(B)$ in each asymmetric circular loop, nonharmonicity of loop circulating currents should result in the appearance of higher harmonics of loop fundamental frequencies in the FFT spectra of the total dc voltage $V_{dc}(B)$ measured in the structure. At temperatures $T = 1.307$, 1.306, 1.300, and 1.297 K, the maximum magnitudes of the calculated circulating currents were equal to $|I_{RS}| = 0.14, 0.22, 0.66$, and 0.83 and $|I_{RL}| = 0.47, 0.55, 1.06$, and 1.25 in the smaller and the larger loops, respectively. Here, the current values are given in microampere.

So at these temperatures, a higher circulating current corresponds to a larger diameter of the loop. On the contrary, for circular loops (cylinders) with infinitely thin walls,⁷ the smaller the structure diameter, the higher the circulating current is. In our case, both walls of each asymmetric loop have two finite thicknesses, w_n and w_w . Therefore, nonzero terms containing $w/2R$ are included in the expression for circulating current.⁴ A larger value of $w/2R$ can result in a smaller value of the circulating current.

Using the maximum values of the calculated circulating currents and experimental magnitudes of the critical current $I_c(T, B=0)$, we can estimate how close the average resistive state of each loop with all parameters shown in Fig. 2 and the right inset of Fig. 2(a) can be to the midpoint of the $S-N$ transition. Then, presupposed relative contributions of both loops to the total rectified dc voltage can be evaluated. The estimations showed that for each average resistive state of the loop, there is a certain point in the $S-N$ transition.

At all parameters [except the parameters for the right inset of Fig. 2(a)], the point in the $S-N$ transition corresponding to the average resistive state of the smaller loop is nearer to the midpoint of the $S-N$ transition than the point corresponding to the average resistive state of the larger loop. Therefore, the contribution from the smaller loop can be expected to be higher than that from the larger one. With the parameters shown in the right inset of Fig. 2(a), the point corresponding to the average resistive state of the larger loop is closer to the midpoint of the $S-N$ transition, whereas the point corresponding to the average resistive state of the

smaller loop is closer to the region of the superconducting state. Then, the contribution of the larger loop should be much greater than that of the smaller one. With the parameters given in Fig. 2(d), the bias ac amplitude I_{bias} considerably exceeds the critical current $I_c(T, B=0)$, and the average resistive states of both loops are very close to the normal state. This results in both a radical decrease in absolute values of both loop contributions to the total rectified voltage and a decrease in the difference between relative contributions of both loops. Indeed, these estimated relative contributions of the loops forming the structure to the total rectified dc voltage agree with experimental relative contributions [Fig. 2 and the right inset of Fig. 2(a)].

The FFT spectra of the $V_{dc}(B)$ functions contain frequencies close to the summation f_Σ and difference f_D frequencies. Because $f_\Sigma = 1/\Delta B_\Sigma = f_{S1} + f_{L1} = (S_S + S_L)/\Phi_0$ and $f_D = 1/\Delta B_D = f_{S1} - f_{L1} = (S_L - S_S)/\Phi_0$, the f_Σ and f_D frequencies are directly proportional to the sum and difference of the loop effective areas, respectively. The presence of the f_Σ and f_D frequencies in the spectra points out to the interaction (non-linear coupling) between the loops.

Let us consider possible mechanisms of the interaction. The magnetic inductive coupling between both circular loops of the figure-of-eight structure can qualitatively explain the appearance of the f_D frequency. However, the interaction should be weak because of the structure geometry. So, the inductive coupling between the two loops composing the figure-of-eight structure is ten times weaker than that between coaxial loops of the same dimensions.

Apart from circulating currents of both circular loops, an additional periodic magnetic-field-dependent closed current can appear along a figure-of-eight contour for which the requirement of superconducting fluxoid quantization is also valid. This additional closed current can qualitatively explain the appearance of the f_Σ frequency. The magnitude of the additional current should be small. Indeed, the magnitude of the FFT spectral peak corresponding to the f_Σ frequency was lower than the peak magnitude corresponding to the f_D frequency.

Another possible reason is that the difference and summation frequencies can be due to electrodynamic interaction between the circular loops, realized through a common bias ac. A characteristic longitudinal scale of the interaction should be expected to equal the penetration depth of a non-uniform electric field¹⁸ into the superconducting structure. The scale considerably exceeds the superconducting coherence length $\xi(T)$. Together with the oscillations of the loop circulating currents, weak oscillations of time-averaged absolute value of the order parameter versus magnetic field occur in both loops. When the bias ac with an amplitude close to critical passes through the figure-of-eight structure, both superconducting and normal components of the current become doubly modulated with the fundamental frequencies corresponding to both circular loops. Because of electrodynamic coupling between the bias ac and both loop circulating currents, difference, summation, and other combination frequencies can arise. To better understand the mechanisms of this interaction, further experiments will be carried out.

IV. CONCLUSION

In conclusion, quantum oscillations of a rectified dc voltage $V_{dc}(B)$ as a function of magnetic field were measured in a superconducting circular-asymmetric figure-of-eight structure. The Fourier analysis of the $V_{dc}(B)$ oscillations revealed relative contributions of both loops, forming the structure, into the total dc voltage. These contributions varied with the bias ac and temperature both in absolute magnitude and with respect to each other. The contribution of the loop is maximum when the average resistive state of the loop corresponds to the midpoint of the $S-N$ phase transition. An interaction between quantum states corresponding to the two circular loops was found. Magnetic coupling, formation of an additional figure-of-eight contour for a periodic magnetic-

field-dependent closed current, and electrodynamic coupling through a common bias ac can be the possible mechanisms of the interaction between the loops.

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