Large voltage from spin pumping in magnetic tunnel junctions

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We studied the response of a ferromagnet–insulator–normal metal tunnel structure under an external oscillating radio frequency (rf) magnetic field. The dc voltage across the junction is calculated and is found not to decrease despite the high resistance of the junction; instead, it is of the order of 1–100 μV, much larger than the experimentally observed value (100 nV) in the "strongly coupled" Ohmic ferromagnet–normal metal bilayers. This is consistent with recent experimental results in tunnel structures, where the voltage is larger than microvolts. The damping and loss of an external rf field in this structure are calculated.

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I. INTRODUCTION

There has been much recent interest in the spin dynamics in hybrid nanostructures composed of ferromagnetic (FM) and normal metal (NM) layers.[1–5] Following earlier work on the spin torque effect, the spin pumping effect[6] has been demonstrated for "strongly coupled" Ohmic metallic multilayers as an additional contribution to the ferromagnetic resonance (FMR) linewidth in FM/NM multilayers (where NM is Pt, Pd, Cu, etc.[7]) and, more recently, as a difference in voltages of the order of 100 nV between two FM/NM interfaces of a NM1/FM/NM2 trilayer.[8] Two types of metallic structures are commonly studied. In addition to the strongly coupled Ohmic multilayer systems, ″weakly coupled″ tunnel structures have also been extensively studied. The physics of these two types of systems can be very different.[9] In particular, for the tunnel structures, the coupling of the longitudinal magnetization and the charge lead to magnetization and charge dipole layers at the interface. After the effect of the electron-electron interaction is included, it is found that because of the large difference of the length scales associated with the charge (screening length ≈1 Å) and the spin (spin diffusion length ≈100 Å) fluctuations, there is a larger splitting of the chemical potentials than that predicted by the conventional spin accumulation picture.[10] Whereas the conventional picture suggests that the splitting scales with the current and will decrease with an increase in resistance, this is no longer true in the more complete picture.

Recently, Moriyama *et al.*[11] reported measurements of the dc voltage attributed to the spin pumping effect in different tunnel junctions and demonstrated that the voltage is larger than microvolts, enhanced orders of magnitude compared to that for metallic trilayers. In this paper, we generalize our recent work on spin torque[13] to the spin pumping situation and found an enhanced voltage for the tunnel structures, which is in agreement with the experimental results. We now describe our results in detail.

The system we have in mind is a ferromagnet–normal metal tunnel junction, where the two interfaces between the ferromagnet-insulator-metal sandwich structure are assumed to be at $z=\pm d/2$. We assume the z axis to be perpendicular to the faces of the tunnel junction. The initial magnetization is assumed to be in the $x-y$ plane, with an orientation given by $\mathbf{p}_0^L=\mathbf{e}_x$ for the ferromagnet on the left hand side of the sandwich structure.

Because the work functions of the metals on opposite sides of the junction may not be equal, at zero external radio frequency (rf) field there will be a charged dipole layer formed at the interfaces. What we are calculating here are the changes from the zero field situation. This surface inhomogeneity can lead to an additional contribution to the increase in the FMR damping, as we explain below. The experimental structures usually possess edge domains where the switching starts. The magnetization is thus not completely uniform in the $x-y$ plane. To bring out the essential physics, we shall not consider this complication in the present paper, but we hope to come back to this in the future.

Under an external time varying rf field, we expect the magnetization in the ferromagnet to be the sum of a uniform magnetization, which is a solution of the inhomogeneous Bloch (Landau-Gilbert) equation due to the external field, and a spatially varying solution of the homogeneous equation so that the boundary condition can be satisfied. This spatially varying part provides for the additional damping and the voltage observed in the experiments. Our approach is to obtain general solutions in each part of the junction [Eqs. (9), (10), (13), and (14)]. The amplitudes of these solutions are determined by the boundary conditions [Eq. (5)]. From these amplitudes, the voltage and the damping can be derived. We first describe the general solution of the magnetization in a ferromagnet.

II. STEADY-STATE MAGNETIZATION IN A FERROMAGNET

Our starting point is the equation of motion of the charge and the magnetization. For the charge, it is just the equation of charge current conservation

$$\nabla \cdot \mathbf{J}_e = -\frac{\partial \delta n}{\partial t}, \tag{1}$$

where $\mathbf{J}_e$ is the total current. The equation for the magnetization $\mathbf{M}$ has been extensively discussed in the past.[12] The equation takes the form of the phenomenological classical Landau–Lifshitz (Bloch) equation with longitudinal and transverse damping and an additional source term

$$\frac{\partial \mathbf{M}}{\partial t} - \gamma \mathbf{M} \times \mathbf{H} - \alpha \mathbf{M} \times (\mathbf{M} \times \mathbf{H}) + \nabla \cdot \hat{\mathbf{J}}_{\mathrm{M}} = - \frac{\delta \mathbf{M}}{\tau}, \quad (2)$$

where γ is the gyromagnetic ratio, and $\mathbf{H}$ is the effective field describing the precession of the magnetic moments given by $\mathbf{H}=\mathbf{H}_{\mathrm{e}}+\mathbf{H}_{\mathrm{an}}+\mathbf{H}_{\mathrm{dip}}+\mathbf{H}_{\mathrm{ex}}$. $\mathbf{H}_{\mathrm{ex}}=J\nabla^2\mathbf{M}$ is the effective field due to direct exchange; the anisotropy term includes a bulk and a surface anisotropy energy $\mathbf{H}_{\mathrm{an}}=H_{ab}+H_{as}$, where $H_{ab}=K\mathbf{M}_0$, and $H_{as}=K_sM_s$, with $M_s=\mathbf{M}_0\delta(z+d/2)$. Here, we have separated a bulk and a surface contribution that acts on the surface magnetization M_s. For simplicity, we have assumed this surface contribution to be localized at the interface. The other terms can also contain a surface contribution and can be treated in a similar manner as this ansiotropy contribution. For simplicity of presentation, we illustrate our results with just this term. $\mathbf{H}_{\mathrm{e}}$ represents the external field and $\mathbf{H}_{\mathrm{dip}}$ denotes the dipole-dipole interaction. $\hat{\mathbf{J}}_{\mathrm{M}}$ is a spin current (tensor). The currents are driven by density gradients (diffusion) and external forces as follows:

$$\mathbf{J}_e = - \sigma \nabla V - eD \nabla \delta n - D_M \nabla (\Delta \mathbf{M} \cdot \mathbf{p}_0),$$

$$\hat{\mathbf{J}}_{\mathrm{M}} = - \sigma_M \nabla (V\mathbf{p}_0) - D'_M \nabla \Delta \mathbf{M} - D' \nabla (\delta n \mathbf{p}_0), \quad (3)$$

where σ and σ_M are the effective conductivities for the charge and magnetization. $\mathbf{p}_0$ is a unit vector along the direction of the equilibrium magnetization: $\mathbf{p}_0=\mathbf{M}_0/|\mathbf{M}_0|$, with $\mathbf{M}_0$ the local equilibrium magnetization and $\Delta\mathbf{M}=\mathbf{M}[1-(|\mathbf{M}|/M_0)]$ the change in magnetization. D, D', D_M, and D'_M are the effective diffusion constants. $V=V_e+W$, with V_e the electric potential describing the external electric field and W the local electric (screening) potential due to the other electric charges determined self-consistently by

$$W(\mathbf{r}) = \int d^3\mathbf{r}' U(\mathbf{r}-\mathbf{r}')\delta n(\mathbf{r}'), \quad (4)$$

with U the Coulomb potential. The total number density of charge carriers and x component of magnetization are given by $n=\Sigma_s n_s$, and $M_x=\Sigma_s s n_s$, respectively. In the coordinate system with one of the coordinate axis along the direction of the magnetization, the spin current can be understood as the difference of the spin up current and the spin down current. The vector dependence is such that the equation is covariant. The Landau–Liftshitz equation without the source term $\nabla \cdot J_M$ is believed to describe the physics of ordinary domain walls, where the direction of the magnetization changes but its magnitude remains fixed. Equation (3) is consistent with this belief. For ordinary domain walls, $J_M=0$. τ is the longitudinal relaxation time, which describes the relaxation of the system toward its local equilibrium value of magnetization. α measures the transverse (Gilbert) damping term.

We assume that the transverse magnetization $\delta m_\perp$ is small and we linearized Eq. (2) with respect to it. The details are described in Appendix A. We summarize the results next.

The solution of the linearized equation can be written as a sum of two terms,

$$\delta \mathbf{M} = \delta \mathbf{M}_0 + \delta \mathbf{m}^i,$$

a spatially uniform ($\delta\mathbf{m}^i$) solution of the bulk inhomogeneous equation with the source term $\mathbf{M}_0\times\mathbf{H}_1$ and a spatially varying solution ($\delta\mathbf{M}_0$) of the homogeneous equation. $\delta\mathbf{M}_0$ is a linear combination of "eigenfunctions" with coefficients picked to satisfy the boundary conditions. By integrating the linearized equation over a small region of space at the boundary, we arrive at the condition that the difference between the tunneling and the ferromagnet pseudospin current is equal to the surface anisotropy term:

$$J^t_{\mathbf{M}} - I^L_{\mathbf{M}} = \gamma K_s \mathbf{M}_0 \times (\delta \mathbf{m}^i + \delta \mathbf{M}_{0s}), \quad (5)$$

where $J^t_{\mathbf{M}}$ is the tunneling magnetization current and the pseudospin current[14] $I_{\mathbf{M}}=J_{\mathbf{M}}-\gamma J\mathbf{M}_0\times\partial_z\delta\mathbf{M}$ includes an extra term involving the exchange that affects only the transverse magnetization current. We expect this extra term to be also present for Ohmic junctions, but so far it has not been included. In previous spin pumping studies[6] on Ohmic junctions, a term of a similar functional form $g\mathbf{n}\times\partial\mathbf{n}/\partial t$ ($\mathbf{n}=\mathbf{M}/|M|$) has been discussed. However, the coefficient was interpreted as a spin mixing conductance. These previous calculations on spin pumping did not consider the coupling to the charge degrees of freedom. Their equation describing the dynamic degrees of freedom also differs from our Eq. (2). We next describe the form of the magnetization.

The solution of the inhomogeneous equation is the same as that described in ferromagnetic resonance in textbooks and with $\mathbf{e}_\pm=\mathbf{e}_z\pm i\mathbf{e}_y$ and $\delta\mathbf{m}^i_\perp=\Sigma\,\delta m_\pm\mathbf{e}_\pm$, we find,

$$\delta m_\pm = \chi^0_\pm H_{1,\pm},$$

where the susceptibility $1/\chi^0_\pm=([-\pm i(1/l^2_{sf})+[(i\omega-\alpha')/D'_M]]+\zeta(\kappa/l^2_{sf})+\gamma H_0)/\gamma M_0$. Associated with this transverse magnetization, there is a change of the longitudinal magnetization given by

$$\delta m^i_x = M_0 - (M^2_0 - \delta m^{i2}_\perp)^{1/2} \approx 0.5\,\delta m^{i2}_\perp/M_0.$$

This is the lowest order correction to the longitudinal magnetization. Higher order nonlinear corrections to the transverse magnetization will produce changes in the longitudinal component that is higher than third order.

The solution of the homogeneous linearized equation is similar to that in our previous studies.[13] The solution involves a change in density of the charge and magnetization (dipole layers) near the interface.

We expect the charge and magnetization dipole layers to decay away from the interface, with length scales controlled by the spin diffusion length and the screening length. Because of the vector nature of the magnetization, there are three normal modes by which they can decay away from the interface. Including the charge degree of freedom, there are four normal modes that one can consider. For the ferromagnetic metal on the left hand side, we thus consider the following ansatz:

$$\delta n^L = \sum_{i=1}^{4} \delta n^L_{i0} e^{[z+(d/2)]/l_i}, \quad \delta \mathbf{M}^L_0 = \sum_{i=1}^{4} \delta \mathbf{M}^L_{i0} e^{[z+(d/2)]/l_i}, \quad (6)$$

where the superscript L denotes the left hand side.

We substitute this into the homogeneous linearized form of Eq. (2), let the coefficients before the exponential scaling functions vanish for steady-state solutions, and get for small ω the renormalized screening length

$$l_1 = \lambda_0 \xi_1^{1/2}, \tag{7}$$

the renormalized spin diffusion length

$$l_2 = l_{sf} \xi_2^{1/2},$$

and a combination of the exchange length and the spin diffusion length

$$l_{3,4} = l_{sf}/\{[(1 - \pm i\zeta\kappa_r)/(1 - \pm i\zeta)] + [(i\omega - \alpha')l_{sf}^2/D_M'^2(1 - \pm i\zeta)]\}^{1/2}.$$

The ξ's and β are measures of the asymmetry of the spin up and spin down conductivities of the ferromagnet: $\xi_1=[1-(D'D_M/DD_M')]/[1-(\sigma_M D'D_M/\sigma DD_M')+(i\omega\lambda_0^2/D)]$. $\xi_2=(1-(\beta^2)/(1-i\omega l_{sf}^2/D_M'))$, $\beta^2=[1-(D'D_M\sigma_M/\sigma DD_M')]$. $\kappa_r=\kappa+(\gamma H_0 l_{sf}^2/\zeta M_0)$. As we shall see below, l_3 and l_4 correspond to length scales with which the "precession" dies away from the interface. The additional term $\gamma\delta M\times H_0$ modifies these two lengths accordingly. The screening length and the spin diffusion length are renormalized. From Eqs. (1) and (3), we find that the charge densities can be related to the magnetization densities by

$$\delta n_{10}^L = e(\xi_1^L - 1)\delta M_{10}^L/\mu_B, \quad \delta n_{20}^L = \frac{e\lambda_0^2 D_M^L}{\mu_B l_2^2 D^L}\delta M_{20}^L,$$

$$\delta n_{30}^L = \delta n_{40}^L = 0. \tag{8}$$

Because $l_2 \gg \lambda_0$, it follows from the above equations that $\delta n_{20}/e \ll \delta M_{20}/\mu_B$. As we see below, generally δM_{20} is much less than δM_{10}. By inserting the "eigensolutions" into Eq. (6), we finally obtain the following analytic expressions for the dipole layers:

$$\delta n^L = \delta n_{10}^L e^{[z+(d/2)]/l_1} + \delta n_{20}^L e^{[z+(d/2)]/l_2}, \tag{9}$$

$$\delta \mathbf{M}^L = \mathbf{p}_0^L \delta M_{10}^L e^{[z+(d/2)]/l_1} + \mathbf{p}_0^L \delta M_{20}^L e^{[z+(d/2)]/l_2} + \mathbf{e}_+^L \delta M_{30}^L e^{[z+(d/2)]/l_3} + \mathbf{e}_-^L \delta M_{40}^L e^{[z+(d/2)]/l_4}. \tag{10}$$

The spatial dependence of the different contributions to the charge and magnetization densities is illustrated in Fig. 1. The charge is not coupled to the transverse magnetizations. The two transverse magnetization modes correspond to the left and right circularly polarized modes $\mathbf{e}_\pm$. δM_{i0}^L, with $i=1,2,3,4$, are to be determined later. Terms of the order $(\lambda_0/l_{sf})^2$ or higher have been neglected since $l_{sf}^2 \gg \lambda_0^2$. Also, to simplify the algebra, we have assumed the ferromagnetic thickness d_F to be larger than the spin diffusion length so that we do not need to worry about "reflection" effects from the leads. As advertised, the charge dipole layer is the sum of two terms, one decaying with a length scale of the screening length; the other, the spin diffusion length. The vector magnetization dipole is now a sum of four terms. The first two ($\delta\mathbf{M}_{10}^L$, $\delta\mathbf{M}_{20}^L$) are along the direction of the original magnetization; the last two are perpendicular to the direction of the original magnetization and describes the precession of the magnetization around the original axis. Again, the first two terms correspond to decay lengths of the order of the spin diffusion length and the screening length, while the precession term only decays with a length scale that is a combination of the exchange length and the spin diffusion length.

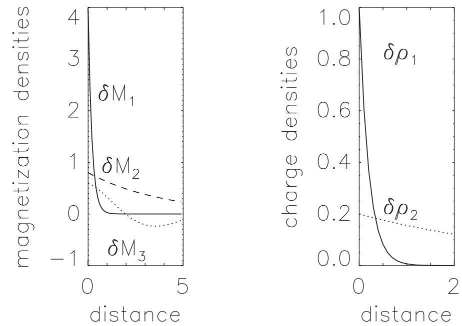

FIG. 1. The spatial dependence of the different components of magnetization (left) and the charge densities (right). For the magnetization densities, the solid, dashed, and dotted lines are for δM_1, δM_2, and δM_3, respectively. For the charge densities, the solid and dotted lines are for $\delta\rho_1$ and $\delta\rho_2$, respectively.

With Eqs. (9) and (10), the charge and magnetization currents $\mathbf{J}_e$ and $\hat{\mathbf{J}}_M$ can be worked out as

$$\mathbf{J}_e^L = \sigma\mathbf{E}_{ext}, \tag{11}$$

$$\hat{\mathbf{J}}_M^L = \sigma_M \mathbf{E}_{ext}\mathbf{p}_0^L/e + \frac{(1-\beta^2)D_M'}{l_2}\mathbf{e}_z\mathbf{p}_0^L\delta M_{20}^L e^{\delta z/l_2} + \frac{D_M'}{l_3}\mathbf{e}_z\mathbf{e}_+\delta M_{30}^L e^{\delta z/l_3} + \frac{D_M'}{l_4}\mathbf{e}_z\mathbf{e}_-\delta M_{40}^L e^{\delta z/l_4}, \tag{12}$$

where $\delta z = z + \frac{d}{2}$ and $\mathbf{E}_{ext}=E_{ext}\mathbf{e}_z$ is the external electric field inside the conductor. Note that the magnetization current is not a function of the rapidly varying part of the charge and magnetization densities δn_{10} and δM_{10}. In principle, the magnetization current can contain a term of the form $J_{M1}\exp(z/l_1)$. In the generalized Landau–Gilbert equation [Eq. (2)], terms of different functional dependences are each equal to zero. The only terms that are proportional to $\exp(z/l_1)$ comes from $\nabla\cdot J_M$ and is proportional to J_{M1}/l_1. This term and, hence, its contribution to the magnetization current are equal to zero. To match the quantities at the boundaries, we next consider the charge and magnetization in a normal metal (N).

III. NORMAL METAL

On the N side, the charge and magnetization are not coupled. The charge is given by

$$\delta n^R = \delta n_0^R \exp(-z/\lambda).$$

The magnetization satisfies the equation $\partial_t\mathbf{M}=(D_n\partial_z^2-1/\tau_{sf}^N)\mathbf{M}=0$. From this we obtain

$$\delta\mathbf{M}^R = \delta\mathbf{M}_0^R \exp[-(z-d/2)/l_R].$$

The longitudinal magnetization current at the interface $(z=d/2)$ is given by $J_M^R=-D_N\delta M_{0x}^R/l_R$.

IV. BOUNDARY CONDITIONS

For the longitudinal spin current, the quantity on the right hand side of Eq. (5) is zero. Also the longitudinal pseudospin current is equal to the longitudinal spin current. We obtain

$$J^t_M = J^L_M = J^R_M.$$

We first determine the charge and magnetization on the right in terms of those on the left. The longitudinal magnetization current at the left interface is given by Eq. (12). By equating J^R_M to J^L_M, we get

$$\delta M^R_{0x} = -(1-\beta^2)D'^L_M \delta M^L_{20} l_R/[D_N l_2]. \quad (13)$$

The magnetization on the right is proportional to δM^L_{20} and is not a function of δM^L_{10}. As we shall see below, $\delta M^L_{10} \gg \delta M^L_{20}$; hence, the longitudinal magnetization change on the right is much less than that on the left at the boundary. The charge neutrality condition $\int_{d/2}^{\infty}\delta n^L dz + \int_{-\infty}^{-d/2}\delta n^L dz = 0$ yields

$$\delta n^R_0 = -(l_1 \delta n^L_{10} + l_2 \delta n^L_{20})/\lambda. \quad (14)$$

These two equations express the quantities on the right in terms of quantities on the left. We now determine the amplitudes of the physical quantities on the left by matching the boundary condition involving the tunneling current.

The longitudinal magnetization tunneling current is equal to the difference of the spin up and spin down tunneling currents. From standard calculations of the tunneling current,[15] we get

$$J^t_M = \sum_s s|T_{ss'}|^2[\delta n_{Ls}(E+\delta\mu^L_s) - \delta n_{Rs}(E+\delta\mu^R_s)].$$

Here, δn_{Ls} contains contributions from the electric potential due to the charges at the interface[9] and that from the accumulation due to the bottleneck effect.[10] The change of the electron density of spin s can be related to the change of the total charge and magnetization densities by (we use units so that $\mu_B=1$) $\delta n_s = 0.5(\delta n + s\delta M_x)$. The longitudinal magnetization density is the sum of contributions from the solutions of the homogeneous and inhomogeneous equations: $\delta M_x = \delta m^i_x + \delta M_{x0}$. From Eq. (12),

$$J^L_M = \frac{(1-\beta^2)D'_M}{l_2}\delta M^L_{20}.$$

The inhomgeneous term δm^i is uniform. It contributes to the tunneling current but does not contribute to the magnetization current J^L_M inside the ferromagnet. From $J^t_M = J^L_M$, we get

$$(1-\beta^2)D'^L_M \delta M_{20}/l_2 = \sum_s s|T_{ss}|^2(\delta n_{Ls} - n_{Rs}).$$

All variables of this equation can be written in terms of the two independent variables $\delta M_{10,20}$. Now $\delta n_{0Ls} = 0.5[\delta n_{10,L} + \delta n_{20,L} + s(\delta M_{10,L} + \delta M_{20,L})]$. By using Eqs. (7) and (8), we get

$$(1-\beta^2)D'^L_M \delta M_{20}/l_2 = \sum_s s|T_{ss}|^2(\delta M^L_{10}\{\xi^L - 1 + s + [l_1(\xi^L-1)/\lambda]\} + s\delta m^i_x). \quad (15)$$

This equation implies that δM_{20} is of the order of $c_t \delta M^L_{10}/c_m$, where c_t (c_m) is the tunneling (metallic) conductance. c_t is much smaller than the metal conductance c_m. Thus, δM_{20} is much smaller than δM_{10}.

For an open circuit, the total charge tunneling current is zero. We get $J = \Sigma_s |T_{ss}|^2(\delta n_{Ls} - n_{Rs}) = 0$. By substituting in the expresssions for the charge densities and using the condition that $\delta M_{20} \ll \delta M_{10}$, we get

$$\sum_s |T_{ss}|^2(\delta M^L_{10}\{\xi^L - 1 + s + [l_1(\xi^L-1)/\lambda]\} + s\delta m^i_x) = 0.$$

By solving this equation, we finally obtain

$$\delta M^L_{10} = -f\delta m^i_x, \quad (16)$$

where $f = (\Sigma_s |T_{ss}|^2 s)/[\Sigma_s |T_{ss}|^2\{\xi^L - 1 + s + [l_1(\xi^L-1)/\lambda\ ]\}$. From Eq. (10) the corresponding charge is $\delta n^L_{10} = (\xi^L - 1)f\delta m^i_x$. The charge and the magnetization densities are proportional only to the ratio of the conductances. Hence, they are not necessarily small for tunnel junctions. As we emphasized before,[9] this comes about because $\lambda \ll l_{sf}$.

V. ELECTROMOTIVE FORCE

In this section, we shall estimate the voltage across the junction. Because the junction is not well characterized (the metallic part is polycrystalline; the quality of the insulator and the nature of the interface are not known), we feel that it is premature to perform a detailed calculation at this time. Instead, we shall settle for an order of magnitude estimate. The dc voltage is estimated as the change of the mean chemical potential across the interface, which is given by $\Delta V = 0.5\Sigma_s \Delta(\delta\mu_s) = 0.5\Sigma_s \Delta(\delta n_s/N_s)$, where N_s is the density of states. This drop can be written as ($\delta M^L \gg \delta M^R$)

$$\Delta V = 0.25(e\delta m^i_x/\mu_B)f[(\xi^L - 1)\{(1/N^L_+) + [2l_1/(\lambda^R N^R)] + (1/N^L_-)\} + (1/N^L_+) - (1/N^L_-)].$$

The longitudinal magnetization density is $\delta m^i_x = 0.5(\delta m^i_\perp)^2/M_0 v = 0.5\theta^2 M_0/v$, where v is the atomic volume and $\theta = \delta m^i/M_0$ is the precession angle. Hence,

$$\Delta V = 0.125e\theta^2 M_0/(v\mu_B)f\{(\xi^L - 1)[(1/N^L_+) + (2l_1/\lambda N^R) + (1/N^L_-)] + (1/N^L_+) - (1/N^L_-)\}. \quad (17)$$

As expected, this dc voltage is proportional to θ^2, as is observed experimentally. Most importantly, it is proportional only to a ratio of the conductances. Hence, its magnitude is not small. The factor f, as given after Eq. (16), depends on the asymmetry between the majority spin and minority spin conductances in the insulator. The larger the difference, the larger the value of $|f|$. We next estimate the order of magnitude of ΔV.

We expect $M_0/v\mu_B$ to be of the order of unity and e/N (N is the average density of states) to be of the order of 0.1 V. Depending on the asymmetry between the majority and minority spin tunnel conductances in the insulator, the value of f can range between 1 and 0.1. Similarly, depending on the asymmetry between the majority and minority spin conductances in the ferromagnet, $\xi^L - 1$ can range in value between 1 and 0.1; $(1/N_+) - (1/N_-)$ to be of the order of $1/N$

to $0.1/N$. Hence, $\Delta V \approx (10^{-2}-10^{-4})\theta^2$ V. For $\theta \approx 0.1$, $\Delta V \approx 10^{-4}-10^{-6}$ V, which is in agreement with the experimental results, which is larger than microvolts. We next address the issue of damping.

VI. DAMPING

The loss can come from three sources: (1) from the interface inhomogeneity, (2) from the loss of the transverse magnetization current through the barrier, and (3) from the loss of the longitudinal magnetization current. As we explain below, these contributions have different dependences on the external rf magnetic field. The contributions for the first two sources to the damping coefficient are independent of the field strength; that from the last source is proportional to the input power. The contributions from the last two sources are inversely proportional to the junction resistance and, thus, are much smaller for tunnel junctions.

We first estimate the loss connected with the longitudinal magnetization. This loss is equal to $\Sigma_s j_s^2 r_s$, where j_s and r_s are the current and junction resistance for spin s. This is of the order of $(\delta m_x^i)^2|T|^2$. Since δm_x is proportional to the input power, this loss is proportional to the power squared. Its contribution to the damping coefficient is obtained by normalizing the loss by the energy density and, hence, is proportional to the power. Because this loss is proportional to $|T|^2$, its contribution is small for tunnel junctions. Similarly, we expect the transverse magnetization current to incur a loss of the order of $(\delta m_\perp^i)^2|T|^2$. Since $\delta m_\perp$ is proportional to the field, this loss is proportional to the power. Its contribution to the damping coefficient, again obtained by normalizing with respect to the energy density, is thus independent of the power. This loss is also proportional to $|T|^2$ and will be small for tunnel junctions.

We next estimate the loss connected with the interface inhomogeneity. This requires knowledge of $\delta M_{30,40}$, which we now determine. Again, we expect the transverse magnetization to be the sum of a term that is the solution of the inhomogeneous equation (δm^i) and terms that are solutions of the homogeneous equation ($\delta M_{3,4}$). We calculate $\delta M_{3,4}$ by using the boundary condition given by Eq. (5). From Eq. (12), the transverse magnetization current at the boundary is

$$\hat{\mathbf{J}}_M^L = -\frac{D'_M}{l_3}\mathbf{e}_+\delta M_{30}^L - \frac{D'_M}{l_4}\mathbf{e}_-\delta M_{40}^L.$$

The pseudospin current in Eq. (5) is thus given by

$$\hat{\mathbf{I}}_M^L = -(D'_M + i\gamma J M_0)(\delta M_{30}^L \mathbf{e}_+/l_3) - (D'_M - i\gamma J M_0) \times (\mathbf{e}_- \delta M_{40}^L/l_4).$$

Equation (5) also involves the tunneling transverse current J_m^t. To evaluate this, we follow standard practice[15] and calculate the rate of change of the transverse magnetization due to tunneling. The details are described in Appendix B. We found that the tunneling current for the transverse magnetization can be written as $J_{M+}^t = M_+^L(g_1+ig_2)+M_+^R(g_3+ig_4)$, where[16] $g_{1,2,3,4}$ are proportional to $|T|^2$. A similar equation for J_{M-} can be written down. Thus, the contribution from the tunneling current is smaller than the other terms in Eq. (5) and will be treated by perturbation theory. We finally obtain, to lowest order, $-I_{\mathbf{M}}^L = \gamma K_s \mathbf{M}_0 \times (\delta\mathbf{m}^i + \delta\mathbf{M}_{0s})$. By substituting in the expression for I_m, this equation becomes

$$[\pm(iD'_M/l_\pm \gamma K_s M_0) - 1 - (J/l_\pm K_s)]\delta M_{\pm 0}^L = \delta m_\pm^i.$$

Here, $l_+ = l_3$, $l_- = l_4$, $\delta M_+ = \delta M_{30}$, and $\delta M_- = \delta M_{40}$. As we go away from the interface, the transverse magnetization density dies off exponentially. The total magnetization is given by $\delta M_\pm l_\pm = \delta m_\pm^i / X_\pm$, where $X_\pm = [\pm(iD'_M/\gamma K_s M_0) - l_\pm - (J/K_s)]$. The correction term due to the tunneling magnetization current is equal to $-X^{-1}I_m^t/\gamma K_s M_0$.

The magnetic susceptibility χ, given by $\chi = [(\delta M_\pm^L l/d_F) + \delta m_\pm]/H_{1,\pm}$, becomes $\chi = \chi^0[1+(X^{-1}/d_F)]$. The additional damping comes from the imaginary part of χ, which now contains a term proportional to $\mathrm{Re}(\chi^0)\mathrm{Im}\, X^{-1}/d_F$. This term is proportional to the metallic "resistance" D'_M, which in turn comes from the spatially varying part of the magnetization induced by the surface, as we have anticipated. This contribution is not a function of the junction resistance and will be of the same order of magnitude for multilayers as well as for tunnel barriers.

In conclusion, we discussed in this paper the voltage and the damping of a rf field in ferromagnetic tunnel junctions. The voltage is controlled by changes of the longitudinal magnetization, whereas the damping seems mainly associated with the transverse magnetization. Additional sources that can induce transverse magnetization localized near the interface can come from localized changes of the Hamiltonian such as the surface anisotropy. The calculation in this paper can be trivially extended to junctions with ferromagnets on both sides. For junctions involving two ferromagnets on opposite sides (F1-I-F2 or F1-F2), the interface anisotropy K_s will contain a term from the dipolar interaction between F1 and F2. The loss will then be a function of the orientation of the magnetizations of F1 and F2, which is consistent with experimental results.

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APPENDIX A

In this appendix, we provide some details in the derivation for the magnetization in the ferromagnet. By substituting the expression for $\hat{\mathbf{J}}_M$ into the modified Landau–Lifshitz equation (2), we obtain the following linearized relaxation equation for $\mathbf{M}$:

$$\nabla^2\delta\mathbf{M} - \left(\frac{1}{l_{sf}^2} + \frac{i\omega}{D'_M}\right)\delta\mathbf{M} + \zeta\mathbf{p}_0 \times \left(\nabla^2\delta\mathbf{M} - \frac{\kappa}{l_{sf}^2}\delta\mathbf{M} - \frac{\kappa_s\delta\mathbf{M}_s}{l_{sf}^2}\right) + \gamma(\delta\mathbf{M}\times\mathbf{H}_0 + \mathbf{M}_0\times\mathbf{H}_1) + \frac{\alpha\delta\mathbf{M}_\perp M_0 H_0}{D'_M} = -\left(\frac{D'}{D'_M}\right)\mathbf{p}_0\left(\nabla^2\delta n - \frac{\delta n}{\lambda_0^2}\right), \tag{A1}$$

where only $\mathbf{H}_{\rm ex}=J\nabla^2\mathbf{M}=J\nabla^2\delta\mathbf{M}$ and $\mathbf{H}_{\rm an}=K\mathbf{M}_0$ are kept in the precession term $\gamma\mathbf{M}\times\mathbf{H}$, and use has been made of Gauss' law: $\nabla^2 V=\nabla^2 W=-(e/\epsilon_0)\delta n$. The bare spin diffusion length l_{sf} and the bare screening length λ_0 are given by $l_{sf}^2=\tau D'_M$ and $\lambda_0^2=\frac{\epsilon_0 D'}{\sigma}$ respectively. Other dimensionless parameters are $\zeta=\gamma|\mathbf{M}_0|J/D'_M$ and $\kappa=l_{sf}^2K/J$.

The charge current conservation (1) yields, for the steady state without linearization,

$$\frac{1}{\lambda_0^2}\delta n-\left(\nabla^2-\frac{i\omega}{D}\right)\delta n-\left(\frac{D_M}{D}\right)\nabla^2(\delta\mathbf{M}\cdot\mathbf{p}_0)=0, \quad \text{(A2)}$$

which, together with Eq. (A1), describes the distribution of the charge and magnetization away from the tunnel junction in terms of their values at the junction. (To simplify the algebra, we have made the approximation that $D=D'$). The values of the charge and magnetization densities at the junction can be determined by matching boundary conditions across the barrier. We first solve these equations in the metal part of the junction. These solutions determine the charge and magnetization dipole layers.

The solution of Eq. (A1) can be written as a sum of two terms,

$$\delta\mathbf{M}=\delta\mathbf{M}_0+\delta\mathbf{m}^i,$$

a spatially uniform ($\delta\mathbf{m}^i$) solution of the bulk inhomogeneous equation with the source term $\mathbf{M}_0\times\mathbf{H}_1$ and a spatially varying solution ($\delta\mathbf{M}_0$) of the homogeneous equation. The inhomogeneous bulk equation is

$$-\left(\frac{1}{l_{sf}^2}+\frac{(i\omega-\alpha')}{D'_M}\right)\delta\mathbf{m}-\zeta\mathbf{p}_0\times\frac{\kappa}{l_{sf}^2}\delta\mathbf{m}+\gamma(\delta\mathbf{m}\times\mathbf{H}_0+\mathbf{M}_0\times\mathbf{H}_1)=0,$$

where $\alpha'=\alpha M_0H_0$. This is the conventional FMR equation, which can be readily solved. Define $\mathbf{e}_\pm=\mathbf{e}_z\pm i\mathbf{e}_y$, then $\mathbf{e}_x\times\mathbf{e}_\pm=\pm i\mathbf{e}_\pm$. We write the transverse magnetization as $\delta\mathbf{m}_\perp^i=\Sigma\,\delta m_\pm\mathbf{e}_\pm$ and obtain

$$\delta m_\pm=\chi_\pm^0H_{1,\pm},$$

where $1/\chi_\pm^0=\{-\pm i(1/l_{sf}^2)+[(i\omega-\alpha')/D'_M]+\zeta(\kappa/l_{sf}^2)+\gamma H_0\}/\gamma M_0$. Associated with this transverse magnetization, there is a change of the longitudinal magnetization given by

$$\delta m_x^i=M_0-(M_0^2-\delta m_\perp^{i2})^{1/2}\approx 0.5\,\delta m_\perp^{i2}/M_0.$$

This is the lowest order correction to the longitudinal magnetization. Higher order nonlinear corrections to the transverse magnetization will produce changes in the longitudinal component that is higher than third order in H_1. In the equation of motion (2), no lower order correction are produced.

The equation for the spatially varying term becomes

$$\nabla^2\delta\mathbf{M}_0-\left(\frac{1}{l_{sf}^2}+\frac{i\omega}{D'_M}\right)\delta\mathbf{M}_0+\zeta\mathbf{p}_0\times\left[\nabla^2\delta\mathbf{M}_0-\frac{\kappa}{l_{sf}^2}\delta\mathbf{M}_0-\frac{\kappa_s(\delta\mathbf{m}^i+\delta\mathbf{M}_{0s})}{l_{sf}^2}\right]+\gamma\delta\mathbf{M}_0\times\mathbf{H}_0+\frac{\alpha\delta\mathbf{M}_{0\perp}M_0H_0}{D'_M}$$
$$=-\left(\frac{D'}{D'_M}\right)\mathbf{p}_0\left(\nabla^2\delta n-\frac{\delta n}{\lambda_0^2}\right).$$

The solution of this equation has been previously described[13] and summarized in Sec. II.

APPENDIX B: TRANSVERSE MAGNETIZATION TUNNELING

In this section, we describe the calculation of the tunneling current for the transverse magnetization. As usual, we start by looking at the time rate of change of the transverse magnetization at the boundary. To illustrate, we look at $J_{M+}=\langle dM_{+,L}/dt\rangle=i[H_t(t),M_{+,L}]$ ($\hbar=1$). Here H_t is the tunneling Hamiltonian $\Sigma T_{ss}c_{R,s}^+c_{L,s}+\text{c.c.}M_{+L}=c_{+,L}^+c_{-L}$ and the square bracket represents the commutator. We obtain

$$\langle dM_{+,L}/dt\rangle=i(T_{++}c_{+R}^+c_{-L}-T_{--}^*c_{+L}^+c_{-R}). \quad \text{(B1)}$$

By expanding $O(t)=\exp[-i(H+H_t)t]O\exp[i(H+H_t)t]$ to the lowest order, we get from a typical linear response calculation

$$\langle dM_+/dt\rangle=-ei\int_{-\infty}^t dt'\langle[dM_{+L}(t)/dt,H_t(t')]\rangle,$$

where $F(t)=\exp(-iHt)F\exp(iHt)$ for any dynamical variable F. This involves expectation values of the form $\langle[(T_{++}c_{+R}^+c_{-L}-T_{--}^*c_{+L}^+c_{-R})(t),(T_{ss}c_{R,s}^+c_{L,s}+\text{c.c.})(t')]\rangle$ and can be written as $\langle-|T_{--}|^2[c_{+L}^+c_{-R}(t),c_{R,-}^+c_{L,-}(t')]+|T_{++}|^2[c_{+R}^+c_{-L}(t),c_{L,+}^+c_{R,+}(t')]+[(T_{++}c_{+R}^+c_{-L},T_{--}^*c_{L,-}^+c_{R,-}(t')]-[T_{--}^*c_{+L}^+c_{-R})(t),T_{++}c_{R,+}^+c_{L,+}(t')]\rangle$.

By carrying out the commutator, we get for the tunneling transverse magnetization current

$$\begin{aligned}J_{M+}\propto\int dt'&-|T_{--}|^2[\langle c_{+L}^+(t)c_{L,-}(t')\rangle\langle c_{-R}(t)c_{R,-}^+(t')\rangle\\&-\langle c_{R,-}^+(t')c_{-R}(t)\rangle\langle c_{L,-}(t')c_{+L}^+(t)\rangle]\\&+|T_{++}|^2[\langle c_{+R}^+(t)c_{R,+}(t')\rangle\langle c_{-L}(t)c_{L,+}^+(t')\rangle-\langle c_{L,+}^+(t')c_{-L}(t)\rangle\\&\times\langle c_{R,+}(t')c_{+R}^+(t)\rangle]+T_{++}T_{--}^*[\langle c_{+R}^+(t)c_{R,-}(t')\rangle\\&\times\langle c_{-L}(t)c_{L,-}^+(t')\rangle-\langle c_{L,-}^+(t')c_{-L}(t)]\rangle\langle c_{R,-}(t')c_{+R}^+(t)\rangle\\&-T_{++}^*T_{--}[\langle c_{+L}^+c_{L,+}(t')\rangle\langle c_{-R}(t)c_{R,+}^+(t')\rangle-\langle c_{R,+}^+(t')c_{-R}(t)\rangle\\&\times\langle c_{L,+}(t'^+)c_{+L}(t)\rangle].\end{aligned} \quad \text{(B2)}$$

We next evaluate the expectation values.

Now $\langle c_{sR}(t)c_{R,s}^+(t')\rangle=(1-n_{R,s})\exp[ie_{Rs}(t'-t)]$. $\langle c_{R,s}^+(t')c_{sR}(t)\rangle=n_{R,s}\exp[ie_{Rs}(t'-t)]$. In the present problem, there is a finite transverse magnetization due to the external transverse field: $M_+^i(t)=M_{+0}^i\exp(i\omega_c t)$. Now $\langle c_{L,+}(t)c_{-L}(t')\rangle=\langle c_{+L}(t)c_{L,-}(t)\rangle\exp[ie_{L-}(t-t')]$. The expectation value $\langle c_{+L}(t)c_{L,-}(t)\rangle$ can be related to $M_\pm^i(t)$. Similarly, we get

$$\langle c_{L,-}(t')c_{+L}^{+}(t)\rangle = \langle c_{L,-}(t)c_{+L}^{+}(t)\rangle \exp[ie_{L-}(t-t')].$$

$$< c_{L,+}^{+}(t')c_{-L}(t) > = \langle c_{+L}^{+}(t)c_{L,-}(t)\rangle \exp[ie_{L-}(t'-t)]$$

.

Carrying out the t' integration. We get

$$J_M \propto \langle c_{+L}^{+}(t)c_{L,-}(t)\rangle(-|T_{--}|^2\{[(1-n_{R,-})/(-ie_{L-}+ie_{R-}+\delta)] + [n_{R,-}/(ie_{R-}-ie_{L-}+\delta)]\} + |T_{++}|^2\{[n_{R,+}/(-ie_{R+}+ie_{L-}+\delta)] + (1-n_{R,+})/(ie_{L-}-ie_{R+}+\delta)\}) + \langle c_{+R}^{+}(t)c_{R,-}(t)\rangle T_{++}T_{--}^{*}\{[(1-n_{L,-})/(ie_{L-}+\delta-ie_{R-})] + [n_{L,-}/(ie_{L-}+\delta-ie_{R-})]\} + \langle[n_{L,+}/(ie_{R+}+\delta-ie_{L+})] + [(1-n_{L,+})1/(ie_{R+}+\delta-ie_{L+})]\rangle. \quad \text{(B3)}$$

Simplifying the algebra, we get for the real part of the current $\mathrm{Re}[J_M] \propto \langle c_{+L}^{+}(t)c_{L,-}(t)\rangle[-|T_{--}|^2/(-ie_{L-}+ie_{R-}) + |T_{++}|^2/(-ie_{R+}+ie_{L-}) + \langle c_{+R}^{+}(t)c_{R,-}(t)\rangle T_{++}T_{--}^{*}[1/(ie_{L-}-ie_{R-}) + 1/(ie_{R+}-ie_{L+})$. If the right hand side is a nonmetal, $e_{R+} = e_{R-}$, we get $\mathrm{Re}[J_M] \propto \langle c_{+L}^{+}(t)e_{L,-}(t)\rangle(-|T_{--}|^2+|T_{++}|^2)/(-ie_{R+}+ie_{L-}) + \langle c_{+R}^{+}(t)c_{R,-}(t)\rangle T_{++}T_{--}^{*}[1/(ie_{L-}-ie_{R})+1/(ie_{R}-ie_{L+})]$. There is a change in the correlation because the up and down electrons tunnel at different rates. Thus, the transverse magnetization current possesses an imaginary part and is damped.

For the imaginary part, we get

$$\mathrm{Im}(J_M)/\pi \propto \langle c_{+L}^{+}(t)c_{L,-}(t)\rangle[-|T_{--}|^2\delta(-e_{L-}+e_{R-}) - |T_{++}|^2\delta(-e_{L-}+e_{R+})] + \langle c_{+R}^{+}(t)c_{R,-}(t)\rangle T_{++}T_{--}^{*}[\delta(e_{L-}-e_{R-}) + + \delta(e_{L+}-e_{R+})].$$

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