Electron scattering in quantum wells subjected to an in-plane magnetic field

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It is shown that the electron scattering by static defects and acoustic or optical phonons in quantum wells subjected to an in-plane magnetic field is asymmetric. The probability of scattering contains terms which are proportional to both the electron wave vector and the magnetic field components. The terms under study are caused by the lack of an inversion center in quantum wells due to structure or bulk inversion asymmetry, although they are of pure diamagnetic origin. Such a magnetic field induced asymmetry of scattering can be responsible for a number of phenomena. In particular, the asymmetry of the electron scattering by phonons contributes to the electric current arising in quantum wells in the in-plane magnetic field when the electron gas is driven out of thermal equilibrium with the crystal lattice.

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I. INTRODUCTION

The processes of scattering of charge carriers by static defects, acoustic and optical phonons, plasmons, and other quasiparticles play an important role in solid states. They govern various transport and optical properties of both bulk materials and low-dimensional structures and determine, to a great extent, the performance of semiconductor devices. Significant information on the electron scattering in semiconductors is obtained by magnetotransport and magneto-optical measurements. In quantum well (QW) structures, the external magnetic field can be applied in the interface plane or perpendicular to the plane. The effect of the in-plane magnetic field on electron states in OWs is not so pronounced as that of the perpendicular field because, due to the strong quantum confinement, the in-plane field does not form the Landau levels. However, it has been established that the inplane magnetic field leads not only to spin splitting of electron states (the Zeeman effect) but also affects the orbital motion of free carriers.¹ In asymmetrical QW structures, the in-plane field induces a diamagnetic shift of each electron subband in k space,¹ which becomes important in tunneling between coupled QWs (see, e.g., Refs. 2-6) and direct optical transitions⁷⁻¹¹ and can also modify the electron-phonon interaction.^{12,13}

Here, we address the effect of an external magnetic field applied in the interface plane on the electron scattering in quantum wells. We show that the magnetic field leads to an in-plane asymmetry of the electron scattering by static defects or phonons, so that the scattering probability contains additional terms which are proportional to both the electron wave vector and the magnetic field components. Taking into account this contribution, one can write for the scattering rate

$$W_{k'k} = W_0 + \sum_{\alpha\beta} w_{\alpha\beta} B_{\alpha}(k_{\beta} + k'_{\beta}), \qquad (1)$$

where k and k' are the initial and scattered wave vectors, respectively, and B is the magnetic field. The scattering rate at zero field W_0 and the pseudotensor components $w_{\alpha\beta}$ are determined by the details of scattering processes and, owing to the time inversion symmetry, are even functions of the electron wave vector. Note that, in fact, the asymmetric part of the scattering rate can contain all terms odd in the wave vector including cubic terms. Although the scattering asymmetry does not modify the energy spectrum, it can affect kinetics of the carriers.

Phenomenologically, asymmetric terms in the scattering rate given by Eq. (1) are caused by the lack of a spatial inversion center in quantum wells due to structure and/or bulk inversion asymmetry. This follows from the symmetry analysis which does not require knowledge of microscopical mechanisms of the scattering. Indeed, the point groups of noncentrosymmetrical QWs make no difference between certain components of the axial vector \boldsymbol{B} and the polar vectors \boldsymbol{k} and k' allowing for the coupling $B_{\alpha}(k_{\beta}+k'_{\beta})$. The sign "+" in the parentheses is in accordance with the time inversion symmetry imposing the condition $W_{k'k}(B) = W_{-k-k'}(-B)$. We note that, from the point of view of the symmetry analysis, the terms under study are similar to spin-orbit-induced contributions to the matrix element of scattering $\propto \sigma_{\alpha}(k_{\beta}+k'_{\beta})$,^{14,15} where the Pauli spin matrices σ_{α} are replaced by the magnetic field components B_{α} . However, odd-in-k terms in Eq. (1) are of pure diamagnetic origin being not related to the spin of carriers.

To be specific, we consider below (001)-oriented QWs grown from zinc-blende-type compounds. In such structures, the pseudotensor $w_{\alpha\beta}$ contains two linearly independent components and the scattering rate [Eq. (1)] assumes the form

$$W_{k'k} = W_0 + w_{SIA} [B_x(k_y + k'_y) - B_y(k_x + k'_x)] + w_{BIA} [B_x(k_x + k'_x) - B_y(k_y + k'_y)],$$
(2)

where the coefficients w_{SIA} and w_{BIA} are caused by structure and bulk inversion asymmetries, respectively, $x \parallel [100]$ and $y \parallel [010]$ are the in-plane coordinates, and $z \parallel [001]$ is the QW normal.

Microscopically, the scattering asymmetry originates from the Lorentz force, which acts on moving charge carriers and modifies their wave functions. The effect is most easily conceivable for the terms in the scattering rate caused by the structure inversion asymmetry, i.e., terms proportional to w_{SIA} in Eq. (2). This case is illustrated in Fig. 1 for the



FIG. 1. Microscopic origin of the in-plane asymmetry of the electron scattering by the δ layer of impurities (dotted line) shifted with respect to the QW center. Due to the Lorentz force, the magnetic field $B \parallel y$ pushes electrons with the positive (negative) velocity v_x to the lower (upper) interface increasing (decreasing) the scattering rate.

electron scattering by impurities. The structure inversion asymmetry is modeled here by placing the δ layer of impurities (dotted line) closer to the lower interface rather than exactly in the QW center. The magnetic field B is applied along the y axis. Electrons with different velocities $v_x = \hbar k_x / m^*$ (m^{*} is the effective mass) move in the QW plane. Due to the Lorentz force $F_L = (e/c)[v \times B]$, where e is the electron charge and c is the light velocity, the magnetic field $B \parallel y$ pushes electrons to the lower or upper interface depending on the sign of their velocity v_x . This leads to the modification of the function of size quantization of electrons along the QW normal $\varphi(z)$ which becomes v_x dependent, as shown in Fig. 1. Since the δ layer of impurities is shifted from the QW center to the lower interface, the wave function of electrons with the positive velocity $v_{\rm r}$ is better overlapped with the impurity potentials than the function of electrons with the negative v_x . As a result, the electrons with $v_x > 0$ are scattered by impurities at higher rate than the carriers with $v_x < 0$ leading to the in-plane asymmetry of the scattering. Since the Lorentz force is proportional to both the magnetic field and the electron velocity, the small corrections to the scattering rate are linear in k and linear in B. In the above model, we assumed that the structure inversion asymmetry and, consequently, k-linear terms in the scattering rate are caused by the asymmetry of the doping profile with respect to the QW center. Obviously, the same arguments are valid for structures where nonequivalence of z and -z directions is achieved by the asymmetry of the QW confinement potential.

The term given by w_{BIA} in Eq. (2) is related to the lack of an inversion center of the QW host crystal with the zincblende lattice. Microscopically, it is caused by modification of the conduction-band wave function due to the magnetic field induced interband mixing. We leave such contributions out of the scope of the present paper focusing below on the terms caused by the structure inversion asymmetry.

The rest of the paper is organized as follows. Section II is devoted to microscopic calculations of asymmetric terms in the scattering probability for different mechanisms of the electron scattering including scattering by static defects and acoustic and optical phonons. In Sec. III, we consider one of the manifestations of the scattering asymmetry: we show that the asymmetry of the electron scattering by phonons contributes to the electric current arising in quantum wells in the in-plane magnetic field when the electron gas is driven out of thermal equilibrium with the crystal lattice. We shall focus on diamagnetic effects only and, therefore, neglect spin related contributions for simplicity.

II. MICROSCOPIC THEORY

We consider linear in the magnetic field terms in the scattering probability caused by the structure inversion asymmetry. To calculate such terms, it is sufficient to consider a one-band model and assume that the contribution to the effective Hamiltonian induced by the in-plane magnetic field is given by

$$H_{B} = \frac{e\hbar}{m^{*}c} (B_{x}k_{y} - B_{y}k_{x})z, \qquad (3)$$

where z is the coordinate operator.¹ The Hamiltonian [Eq. (3)] corresponds to the magnetic field $B = \operatorname{rot} A$ with the vector potential chosen in the form $A = (B_y z, -B_x z, 0)$.

The in-plane magnetic field intermixes electron states from different quantum subbands at nonzero wave vector k, leading to the dependence of the functions of size quantization on the wave vector.¹⁶ To first order in the perturbation theory, the electron wave function of the first level of size quantization e1 has the form

$$\psi_{1k}(\mathbf{r}) = \varphi_{1k}(z) \exp(i\mathbf{k} \cdot \boldsymbol{\rho}), \qquad (4)$$

where ρ is the in-plane coordinate, $\varphi_{1k}(z)$ is the function of size quantization,

$$\varphi_{1k}(z) = \varphi_1(z) - (B_x k_y - B_y k_x) \frac{e\hbar}{m^* c} \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \varphi_{\nu}(z), \quad (5)$$

 ν is the subband index [in Eq. (5) it runs over all conduction subbands except the subband e_1], $\varepsilon_{\nu 1} = \varepsilon_{\nu} - \varepsilon_1$ is the energy separation between the subbands, ε_{ν} is the bottom energy of the subband ν , $z_{\nu 1} = \int \varphi_{\nu}(z) z \varphi_1(z) dz$ is the coordinate matrix element, and φ_{ν} is the function of size quantization of the subband ν at zero magnetic field.

Taking into account the dependence of the electron wave functions on the magnetic field [Eq. (5)], one can derive that the scattering rate $W_{k'k}$ has the form of Eq. (2), with the ratio between w_{SIA} and W_0 being

$$w_{SIA} = -2W_0 \frac{e\hbar}{m^* c} \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \xi_{\nu 1}, \qquad (6)$$

where $\xi_{\nu 1}$ are dimensionless parameters. The explicit form of the scattering rate at zero magnetic field W_0 and the parameters $\xi_{\nu 1}$ depend on the details of scattering and are calculated below.

A. Scattering by static defects

At low temperatures, the electron scattering is dominated by elastic processes from static defects such as impurities, imperfections in the QW interfaces, etc. In the case of shortrange static defects, the matrix element of scattering has the form ELECTRON SCATTERING IN QUANTUM WELLS...

$$V_{k'k} = V_0 \sum_{j} \int \psi_{1k'}^*(\mathbf{r}) \,\delta(\mathbf{r} - \mathbf{r}_j) \,\psi_{1k}(\mathbf{r}) d\mathbf{r}, \tag{7}$$

where V_0 is a parameter characterizing the impurity strength, r_j is the impurity position, and the index *j* enumerates impurities contributing to the scattering.

The transition rate between electron states with the wave vectors k and k' is defined, in the Born approximation, by the standard equation

$$W_{k'k} = \frac{2\pi}{\hbar} |V_{k'k}|^2 \delta(\varepsilon_{k'} - \varepsilon_k), \qquad (8)$$

where $\varepsilon_k = \hbar^2 k^2 / (2m^*)$ is the electron kinetic energy. We note that the small diamagnetic correction to the kinetic energy, which is given by the diagonal matrix element of the Hamiltonian [Eq. (3)], is neglected in our analysis since it leads to no essential contribution to the scattering asymmetry. In fact, to first order in the magnetic field, it results only in a displacement of the subband spectrum in k space. This shift does not disturb the symmetric distribution of carriers within the subband and can be excluded by a proper choice of the coordinate origin.

Squaring the matrix element of scattering [Eq. (7)] and averaging it over the positions of impurities, one derives

$$W_0 = \frac{2\pi}{\hbar} |V_0|^2 N_d \delta(\varepsilon_{k'} - \varepsilon_k) \int_{-\infty}^{\infty} \varphi_1^4(z) u(z) dz, \qquad (9)$$

$$\xi_{\nu 1} = \frac{\int_{-\infty}^{\infty} \varphi_1^3(z) \varphi_{\nu}(z) u(z) dz}{\int_{-\infty}^{\infty} \varphi_1^4(z) u(z) dz},$$
 (10)

where N_d is the sheet density of impurities and u(z) is their distribution function along the growth direction, $\int u(z)dz=1$.

B. Scattering by acoustic phonons

At finite temperatures, electron-phonon interaction can predominate over the electron collisions with static defects. In the case of scattering from bulk acoustic phonons, the squared matrix element of the scattering assisted by emission or absorption of a longitudinal phonon has the form¹⁷

$$|V_{k'k}^{\pm}(q)|^{2} = \Xi_{c}^{2} \frac{\hbar q^{2} N_{q}^{\pm}}{2\rho_{c} \Omega_{q}} \left| \int \psi_{1k'}^{*}(r) e^{\mp i q \cdot r} \psi_{1k}(r) dr \right|^{2}.$$
(11)

Here, the upper and lower signs correspond to the phonon emission and absorption, respectively, Ξ_c is the conductionband deformation-potential constant, ρ_c is the crystal density, $N_q^- = N_q$, $N_q^+ = N_q + 1$, $N_q = 1/[\exp(\hbar\Omega_q/k_BT_0) - 1]$ is the phonon occupation number, $\Omega_q \approx s_L q$ is the frequency of the longitudinal acoustic wave, k_B is the Boltzmann constant, T_0 is the lattice temperature, s_L is the sound velocity, $q = |\mathbf{q}|$, and \mathbf{q} is the three-dimensional wave vector of the phonon involved, $\mathbf{q} = \pm (\mathbf{k} - \mathbf{k}', q_z)$.

The rate of the electron scattering assisted by emission or absorption of phonons is given by

$$W_{k'k}^{\pm} = \frac{2\pi}{\hbar} \sum_{q} |V_{k'k}^{\pm}(q)|^2 \delta(\varepsilon_{k'} - \varepsilon_k \pm \hbar \Omega_q).$$
(12)

We assume that electrons populate the bottom of the ground subband e1. Then, one derives

$$W_0^{\pm} = \frac{\Xi_c^2}{2s_L \rho_c} \int_{-\infty}^{\infty} |Q_{11}|^2 N_q^{\pm} \,\delta(\varepsilon_{k'} - \varepsilon_k \pm \hbar \Omega_q) q dq_z, \quad (13)$$

$$\xi_{\nu 1}^{\pm} = \frac{\int_{-\infty}^{\infty} \operatorname{Re}[Q_{11}Q_{\nu 1}^{*}] N_{q}^{\pm} \,\delta(\varepsilon_{k'} - \varepsilon_{k} \pm \hbar\Omega_{q}) q dq_{z}}{\int_{-\infty}^{\infty} |Q_{11}|^{2} N_{q}^{\pm} \,\delta(\varepsilon_{k'} - \varepsilon_{k} \pm \hbar\Omega_{q}) q dq_{z}}, \quad (14)$$

where $Q_{\nu 1} = \int \varphi_{\nu}(z) \varphi_{1}(z) \exp(iq_{z}z) dz$.

The electron scattering by acoustic phonons is usually considered as a quasielastic process because the energy of the phonon involved $\hbar\Omega_q$ is small, as compared to the electron kinetic energy ε_k . If one is interested in the momentum scattering only neglecting energy transfer between the electron and phonon systems, one can omit the term $\hbar\Omega_q$ in the δ functions in Eqs. (13) and (14). Under this approximation and provided that $N_q^{\pm} \approx k_B T_0 / (\hbar\Omega_q) \ge 1$, the transition rate W_0^{\pm} and the parameters $\xi_{\nu 1}^{\pm}$ assume the form

$$\begin{split} W_0^{\pm} &\approx \frac{\pi k_B T_0}{\hbar} \Xi_c^2 \delta(\varepsilon_{k'} - \varepsilon_k) \int_{-\infty}^{\infty} \varphi_1^4(z) dz, \\ &\xi_{\nu 1}^{\pm} \approx \frac{\int_{-\infty}^{\infty} \varphi_1^3(z) \varphi_{\nu}(z) dz}{\int_{-\infty}^{\infty} \varphi_1^4(z) dz}, \end{split}$$

which is similar to Eqs. (9) and (10) with u(z)=const. It is reasonable that acoustic phonons in the deformationpotential model behave as an ensemble of short-range scatterers uniformly distributed along the QW growth direction.

C. Scattering by optical phonons

At even higher temperatures, the electron scattering is governed by interaction with optical phonons. For the Frölich mechanism of electron-phonon interaction,¹⁷ which predominates in crystals such as GaAs at room temperature, the squared matrix element of the scattering assisted by emission or absorption of a longitudinal optical (LO) phonon has the form

$$|V_{k'k}^{\pm}(q)|^{2} = \frac{2\pi e^{2}\hbar\Omega_{\rm LO}N_{\rm LO}^{\pm}}{\epsilon^{*}q^{2}} \left| \int \psi_{1k'}^{*}(r)e^{\mp iq\cdot r}\psi_{1k}(r)dr \right|^{2},$$
(15)

where Ω_{LO} is the phonon frequency, $1/\epsilon^* = 1/\epsilon_{\infty} - 1/\epsilon_0$, ϵ_0 and ϵ_{∞} are the dielectric constants at low and high frequencies, respectively, $N_{LO}^- = N_{LO}$, $N_{LO}^+ = N_{LO} + 1$, and N_{LO} is the phonon occupation number. We assume that electrons populate the bottom of the ground subband only and the phonon energy $\hbar\Omega_{LO}$ is much smaller than the energy separation between quantum subbands. In this particular case, one obtains

$$W_0^{\pm} = \frac{2\pi^2 e^2}{\epsilon^*} \frac{\Omega_{\rm LO}}{|\mathbf{k}' - \mathbf{k}|} N_{\rm LO}^{\pm} \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} \pm \hbar \Omega_{\rm LO}), \quad (16)$$

$$\xi_{\nu 1} = -|\mathbf{k}' - \mathbf{k}| \int \int \varphi_1^2(z)\varphi_1(z')\varphi_\nu(z')|z - z'|dzdz'.$$
(17)

In accordance with general symmetry arguments, the asymmetric terms in the scattering rate proportional to w_{SIA} are related to the inversion asymmetry of the heterostructure and vanish for the absolutely symmetrical quantum well. This follows also from Eq. (6) together with Eq. (10) and (14), or (17), which demonstrates that the sign and magnitude of w_{SIA} are determined by the products $z_{\nu 1}\xi_{\nu 1}$. The products are nonzero to the extent of asymmetry of the confinement potential and/or the doping profile and vanish for the absolutely symmetrical structure, where u(z) is an even function and φ_{ν} is either even or odd function with respect to the QW center.

Following Eqs. (2) and (6), we can estimate the ratio between the asymmetric and symmetric parts of the scattering rate as $w_{SIA}Bk/W_0 \propto (\hbar \omega_c/\varepsilon_{21})kz_{21}\xi_{21}$, where $\omega_c = |e|B/(m^*c)$. The estimate gives $w_{SIA}Bk/W_0 \sim 10^{-3}$ for GaAs-based QW structure with the asymmetry degree $\xi_{21}=0.1$, the intersubband energy separation $\varepsilon_{21}=100$ meV, and $kz_{21}=1$ in the magnetic field B=1 T.

III. ELECTRIC CURRENT CAUSED BY ENERGY RELAXATION OF CARRIERS

The magnetic field induced asymmetry of the electronphonon interaction can lead to the generation of an electric current if the electron gas is driven out of thermal equilibrium with the crystal lattice. Such an effect was described by Ivchenko and Pikus,¹⁴ who proposed a paramagnetic (spindependent) mechanism of the electron-phonon interaction asymmetry. Kibis¹³ demonstrated that there also exists a diamagnetic contribution to the effect in two-dimensional electron systems, which is related to the asymmetry of the electron energy spectrum in the in-plane magnetic field. Experimentally, the magnetoinduced electric current was observed in Refs. 18 and 19, where the electron gas was heated by a low-frequency electric current or far infrared radiation. It was shown that different microscopic mechanisms of both diamagnetic^{13,18,19} and paramagnetic^{14,19,20} origins can be responsible for the effect. The magnetic field induced asymmetry of the inelastic electron scattering by phonons given by Eq. (1) also gives rise to an electric current if the electron gas temperature T_{e} differs from the lattice temperature T_{0} . This mechanism seems to be the dominant diamagnetic contribution because the current originating from the magnetic field induced modification of the energy spectrum vanishes in k-linear approximation and arises in high-order approximations only.

The electric current in the relaxation time approach is given by

$$\boldsymbol{j} = 2e \sum_{\boldsymbol{k}} \tau_p \boldsymbol{v} \boldsymbol{f}_{\boldsymbol{k}}, \tag{18}$$

where τ_p is the moment relaxation time, f_k stands for the generation function stemming from the electron scattering by phonons, and the factor 2 in Eq. (18) accounts for the spin degeneracy. The function f_k has the form

$$\dot{f}_{k} = \sum_{k',\pm} \left[W_{kk'}^{\pm} f_{k'}(1 - f_{k}) - W_{k'k}^{\pm} f_{k}(1 - f_{k'}) \right], \quad (19)$$

where $f_k = 1/\{\exp[(\varepsilon_k - \mu)/(k_B T_e)] + 1\}$ is the distribution function of carriers and μ is the chemical potential. Combining Eqs. (18) and (19), one obtains for the current

$$j = 2e \sum_{kk'} \tau_p(\boldsymbol{v}_k - \boldsymbol{v}_{k'}) [W_{kk'}^{\dagger} f_{k'}(1 - f_k) - W_{k'k}^{-} f_k(1 - f_{k'})].$$
(20)

In thermal equilibrium, when the electron and lattice temperatures coincide, the expression in square brackets in Eq. (20) vanishes because the processes of phonon emission and absorption compensate each other. In contrast, if $T_e \neq T_0$, the absorption and emission rates become nonequal, leading, due to scattering asymmetry in k space, to the electric current.

We assume that the electron temperature differs slightly from the lattice temperature, so that the inequality $|\Delta T|\hbar\Omega_q \ll k_B T_e T_0$ is fulfilled, where $\Delta T = T_e - T_0$. Then, taking into account that the rate of electron scattering assisted by emission or absorption of a phonon with the wave vector q is proportional to N_q^{\pm} and

$$\begin{split} N_q^+ f_{k'}(1-f_k) &- N_q^- f_k(1-f_{k'}) \big|_{\varepsilon_{k'}=\varepsilon_k+\hbar\Omega_q} \\ &\approx N_q^- f_k(1-f_{k'}) \frac{\Delta T}{T_q} \frac{\hbar\Omega_q}{k_B T_0}, \end{split}$$

one can derive the expression for the electric current valid in the linear approximation in $\Delta T/T_e$.

Calculations show that the electric current caused by the structure inversion asymmetry flows in the direction perpendicular to the applied magnetic field and is given by

$$j_x = jB_y/B, \quad j_y = -jB_x/B,$$
 (21)

where j is the electric current magnitude. We assume that the temperature is sufficiently high and the carriers obey the Boltzmann statistics. Then, for the scattering by acoustic phonons, one derives

$$j = \tau_p B N_e \frac{e^2 \Xi_c^2}{c \hbar \rho_c} \frac{\Delta T}{T_e} \sum_{\nu \neq 1} \frac{z_{1\nu}}{\varepsilon_{\nu 1}} \int \varphi_1(z) \varphi_\nu(z) \frac{d^2 \varphi_1^2(z)}{dz^2} dz,$$
(22)

where $N_e = 2\Sigma_k f_k$ is the electron density. In the case of the electron scattering by optical phonons, the calculation yields

$$j = \tau_p B N_e \frac{2\pi e^4}{c \epsilon^*} \frac{\Omega_{\rm LO}^3 N_{\rm LO}}{k_B T_0} \frac{\Delta T}{T_e} \sum_{\nu \neq 1} \frac{z_{1\nu}}{\varepsilon_{\nu 1}} \int \int \varphi_1^2(z) \varphi_1(z') \\ \times \varphi_{\nu}(z') |z - z'| dz dz'.$$
(23)

The estimation after Eq. (22) gives $j \sim 10^{-7}$ A/cm in the

magnetic field B=1 T for GaAs-based structures with the momentum relaxation time $\tau_p = 10^{-12}$ s, the carrier density $N_e = 10^{12}$ cm⁻², the relative temperature difference $\Delta T/T_e = 0.1$, and the QW asymmetry degree $\xi = 0.1$.

In conclusion, we have shown that the magnetic field applied in the quantum well plane has a diamagnetic influence on the scattering of charge carriers. The magnetic field leads to odd wave vector terms in the scattering rate resulting in the in-plane asymmetry of the scattering. The scattering rate has been calculated for the electron interaction with impurities as well as acoustic and optical phonons.

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