

# Spin relaxation due to the Bir-Aronov-Pikus mechanism in intrinsic and $p$ -type GaAs quantum wells from a fully microscopic approach

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We study the electron spin relaxation in intrinsic and  $p$ -type (001) GaAs quantum wells by constructing and numerically solving the kinetic spin Bloch equations. All the relevant scatterings are explicitly included, especially the spin-flip electron-heavy-hole exchange scattering which leads to the Bir-Aronov-Pikus spin relaxation. We show that, due to the neglect of the nonlinear terms in the electron-heavy-hole exchange scattering in the Fermi-golden-rule approach, the spin relaxation due to the Bir-Aronov-Pikus mechanism is greatly exaggerated at moderately high electron density and low temperature in the literature. We compare the spin relaxation time due to the Bir-Aronov-Pikus mechanism with that due to the D'yakonov-Perel' mechanism which is also calculated from the kinetic spin Bloch equations with all the scatterings, especially the spin-conserving electron-electron and electron-heavy-hole scatterings, included. We find that, in intrinsic quantum wells, the effect from the Bir-Aronov-Pikus mechanism is much smaller than that from the D'yakonov-Perel' mechanism at low temperature, and it is smaller by no more than 1 order of magnitude at high temperature. In  $p$ -type quantum wells, the spin relaxation due to the Bir-Aronov-Pikus mechanism is also much smaller than the one due to the D'yakonov-Perel' mechanism at low temperature and becomes comparable to each other at higher temperature when the hole density and the width of the quantum well are large enough. We claim that unlike in the bulk samples, which still require reexamination, the Bir-Aronov-Pikus mechanism hardly dominates the spin relaxation in two-dimensional samples.

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## I. INTRODUCTION

Much attention has been given to semiconductor spintronics both theoretically and experimentally due to great prospect of the potential applications.<sup>1-3</sup> The study of the spin relaxation/dephasing (R/D) in semiconductors contains rich physics and is of great importance for the device application. Three spin R/D mechanisms have long been proposed in zinc-blend semiconductors, i.e., the Elliott-Yafet (EY) mechanism,<sup>4</sup> caused by the spin-flip electron-impurity scattering due to the spin-orbit coupling; the D'yakonov-Perel' (DP) mechanism<sup>5</sup> which is due to the momentum-dependent spin splitting in crystal without a center of symmetry; and the Bir-Aronov-Pikus (BAP) mechanism<sup>6</sup> which originates from the spin-flip electron-hole exchange interaction. Previous researches have shown that, in bulk systems, the EY mechanism is important in narrow-band-gap and high impurity semiconductors, the DP mechanism is dominant in  $n$ -type semiconductors, and the BAP mechanism can have significant effect in  $p$ -doped semiconductors.<sup>7-9</sup> It is known that, in heavily  $p$ -doped bulk samples, the BAP mechanism is dominant at low temperature, whereas the DP mechanism is dominant at high temperature with the crossover temperature determined by the doping level. In bulk samples with low hole density, the BAP mechanism has been shown to be irrelevant.<sup>7-9</sup> In addition, the hyperfine interaction induced spin relaxation is another possible mechanism.<sup>10</sup>

In contrast to the bulk systems, the relative importance of the BAP and DP mechanisms for the electron spin R/D in two dimensional (2D) systems, especially in  $p$ -type 2D systems, is still not very clear, sometimes even confusing. In

Ref. 11, extremely long spin relaxation time (SRT), which is 2 orders of magnitude longer than that in the bulk sample with corresponding acceptor concentrations, was reported by Wagner *et al.* in  $p$ -type GaAs quantum wells (QWs). The authors argued that the BAP mechanism is dominant at low temperature. However, in Ref. 12, the SRT in  $p$ -type QWs was reported to be a factor of 4 shorter than that in comparably bulk GaAs by Damen *et al.* at low temperature. The authors also referred the BAP mechanism as a cause for the decrease of the SRT. Hence, two opposite experimental results arrive at the same conclusions regarding the importance of the BAP mechanism. Moreover, Gotoh *et al.* further pointed out that the BAP mechanism should not be ignored even at room temperature.<sup>13</sup> They investigated the electric field dependence of SRT and found that the SRT decreases with the increase of the bias. They concluded that the decrease is from the BAP mechanism as the SRT due to the DP mechanism does not change with electric field. Actually, they overlooked the fact that the Rashba spin-orbit coupling<sup>14</sup> can also lead to the spin R/D due to the DP mechanism. Therefore, we believe that the decrease of SRT in their experiment cannot be a proof of the importance of the BAP mechanism. Very recently, it was shown that the SRT at room temperature can be increased at the (100) GaAs surface due to the relatively lower concentration of holes at the surface and the mechanism for the SRT was referred to as the BAP mechanism.<sup>15</sup> Theoretically, Maialle<sup>16</sup> pointed out that the effect of the BAP mechanism in 2D systems is a little smaller than that of the DP mechanism at zero temperature by using the Fermi golden rule to calculate the SRT in which the elastic scattering approximation was applied and conse-

quently the nonlinear terms of the electron-hole Coulomb scattering were neglected. The SRT due to the DP mechanism ( $\tau_{DP}$ ) was also calculated by using the single particle approach.<sup>1,5</sup> The author compared  $\tau_{DP}$  and  $\tau_{BAP}$  for different electron momenta (kinetic energies) and showed that these two SRTs have nearly the same order of magnitude in heavily doped QWs. However,  $\tau_{DP}$  calculated in Ref. 16 is quite cursory because, under the framework of single particle theory, the carrier-carrier Coulomb scattering, which is very important to spin R/D,<sup>17-21</sup> is not included. Also, the counter-effect of the scattering to the spin R/D is also not fully accounted.<sup>18-20,22,23</sup> Moreover, it is also important to calculate the spin-flip electron-hole exchange scattering explicitly in order to find out the effect of the nonlinear terms ignored in the Fermi golden rule approach by Maialle and co-workers<sup>16,26,27</sup> We also want to find out the temperature dependence of the relative importance of both mechanisms in 2D systems, which to the best of our knowledge is still absent in the literature.

In order to accurately investigate the relative importance of the DP and the BAP mechanisms beyond the single-particle Fermi golden rule approach, we use the fully microscopic approach established by Wu *et al.*<sup>24</sup> by constructing and numerically solving the kinetic spin Bloch equations.<sup>17-20,22,23,25</sup> In this approach, all the corresponding scatterings such as the electron-acoustic (AC) phonon, electron-longitudinal optical (LO) phonon, electron-nonmagnetic impurity, and electron-electron Coulomb scatterings are explicitly included. The results and/or predictions obtained from this approach are in very good agreement with various experiments.<sup>20,28-30</sup> It was previously pointed out that, in the presence of inhomogeneous broadening, any type of scattering, including the Coulomb scattering, can give rise to the spin R/D.<sup>17-20,23</sup> In this paper, in addition to the all the above mentioned scatterings in *n*-type QWs as considered in Ref. 20, we further add the spin-conserving and spin-flip electron-heavy-hole Coulomb scatterings, both contributing to the DP mechanism and the latter further leading to the spin R/D due to the BAP mechanism. By solving the kinetic spin Bloch equations self-consistently, we obtain the SRT from the BAP mechanism from a fully microscopic fashion. We further investigate the relative importance of the BAP and DP mechanisms in 2D systems.

This paper is organized as follows. In Sec. II, we construct the kinetic spin Bloch equations and present the scattering terms from the spin-conserving and spin-flip electron-hole Coulomb scatterings. We also discuss the SRTs due to the BAP mechanism from different approaches. Then, we present our numerical results in Sec. III. We study the SRT due to both the DP and the BAP mechanisms under various conditions such as temperatures, electron and/or hole densities, impurity densities, and well widths. We conclude in Sec. VI.

## II. KINETIC SPIN BLOCH EQUATIONS

We construct the kinetic spin Bloch equations in intrinsic and *p*-type (001) GaAs QWs by using the nonequilibrium Green's function method,<sup>31</sup>

$$\dot{\rho}_{\mathbf{k},\sigma\sigma'} = \dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{\text{coh}} + \dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{\text{scatt}}, \quad (1)$$

with  $\rho_{\mathbf{k},\sigma\sigma'}$  representing the single-particle density matrix elements. The diagonal and off-diagonal elements of  $\rho_{\mathbf{k},\sigma\sigma'}$  give the electron distribution functions  $f_{\mathbf{k}\sigma}$  and the spin coherence  $\rho_{\mathbf{k},\sigma-\sigma}$ , respectively. The coherent terms  $\dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{\text{coh}}$  describe the precession of the electron spin due to the effective magnetic field from the Dresselhaus term<sup>32</sup>  $\Omega(\mathbf{k})$  and the Hartree-Fock Coulomb interaction. The expression of the coherent terms can be found in the Appendix (and also Ref. 18). The Dresselhaus term can be written as<sup>33</sup>

$$\Omega_x(\mathbf{k}) = \gamma k_x (k_y^2 - \langle k_z^2 \rangle), \quad (2)$$

$$\Omega_y(\mathbf{k}) = \gamma k_y (\langle k_z^2 \rangle - k_x^2), \quad (3)$$

$$\Omega_z(\mathbf{k}) = 0, \quad (4)$$

in which  $\langle k_z^2 \rangle$  represents the average of the operator  $-(\partial/\partial z)^2$  over the electronic state of the lowest subband,<sup>20</sup> and  $\gamma$  is the spin splitting parameter<sup>1</sup> which is chosen to be 11.4 eV  $\text{\AA}^3$  all through the paper.<sup>34</sup>  $\dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{\text{scatt}}$  in Eq. (1) denote the electron-LO-phonon, electron-AC-phonon, electron-nonmagnetic impurity, and the electron-electron Coulomb scatterings whose expressions are given in detail in the Appendix (see also Refs. 18-20). All these scatterings are calculated *explicitly* without any relaxation time approximation. Moreover, we further include the spin-conserving and spin-flip electron-heavy-hole scatterings as what follows.

The Hamiltonian of electron-heavy hole interaction is given by

$$H_{eh} = \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}, \sigma = \pm 1, \sigma' = \pm 1} V_{eh, \mathbf{q}} c_{\mathbf{k}+\mathbf{q}, \sigma}^\dagger c_{\mathbf{k}, \sigma/2} c_{\mathbf{k}, \sigma/2} b_{\mathbf{k}'-\mathbf{q}, 3\sigma'/2}^\dagger b_{\mathbf{k}', 3\sigma'/2}, \quad (5)$$

where  $c$  ( $c^\dagger$ ) and  $b$  ( $b^\dagger$ ) are the annihilation (creation) operators of electrons in conduction (heavy-hole valence) band, respectively. We denote  $\sigma$  ( $\sigma'$ ) to be  $\pm 1$  throughout the paper. The screened Coulomb potential under the random-phase approximation reads<sup>31</sup>

$$V_{eh, \mathbf{q}} = \frac{\sum v_Q f_{eh}(q_z)}{\epsilon(\mathbf{q})}, \quad (6)$$

with the bare Coulomb potential  $v_Q = 4\pi e^2/Q^2$  and

$$\begin{aligned} \epsilon(\mathbf{q}) = & 1 - \sum_{q_z} v_Q f_e(q_z) \sum_{\mathbf{k}, \sigma} \frac{f_{\mathbf{k}+\mathbf{q}, \sigma} - f_{\mathbf{k}, \sigma}}{\epsilon_{\mathbf{k}+\mathbf{q}}^e - \epsilon_{\mathbf{k}}^e} \\ & - \sum_{q_z} v_Q f_h(q_z) \sum_{\mathbf{k}', \sigma} \frac{f_{\mathbf{k}'+\mathbf{q}, \sigma} - f_{\mathbf{k}', \sigma}}{\epsilon_{\mathbf{k}'+\mathbf{q}}^h - \epsilon_{\mathbf{k}'}^h} \end{aligned} \quad (7)$$

is the electron-hole plasma screening.<sup>35</sup> In these equations,  $Q^2 = \mathbf{q}^2 + q_z^2$  and  $f_{\mathbf{k}, \sigma}^h$  ( $f_{\mathbf{k}, \sigma}$ ) denotes the heavy-hole (electron) distribution function with spin  $\frac{3}{2}\sigma$  ( $\frac{1}{2}\sigma$ ). The form factors can be written as

$$f_e(q_z) = \int dz dz' \xi_c(z) \xi_c(z') e^{iq_z(z-z')} \xi_c(z') \xi_c(z), \quad (8)$$

$$f_h(q_z) = \int dz dz' \eta_h(z) \eta_h(z') e^{iq_z(z-z')} \eta_h(z') \eta_h(z), \quad (9)$$

$$f_{eh}(q_z) = \int dz dz' \xi_c(z) \eta_h(z') e^{iq_z(z-z')} \eta_h(z') \xi_c(z), \quad (10)$$

where  $\xi_c(z)$  [ $\eta_h(z)$ ] is the envelope function of the electron (heavy hole) along the growth direction  $z$ .<sup>20</sup> The scattering term of this spin-conserving electron-hole Coulomb scattering can be written as

$$\begin{aligned} \left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{eh} = & -2\pi \sum_{\mathbf{k}',\mathbf{q},\sigma'} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) V_{eh,\mathbf{q}}^2 \\ & \times \{ (1 - f_{\mathbf{k}',\sigma'}^h) f_{\mathbf{k}'-\mathbf{q},\sigma'}^h \\ & \times [f_{\mathbf{k},\sigma}(1 - f_{\mathbf{k}-\mathbf{q},\sigma}) - \text{Re}(\rho_{\mathbf{k}} \rho_{\mathbf{k}-\mathbf{q}}^*)] \\ & - f_{\mathbf{k}',\sigma'}^h (1 - f_{\mathbf{k}'-\mathbf{q},\sigma'}^h) [f_{\mathbf{k}-\mathbf{q},\sigma}(1 - f_{\mathbf{k},\sigma}) \\ & - \text{Re}(\rho_{\mathbf{k}} \rho_{\mathbf{k}-\mathbf{q}}^*)] \}, \end{aligned} \quad (11)$$

$$\begin{aligned} \left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{eh} = & -\pi \sum_{\mathbf{k}',\mathbf{q},\sigma,\sigma'} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) V_{eh,\mathbf{q}}^2 \\ & \times \{ (1 - f_{\mathbf{k}',\sigma'}^h) f_{\mathbf{k}'-\mathbf{q},\sigma'}^h \\ & \times [(1 - f_{\mathbf{k}-\mathbf{q},\sigma}) \rho_{\mathbf{k}} - f_{\mathbf{k},\sigma} \rho_{\mathbf{k}-\mathbf{q}}] \\ & + f_{\mathbf{k}',\sigma'}^h (1 - f_{\mathbf{k}'-\mathbf{q},\sigma'}^h) [f_{\mathbf{k}-\mathbf{q},\sigma} \rho_{\mathbf{k}} - (1 - f_{\mathbf{k},\sigma}) \rho_{\mathbf{k}-\mathbf{q}}] \}, \end{aligned} \quad (12)$$

where  $\rho_{\mathbf{k}} \equiv \rho_{\mathbf{k},(1/2)-(1/2)} \equiv \rho_{\mathbf{k},-(1/2)(1/2)}$ . This spin-conserving scattering only enhances the total scattering strength moderately and contributes to the spin R/D due to the DP mechanism.

The Hamiltonian of the spin-flip electron-heavy hole exchange interaction reads

$$H_{\text{BAP}} = \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma} M_{\sigma}(\mathbf{k},\mathbf{k}') c_{\mathbf{k}+\mathbf{q},\sigma/2}^{\dagger} b_{\mathbf{k}'-\mathbf{q},-3\sigma/2}^{\dagger} c_{\mathbf{k},-\sigma/2} b_{\mathbf{k}',3\sigma/2}. \quad (13)$$

The matrix elements in the Hamiltonian are given by<sup>27</sup>

$$M_{\sigma}(\mathbf{k},\mathbf{k}') = \frac{3}{8} \frac{\Delta E_{LT}}{|\phi_{3D}(0)|^2} \sum_{q_z} \frac{f_{ex}(q_z)(k_{\sigma}^2 + k_{\sigma'}^2)}{q_z^2 + |\mathbf{k} + \mathbf{k}'|^2}, \quad (14)$$

where  $\Delta E_{LT}$  is the longitudinal-transverse splitting in bulk,  $|\phi_{3D}(0)|^2 = 1/(\pi a_0^3)$  is the three-dimensional (3D) exciton state at zero relative distance, and  $k_{\sigma} = k_x + i\sigma k_y$ . For GaAs,  $\Delta E_{LT} = 0.08$  meV and  $a_0 = 146.1$  Å.<sup>37</sup> The form factor can be written as

$$f_{ex}(q_z) = \int dz dz' \xi_c(z') \eta_h(z') e^{iq_z(z-z')} \eta_h(z) \xi_c(z). \quad (15)$$

The scattering term from this Hamiltonian reads

$$\begin{aligned} \left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{\text{BAP}} = & -2\pi \sum_{\mathbf{k}',\mathbf{q}} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) \\ & \times M_{\sigma}(\mathbf{k} - \mathbf{q}, \mathbf{k}') M_{-\sigma}(\mathbf{k}, \mathbf{k}' - \mathbf{q}) \\ & \times [(1 - f_{\mathbf{k}',\sigma'}^h) f_{\mathbf{k}'-\mathbf{q},-\sigma}^h f_{\mathbf{k},\sigma}(1 - f_{\mathbf{k}-\mathbf{q},-\sigma}) \\ & - f_{\mathbf{k}',\sigma'}^h (1 - f_{\mathbf{k}'-\mathbf{q},-\sigma}^h) (1 - f_{\mathbf{k},\sigma}) f_{\mathbf{k}-\mathbf{q},-\sigma}], \end{aligned} \quad (16)$$

$$\begin{aligned} \left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{\text{BAP}} = & -\pi \sum_{\mathbf{k}',\mathbf{q},\sigma} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) \\ & \times M_{\sigma}(\mathbf{k} - \mathbf{q}, \mathbf{k}') M_{-\sigma}(\mathbf{k}, \mathbf{k}' - \mathbf{q}) \\ & \times [(1 - f_{\mathbf{k}',\sigma'}^h) f_{\mathbf{k}'-\mathbf{q},-\sigma}^h (1 - f_{\mathbf{k}-\mathbf{q},-\sigma}) \rho_{\mathbf{k}} \\ & + f_{\mathbf{k}',\sigma'}^h (1 - f_{\mathbf{k}'-\mathbf{q},-\sigma}^h) f_{\mathbf{k}-\mathbf{q},\sigma} \rho_{\mathbf{k}}]. \end{aligned} \quad (17)$$

If we denote  $\mathbf{K} = \mathbf{k} + \mathbf{k}'$  as the center-of-mass momentum of the electron-hole pair, the product of the matrix elements in Eqs. (16) and (17) can be reduced to

$$\begin{aligned} |M(\mathbf{K} - \mathbf{q})|^2 = & M_{\sigma}(\mathbf{k} - \mathbf{q}, \mathbf{k}') M_{-\sigma}(\mathbf{k}, \mathbf{k}' - \mathbf{q}) \\ = & \frac{9\Delta E_{LT}^2}{16|\phi_{3D}(0)|^4} \left[ \sum_{q_z} \frac{f_{ex}(q_z)(\mathbf{K} - \mathbf{q})^2}{q_z^2 + (\mathbf{K} - \mathbf{q})^2} \right]^2. \end{aligned} \quad (18)$$

It is noted that the spin R/D of the photoexcited holes is very fast<sup>23</sup> and the electron-hole recombination is very slow compared to the electron spin R/D. Therefore, we take the hole distribution in equilibrium Fermi distribution and  $f_{\mathbf{k}\sigma}^h = f_{\mathbf{k}-\sigma}^h \equiv f_{\mathbf{k}}^h$ . Further, by subtracting  $\left. \frac{\partial f_{\mathbf{k},-1}}{\partial t} \right|_{\text{BAP}}$  from  $\left. \frac{\partial f_{\mathbf{k},+1}}{\partial t} \right|_{\text{BAP}}$  in Eq. (16), one obtains

$$\begin{aligned} \left. \frac{\partial \Delta f_{\mathbf{k}}}{\partial t} \right|_{\text{BAP}} = & \left. \frac{\partial (f_{\mathbf{k},+1} - f_{\mathbf{k},-1})}{\partial t} \right|_{\text{BAP}} \\ = & -2\pi \sum_{\mathbf{k}',\mathbf{q}} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) \\ & \times |M(\mathbf{K} - \mathbf{q})|^2 \left\{ \Delta f_{\mathbf{k}} \left[ (1 - f_{\mathbf{k}'}^h) f_{\mathbf{k}'-\mathbf{q}}^h \right. \right. \\ & + \frac{1}{2} (f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h) (f_{\mathbf{k}-\mathbf{q},+1} + f_{\mathbf{k}-\mathbf{q},-1}) \left. \right] \\ & + \Delta f_{\mathbf{k}-\mathbf{q}} \left[ f_{\mathbf{k}'}^h (1 - f_{\mathbf{k}'-\mathbf{q}}^h) \right. \\ & \left. \left. - \frac{1}{2} (f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h) (f_{\mathbf{k},+1} + f_{\mathbf{k},-1}) \right] \right\}. \end{aligned} \quad (19)$$

In above equation, the terms  $\Delta f_{\mathbf{k}} [(1 - f_{\mathbf{k}'}^h) f_{\mathbf{k}'-\mathbf{q}}^h + \frac{1}{2} (f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h) (f_{\mathbf{k}-\mathbf{q},+1} + f_{\mathbf{k}-\mathbf{q},-1})]$  describe the forward scattering and correspondingly the terms  $\Delta f_{\mathbf{k}-\mathbf{q}} [f_{\mathbf{k}'}^h (1 - f_{\mathbf{k}'-\mathbf{q}}^h) - \frac{1}{2} (f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h) (f_{\mathbf{k},+1} + f_{\mathbf{k},-1})]$  describe the backward scattering. The SRT due to the BAP mechanism from the Fermi golden rule<sup>16</sup> can be recovered from Eq. (19) by applying the elastic scattering approximation:  $\varepsilon_{\mathbf{k}-\mathbf{q}}^e \approx \varepsilon_{\mathbf{k}}^e$  and  $\varepsilon_{\mathbf{k}'}^h \approx \varepsilon_{\mathbf{k}'-\mathbf{q}}^h$ . Under

this approximation, the nonlinear terms (in the sense of the electron distribution function)  $\frac{1}{2}\Delta f_{\mathbf{k}}(f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h)(f_{\mathbf{k}-\mathbf{q},+1} + f_{\mathbf{k}-\mathbf{q},-1})$  in the forward scattering and  $\frac{1}{2}\Delta f_{\mathbf{k}-\mathbf{q}}(f_{\mathbf{k}'}^h - f_{\mathbf{k}'-\mathbf{q}}^h) \times (f_{\mathbf{k},+1} + f_{\mathbf{k},-1})$  in the backward scattering tend to zero. In the remaining linear terms,  $\Delta f_{\mathbf{k}} = -\frac{\partial f_{0\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}}(\phi_{1/2} - \phi_{-1/2}) \approx \Delta f_{\mathbf{k}-\mathbf{q}}$  with  $f_{0\mathbf{k}} = \frac{1}{e^{\beta(\varepsilon_{\mathbf{k}} - \mu)} + 1}$  by choosing  $f_{\mathbf{k},\sigma} = \frac{1}{e^{\beta(\varepsilon_{\mathbf{k}} - \mu - \phi_{\sigma})} + 1}$ . Therefore, one recovers the SRT due to the BAP mechanism from the Fermi golden rule approach,<sup>16</sup>

$$\frac{1}{2\tau_{\text{BAP}}^1(\mathbf{k})} = 2\pi \sum_{\mathbf{k}',q} \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^h - \varepsilon_{\mathbf{k}'-\mathbf{q}}^h) \times |M(\mathbf{K}-\mathbf{q})|^2 [(1 - f_{\mathbf{k}'}^h) f_{\mathbf{k}'-\mathbf{q}}^h]. \quad (20)$$

In the next section, we will discuss the applicability of above equation which relies on the elastic scattering approximation.

In this work, we do not use the SRTs from the single-particle approach for both the BAP and DP mechanisms. Instead, we solve the kinetic spin Bloch equations self-consistently with all the scattering explicitly included. The detail of the numerical scheme is given in Refs. 19 and 20. The spin relaxation and dephasing times can be obtained from the temporal evolutions of the electron distribution functions  $f_{\mathbf{k},\sigma}$  and the spin coherence  $\rho_{\mathbf{k},\sigma-\sigma'}$ , respectively.<sup>25,36</sup> We will show that the SRT due to the BAP mechanism obtained from the kinetic spin Bloch approach can give markedly different results compared to the one calculated from Eq. (20) by using the elastic scattering approximation, similar to the situation of the SRT due to the DP mechanism which has been discussed in great detail in our previous works.<sup>18–20</sup>

### III. NUMERICAL RESULTS AND ANALYSIS

The SRTs calculated from the kinetic spin Bloch equations are plotted in Figs. 1–6. In these figures, the solid curves represent the SRTs due to the BAP mechanism ( $\tau_{\text{BAP}}$ ) which are calculated from the kinetic spin Bloch equations by setting the DP term  $\Omega(\mathbf{k})=0$ , the dashed curves are the SRTs due to the DP mechanism ( $\tau_{\text{DP}}$ ) which are calculated by setting  $\partial \rho_{\mathbf{k},\sigma\sigma'}/\partial t|_{\text{BAP}}=0$ , and the dash-dotted curves represent the total SRTs ( $\tau_{\text{total}}$ ) obtained from Eq. (1) with all the terms explicitly included. We always use different colors and widths of curves for different conditions.

We first discuss the SRT in an intrinsic GaAs QW confined by  $\text{Al}_{0.4}\text{Ga}_{0.6}\text{As}$  barriers. In Fig. 1(a), we plot the temperature dependence of the SRT for a QW with well width  $a=20$  nm. The electron (heavy hole) density  $n$  ( $p$ ) is  $2 \times 10^{11} \text{ cm}^{-2}$  and the impurity density  $n_i=n$ . It is seen from the figure that the SRT due to the BAP mechanism is much larger than that due to the DP mechanism. Moreover,  $\tau_{\text{BAP}}$  decreases dramatically with  $T$  at low temperature, followed by a more moderate decrease at high temperature. The temperature dependence of  $\tau_{\text{BAP}}$  can be understood as follows. When the temperature increases, more electrons and holes tend toward the larger momentum, hence the larger center-of-mass momentum  $\mathbf{K}$ . This leads to a larger the matrix element in Eq. (18), and consequently a larger scattering rate. Fur-

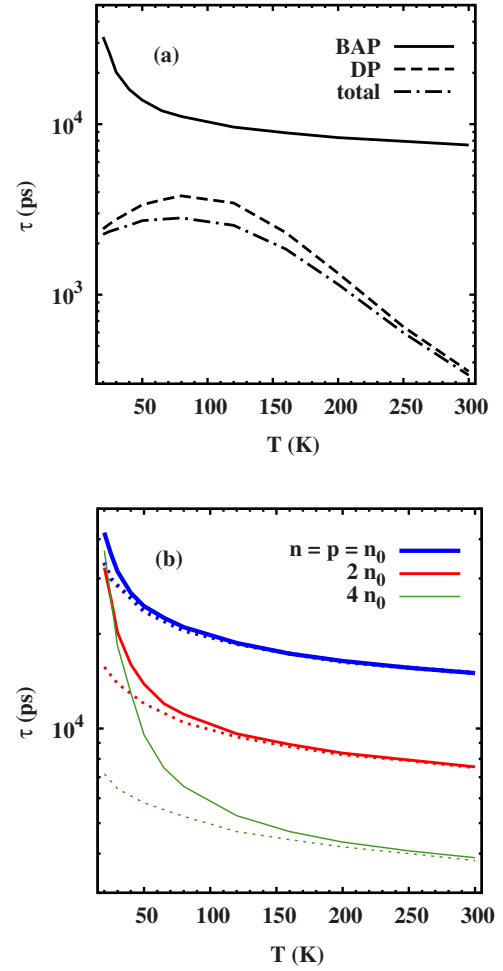


FIG. 1. (Color online) (a) SRT due to the BAP (solid curve) and DP (dashed curve) mechanisms and the total SRT (dash-dotted curve) vs temperature  $T$  in intrinsic QW when  $a=20$  nm, electron and hole densities  $n=p=2n_0$ , and impurity density  $n_i=n$ . (b) SRT due to the BAP mechanism with full spin-flip electron-hole exchange scattering (solid curves) and with only the linear terms in the spin-flip electron-hole exchange scattering (dotted curves) at different electron densities against temperature  $T$ .  $n_0=10^{11} \text{ cm}^{-2}$ .

thermore, the Pauli blocking which suppresses the scattering decreases with the increase of temperature. Both leads to the decrease of the SRT due to the BAP mechanism. The temperature dependence of the SRT due to the DP mechanism has been well discussed in Refs. 18–20 and 22. Therefore, we will not discuss the DP mechanism in detail in this paper.

In order to see the difference of the SRT due to the BAP mechanism calculated from the full spin-flip scattering [Eq. (17)] and the one from the Fermi golden rule [Eq. (20)], i.e., neglecting the nonlinear terms in Eq. (17), we plot the BAP SRT calculated from the Bloch equations with only the linear terms in the spin-flip scattering as dotted curves for two different electron (hole) densities in Fig. 1(b). It is noted that for high electron density, the SRT due to the BAP mechanism from the Fermi golden rule is much smaller than  $\tau_{\text{BAP}}$  at low temperature. Furthermore, the lower the temperature and/or the larger the electron density, the larger the difference is due to the *breakdown* of the elastic scattering ap-

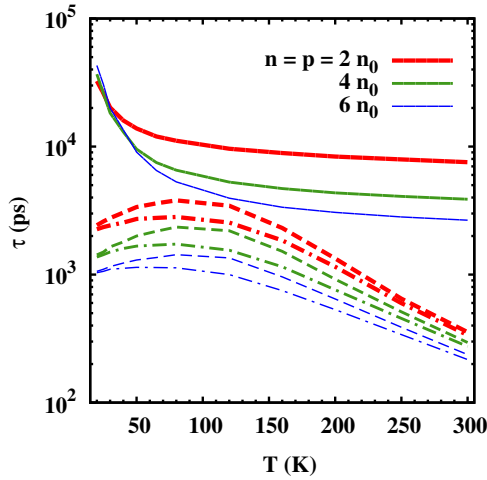


FIG. 2. (Color online) SRT due to the BAP (solid curves) and DP (dashed curves) mechanisms and the total SRT (dash-dotted curves) vs temperature  $T$  in intrinsic QWs at different densities ( $n = p = 2n_0, 4n_0,$  and  $6n_0$ ) when  $a = 20$  nm and  $n_i = n$ .  $n_0 = 10^{11}$  cm $^{-2}$ .

proximation at low temperature and/or high density. This is in good agreement with the condition for the elastic scattering. The difference can be very small when the electron density is smaller than  $5 \times 10^{10}$  cm $^{-2}$  according our calculation. Consequently, the SRT for high electron density obtained in Ref. 16 at zero temperature is much smaller than the actual one. Therefore, the effect of the BAP mechanism for high electron density at very low temperature is smaller than that claimed by Maialle *et al.* In fact, it can even be ignored. We further stress that *the effect of the BAP mechanism at low temperature and high electron density is far exaggerated in the literature* due to the neglect of the nonlinear terms in the spin-flip electron-hole exchange scattering.

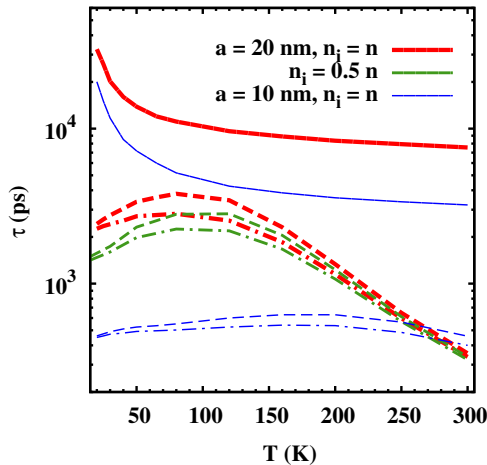


FIG. 3. (Color online) SRT due to the BAP (solid curves) and DP (dashed curves) mechanisms and the total SRT (dash-dotted curves) vs temperature  $T$  in intrinsic QWs for different well widths ( $a = 10$  and  $20$  nm).  $n = p = 2n_0$  and impurity densities ( $n_i = 0.5n$  and  $n$ ). Note that the solid curves with the same well width but different impurity densities exactly coincides with each other.  $n_0 = 10^{11}$  cm $^{-2}$ .

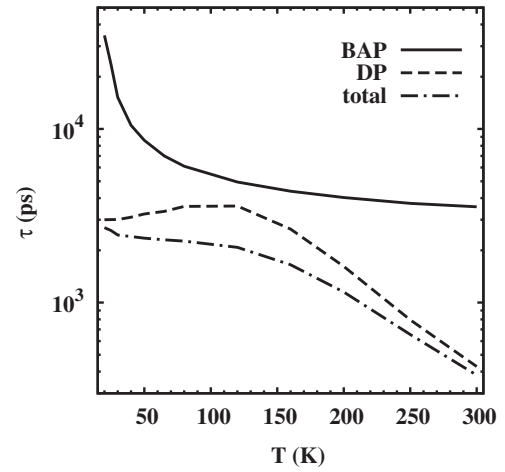


FIG. 4. SRT due to the BAP (solid curve) and DP (dashed curve) mechanisms and the total SRT (dash-dotted curve) vs temperature  $T$  in  $p$ -type QW when  $a = 20$  nm,  $n = 0.5n_0$ ,  $p_0 = 4n_0$ , and  $n_i = n$ .  $n_0 = 10^{11}$  cm $^{-2}$ .

In addition, in the presence of inhomogeneous broadening, any scattering can give rise to spin R/D.<sup>17-19,23,24</sup> It is intuitive that the SRTs should satisfy

$$\frac{1}{\tau_{\text{total}}} = \frac{1}{\tau'_{\text{DP}}} + \frac{1}{\tau_{\text{BAP}}} = \frac{1}{\tau_{\text{DP}}} + \frac{1}{\tau_{\text{BAP}}} + \frac{1}{\tau_{\text{differ}}}, \quad (21)$$

where  $\tau_{\text{BAP}}$  is directly caused by the spin-flip electron-hole exchange interaction,  $\tau_{\text{DP}}$  is from the inhomogeneous broadening when there is no spin-flip electron-hole exchange interaction, and  $\tau'_{\text{DP}}$  corresponds to case with the presence of the spin-flip electron-hole exchange scattering. The difference between  $\frac{1}{\tau_{\text{DP}}}$  and  $\frac{1}{\tau'_{\text{DP}}}$  is noted as  $\frac{1}{\tau_{\text{differ}}}$ . In our calculation, we found that  $\frac{1}{\tau_{\text{differ}}}$  is so small that can be totally ignored. This is because the spin-flip electron-hole scattering is much smaller than the other scatterings.

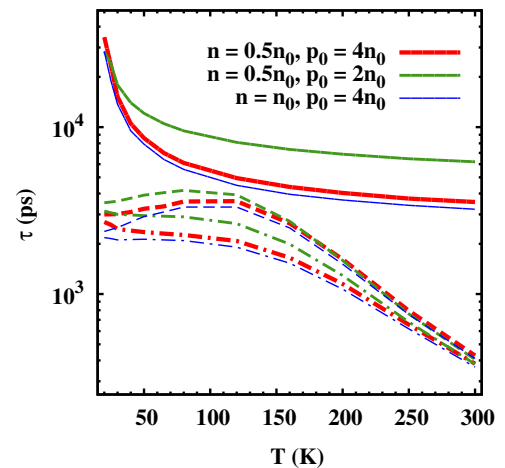


FIG. 5. (Color online) SRT due to the BAP (solid curves) and DP (dashed curves) mechanisms and the total SRT (dash-dotted curves) vs temperature  $T$  in  $p$ -type QWs with  $a = 20$  nm at different electron densities ( $n = 0.5$  and  $1n_0$ ) and hole densities  $p_0 = 2$  and  $4n_0$ .  $n_i = n$ .  $n_0 = 10^{11}$  cm $^{-2}$ .

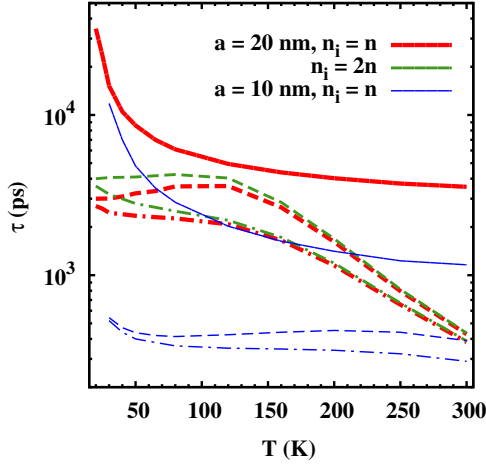


FIG. 6. (Color online) SRT due to the BAP (solid curves) and DP (dashed curves) mechanisms and the total SRT (dash-dotted curves) vs temperature  $T$  in  $p$ -type QWs with  $n=0.5n_0$ ,  $p_0=4n_0$  at different well widths ( $a=10$  and  $20$  nm) and impurity densities ( $n_i=n$  and  $2n$ ).  $n_0=10^{11}$  cm $^{-2}$ .

Then, we discuss the temperature dependence for different electron densities in intrinsic QWs in Fig. 2. One can see that  $\tau_{\text{BAP}}$  decreases with increasing densities at high temperature but it behaves oppositely at low temperature. On the other hand,  $\tau_{\text{DP}}$  decreases with increasing densities at all temperatures. We again interpret the density dependence of BAP mechanism by using the previous arguments: at low temperature regime, i.e., in the degenerate limit, the Pauli blocking is enhanced by increasing the carrier density and/or lowering the temperature. Therefore, the scattering can be suppressed by increasing density. This causes an increase of  $\tau_{\text{BAP}}$ . At high temperature regime, i.e., in the nondegenerate case, higher momentum states are occupied for larger density. This leads to a stronger scattering and hence  $\tau_{\text{BAP}}$  decreases with electron density. From this, we find that the relative importance of the DP and the BAP mechanisms does not change so much by changing the electron density.

In Fig. 3, we plot the temperature dependence of the SRTs in intrinsic QWs for different impurity densities and well widths. It is clear that  $\tau_{\text{BAP}}$  does not depend on impurity density; in other words, the curves corresponding to different impurities concentrations exactly coincide. However,  $\tau_{\text{DP}}$  can be enhanced due to the increased impurity scattering strength. If we enlarge the well width, both  $\tau_{\text{DP}}$  and  $\tau_{\text{BAP}}$  become larger. This is because the electron-hole exchange strength is weakened by the form factor [Eq. (15)] in the scattering matrix elements in the BAP mechanism for wider QWs. The leading term (linear term) of the Dresselhaus spin-orbit coupling in Eqs. (2)–(4) is smaller for wider QWs in the DP mechanism. The variation of  $\tau_{\text{DP}}$  is larger than  $\tau_{\text{BAP}}$ ; that is to say, the relative influence of the BAP mechanism becomes more important for wider QWs.

From our detailed investigations, we conclude that  $\tau_{\text{BAP}}$  in intrinsic GaAs QWs is always larger than  $\tau_{\text{DP}}$ . At very low temperatures, the BAP mechanism can be ignored. However, it should be considered at higher temperatures for accurate calculating. Moreover, the relative importance of the BAP mechanism is increased by raising the impurity density and the well width.

We now turn to study the SRT in  $p$ -type QWs. In Fig. 4, we choose the well width  $a=20$  nm,  $n=0.5 \times 10^{11}$  cm $^{-2}$ ,  $p=n+p_0=n+4 \times 10^{11}$  cm $^{-2}$ , and  $n_i=n$ . One can see that the magnitudes of  $\tau_{\text{DP}}$  and  $\tau_{\text{BAP}}$  are very close around  $T=150$  K. In  $p$ -type QWs, both the spin-conserving and spin-flip electron-hole scatterings are greatly enhanced by increasing the hole density. The former gives rise to the increase of  $\tau_{\text{DP}}$  in the strong scattering limit<sup>20,23</sup> and the latter gives rise to the decrease of  $\tau_{\text{BAP}}$ . Therefore, both SRTs are getting closer for larger hole concentration. In the case of Fig. 4, the contributions from the DP and BAP mechanisms are nearly the same around 150 K, and at lower and higher temperatures, the contribution from the DP mechanism is no more than 1 order of magnitude larger than the BAP one. In addition,  $1/\tau_{\text{differ}}$  is still very small and can be totally ignored.

We now analyze the temperature dependence of the SRT for different electron and hole densities in  $p$ -type QWs. In Fig. 5, the calculated SRT for different electron and hole densities are shown. In Fig. 6, a similar analysis is made for different well widths and impurity densities. The general features can be understood from the following. When the electron density becomes larger, both  $\tau_{\text{DP}}$  and  $\tau_{\text{BAP}}$  become smaller with similar amplitude. (Note that  $n=0.5n_0$  and  $n_0$  are both within the nondegenerate limit.) When hole density gets larger, both  $\tau_{\text{DP}}$  and  $\tau_{\text{BAP}}$  become smaller with the amplitude of the latter being larger than the former (i.e., the importance of the BAP mechanism gets increased). This is because the electron-heavy hole scattering is markedly enhanced with the hole density. As the BAP mechanism is determined by the hole density,  $\tau_{\text{BAP}}$  is very sensitive to the hole density. Nevertheless,  $\tau_{\text{DP}}$  is less sensitive as it is also determined by all the other scatterings. When the well width gets larger,  $\tau_{\text{DP}}$  is enhanced with a larger amplitude at low temperature and with a small amplitude at high temperature, whereas  $\tau_{\text{BAP}}$  becomes larger moderately. These results are similar to Fig. 3. Consequently, the BAP mechanism becomes more important, especially around  $T=150$  K in the present case. When the impurity density gets larger,  $\tau_{\text{DP}}$  becomes larger and  $\tau_{\text{BAP}}$  does not change. This makes the relative effect of BAP mechanism become larger.

From above features, we emphasize that the BAP mechanism is important in  $p$ -type QWs, especially for large well width and/or large hole densities (i.e., heavily doped) and large impurity densities. It is very different from the bulk systems in which the BAP mechanism is absolutely dominant at low temperature. Therefore, both the BAP and the DP mechanisms should be considered to get the right SRT in QWs.

#### IV. SUMMARY

In summary, we have investigated the SRT due to both the DP and BAP mechanisms in intrinsic and  $p$ -type GaAs (001) QWs by constructing and numerically solving the fully microscopic kinetic spin Bloch equations. We consider all the relevant scatterings such as the electron-AC-phonon, electron-LO-phonon, electron-nonmagnetic impurity, and electron-electron Coulomb scatterings. Furthermore, the spin-conserving electron-heavy hole scattering, which en-

hances the total scattering strength and therefore  $\tau_{DP}$ , and the spin-flip electron-hole exchange scattering, which induces the BAP SRT, are also included.

We stress it is very important to calculate the SRT from our fully microscopic approach, especially at high electron density and low temperatures where the nonlinear terms in the electron-hole exchange scattering becomes very important. The SRT obtained from our fully microscopic approach is much larger than that from the Fermi golden rule. This means that the BAP mechanism is *negligible at very low temperature and high electron density*. We speculate this is also true in the bulk case. This is very different from the predictions in the literature.

We investigate the temperature dependence of the SRTs: The SRT due to the BAP mechanism  $\tau_{BAP}$  decreases rapidly with increasing temperature at very low temperature and slowly at higher temperature for both intrinsic and  $p$ -type QWs. It also decreases with electron density for both intrinsic and  $p$ -type QWs. For  $p$ -type semiconductors, it further decreases with hole density. We also compare the relative importance of the SRTs from the BAP and DP mechanisms. The SRT from the DP mechanism is also calculated from the kinetic spin Bloch equations which give the SRT also quite different from that from the single-particle approach as discussed extensively in our previous works.<sup>18–20,24,28</sup> We find that in intrinsic QWs, the effect of the BAP mechanism is much smaller than that from the DP mechanism at low temperature and it is smaller by nearly 1 order of magnitude at higher temperature. In  $p$ -type QWs, the SRT from the BAP mechanism is comparable with the one from the DP mechanism around certain temperature (such as 150 K in the case we study), especially when the hole density and/or the width of the QWs are large. For both the intrinsic and  $p$ -type QWs, the contribution from the BAP mechanism at very low temperature are negligible. We conclude that the spin R/D in QWs is very different from the bulk samples. In 2D case, the BAP mechanism hardly dominates the spin relaxation. Instead, it is either smaller or comparable to the DP mechanism.

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#### APPENDIX: COHERENT AND SPIN-CONSERVING SCATTERING TERMS IN KINETIC SPIN BLOCH EQUATIONS

The coherent terms can be written as

$$\left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{\text{coh}} = -\sigma [\Omega_x(\mathbf{k}) \text{Im} \rho_{\mathbf{k}} + \Omega_y(\mathbf{k}) \text{Re} \rho_{\mathbf{k}}] + 2\sigma \text{Im} \sum_{\mathbf{q}} V_{ee,q} \rho_{\mathbf{k}+\mathbf{q}}^* \rho_{\mathbf{k}}, \quad (\text{A1})$$

$$\left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{\text{coh}} = \frac{1}{2} [i\Omega_x(\mathbf{k}) + \Omega_y(\mathbf{k})] (f_{\mathbf{k},+1} - f_{\mathbf{k},-1}) + i \sum_{\mathbf{q}} V_{ee,q} [(f_{\mathbf{k}+\mathbf{q},+1} - f_{\mathbf{k}+\mathbf{q},-1}) \rho_{\mathbf{k}} - \rho_{\mathbf{k}+\mathbf{q}} (f_{\mathbf{k},+1} - f_{\mathbf{k},-1})], \quad (\text{A2})$$

$$\text{where } V_{ee,q} = \frac{\sum_{q_z} v_{qz} f_e(q_z)}{\epsilon(\mathbf{q})},$$

The electron-impurity scattering terms read

$$\left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{\text{im}} = \left\{ -2\pi n_i \sum_{\mathbf{q}} U_q^2 \delta(\epsilon_{\mathbf{k}}^e - \epsilon_{\mathbf{k}-\mathbf{q}}^e) [f_{\mathbf{k},\sigma} (1 - f_{\mathbf{k}-\mathbf{q},\sigma}) - \text{Re}(\rho_{\mathbf{k}} \rho_{\mathbf{k}-\mathbf{q}}^*)] \right\} - \{\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}\}, \quad (\text{A3})$$

$$\left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{\text{im}} = \left\{ \pi n_i \sum_{\mathbf{q}} U_q^2 \delta(\epsilon_{\mathbf{k}}^e - \epsilon_{\mathbf{k}-\mathbf{q}}^e) [(f_{\mathbf{k},+1} + f_{\mathbf{k},-1}) \rho_{\mathbf{k}-\mathbf{q}} - (2 - f_{\mathbf{k}-\mathbf{q},+1} - f_{\mathbf{k}-\mathbf{q},-1}) \rho_{\mathbf{k}}] \right\} - \{\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}\}, \quad (\text{A4})$$

in which  $\{\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}\}$  stands for the same terms previously in  $\{\}$  but interchanging  $\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}$ . In these equations,  $U_{\mathbf{q}}^2 = \sum_{q_z} [Z_i v_{qz} / \epsilon(\mathbf{q})]^2 f_e(q_z)$  with  $Z_i$  (assumed to be 1 in our calculation) the charge number of the impurity. The electron-phonon scattering terms are

$$\left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{\text{ph}} = \left\{ -2\pi \sum_{\mathbf{q}q_z\lambda} g_{\mathbf{q}q_z\lambda}^2 \delta(\epsilon_{\mathbf{k}}^e - \epsilon_{\mathbf{k}-\mathbf{q}}^e - \Omega_{\mathbf{q}q_z\lambda}) \times [N_{\mathbf{q}q_z\lambda} (f_{\mathbf{k},\sigma} - f_{\mathbf{k}-\mathbf{q},\sigma}) + f_{\mathbf{k},\sigma} (1 - f_{\mathbf{k}-\mathbf{q},\sigma}) - \text{Re}(\rho_{\mathbf{k}} \rho_{\mathbf{k}-\mathbf{q}}^*)] \right\} - \{\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}\}, \quad (\text{A5})$$

$$\left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{\text{ph}} = \left\{ \pi \sum_{\mathbf{q}q_z\lambda} g_{\mathbf{q}q_z\lambda}^2 \delta(\epsilon_{\mathbf{k}}^e - \epsilon_{\mathbf{k}-\mathbf{q}}^e - \Omega_{\mathbf{q}q_z\lambda}) \times [\rho_{\mathbf{k}-\mathbf{q}} (f_{\mathbf{k},+1} + f_{\mathbf{k},-1}) + (f_{\mathbf{k}-\mathbf{q},+1} + f_{\mathbf{k}-\mathbf{q},-1} - 2) \rho_{\mathbf{k}} - 2N_{\mathbf{q}q_z\lambda} (\rho_{\mathbf{k}} - \rho_{\mathbf{k}-\mathbf{q}})] \right\} - \{\mathbf{k} \leftrightarrow \mathbf{k} - \mathbf{q}\}, \quad (\text{A6})$$

where  $\lambda$  represents the phonon mode. For the electron-longitudinal-optic (LO) phonon scattering, the matrix element  $g_{\mathbf{Q},\text{LO}}^2 = \{2\pi^2 \Omega_{\text{LO}} / [(q^2 + q_z^2)]\} (\kappa_{\infty}^{-1} - \kappa_0^{-1}) f_e(q_z)$ ; for electron-acoustic-phonon scattering due to the deformation potential,  $g_{\mathbf{Q},\text{def}}^2 = \frac{\hbar \Xi^2 Q}{2d v_{st}} f_e(q_z)$ ; and for that due to the piezoelectric coupling,  $g_{\mathbf{Q},\text{pe}}^2 = \frac{32\pi^2 \hbar^2 e_{14}^2 (3q_x q_y q_z)^2}{\kappa_0^2 d v_{st} Q^7} f_e(q_z)$  for the longitudinal phonon and  $g_{\mathbf{Q},\text{pe}}^2 = \frac{32\pi^2 \hbar^2 e_{14}^2}{\kappa_0^2 d v_{st} Q^5} (q_x^2 q_y^2 + q_y^2 q_z^2 + q_z^2 q_x^2 - \frac{(3q_x q_y q_z)^2}{Q^2}) f_e(q_z)$  for the transverse phonon. Here,  $\Xi = 8.5$  eV is the deformation potential,  $d = 5.31$  g/cm<sup>3</sup> is the mass density of the crystal,  $v_{st} = 5.29 \times 10^3$  m/s ( $v_{st} = 2.48 \times 10^3$  m/s) is the velocity of the longitudinal (transverse) sound wave,  $\kappa_0 = 12.9$  denotes the static dielectric constant and  $\kappa_{\infty} = 10.8$  denotes the optical dielectric constant, and  $e_{14} = 1.41 \times 10^9$  V/m represents the piezoelectric constant.

$\Omega_{LO}=35.4$  meV is the LO phonon frequency, and the AC phonon spectra  $\Omega_{Q\lambda}$  are given by  $\Omega_{Ql}=v_{sl}Q$  for the longitudinal mode and  $\Omega_{Qt}=v_{st}Q$  for the transverse mode.<sup>38</sup>  $N_{q\lambda}=[\exp(\beta\Omega_{q\lambda})-1]^{-1}$  represents the Bose distribution.

The spin-conserving electron-electron Coulomb scattering terms are given by

$$\left. \frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \right|_{ee} = \left\{ -2\pi \sum_{\mathbf{q},\mathbf{k}',\sigma'} V_{ee,q}^2 \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^e - \varepsilon_{\mathbf{k}'-\mathbf{q}}^e) \right. \\ \times \left[ (1-f_{\mathbf{k}-\mathbf{q},\sigma})f_{\mathbf{k},\sigma}(1-f_{\mathbf{k}',\sigma'})f_{\mathbf{k}'-\mathbf{q},\sigma'} \right. \\ \left. \left. + \frac{1}{2}\rho_{\mathbf{k}}\rho_{\mathbf{k}-\mathbf{q}}^*(f_{\mathbf{k}',\sigma'} - f_{\mathbf{k}'-\mathbf{q},\sigma'}) \right] \right\}$$

$$\left. + \frac{1}{2}\rho_{\mathbf{k}'}\rho_{\mathbf{k}'-\mathbf{q}}^*(f_{\mathbf{k}-\mathbf{q},\sigma} - f_{\mathbf{k},\sigma}) \right\} \\ - \{\mathbf{k} \leftrightarrow \mathbf{k}-\mathbf{q}, \mathbf{k}' \leftrightarrow \mathbf{k}'-\mathbf{q}\}, \quad (\text{A7})$$

$$\left. \frac{\partial \rho_{\mathbf{k}}}{\partial t} \right|_{ee} = \left\{ -\pi \sum_{\mathbf{q},\mathbf{k}',\sigma'} V_{ee,q}^2 \delta(\varepsilon_{\mathbf{k}-\mathbf{q}}^e - \varepsilon_{\mathbf{k}}^e + \varepsilon_{\mathbf{k}'}^e - \varepsilon_{\mathbf{k}'-\mathbf{q}}^e) \right. \\ \times [(f_{\mathbf{k}-\mathbf{q},+1}\rho_{\mathbf{k}} + f_{\mathbf{k},-1}\rho_{\mathbf{k}-\mathbf{q}})(f_{\mathbf{k}',\sigma'} - f_{\mathbf{k}'-\mathbf{q},\sigma'}) \\ + \rho_{\mathbf{k}}[(1-f_{\mathbf{k}',\sigma'})f_{\mathbf{k}'-\mathbf{q},\sigma'} - \text{Re}(\rho_{\mathbf{k}'}\rho_{\mathbf{k}'-\mathbf{q}}^*)] \\ - \rho_{\mathbf{k}-\mathbf{q}}[f_{\mathbf{k}',\sigma'}(1-f_{\mathbf{k}'-\mathbf{q},\sigma'}) - \text{Re}(\rho_{\mathbf{k}'}\rho_{\mathbf{k}'-\mathbf{q}}^*)]] \left. \right\} \\ - \{\mathbf{k} \leftrightarrow \mathbf{k}-\mathbf{q}, \mathbf{k}' \leftrightarrow \mathbf{k}'-\mathbf{q}\}. \quad (\text{A8})$$

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