Donor-donor interaction mediated by cavity photons and its relation to interactions mediated by excitons and polaritons

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I report theoretical predictions of two models of donor-donor indirect interaction mediated by photons in zero- and two-dimensional cavities. These results are compared to previously studied cases of indirect interactions mediated by excitons and/or polaritons in bulk semiconductor and two-dimensional cavities. I find that photons mediate an Ising-like interaction between donors in the same manner polaritons do, in contrast to the Heisenberg-like interaction mediated by exciton. For the particular case of a two-dimensional cavity, the model shows that the dependence on distance of the donor-donor coupling constant is the same for photons and polaritons when the donor-donor distance is large. Then, it becomes clear that photons are responsible for the long-range behavior of the polariton indirect interaction.

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I. INTRODUCTION

The advances in the fabrication of low-dimensional structures have opened new areas of research in condensed matter physics. These new man-made systems not only pose challenges in terms of basic science but also permit further technological developments. The interest and current progress in the fabrication and characterization of zero-dimensional (0D) and two-dimensional (2D) cavities containing quantum dots (QDs) and impurities is evidenced from the work of several experimental groups around the world.[1]

Indirect interactions (II) in solid state physics are a well-known phenomenon. The first investigations date back to the 1950s with the study of the II between nuclear spins mediated by conduction electrons in metals[2] and in insulators,[3] process called RKKY (Ruderman-Kittel-Kasuya-Yosida). Closer to present days, we find the optical RKKY,[4,5] where the II is mediated by optically excited excitons in semiconductors. As already mentioned, the developments in nanoscience call for the revision and extension of previous works. Based on the ideas mentioned above and other related studies, investigations in the II in low-dimensional structures are being carried out to answer basic questions as well as to suggest applications. Examples of these works are the II in QDs embedded in microdisk structures as a scheme for quantum computing,[6] the ferromagnetic ordering induced by the II,[7] the II between impurities in a QD inside a 0D cavity,[8] the II between impurities close to QDs embedded in quantum wells as a scheme for quantum operations,[9] and the II in 2D cavities mediated by polaritons.[10] As can be seen, a major field of application of these concepts is quantum computing since the optical control of single impurities or quantum dots promises to be a fast and suitable tool for implementing quantum operations.

This paper presents theoretical predictions for two closely related systems exhibiting donor-donor II mediated by photons. The systems are 0D and 2D cavities with embedded QDs each coupled to a donor. To the best of the author's knowledge, no previous works report on either the optical RKKY-like interaction mediated by virtual photons in 2D cavities with QDs or the exact solution of the II mediated by photons in 0D cavities. In addition, the results are compared to previously studied cases of indirect interactions mediated by excitons and/or polaritons in bulk semiconductor and 2D cavities. This paper is organized as follows. First, a common framework for both models is given in Sec. II. Sections III and IV describe, respectively, the 2D and 0D models together with their results. Section V presents the general conclusions of this work.

II. GENERAL CONSIDERATIONS

Here, I summarize the main features common to both theoretical models. The systems consist of a cavity, either 0D or 2D, containing two QDs each one coupled to a donor impurity.[13] The neutral QDs may be excited by a cavity photon, and the resulting exciton may interact with the impurity via an exchange interaction between the electron in the donor and the electron in the exciton—for donors, the hole-electron exchange is typically smaller than the electron-electron exchange and is thus neglected. Due to quantum confinement, the heavy hole and light hole levels split; then, one can restrict the study to the heavy hole level only which spans a four-dimensional space for the exciton states in the QD: two optically active and two dark states.

I first treat the case of a 2D cavity and solve the problem analytically by perturbation theory. Then, the 0D cavity model is studied by an exact numerical diagonalization of its Hamiltonian.

III. DONOR-DONOR INTERACTION MEDIATED BY VIRTUAL PHOTONS IN A TWO-DIMENSIONAL CAVITY

The model consists of a 2D cavity with two embedded QDs, each of them coupled to a donor. The system is optically excited by an off-resonance monochromatic laser field

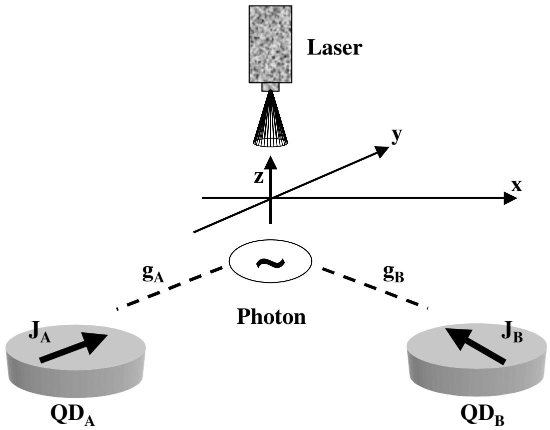

FIG. 1. The system consists of a 2D cavity (not drawn) and two QDs each coupled to a donor impurity. The cavity is excited from outside by an off-resonance monochromatic electromagnetic field propagating normal to the cavity in the z direction.

propagating parallel to the z axis (normal to the cavity), which excites virtual cavity photons (Fig. 1).

Inside the cavity, the quantized electric field with wave vector $\mathbf{m}=\mathbf{k}+\mathbf{q}$ of canonical spherical angles $\{\phi,\theta\}$ having in-plane/z component $\mathbf{k}/\mathbf{q}$ is

$$\mathbf{E}=i\sum_{\chi kq}\left(\frac{\hbar\omega_k}{2\epsilon V}\right)^{1/2}\mathbf{n}_{\chi kq}c_{\chi kq}e^{-i(\omega_k t-\mathbf{kR})}+\text{H.c.},\qquad(1)$$

where $\mathbf{n}_{\chi kq}=R_{zy}(\phi,\theta)\varepsilon_\chi$ is the circular polarization vector resulting from a rotation of the vector $\varepsilon_\pm=\hat{x}\pm i\hat{y}$, $c_{\chi kq}$ is the annihilation operator for cavity photons with frequency $\omega_k=\frac{c}{n}\sqrt{k^2+q^2}$, and V is the cavity volume. Due to the spatial confinement in the z direction, the corresponding momentum is restricted to two possible values $\mathbf{q}=\pm q\hat{z}$—inversely proportional to the width of the cavity—while the in-plane momentum $\mathbf{k}$ has a quasicontinuum spectrum. The phase factor e^{iqz} has been eliminated by taking the quantum well at $z=0$.

A. Hamiltonian

The complete Hamiltonian involves the bare energy (H_0) and the interactions: laser–cavity photon (H_{LC}), cavity photon–QD exciton (H_{CX}), and QD exciton–donor (H_{XS}).

Bare Hamiltonian.

$$H_0=E_X^A+E_X^B+\hbar\sum_{\chi kq}\omega_k c_{\chi kq}^\dagger c_{\chi kq},\qquad(2)$$

with exciton energy $E_X^i<\hbar\omega_k$ for $i=A/B$ QD; the ground state energy of the donors is taken equal to zero.

Laser–Cavity photon Hamiltonian. In accordance with the quasimode approach, useful for high-Q cavities,[14] the interaction between the laser and the cavity photon is represented by a semiclassical field that creates and/or annihilates cavity photons with momentum in z equal to $\pm q$,

$$H_{LC}=\hbar\sqrt{\mathcal{A}}\sum_{\sigma q}\mathcal{V}_\sigma e^{i\omega_L t}c_{\sigma 0q}+\text{H.c.},\qquad(3)$$

where $\mathcal{A}$ is the area of the 2D cavity and $\mathcal{V}_\sigma$ the coupling constant.

Cavity photon–QD exciton Hamiltonian. The Schrödinger representation of the interaction between the cavity mode Eq. (1) and the exciton taken in the dipole moment approximation is

$$H_{CX}=i\frac{g}{\sqrt{\mathcal{A}}}\sum_{\chi\sigma kqp}(\mathbf{n}_{\chi kq}\cdot\mathbf{d}_\sigma^*)m^{1/2}e^{i\mathbf{kR}_p}b_\sigma^{(p)\dagger}c_{\chi kq}+\text{H.c.},\qquad(4)$$

with g the coupling constant and $m=|\mathbf{m}|$. $b_\sigma^{(p)\dagger}$ is the creation operator for excitons in the $p=A/B$ QD with the condition: $\sigma+\equiv(e\downarrow,h\uparrow)$ and $\sigma-\equiv(e\uparrow,h\downarrow)$ for the spins of the electron (e) and the hole (h). Assuming the quantization axis for the excitons in z, the dipole moment is $\mathbf{d}=\mp\frac{1}{\sqrt{2}}(\hat{x}\pm i\hat{y})$. The scalar product is

$$\mathbf{n}_{\pm kq}\cdot\mathbf{d}_\pm^*=\frac{1}{2}e^{\pm i\phi}\left(1+\frac{\mathbf{q}\cdot\hat{z}}{m}\right),$$

$$\mathbf{n}_{\pm kq}\cdot\mathbf{d}_\mp^*=\frac{1}{2}e^{\mp i\phi}\left(1-\frac{\mathbf{q}\cdot\hat{z}}{m}\right),$$

Notice that the interaction is valid for all angles $\{\theta,\phi\}$.

QD exciton–Donor Hamiltonian.

$$H_{XS}=\sum_{p\alpha\alpha'\beta}J^{(p)}s_{\alpha\alpha'}\cdot s^{(p)}b_{\alpha\beta}^{(p)\dagger}b_{\alpha'\beta}^{(p)}+\text{H.c.},\qquad(5)$$

where $b_{\alpha\beta}^{(p)\dagger}$ has electron spin α and hole spin β [same operator as in Eq. (4), but with the spin index explicitly written for the electron and the hole], $s^{(p)}$ is the spin of the donor, and the tensor

$$[\mathbf{s}_{\alpha\alpha'}]=\frac{\hbar}{2}\begin{pmatrix}\hat{z} & \hat{x}-i\hat{y}\\ \hat{x}+i\hat{y} & -\hat{z}\end{pmatrix},$$

with, for example, $s_{12}=s_{\uparrow\downarrow}$.[10] Notice that the interaction does not change the spin of the hole.

B. Effective donor-donor Hamiltonian

I seek to derive a Hamiltonian expression for the donor-donor interaction that contains operators only for the spin degree of freedom of each impurity. The procedure involves moving to a rotating frame to eliminate the time dependence from H_{LC}, applying second order perturbation theory to H_{LC}, and finding the correction to the unperturbed Hamiltonian H_0, keeping the lowest order in H_{XS} and H_{XC} (see Appendix for details). It is worth mentioning that the off-resonance excitation of the system makes possible the use of perturbation theory since the detuning—which is a controllable parameter—enters the expansion coefficient.

The first relevant term that gives rise to a process that correlates both spins is (the same process exchanging $A\leftrightarrow B$ must also be considered): (1) The extracavity EM field shines on the cavity coherently exciting a virtual cavity photon, (2) the cavity photon creates an exciton in QD A, (3) the exciton interacts with the donor spin A, (4) the exciton in QD A annihilates and a cavity photon is created, (5) the cavity photon creates an exciton in QD B, (6) the exciton interacts with the donor spin B, (7) the exciton in QD B annihilates and creates a cavity photon, and (8) the cavity photon is deexcited (see Fig. 2). It should be kept in mind that the whole process is driven by an off-resonance excitation and thus no energy absorption occurs. The final expression for the effective Hamiltonian corresponding to this process is

$$H_{eff} = \left\{ \frac{2\hbar^3 qn}{c} \left(\frac{|\mathcal{V}_{\sigma+}|^2 + |\mathcal{V}_{\sigma-}|^2}{\delta_C^2} \right) \frac{J^{(A)} g^{(A)2} J^{(B)} g^{(B)2}}{\delta_A^2 \delta_B^2} F(\omega_L;R) \right\} s_Z^{(B)} s_Z^{(A)}, \tag{6}$$

with $\delta_{A/B} = \hbar\omega_L - E_X^{A/B}$ the detuning for exciton A/B and $\delta_C = \hbar\omega_L - (\hbar c q)/n$ the detuning for photons, with c the speed of light and n the index of refraction, $\mathbf{R} = \mathbf{R}_B - \mathbf{R}_A$ the distance between donors, and

$$F(\omega_L;R) = (2\pi\hbar)^2 \int d^2\mathbf{k} \frac{\omega_k^2 + (qc/n)^2}{\omega_k(\omega_L - \omega_k)} \cos(\mathbf{k}\mathbf{R}).$$

The quantity within curly brackets in Eq. (6) is the so-called *effective* coupling constant J_{eff}.

C. Results

For a comprehensive understanding of the results, I first recall previous similar research works. Theoretical studies have demonstrated that impurities embedded in a semiconductor host either in bulk or inside 2D cavities can be made to interact using virtual excitons or polaritons+excitons as intermediate particles, respectively.[4,5,10] These reports show that the spin-spin interaction mediated by excitons in bulk is Heisenberg, and the interaction mediated by polaritons +excitons in 2D cavities is anisotropic (with the polaritons and excitons providing Ising and transverse terms, respectively). In all cases, the coupling strength J_{eff} falls off exponentially as a function of the separation R between impurities, having a different range in each case. This exponential decay is a consequence of virtual (off-resonance) excitations.

Equation (6) shows that the donor-donor interaction mediated by photons is of the Ising type, which can be explained as follows. The essential ingredient for the correlation between different donors is, clear enough, the existence of a cavity photon that travels from one QD to another. The cavity photon acts upon (and is acted on by) the donors only indirectly, meaning that the photon interacts with each donor through an *optically active* exciton in the corresponding QD. No matter how complicated the interaction within the QD may be (donor's electron-exciton's electron of higher order or even an interaction that includes a donor's electron-exciton's hole term), the initial and final states that connect to the cavity photon must be optically active excitons. This ensures that spin flips within the QD only occur in pairs of the type s_+s_- or s_-s_+. According to the algebra of SU(2), a transformation that is a product of those pairs is of the form $a+bs_Z$. Therefore, I conclude that the restriction of having an interaction mediated only by photons causes the donor-donor coupling to be Ising-like (to all order). This is in agreement with the polariton case in 2D cavities; it is in contrast to the situation of excitons in bulk or polaritons+excitons in a 2D cavity, where the dark excitons also propagate the interaction and so do not force the spin flip to come in pairs of raising and lowering operators for each impurity.

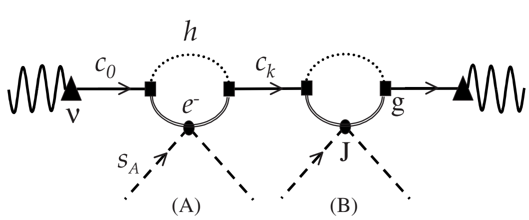

FIG. 2. Diagrammatic representation of the lowest order contribution ($\mathcal{O}[(G_X^{02} g^2 JF)^2]$) to the indirect interaction between two donors. Loops are for excitons in each (A/B) QD, solid lines are for cavity photons, wavy lines are for the laser field, and dash lines are for donors. The coupling constants $\{\nu, J, g\}$ are represented by triangle, circle, and square vertices, respectively.

The integration of $F(\omega_L;R)$ is performed numerically after a cutoff k_c for the in-plane momentum of the photon is prescribed. The coupling between a cavity photon and a QD exciton [Eq. (4)] is taken in the dipole moment approximation. For this approximation to be valid, the wavelength of the cavity photon must be larger than the size s of the excitonic wave function—roughly equal to the size of a QD—or, equivalently, the total momentum m of the photon must be smaller than the inverse of this size. This imposes a condition on the maximum value of the in-plane momentum k and yields a cutoff k_c that must satisfy $\sqrt{k_c^2+q^2} \sim 1/s$—for $k=0$, the dipole moment approximation is valid since the width of the cavity is larger than the size s of the QD.

The effective coupling constant J_{eff} is evaluated using numerical values for the parameters that represent a system of Si donors in InAs/GaAs QDs inside a planar microcavity. A plot comparing the effective coupling J_{eff} for the II mediated by polaritons, excitons, and photons in 2D cavities is shown in Fig. 3. The plot suggests that for large separation R between the donors, (i) the II decays exponentially—a consequence of the off-resonance excitation of the system—and (ii) the slopes of the polariton and photon coupling constants are very similar. In order to verify these ideas, let us consider the function $F(\omega_L;R)$ containing the spatial dependence. Its angular integration yields a Bessel function $J_0(kR)$, which suppresses high momenta k for large R. Therefore, the re-

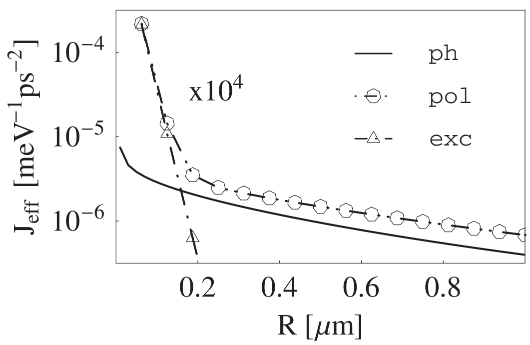

FIG. 3. Effective donor-donor coupling constant $J_{eff}(R)$ for the interaction mediated by excitons (triangle), polaritons (circle), and photons (solid) in a 2D cavity. The parameters have been chosen in order to make the correspondence among the three cases the most compatible and to represent a system of Si donors in InAs/GaAs QDs: $\hbar=0.625$ meV ps, $q=0.021$ nm^{-1}, $c=3.04\ 10^5$ nm ps^{-1}, $\delta_C=-0.66$ meV, $\mathcal{V}=0.9$ meV, $\delta_A=\delta_B=-0.1$ meV, $g_A=g_B=0.3$ meV, $n=3$, $J_A=J_B=1$ meV^{-1} ps^{-2}, and $s=10$ nm. (For the sake of comparison, the polaritons and excitons curves are plotted a factor 10^{-4} of their actual value.)

maining of the integrand can be expanded in powers of k/q, and the integral can be solved for the lowest order contribution. Then, for large R, $J_{eff}(R)\propto R^{-1/2}\exp(-R/R_0)$, with $R_0=(2M\delta_C/\hbar^2)^{-1/2}$ and $M=\hbar nq/c$ the "mass" of the photon. The fitting of $J_{eff}(R)$ by such a Yukawa 2D function is excellent and allows us to determine the parameter M from the slope; the mass obtained from the fit $M=\hbar^2/(2R_0^2\delta_C)$ deviates about 5% from its theoretical value $M=\hbar nq/c$. The slope of polaritons can also be calculated using the Yukawa approximation. The slopes of both polaritons and photons curves agree to 15%, and this mismatch is explained by the slightly different masses of the two particles. Together with the fact that the polariton and photon interactions are of the Ising type, I conclude that the long-range coupling of polaritons is of photon nature.

Finally, using an optimistic set of parameters compatible with experimental studies, I find that the strength of the interaction mediated by photons is about 2 orders of magnitude smaller than that of polaritons. In both cases, the strength of the coupling can be modified by the detuning δ_C. For the photon mediated interaction, Eq. (6) shows that δ_A and δ_B play a role too; however, these detunings depend on the exciton energy of each QD, and they are not simultaneously controllable in an easy way, and so their manipulation to enhance the photon mediated II is nowadays doubtful.

IV. DONOR-DONOR INTERACTION MEDIATED BY PHOTONS IN A ZERO-DIMENSIONAL CAVITY

The system shown in Fig. 4 consists of two QDs each coupled to a donor, all embedded in a 0D cavity supporting one photon mode. The axis normal to the surface of the cavity is named z and is the quantization axis for the excitons in the QDs for which the heavy hole (hh) and light hole levels split; I retain the hh level only.

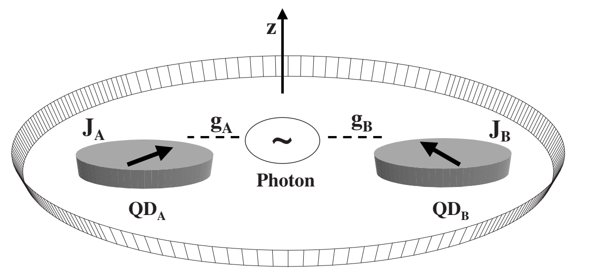

FIG. 4. The 0D cavity model consisting of a 0D cavity and two QDs each coupled to a donor impurity.

I seek to determine the correlation between the spins and use this information to infer the indirect interaction in thermal equilibrium.[8,15] To this end, the exact eigenvalues and eigenvectors of the complete Hamiltonian

$$H=H_0+\sum_{\sigma p} g^{(p)}c_\sigma^\dagger b_\sigma^{(p)}+\sum_{\alpha\alpha' p} J^{(p)}s_{\alpha\alpha'}\cdot s^{(p)}b_{\alpha\beta}^{(p)\dagger}b_{\alpha'\beta}^{(p)}+\text{H.c.} \tag{7}$$

(all orders in g and J) were determined by numerical diagonalization in the subspace of one particle (photon/A exciton/B exciton).[16] In Eq. (7), c and b stand for the operators for cavity photons and excitons, respectively. The bare energy H_0 contains the exciton energy E_X^p of QD $p=A/B$, and the energy E_c of the cavity photon. Note the similarities between this model and the 2D cavity presented in the previous section.

The eigenvectors and energies were then used to calculate the correlation in a thermal distribution,

$$\langle s_j^{(A)}s_j^{(B)}\rangle=\frac{1}{Z}\sum_i e^{-\beta E_i}\langle n_i|s_j^{(A)}s_j^{(B)}|n_i\rangle,$$

where n_i stands for the eigenvectors of the complete Hamiltonian of the system, Z is the partition function, and $s_j^{(D)}$ is the spin operator in the j direction corresponding to the donor inside QD D. This correlation allows the calculation of the effective coupling between donors.

A. Results

First, I discuss the validity and scope of this 0D cavity model. Consider a typical structure, a micropilar of height $h=0.1$ μm and diameter $\phi=2$ μm. In this case, the energy levels of cavity photons are separated by about $\Delta E\simeq 10$ meV, difference much larger than any energy of the system ($g,J<1$ meV). Therefore, the system presents only one cavity mode, and the model is applicable to a realistic situation. In addition, the maximum spatial separation between donors, given by the size of the micropilar, is at least the separation considered in the case of a 2D cavity, so the values of the effective coupling constant for both models may be compared. Finally, it is worth noticing that a thorough study of strong and weak coupling between the QD exciton and the cavity photon would require the inclusion of decoherence. The model I consider disregards decoherence and so will only represent the situation of strong coupling (g

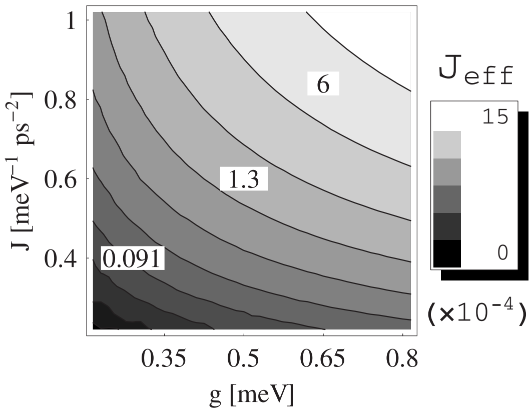

FIG. 5. Contour plot of the Ising effective coupling constant J_{eff} (meV^{-1} ps^{-2}) ($\times 10^{-4}$) as a function of couplings J and g for a detuning $\delta = E_C - E_X^A = 0.5$ meV with $E_X^A = E_X^B$, $g = g_A = g_B$, $J = J_A = J_B$, and $T = 5$ K.

larger than any decay constant). In view of the available experimental data,[1] systems of the sort I study here are in strong coupling regime for values of g in the order of tens of μeV or larger.

Numerical calculations performed for all admissible values of the physical constants[1,8,17] reveal that the in-plane or xy correlation is zero, consistent with the results of Sec. III C. Thus, I am led to assume an Ising donor-donor effective Hamiltonian. The numerical data for the normal or z correlation can be used to infer the value of the effective coupling constant. A model of two 1/2 spins interacting through an Ising Hamiltonian $H = J_{eff} s_z^{(A)} s_z^{(B)}$ yields the correlation $\langle s_z^{(A)} s_z^{(B)} \rangle = \hbar^2/4 \tanh[\hbar^2 J_{eff}/(4kT)]$ in thermal equilibrium. By direct inversion of this relation or by fitting the data for different temperatures, the value of the effective coupling constant is computed.

Figure 5 shows a contour plot of J_{eff} as a function of the exciton-donor coupling J and photon-exciton coupling g for an off-resonance situation at a typical temperature at which experiments are done. The plot shows, as expected, that the effective coupling between donors becomes stronger as the coupling g and J is increased. In addition, I find that the change of J_{eff} with temperature is very pronounced, for example, at $T = 0.1$ K, the coupling becomes $J_{eff} = 2 \times 10^{-1}$ meV^{-1} ps^{-2} for $g = 0.8$ meV and $J = 1$ meV^{-1} ps^{-2}.

Even though the transition from weak to strong coupling cannot be addressed here, it is worth considering what occurs for different values of the exciton-photon detuning δ. This parameter causes the excitation in the system to be a photon, an exciton, or an admixture of them: the polariton. To determine the nature of the excitation, let us look at the exact eigenvectors resulting from the diagonalization of Eq. (7). We only need to consider a small number of them because at low temperature, the correlation (and so J_{eff}) is dominated by a small number of low-energy exact eigenvectors. Every exact eigenvectors is a linear superposition of vectors of the uncoupled problem H_0; the way these uncoupled vectors mix to form the dominant exact states tells us the approximate nature of the excitation. To exemplify, take a set of

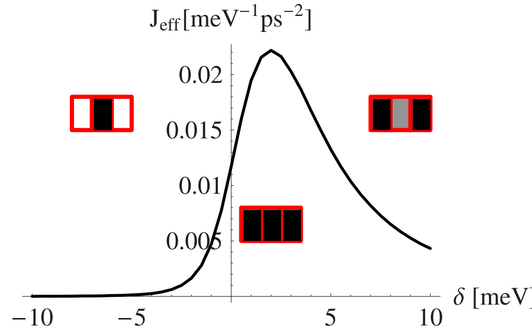

FIG. 6. (Color online) J_{eff} vs detuning $\delta = E_C - E_X^A$ for $J_A = J_B = 0.5$ meV^{-1} ps^{-2}, $g_A = g_B = 0.8$ meV, $T = 1$ K, and $E_X^A = E_X^B$. As represented by the three-block rectangles, the exact eigenstates of the lowest-energy subspace are predominantly: photonlike for $\delta < 0$, polaritonlike for $\delta \simeq 0$, and excitonlike for $\delta > 0$.

parameters ($\delta = E_C - E_X^A = -5$ meV, $E_X^A = E_X^B$, $g = 0.4$ meV, $J = 0.5$ meV^{-1} ps^{-2}, and $T = 0.1$ K): the energy spectrum consists of a set of eight low energy levels well separated from the rest of the upper levels; these eight states dominate the correlation. Every state of this subspace is of the form $\alpha|C\rangle + \cdots$, with $\alpha \simeq 0.99$ and $|C\rangle$ a state with a cavity photon (the spin state of the donors is different for each of the eight eigenvectors). For the probability amplitude, a simple pictorial representation consisting of a three-block rectangle may be used: the left, right, and middle parts of it represent the left QD exciton, the right QD exciton, and the cavity photon, respectively. A gray scale is used, where the largest probability for the corresponding particle in the exact eigenstate appears as the darkest block (for the previous example: □■□). Figure 6 shows the effective coupling constant J_{eff} as a function of the detuning for $E_X^A = E_X^B$. For other choices of the energies $\{E_C, E_X^A, E_X^B\}$, one finds different compositions of excitations, e.g., $\{E_C \simeq E_X^B, E_X^A > E_X^B\}$: □■■, a polariton in the right QD. As can be seen, the largest effective coupling J_{eff} corresponds to a situation where each of the exact states belonging to the dominant subspace is an admixture, in equal proportion, of all the particles of the uncoupled problem H_0.

V. CONCLUSIONS

I have studied the donor-donor indirect interaction mediated by photons in two closely related systems, i.e., 0D and 2D cavities. I found that this spin-spin effective interaction is of the Ising type and no transverse term is present. The reason for this being that the donors can interact only through the exchange of a cavity photon. Photons can only be produced by optically active excitons in each QD; thus, the initial and final states of each QD are always optically active excitons. Any spin flip that may occur within the QD must be compensated by an opposite flip (of the electron or even the hole if an electron-hole exchange is included in the Hamiltonian). Because the spin flips come in pairs of raising and lowering operators, the $s = 1/2$ algebra determines that ultimately the whole operation within each QD is equivalent to s_z. This is analogous to what happens in the case of polaritons in a 2D cavity. In contrast, excitons in bulk or excitons+polaritons in a 2D cavity do not have this restriction and present a transverse spin-spin interaction.

For the 2D model, the dependence of the effective coupling constant (J_{eff}) on the donor-donor separation (R) plus the fact that J_{eff} is of the Ising type make evident that the long-range behavior of polaritons is due to photons. Nevertheless, for an optimistic set of parameters, the photon mediated interaction is much weaker than the polariton mediated interaction. One possible reason for this difference is that the present photon model involves more interaction steps than the model with polaritons—polaritons interact directly with the donor spins, while photons interact only through the intermediate exciton degree of freedom.

In the case of the 0D cavity, the strength of the interaction becomes larger when the exact eigenstate of the system is a full admixture of all particles: the excitons in each QD and the photon. Between the situations of positive and negative detunings, the former seems more effective to maintain correlation among donors. In contrast to the case of the 2D cavity, J_{eff} does not depend on the separation between donors and remains constant up to the values of R analyzed in the 2D model. The coupling constant is very sensitive to changes in temperature and it increases as the temperature decreases.

Although an accurate comparison between the 0D and 2D models is not possible (since the effective coupling is driven in different ways in each case), for typical values of the parameters and temperatures used in experiments, the J_{eff} of the 0D cavity appears to be weaker than that of excitons +polaritons. However, for large separations R and temperatures $T<1$ K, the numerical results indicate that the strength of the 0D cavity may overcome that of the 2D system.

From the point of view of application to quantum information science, these findings suggest that—among the systems compared here—either the 0D-photon or the 2D-polariton schemes presents the most advantageous features: the former for the strength at large spatial separation R, and the latter due to the versatility given by the anisotropic Heisenberg Hamiltonian. Nevertheless, a deep analysis is still needed to determine how other factors, such as decoherence, may condition or limit the applications of this 0D-photon proposal.

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APPENDIX: EFFECTIVE HAMILTONIAN FOR THE TWO-DIMENSIONAL CAVITY

After applying a transformation to a rotating frame of frequency ω_L, one can use time-independent perturbation theory on H_{LC} in the form of projection operators. To second order in H_{LC}, the correction to the unperturbed Hamiltonian is given by the level-shift operator R,[5,18]

$$PRP = \mathcal{P}H_{LC}\mathcal{Q}\frac{1}{z-(H_0+H_{CX}+H_{XS})}\mathcal{Q}H_{LC}\mathcal{P},$$

where $\mathcal{P}$ is the projector onto the subspace of zero cavity photons and excitons, and $\mathcal{Q}=1-\mathcal{P}$. Then,

$$H_{eff,\lambda\lambda'} = \langle\lambda|H_{LC}\frac{\mathcal{Q}}{z-(H_0+H_{CX}+H_{XS})}H_{LC}|\lambda'\rangle,$$

where $|\lambda\rangle$ designates any one of $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ vectors of the spin of both donors, with zero cavity photons and zero excitons. The light-cavity Hamiltonian acting on $|\lambda\rangle$ yields

$$H_{LC}|\lambda\rangle = \hbar\sqrt{\mathcal{A}}\sum_{\sigma}\mathcal{V}_\sigma|\lambda c_0\rangle,$$

with $|\lambda c_0\rangle$ a vector with one cavity photon with polarization σ. Then,

$$H_{eff,\lambda\lambda'} = \hbar^2\mathcal{A}\sum_{\sigma}|\mathcal{V}_\sigma|^2\langle\lambda c_0|\frac{1}{(z-H_0)-(H_{CX}+H_{XS})}|\lambda' c_0\rangle,$$

with $[G_k^0(\omega_L)]^{-1}=z-H_0$. The cavity photon in both sides of the matrix element is the same, for I seek to study a coherent process.

I want to consider the case of one virtual particle, either a cavity photon or an exciton in one or the other QD. That gives three possible particles. I separate the space of one virtual excitation into three subspaces $\{\Im_C,\Im_A,\Im_B\}$ and obtain

$$H_I = \begin{pmatrix} H_{XS}^A & H_{CX}^A & 0 \\ H_{CX}^A & 0 & H_{CX}^B \\ 0 & H_{CX}^B & H_{XS}^B \end{pmatrix},$$

$$G^0 = \begin{pmatrix} G_X^{0A} & 0 & 0 \\ 0 & G_C^0 & 0 \\ 0 & 0 & G_X^{0B} \end{pmatrix}.$$

The bare Green's functions are

$$G_X^{0A/B} = \frac{1}{\hbar\omega_L - E_X^{A/B}},$$

$$G_C^0 = \frac{1}{\hbar\omega_L - \frac{\hbar c}{n}\sqrt{k^2+q^2}}.$$

The first relevant term, in the expansion of the Green's function, that will give rise to a process that correlates both spins belongs to the $G^{(2+4)}=G^0(H_IG^0)^3(H_IG^0)^3$. The process is (1) creation of an exciton in QD A, (2) exciton interacting with the spin, (3) annihilation of the exciton in QD A with the creation of the cavity photon, (4) creation of an exciton in

QD B, (5) interaction between exciton and spin, (6) annihilation of the exciton in QD B, and creation of the cavity photon. I obtain

$$H_{eff,\lambda\lambda'} = \hbar^2 \mathcal{A} |G^0_{Ck=0}|^2 \sum_\sigma |\mathcal{V}_\sigma|^2 \langle \lambda c_0 | \Sigma_{BA} + \Sigma_{AB} | \lambda' c_0 \rangle,$$

with the matrix element written in explicit form,

$$\langle \lambda c_0 | \Sigma_{BA} | \lambda' c_0 \rangle = \langle \lambda c_0 | (H^B_{CX} G^{0B}_X H^B_{XS} G^{0B}_X H^B_{CX}) \times G^0_C (H^A_{CX} G^{0A}_X H^A_{XS} G^{0A}_X H^A_{CX}) | \lambda' c_0 \rangle,$$

which is $\mathcal{O}(J^2)$ and $\mathcal{O}(g^4)$. Straightforward, but lengthy, operator algebra leads to $H_{eff}=J_{eff} s_Z^{(B)} s_Z^{(A)}$ as presented in the main part of this paper.

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