Coupled phonon polaritons in a piezoelectric-piezomagnetic superlattice

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Propagation of electromagnetic (EM) waves in a piezoelectric-piezomagnetic superlattice (PPS) has been investigated. In a PPS, the electric and magnetic vectors of EM waves could simultaneously couple with the identical superlattice vibration, respectively, due to piezoelectric effect and piezomagnetic effect, which results in magnetoelectric effect. Consequently, two orthogonally polarized EM waves could simultaneously couple with the identical vibration, which would give birth to coupled phonon polaritons. Attributed to this mechanism, in a PPS, the propagation of EM waves varies drastically. EM waves perpendicular to the PPS vector can propagate, while the propagation is inhibited along the PPS vector in the original band gap of either the piezoelectric superlattice or the piezomagnetic superlattice. The origin of the differences in propagation is analyzed and some potential applications are discussed.

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I. INTRODUCTION

The interaction of waves with periodic condensed matter, especially artificial superlattices, has attracted much attention. It is well known that the periodic modulation of the given parameter may lead to band structures and make it possible to control the propagation of information carriers, for example, electrons and photons. Due to intense Bragg scatterings at the boundaries of the Brillouin zone, a photonic band gap can be introduced to control the propagation of light in photonic crystals.[1] In addition, an alternative to form band structures is the coupling effect of polaritons. For example, in an ionic crystal, the coupling between EM waves and lattice vibrations leads to phonon polaritons, whose band gaps appear in the infrared range.[2] It should be noticed that, different from the case in photonic crystals, the wave vector corresponding to the intense coupling in ionic crystals is close to the Γ point in the Brilloiun zone.

Polaritons can also exist in artificial superlattices,[3] such as piezoelectric (PE) superlattices[4–6] and piezomagnetic (PM) superlattices,[7] in which the periodicity of the lattice is artificially expanded from atomic scale to nanometers and microns. In PE and PM superlattices, both PE and PM phonon polaritons result from the coupling between EM waves and superlattice vibrations, and the propagation of EM waves is not allowed near resonant frequencies.

In this paper, we will demonstrate another kind of superlattice that alternates between PE and PM stacks, a complex structure made of a PE superlattice[4–6] and a PM superlattice.[7] In a PPS, owing to PE and PM effects, the electric and magnetic fields could simultaneously couple with the identical vibration, represented as PE phonons and PM phonons, which would result in piezoelectric (PE) polaritons, piezomagnetic (PM) polaritons, and magnetoelectric (ME) effect. Furthermore, due to the ME effect, the PE and PM polaritons would further couple with each other. The coupling between PE and PM polaritons would endow the propagation of electromagnetic (EM) waves with novel properties in the original band gaps of both PE polariton and PM polariton.

The organization of this paper is as the follows. In Sec. II, we demonstrate how ME effect occurs naturally from PE and PM effects in a PPS through their mechanical strain by effective-medium expression. In Secs. III and IV, we com-

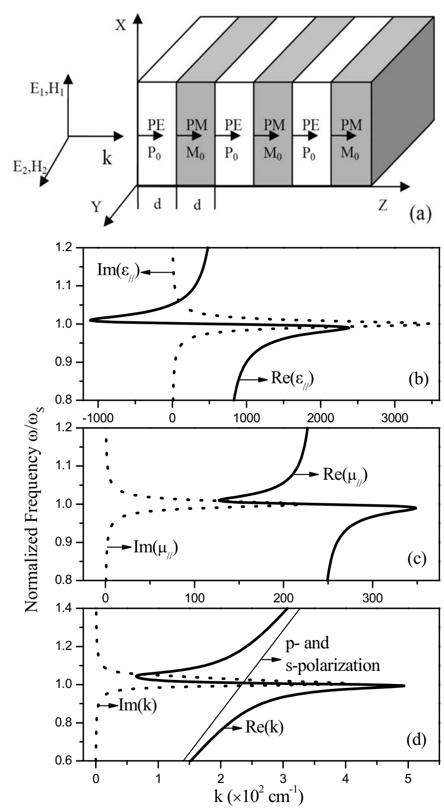

FIG. 1. (a) Schematics of the PPS and the two orthogonally polarized EM waves along the z axis. Due to ME effect, p polarization would be induced by the incident s polarization. (b), (c), and (d) are the calculated dispersion relations of permittivity, permeability, and wave vector k, described in the form of $X = \mathrm{Re}(X) + i\,\mathrm{Im}(X)$. Here, the thinner solid line in (d) represents the propagation of the degenerate p and s polarizations in the case without PE and PM effects.

bine the expression with Maxwell's equations to investigate the propagation of EM waves parallel and/or perpendicular to the stacking direction. In Sec. V, we present the conclusion.

II. THEORETICAL TREATMENT

The piezoelectric-piezomagnetic superlattice (PPS) presented in this paper [shown in Fig. 1(a)] is a one-dimensional periodic structure composed of alternate layers of PE and PM materials along the z axis. To simplify the problem, we assume that PE and PM layers are polycrystalline, and the thickness for each layer is d. This PPS is electrically and magnetically poled along the z direction. Consequently, both PE and PM layers could be considered as the transversely isotropic systems with the same point group of ∞m symmetry.

Let us focus on the modulations of PE and PM constants. The interaction between EM waves and superlattice vibrations is described by the following equations:

$$T_I = C_{IJ}S_J(z) - e_{Ij}f(z)E_j - q_{Ij}g(z)H_j$$
$$D_i = e_{iJ}f(z)S_J(z) + \varepsilon_0\varepsilon^s_{ij}(z)E_j \quad (i,j=1,2,3;\ I,J=1,2,\ldots,6).$$
$$B_i = q_{iJ}g(z)S_J(z) + \mu_0\mu^s_{ij}(z)H_j \tag{1}$$

Here T, S, C, E, H, ε, μ, e, and q are the stress, strain, elastic stiffness coefficient, electric field, magnetic field, permittivity, permeability, and PE and PM constants, respectively. The modulation functions $f(z)$ and $g(z)$ owing to the PE and PM effects in Eq. (1) have the values of 1 and 0 in PE layers, respectively, while 0 and 1 in PM layers. The elastic tensor element is described as $C_{IJ}=c_{IJ}-i\omega\eta_{IJ}$, where c_{IJ} is the real part and η_{IJ} is the damping coefficient. It should be noticed that there is no ME effect involved in this stage. In our derivation, we have not supposed any *a priori* ME term that is used in previous approximations.[8,9] PE and PM effects can inherently interact with each other. ME effect results naturally from the stress that both adjacent PE and PM layers suffer, and it will be revealed in the following discussion.

We also assume that the transverse dimensions are much larger than the acoustic wavelength in the PPS, so a one-dimensional model can be applied properly. Then, taking the long-wavelength approximation into account and substituting T_5 into the equation of motion for superlattice vibrations, we can obtain

$$\rho\frac{\partial^2 S_5}{\partial t^2} - C_{44}\frac{\partial^2 S_5}{\partial z^2} = \frac{\partial^2}{\partial z^2}[-e_{15}f(z)E_1(z) - q_{15}g(z)H_1(z)], \tag{2}$$

where $E_1(z)$ and $H_1(z)$ are the x components of EM fields, the ρ and C_{44} are the effective mass density and the effective elastic tensor element, respectivley. With the Fourier transformation, the modulation functions are written as

$$f(z) = \sum_n f_n e^{iG_nz} = \frac{1}{2} + \frac{1}{2}\sum_{n\neq0}\frac{i(1-\cos n\pi)}{n\pi}e^{iG_nz}$$
$$g(z) = \sum_n g_n e^{iG_nz} = \frac{1}{2} - \frac{1}{2}\sum_{n\neq0}\frac{i(1-\cos n\pi)}{n\pi}e^{iG_nz} \quad \left(G_n = \frac{n\pi}{d}\right). \tag{3}$$

The PPS resembles a forced oscillator. The solution of Eq. (2) is described as

$$S_5(z) = \sum_n \frac{G_n^2}{\rho\omega^2 - C_{44}G_n^2}e^{iG_nz}[-e_{15}f_nE_1(z) - q_{15}g_nH_1(z)]. \tag{4}$$

Substitution of Eq. (4) into Eq. (1) gives

$$D_1(z) = \varepsilon_0[\varepsilon^s_{11}(z) - ae^2_{15}f(z)f_n/\varepsilon_0]E_1(z) - ae_{15}q_{15}f(z)g_nH_1(z)$$
$$= \varepsilon_0\varepsilon_\parallel(z)E_1(z) + \alpha_{11}(z)H_1(z),$$
$$B_1(z) = \mu_0[\mu^s_{11}(z) - aq^2_{15}g(z)g_n/\mu_0]H_1(z)$$
$$- ae_{15}q_{15}g(z)f_nE_1(z)$$
$$= \mu_0\mu_\parallel(z)H_1(z) + \beta_{11}(z)E_1(z), \tag{5}$$

where

$$a = \sum_n \frac{G_n^2}{\rho\omega^2 - C_{44}G_n^2}e^{iG_nz}. \tag{6}$$

If the frequency of the incident EM wave is close to the fundamental resonant frequency of transverse vibration S_5, we can get $a=e^{iG_1z}G_1^2/(\rho\omega^2-C_{44}G_1^2)$ by ignoring the other high-order reciprocal vectors. Owing to the boundary conditions, the transverse electric and magnetic fields are continuous at the interfaces of the layers. In the long-wavelength approximation (the EM wavelength is much larger than the period of the PPS), their variations inside each layer can be neglected. So, $E_1^{pe}=E_1^{pm}=\overline{E}_1$ and $H_1^{pe}=H_1^{pm}=\overline{H}_1$, where $\overline{E}_1$ and $\overline{H}_1$ represent their values averaged over the period; the superscripts pe and pm indicate the physical quantities of PE layers and PM layers, respectively. Thus the x components of electric displacement and magnetic field intensity averaged over the period are

$$\overline{D_1} = \frac{1}{2d}\left\{\int_0^d [\varepsilon_0\varepsilon_\parallel(z)E_1^{pe} + \alpha_{11}(z)H_1^{pe}]dz + \int_d^{2d} \varepsilon_0\varepsilon_\parallel(z)E_1^{pm}dz\right\} = \varepsilon_0\varepsilon_\parallel E_1 + Ae_{15}q_{15}H_1,$$

$$\overline{B_1} = \frac{1}{2d}\left\{\int_0^d \mu_0\mu_\parallel(z)H_1^{pe}dz + \int_d^{2d} [\mu_0\mu_\parallel(z)H_1^{pm} + \beta_{11}(z)E_1^{pm}]dz\right\} = \mu_0\mu_\parallel H_1 + Ae_{15}q_{15}E_1, \quad (7)$$

where

$$\varepsilon_\parallel = (\varepsilon_{11}^{s,pe} + \varepsilon_{11}^{s,pm})/2 - Ae_{15}^2/\varepsilon_0,$$

$$\mu_\parallel = (\mu_{11}^{s,pe} + \mu_{11}^{s,pm})/2 - Aq_{15}^2/\mu_0,$$

$$A = 2/[d^2\rho(\omega^2 - \omega_S^2 + i\omega\gamma_s)]. \quad (8)$$

Here, $\omega_S=G_1v_S$ and $v_S=\sqrt{c_{44}/\rho}$ are the fundamental resonant frequency and the effective velocity of the transverse acoustic wave in the PPS; $\gamma_S=(\eta_{44}/c_{44})\omega_S^2$ is the relevant damping constant. After this, we can easily get the expressions of $\overline{D_2}$ and $\overline{B_2}$ for the transverse isotropy.

Similarly, the averaged values of the z components can be obtained. The dynamic equation for longitudinal vibration S_3 and its solution are

$$\rho\frac{\partial^2 S_3}{\partial t^2} - C_{33}\frac{\partial^2 S_3}{\partial z^2} = \frac{\partial^2}{\partial z^2}[-e_{33}f(z)E_3(z) - q_{33}g(z)H_3(z)],$$

$$S_3(z) = \sum_n \frac{G_n^2}{\rho\omega^2 - C_{33}G_n^2}e^{iG_nz}[-e_{33}f_nE_3(z) - q_{33}g_nH_3(z)]. \quad (9)$$

where $E_3(z)$ and $H_3(z)$ are the z components of EM fields, and ρ and C_{33} are the effective mass density and the effective elastic tensor element, respectively. Substitution of the expression of $S_3(z)$ into Eq. (1) gives

$$D_3(z) = \varepsilon_0[\varepsilon_{33}^s(z) - be_{33}^2f(z)f_n/\varepsilon_0]E_3(z) - be_{33}q_{33}f(z)g_nH_3(z),$$

$$B_3(z) = \mu_0[\mu_{33}^s(z) - bq_{33}^2g(z)g_n/\mu_0]H_3(z) - be_{33}q_{33}g(z)f_nE_3(z), \quad (10)$$

where

$$b = \sum_n \frac{G_n^2}{\rho\omega^2 - C_{33}G_n^2}e^{iG_nz}. \quad (11)$$

If the frequency is close to the fundamental resonant frequency of longitudinal vibration S_3, we can get $b = e^{iG_1z}G_1^2/(\rho\omega^2 - C_{33}G_1^2)$ by ignoring the other high-order reciprocal vectors. In PE and PM layers, Eq. (10) can be expressed, respectively, as

$$D_3^{pe} = \frac{1}{d}\int_0^d [\varepsilon_0(\varepsilon_{33}^{pe} - be_{33}^2f_1/\varepsilon_0)E_3^{pe} - be_{33}q_{33}g_1H_3^{pe}]dz,$$

$$B_3^{pe} = \frac{1}{d}\int_0^d \mu_0\mu_{33}^{pe}H_3^{pe}dz,$$

$$D_3^{pm} = \frac{1}{d}\int_d^{2d} \varepsilon_0\varepsilon_{33}^{pm}E_3^{pm}dz,$$

$$B_3^{pm} = \frac{1}{d}\int_d^{2d} [\mu_0(\mu_{33}^{pm} - bq_{33}^2g_1/\mu_0)H_3^{pm} - bq_{33}e_{33}f_1E_3^{pm}]dz. \quad (12)$$

According to the boundary conditions, we have

$$D_3^{pe} = D_3^{pm} = \overline{D_3},$$

$$B_3^{pe} = B_3^{pm} = \overline{B_3},$$

$$\overline{E_3} = \frac{1}{2d}\left[\int_0^d E_3^{pe}dz + \int_d^{2d} E_3^{pm}dz\right],$$

$$\overline{H_3} = \frac{1}{2d}\left[\int_0^d H_3^{pe}dz + \int_d^{2d} H_3^{pm}dz\right]. \quad (13)$$

Solving the combined equations composed of Eqs. (12) and (13), we can get

$$\overline{D_3} = \varepsilon_0\varepsilon_\perp\overline{E_3} + \alpha_{33}\overline{H_3},$$

$$\overline{B_3} = \mu_0\mu_\perp\overline{H_3} + \beta_{33}\overline{E_3}, \quad (14)$$

where

$$\varepsilon_\perp = 2\varepsilon_{33}^{s,pm}(\varepsilon_0\varepsilon_{33}^{s,pe} - 2Be_{33}^2)(\mu_0\mu_{33}^{s,pe} + \mu_0\mu_{33}^{s,pm} - 2Bq_{33}^2)/C,$$

$$\alpha_{33} = 2\varepsilon_0\varepsilon_{33}^{s,pm}(\mu_0\mu_{33}^{s,pm} - 2Bq_{33}^2)Be_{33}q_{33}/C,$$

$$\mu_\perp = 2\mu_{33}^{s,pe}(\mu_0\mu_{33}^{s,pm} - 2Bq_{33}^2)(\varepsilon_0\varepsilon_{33}^{s,pe} + \varepsilon_0\varepsilon_{33}^{s,pm} - 2Be_{33}^2)/C,$$

$$\beta_{33} = 2\mu_0\mu_{33}^{s,pe}(\varepsilon_0\varepsilon_{33}^{s,pe} - 2Be_{33}^2)Be_{33}q_{33}/C,$$

$$B = 2/[d^2\rho(\omega^2 - \omega_L^2 + i\gamma_L\omega_L)],$$

$$C = (\varepsilon_0\varepsilon_{33}^{s,pe} + \varepsilon_0\varepsilon_{33}^{s,pm} - 2Be_{33}^2)(\mu_0\mu_{33}^{s,pe} + \mu_0\mu_{33}^{s,pm} - 2Bq_{33}^2) - 4B^2e_{33}^2q_{33}^2. \quad (15)$$

Here, $\omega_L=G_1v_L$ and $v_L=\sqrt{C_{33}/\rho}$ are the fundamental resonant frequency and the effective velocity of the longitudinal acoustic wave in the PPS; $\gamma_L=(\eta_{33}/c_{33})\omega_L^2$ is the relevant longitudinal damping constant. If the top labels are omitted, the constitutive equation for the effective medium can be described as

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_0\varepsilon_\parallel & 0 & 0 & Ae_{15}q_{15} & 0 & 0 \\ 0 & \varepsilon_0\varepsilon_\parallel & 0 & 0 & Ae_{15}q_{15} & 0 \\ 0 & 0 & \varepsilon_0\varepsilon_\perp & 0 & 0 & \alpha_{33} \\ Ae_{15}q_{15} & 0 & 0 & \mu_0\mu_\parallel & 0 & 0 \\ 0 & Ae_{15}q_{15} & 0 & 0 & \mu_0\mu_\parallel & 0 \\ 0 & 0 & \beta_{33} & 0 & 0 & \mu_0\mu_\perp \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ H_1 \\ H_2 \\ H_3 \end{bmatrix}. \quad (16)$$

From Eq. (16), we can find that a PPS is another kind of structure-induced bianisotropic medium,[10] in which both polarization and magnetization linearly depend on the magnetic field and the electric field, even though neither of the two layers has ME effect. The ME effect in the PPS has been obviously constructed by the fact that a PE layer is stressed in an electric field, which, in turn, induces the magnetization of the adjacent PM layer by PM effect. Similarly, the polarization in PE layers is also related to the magnetization in PM layers driven by the magnetic field. In other words, the mechanical strain and stress bridge the coupling between the electric field E and the magnetic field H in the PPS. The scenario of the ME effect in the PPS is *electric-field–stress–magnetization and magnetic-field–stress–polarization*.

III. PROPAGATION ALONG THE Z DIRECTION

In general, the propagation of EM waves in a bianisotropic medium is complicated. Here, we just discuss some special cases. First, we assume that the incident light is propagating along the z axis and polarized in the y direction [s polarization (E perpendicular to x-z plane)]. Due to ME effect, p polarization (E parallel to the x-z plane) will be excited by the s-polarized incidence, as shown in Fig. 1(a). From Eqs. (1) and (2) and the transverse isotropy, we can find that a transverse superlattice vibration S_4 oscillating in the y direction will be excited and synchronously induce both the transverse electric polarization P_2 due to PE effect and the transverse magnetic polarization M_2 due to PM effect. Through Maxwell's equations, the two transverse polarizations will, in turn, emit orthogonally polarized EM waves. Both of them interfere with the original EM waves. In fact, the same process will be induced by S_5, which is oscillating in the x direction. This is to say that PE and PM phonon polaritons could be excited, similar to the cases described, respectively, in Refs. 6 and 7. In addition, the transverse vibrations would bridge the coupling between the two polaritons. Corresponding changes would appear in the polariton dispersion.

To get the polariton dispersion, we substitute Eq. (16) into Maxwell's equations. Thus, we can obtain

$$\frac{\partial E_1}{\partial z} - i\omega\mu_0\mu_\parallel H_2 - iA\omega e_{15}q_{15}E_2 = 0,$$

$$\frac{\partial E_2}{\partial z} + i\omega\mu_0\mu_\parallel H_1 + iA\omega e_{15}q_{15}E_1 = 0,$$

$$\frac{\partial H_1}{\partial z} + i\omega\varepsilon_0\varepsilon_\parallel E_2 + iA\omega e_{15}q_{15}H_2 = 0,$$

$$\frac{\partial H_2}{\partial z} - i\omega\varepsilon_0\varepsilon_\parallel E_1 - iA\omega e_{15}q_{15}H_1 = 0. \quad (17)$$

From Eq. (17), a dispersion relation can be obtained:

$$k^2/\omega^2 = \varepsilon_0\mu_0\varepsilon_\parallel\mu_\parallel - A^2e_{15}^2q_{15}^2. \quad (18)$$

The dispersion relation shown in Eq. (18) is a normal phonon polariton[2] with the modulation from the coupling between PE and PM polaritons (the second term of the right side).

A similar model of a PPS was proposed to realize double negative permittivity and permeability without considering elastic damping and ME effect.[11] Due to the neglect of ME effect, PE and PM polaritons would not couple with each other, and accordingly the modulation term would disappear in the case of Ref. 11. In Ref. 11, the excited PE and PM polaritons will lead to individual band gaps near transverse resonant frequency ω_S. Double negativity can, in principle, be obtained in the overlapped band gap and result in a propagating state, which might be the situation of negative refraction as predicted firstly by Veselago[12] and has recently attracted intensive attention. However, for a real heterogeneous PPS, a proper damping constant γ_S and ME effect must be considered, which drastically modify the characters of the EM waves, as described in the following.

In order to have a better understanding of the above theoretical results, we perform a realistic numerical computation for a PPS with ∞m symmetry, such as a $BaTiO_3$-$CoFe_2O_4$ superlattice.[13] In the calculations of this paper, a proper damping constant $\gamma_S=0.02\omega_S$ is chosen. Each layer in this PPS has a length of $d=500$ nm. So, the transverse resonant frequency ω_S is calculated at 18.0 GHz according to Eq. (8). The abnormal dispersions of permittivity and permeability, as functions of normalized frequency (ω/ω_S), are shown in Figs. 1(b) and 1(c). The real part of permeability keeps positive, and the imaginary parts of permittivity and permeability are comparable to their real parts. Moreover, the coupling between the parallel electric and magnetic fields bridged by the transverse vibration gives birth to ME effect. Due to ME effect, PE and PM polaritons will further couple with each other, which leads to a new coupled polariton. The polariton dispersion relations are shown in Eq. (18) and Fig. 1(d). Near ω_S, the imaginary part of the wave vector is large and no propagating mode exists.

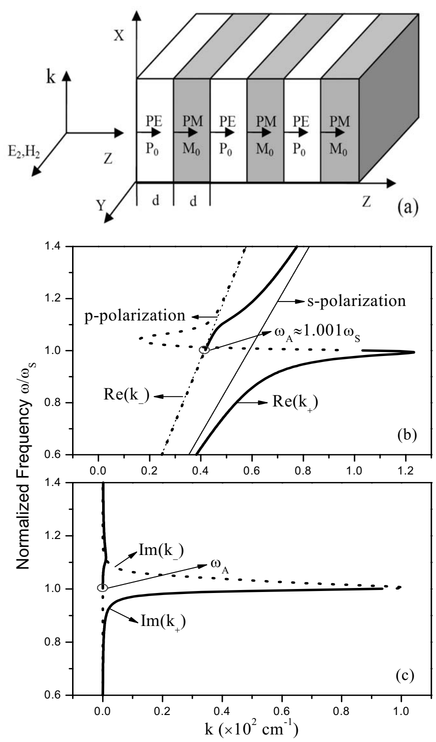

FIG. 2. (a) Schematic of the two orthogonally polarized EM waves along the x axis and the first-order polariton dispersion curves, and (b) the real part and (c) the imaginary part of the wave vector k. The coupling between EM waves and the vibration S_4 results in k_+ and k_- modes that are degenerating into s and p polarizations, respectively, when the frequency is away from ω_A.

The ME coupling term shown in Eq. (18) dramatically changes the inherent characters of original PE and PM polaritons, even though their band gaps are still kept. Hence, it is hard to realize negative refraction via this approach, due to the huge loss and the intense ME coupling near ω_S.

IV. PROPAGATION ALONG THE X DIRECTION

In addition to the coupled polariton, as we described above, that is induced by EM waves propagating along the z direction of the PPS, the case for the propagation of EM wave along the x direction has also been investigated. We assume that the incident EM wave is polarized in the y direction and its frequency is near the transverse resonant frequency, far from the longitudinal one. Thus, the ME effect in the z direction can be negligible. Due to the ME effect in the y direction, p polarization (E parallel to the x-z plane) will be excited by the incident s polarization (E perpendicular to the x-z plane), as shown in Fig. 2(a). Similar to the analysis in Sec. III, both two orthogonally polarized EM waves can contribute to the excitation of S_4. In other words, the orthogonally polarized EM waves will couple with the superlattice vibration S_4, which results in PE and PM phonon polaritons. The two polaritons then couple with each other due to ME effect via S_4, leading to the corresponding coupled polaritons.

Substituting our constitutive equation into Maxwell's equations, we can get

$$-\frac{\partial E_3}{\partial x} + i\omega\mu_0\mu_{\parallel}H_2 + iA\omega e_{15}q_{15}E_2 = 0,$$

$$\frac{\partial E_2}{\partial x} + i\omega\mu_0\mu_{\perp}H_3 = 0,$$

$$\frac{\partial H_3}{\partial x} + i\omega\varepsilon_0\varepsilon_{\parallel}E_2 + iA\omega e_{15}q_{15}H_2 = 0,$$

$$\frac{\partial H_2}{\partial x} - i\omega\varepsilon_0\varepsilon_{\perp}E_3 = 0. \tag{19}$$

From Eq. (19), we obtain the following polariton dispersion relations:

$$2c_0^2k_{\pm}^2/\omega^2 = M_{\pm} = (\varepsilon_{\parallel}\mu_{\perp} + \varepsilon_{\perp}\mu_{\parallel}) \pm \sqrt{(\varepsilon_{\parallel}\mu_{\perp} - \varepsilon_{\perp}\mu_{\parallel})^2 + 4A^2 e_{15}{}^2 q_{15}{}^2 \varepsilon_{\perp}\mu_{\perp}/\varepsilon_0\mu_0}. \tag{20}$$

Here, c_0 is the phase velocity of EM waves in free space. The transverse permittivity and permeability have abnormal dispersion relations, shown in Figs. 1(b) and 1(c); while the longitudinal ones keep constant because the longitudinal ME effect is ignored. The imaginary and real parts of the first-order phonon polariton dispersions are plotted in Figs. 2(b) and 2(c), respectively. The propagations of p polarization and s polarization are illustrated in Fig. 2(b) too, which can be obtained according to our constitutive equations by setting all the elements of PE and PM tensors with the values of 0. It should be noticed that near ω_S, there is an exchange point (see Fig. 2) in the dispersion curves of the first-order polaritons, different from the conventional polaritons.[1] The second term of $M_{\pm}$ can also be expressed as $\pm\sqrt{D'(\omega)+iD''(\omega)}$. If $D'(\omega)$ is kept positive, the discontinuity of the wave vector would appear when $D''(\omega)$ changes its sign. The exchange point is set when $D''(\omega_A)=0$. The first dispersion, named k_+ mode, has the large imaginary values but small values above ω_A. The second one named k_- mode, however, has correspondingly small and large imaginary values. When the frequency departs away from ω_A, k_+ and k_- modes degenerate into s polarization and p polarization, respectively. We can draw a conclusion that below ω_A, k_- mode is supported and k_+ mode is forbidden, while vice versa above ω_A. Such analysis of dispersion relations reveals that, different from the case in the pure PE or PM superlat-

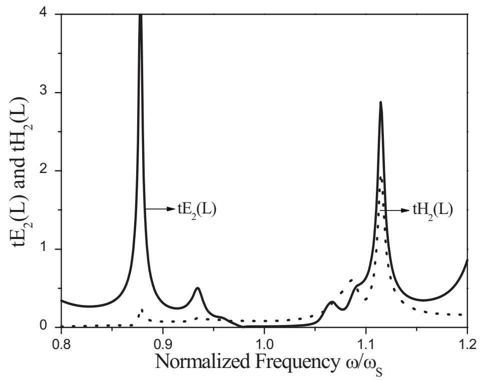

FIG. 3. Variations of the parallel electric and magnetic fields in the y direction. A combination between the two fields occurs and the energy is transferred between electric and magnetic energies bridged by the vibration S_4.

tice, EM waves can travel through the PPS along the x axis even in the original band gap, just by cleverly changing its propagating mode at the exchange point ω_A.

To further investigate the characteristics of the PPS, a quantitative analysis of EM spectra has been calculated. We assume that the PPS has a length of $L=5000d=2.5$ mm in the x direction and is located in the free space. Using the dispersion relations and Maxell's equations, we find that the EM fields in the medium satisfy the following rules:

$$E_2(x) = ae^{-ik_+x} + ce^{ik_+x} + be^{-ik_-x} + de^{ik_-x},$$

$$H_3(x) = (ae^{-ik_+x} - ce^{ik_+x})/Z_+^s + (be^{-ik_-x} - de^{ik_-x})/Z_-^s,$$

$$E_3(x) = P_+(ae^{-ik_+x} - ce^{ik_+x}) + P_-(be^{-ik_-x} - de^{ik_-x}),$$

$$H_2(x) = P_+(ae^{-ik_+x} + ce^{ik_+x})/Z_+^p + P_-(be^{-ik_-x} + de^{ik_-x})/Z_-^p, \tag{21}$$

where

$$P_\pm = \frac{\sqrt{M_\pm/2}(\varepsilon_\parallel\mu_\perp - M_\pm/2)}{Ae_{15}q_{15}\varepsilon_\perp\mu_\perp c_0},$$

$$Z_\pm^s = \sqrt{\frac{2\mu_0\mu_\perp^2}{M_\pm\varepsilon_0}}, \quad Z_\pm^p = \mp\sqrt{\frac{M_\pm\mu_0}{2\varepsilon_0\varepsilon_\perp^2}}. \tag{22}$$

In addition, we set the following boundary conditions:

$$E_2^I(0) = E_{20}, \quad E_3^I(0) = 0,$$

$$E_2^R(0) = e, \quad E_3^R(0) = f,$$

$$E_2^T(L) = g, \quad E_3^T(L) = h, \tag{23}$$

where the superscripts I, R, and T represent the physical quantities of the incidence, reflection, and transmission, respectively. In the expressions of EM fields, time harmonic factor $e^{i\omega t}$ is omitted. We can get eight equations including eight variations, according to the boundary conditions of the continuity of transverse electric and magnetic fields. The eight variations from a to h are first referred in Eqs. (21) and (23). By solving the eight equations, we can obtain eight expressions from a to h, respectively.

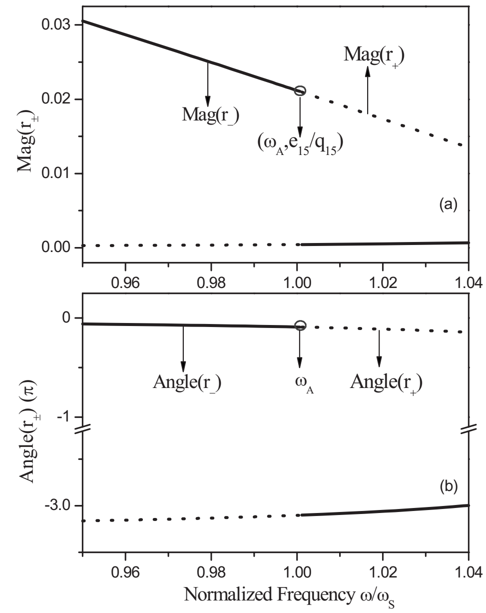

FIG. 4. The ratios of magnetic field to electric field in the y direction. (a) and (b) show the magnitudes and the phase angles of the ratios, respectively. Considering the dispersion curves, it is obvious that at resonance the magnitude and the phase angle of the supported mode have the values of about e_{15}/q_{15} and 0, respectively. With this given ratios of magnetic field to electric field in the y direction, the vibration is weak and the total dissipation is low.

Now we can make a numerical analysis of the EM fields inside and outside PPS. We define the transmission of the EM fields as $tE_2(L)=|E_2(L)/E_{20}|$ and $tH_2(L)=|\sqrt{\mu_0}H_2(L)/(\sqrt{\varepsilon_0}E_{20})|=|E_3(L)/E_{20}|$. Their dependences on normalized frequency are shown in Fig. 3. One can find that at resonance, a very large conversion from the electric field to the magnetic field in the y direction occurs, though the efficiency is not quite high. Seen from Fig. 2(c), if there is a gap for EM waves, the imaginary value of the wave vector is about 100 cm^{-1} near the resonant frequency. For our PPS with the length $L=2.5$ mm, the transmission index should be about e^{-25} that is much less than $tH_2(L)$ of 0.08 shown in Fig. 3. Hence, we can get a conclusion that EM waves can certainly travel through the PPS even in the original forbidden gap, just by wisely changing its propagating mode at the exchange point ω_A.

All above indicate that the coupling between polaritons, induced by PE and PM effects, makes it possible for EM waves to break the ban on the propagation near the resonant frequency. However, the results are quite different for the propagations along the x and z directions. Taking transverse isotropy into account and neglecting the higher-order reciprocal vectors, we can reduce Eq. (4) to $S_4 \propto (-e_{15}E_2+q_{15}H_2)$. For the PE superlattice[4–6] with $q_{15}=0$, the magnitude of this vibration is proportional to the electric field. For the PM superlattice[7] with $e_{15}=0$, it is proportional to the magnetic field. For the PPS, however, it is proportional to the linear combination of electric and magnetic fields. In the PE or the PM superlattice, intense reflective waves are excited to cut the vibration magnitude down, which results in band gaps. However, in the PPS, a route has been proposed above that EM waves can propagate near the resonant frequency by diminishing the magnitude of vibration S_4 with a given combination of electric and magnetic fields.

To see this clearly, we can define the ratios of the magnetic field to the electric field propagating in the y direction with different modes as $r_{\pm}(x)=P_{\pm}/Z_{\pm}^{TM}$, according to Eq. (21). We randomly choose a position far from the incident position along the x axis, such as $x=0.5L$. The ratios at this position are calculated, and their magnitudes and phase angles are plotted in Figs. 4(a) and 4(b). It should be noticed that the identical exchange point ω_A appears again. Near this point, for the propagating modes, the magnitudes of the ratios are about e_{15}/q_{15}, and the phase angles are about 0; for the forbidden modes, they are about 10^{-4} and 3π, respectively. In this way, the vibration in the y direction S_4 is definitely weakened. This is to say that the EM waves propagating along the x axis can travel through the PPS with less dissipation even at resonant frequency. Unfortunately, when EM waves are propagating along the z axis, although one of the two degenerate transverse vibrations, S_4 and S_5, is weakened while the other is enhanced. So, the gap still exists, even though the coupling does occur. Generally, in a PPS, intense coupling between EM waves and superlattice vibrations does not always mean intense vibrations.

V. CONCLUSION

To summarize, we have demonstrated clearly how ME effect occurs naturally from PE and PM effects through their mechanical strain in a PPS and deduced the constitutive equation for this effective medium. With the constitutive equation and Maxwell's equations, we have studied the propagation of EM waves in a PPS. It turns out that the superlattice vibrations can couple synchronously with the electric field and the magnetic field of EM waves, which results in coupled phonon polaritons. Because of the coupling between phonon polaritons, we find that the propagation of EM waves is remodeled, especially near the resonant frequency: The propagation along the x axis is permitted, while it is forbidden along the z axis due to the degenerate transverse phonons. The coupling in a PPS presents an approach for EM waves to break the ban on the propagation near the resonant frequency, exhibiting rich physics in artificial microstructures. The properties can give birth to some practical applications, such as for wave division multiplexing[16] devices in optical communications and for microwave absorption.

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