Universal interacting crossover regime in two-dimensional quantum dots

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(Received 18 December 2007; published 21 February 2008)

Interacting electrons in quantum dots with large Thouless number g in the three classical random matrix symmetry classes are well understood. When a specific type of spin-orbit coupling known to be dominant in two-dimensional semiconductor quantum dots is introduced, we show that an interacting quantum critical crossover energy scale emerges and low-energy quasiparticles generically have a decay width proportional to their energy. The low-energy physics of this system is an example of a universal interacting crossover regime.

DOI: 10.1103/PhysRevB.77.073309 PACS number(s): 73.23.−b, 05.40.−a, 73.50.Jt

The statistics of the single-particle states of mesoscopic systems with disorder or chaotic boundary scattering[1,2] are controlled by random matrix theory (RMT),[3] as long as the states are separated by less than the Thouless energy E_T (related to the ergodicization time for a particle τ_{erg} by the uncertainty principle $E_T=\hbar/\tau_{erg}$). For mean single-particle level spacing δ, the Thouless number is $g=E_T/\delta$.

Since disorder breaks all the spatial symmetries, only time reversal $\mathcal{T}$ and, possibly, Kramers degeneracy remain. There are three classical symmetry classes,[3] the Gaussian orthogonal ensemble (GOE) ($\mathcal{T}$ intact, no spin-orbit coupling), the Gaussian unitary ensemble (GUE) ($\mathcal{T}$ broken), and the Gaussian symplectic ensemble or GSE ($\mathcal{T}$ intact, with spin-orbit coupling). More recently, other classes have been identified for disordered superconductors[4] and quantum dots constructed from two-dimensional semiconductor heterostructures with spin-orbit coupling.[5] We will focus on a symmetry class in the latter case [which we call the Aleiner-Falko (AF) class[5]] in which, after a canonical transformation, the single-particle S_z is conserved, while $\mathbf{S}^2$ is not.[5]

The idea of a crossover between two symmetry classes will play a central role in this paper. Consider a system in which the Hamiltonian is crossing over from the GOE to the GUE,[3] acheived, e.g., by turning on the orbital effects of a magnetic field. For Thouless number $g \gg 1$ of the original GOE, the $g\times g$ crossover Hamiltonian is

$$H_X(\alpha) = H_{GOE} + \frac{\alpha}{\sqrt{g}} H_{GUE}, \tag{1}$$

where α is the crossover parameter. Properties of eigenvector[6,7] correlations have been computed in the crossover. For $g \gg 2\alpha^2 = g_X \gg 1$, the following ensemble-averaged correlations hold for the eigenstates $\psi_\mu(i)$, where $\mu \neq \nu$ labels the states and i,j,k,l the original orthogonal labels,

$$\langle \psi^*_\mu(i)\psi_\nu(j)\rangle = \frac{1}{g}\delta_{\mu\nu}\delta_{ij},$$

$$\langle \psi^*_\mu(i)\psi^*_\nu(j)\psi_\mu(k)\psi_\nu(l)\rangle = \frac{\delta_{ik}\delta_{jl}}{g^2} + \frac{\delta_{ij}\delta_{kl}}{g^2}\frac{E_X\delta/\pi}{E_{X^2}+(\epsilon_\mu-\epsilon_\nu)^2}. \tag{2}$$

The last term on the second line shows the extra correlations induced in the crossover.[7] The crossover scale $E_X=g_X\delta/\pi$ represents a window within which GUE correlations have spread, while GOE correlations remain at high energies.

The crossover from the spin-rotation-invariant GOE to the AF class is a GOE→GUE crossover, where the "magnetic flux" has opposite signs for opposite eigenvalues of S_z.[5] If the linear size of the system is L and the spin-orbit scattering length is $\xi \gg L$, this new AF symmetry class manifests itself below $E_X \simeq (\frac{L}{\xi})^4 E_T$. The crossover to the fully symplectic GSE occurs[5] at the parametrically smaller energy scale of $(\frac{L}{\xi})^6 E_T$, set here to 0.

Turning from single-particle physics to interactions, for small to moderate r_s, the "universal Hamiltonian"[8,9] is known to contain all the relevant couplings[10] at low energies in the renormalization group sense.[11]

$$H_U = \sum_{\alpha,s} \epsilon_\alpha c^\dagger_{\alpha,s}c_{\alpha,s} + \frac{U_0}{2}\hat{N}^2 - J\mathbf{S}^2 + \lambda T^\dagger T. \tag{3}$$

Here, $\hat{N}$ is the total particle number, $\mathbf{S}$ is the total spin, and $T=\Sigma c_{\beta,\downarrow}c_{\beta,\uparrow}$. H_U has a charging energy U, an exchange energy J, and a superconducting coupling λ. This last term is absent in the GUE, while the exchange term disappears in the GSE. In the large-g limit, only interaction terms which are invariant under the symmetries of the one-body Hamiltonian appear in H_U.[8,9] At larger r_s, the system enters a quantum critical regime[12] connected with the impending Pomranchuk transition.[10] The zero-dimensional quantum critical point associated with this transition (for an odd Landau channel) is related to the Caldiera-Leggett model.[13]

We will consider isolated dots with a conserved particle number and will ignore the charging term in what follows. In the GOE, ignoring the Cooper coupling (two-dimensional semiconductor quantum dots do not superconduct) and tuning J, one sees the mesoscopic Stoner effect.[8,9] Take for illustration an evenly spaced set of levels with level spacing δ. Since both $\mathbf{S}^2$ and S_z are conserved, we can focus on the ground state with $S_z=S$ and find its energy for an even number of particles to be $E_{gs}(S)=S^2\delta - JS(S+1)$. Defining $\tilde{J}=J/\delta$ and minimizing with respect to S leads to steps from $S=0$ to $S=1$ at $\tilde{J}=\frac{1}{2}$, from $S=1$ to $S=2$ at $\tilde{J}=\frac{3}{4}$, etc. Including the mesoscopic (sample-to-sample) fluctuations of the energies and the matrix elements leads to probability distributions for spin S.[8,9,14] Note that the $S_z=S$ ground state is electronically uncorrelated (a single Slater determinant, not a superposition).

What happens when spin-orbit coupling of the AF class is introduced into a slightly generalized form of H_U?[15]

$$H=\sum \epsilon_{\mu s} c^{\dagger}_{\mu s} c_{\mu s} - J_z S_z^2 - J(S_x^2+S_y^2). \quad (4)$$

Here, the basis μ labels the eigenbasis of the single-particle AF crossover Hamiltonian[5] and s is the eigenvalue of $S_z = \frac{\hbar}{2}\sum c^{\dagger}_{\mu\uparrow}c_{\mu\uparrow} - c^{\dagger}_{\mu\downarrow}c_{\mu\downarrow}$. $S_{x,y}$ are expressed[15] in terms of the combinations $S_{\pm}=S_x \pm iS_y$ as

$$S_+ = \sum_{\alpha\beta} M_{\alpha\beta} c^{\dagger}_{\alpha\uparrow} c_{\beta\downarrow} \quad (5)$$

with $S_-=(S_+)^{\dagger}$. The matrix $M_{\alpha\beta}$ depends on the particular realization and has the ensemble average[15]

$$\langle |M_{\alpha\beta}|^2\rangle = \frac{E_X\delta/\pi}{E_X^2+(\epsilon_{\alpha\uparrow}-\epsilon_{\beta\downarrow})^2}. \quad (6)$$

At strong spin-orbit coupling,[8,9,15] noting that $-J(S_x^2+S_y^2)$ is irrelevant in the renormalization group (RG) sense,[10,16] we end up with the Hamiltonian of Eq. (4) with $J=0$. The ground and excited states of this Hamiltonian have definite S_z and are electronically uncorrelated.

We are now ready to state our central results. In the crossover to the AF class, a quantum critical regime[12] emerges at a many-body quantum critical crossover scale $E_{QCX}=(1-\tilde{J})E_X$ (a result the author obtained previously[16] in the restricted case $J_z=0$). In contrast to the limits of zero and strong spin-orbit coupling, in the crossover the ground and low-lying states are electronically strongly correlated. Transverse spin fluctuations are controlled by a new zero-dimensional quantum critical point and have a nonzero density at low energies (as long as the ground state $S_z\neq 0$). Under the same conditions, fermionic quasiparticles become very broad at low energy. Finally, the mesoscopic Stoner effect is smoothly pushed to higher J as spin-orbit coupling increases.

We will set $J_z=J$ in Eq. (4) henceforth since that is the correct starting point for a Hamiltonian deep in the Thouless band.[10] We decompose both the interaction terms by introducing the Hubbard-Stratanovich fields $h(t)$, $q(t)=X(t)+iY(t)$, and $q^*(t)$ to get the $T=0$ imaginary time action,

$$A=\int_{-\infty}^{\infty} dt \sum_{\alpha,ss'} \bar{\psi}_{\alpha s}\left[(\partial_t+\epsilon_\alpha)\delta_{ss'} - \frac{h}{2}(\sigma_z)_{ss'}\right]\psi_{\alpha s'} + \frac{h^2+qq^*}{4J} - \frac{q(t)}{2}\sum_{\alpha\beta} M_{\alpha\beta}\bar{\psi}_{\alpha\uparrow}\psi_{\beta\downarrow} - \frac{q^*(t)}{2}\sum_{\alpha\beta} M^*_{\alpha\beta}\bar{\psi}_{\beta\downarrow}\psi_{\alpha\uparrow}. \quad (7)$$

$X(t)$ and $Y(t)$ are fluctuating fields and are integrated out, but $h(t)$ acquires an expectation value (also called h). For our picket fence spectrum $\epsilon_n=\delta(n-\frac{1}{2})$ with chemical potential at 0, at $h=0$ all states $n\leqslant 0$ are occupied while all states $n\geqslant 1$ are empty. When h lies between $h_n=\delta(2n-1)$ and $h_{n+1}=\delta(2n+1)$, at $T=0$ the single-particle states between $-n+1$ and n are singly occupied, and $S_z=n\equiv\mathcal{S}$. We will find saddle points for h in each of the intervals $h_n<h<h_{n+1}$, thereby obtaining the ground state energy as a function of $\mathcal{S}=n$, and look for the lowest one.

First, integrate out the fermion fields and obtain a quadratic effective action for h, X, and Y,

$$A_{eff}=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{|h(\omega)|^2}{4J}+\frac{|X(\omega)|^2+Y(\omega)|^2}{4J}[1-J\chi_R(i\omega)] + \frac{X(\omega)Y(-\omega)-Y(\omega)X(-\omega)}{4J}J\chi_I(i\omega). \quad (8)$$

The cross terms are a consequence of $[S_x,S_y]=iS_z$, and χ_R and χ_I are the real and imaginary parts of χ, the fermionic transverse spin susceptibility,

$$\chi(i\omega)=\sum_{mn}|M_{mn}|^2\frac{N_F(\epsilon_{m\uparrow})-N_F(\epsilon_{n\downarrow})}{i\omega+\epsilon_{n\downarrow}-\epsilon_{m\uparrow}}, \quad (9)$$

where N_F is the Fermi occupation. To make further progress, we assume that $E_X\gg\delta$, which allows us to convert the sums over states into integrals, and also replace the sample-specific value of $|M_{mn}|^2$ by its ensemble average. Such self-averaging occurs naturally in the large-N limit.[10] The dominant contribution to χ is

$$\chi(i\omega)=\frac{1}{\delta}\frac{E_X-iE_{\mathcal{S}}\,\mathrm{sgn}(\omega)}{|\omega|+E_X-ih\,\mathrm{sgn}(\omega)}, \quad (10)$$

where $E_{\mathcal{S}}=2\mathcal{S}\delta$ and $\mathrm{sgn}(\omega)$ is the sign of ω. Next, the integration over X and Y results in a fluctuation contribution to the effective action,

$$A_{fluc}=\int_{-\infty}^{\infty}\frac{d\omega}{4\pi}\log\left(\frac{(|\omega|+E_{QCX})^2+(h-\tilde{J}E_{\mathcal{S}})^2}{(|\omega|+E_X)^2+h^2}\right), \quad (11)$$

where the argument of the logarithm is the determinant of the matrix of the quadratic form of X and Y. A_{fluc} is logarithmically divergent due to a limitation of the ensemble averages Eqs. (2) and (6) for large energy separations $E_X>E_T$. Cutting it off, discarding terms of the form $\log(E_T/E_X)$, which are independent of $\mathcal{S}$, defining $\Delta h=h-\tilde{J}E_{\mathcal{S}}$, and adding the one-body energy, we find

$$A_{eff}(\mathcal{S},h)=\mathcal{S}^2\delta+\frac{h^2}{4J}-h\mathcal{S}+\frac{1}{\pi}\left(\Delta h\tan^{-1}\frac{\Delta h}{E_{QCX}}-h\tan^{-1}\frac{h}{E_X}+\frac{E_X}{2}\log\left(1+\frac{h^2}{E_X^2}\right)-\frac{E_{QCX}}{2}\log\left(1+\frac{(\Delta h)^2}{E_{QCX}^2}\right)\right). \quad (12)$$

Equation (12) is one of the central results of this paper. Note that the term in the brackets is of order $1/\mathcal{S}$ compared to the first three terms (h will turn out to be of order $\mathcal{S}$). From it, we find that the saddle point value of h_0 satisfies

$$h_0=\tilde{J}E_S+\frac{2J}{\pi}\left(\tan^{-1}\frac{\Delta h_0}{E_{QCX}}-\tan^{-1}\frac{h_0}{E_X}\right). \quad (13)$$

We must also ensure that $h_n<h_0<h_{n+1}$ for $\mathcal{S}=n$. Using $h=h_0$ in Eq. (12) gives the ground state energy for that $\mathcal{S}$. Even though this result has been derived for $E_X\gg\delta$ and $\mathcal{S}\gg 1$, note that one recovers the correct spin-rotation-invariant

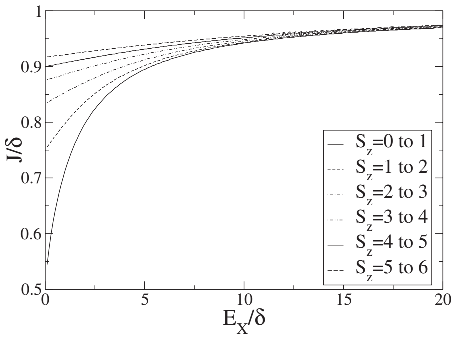

FIG. 1. Phase diagram in the E_X–$\tilde{J}$ plane showing the lines of ground state S_z transitions. Note the smooth approach to the spin-rotation-invariant result as $E_X \to 0$.

result for all $\mathcal{S}$ on taking the $E_X \to 0$ limit first in Eq. (12) and solving it to obtain the ground state energy. Figure 1 shows regions in the E_X, $\tilde{J}$ plane with different ground states $S_z = \mathcal{S}$. Note that the smooth approach to the spin-rotation-invariant results as $E_X \to 0$.

Now, consider the low-lying excitations, which are bosonic spin excitations and fermionic quasiparticles. The imaginary time transverse spin correlator can be used to find the spectral function for transverse spin excitations,

$$B(\omega) = -2\,\mathrm{Im}[D_{ret}(\omega)] = \frac{2}{\delta}\frac{\omega E_X + E_S E_{QCX}}{\omega^2 + E_{QCX}^2}. \tag{14}$$

For $\mathcal{S}=0$, this reproduces the scaling function computed earlier.[16] More importantly, for $\mathcal{S} \neq 0$, there is a nonzero density of spin excitations even as $\frac{\omega}{E_{QCX}} \ll 1$ (but $\omega \gg \delta$). These excitations are related to the $2\mathcal{S}+1$-degenerate ground states of the spin-rotation-invariant system (since $\mathbf{S}^2$ is not conserved here, the ground state contains a superposition of many values of $\mathbf{S}^2$). These low-energy excitations make the system strongly correlated in the electronic sense: that is, the ground and low-lying states are no longer described even approximately by single Slater determinants.

The decay of an electron of energy ε to leading order occurs via the emission of a single spin excitation. Using the Fermi golden rule, assuming a constant fermionic density of states $\rho(\varepsilon) = 1/\delta$, and using Eqs. (6) and (14), we obtain

$$\Gamma(\varepsilon) \simeq \frac{J^2 E_X}{\delta\pi(E_X^2 - E_{QCX}^2)}\left(\frac{E_X}{2}\left[\log\left(1+\frac{\varepsilon^2}{E_{QCX}^2}\right) - \log\left(1+\frac{\varepsilon^2}{E_X^2}\right)\right] + 4E_S E_{QCX}\left[\frac{\tan^{-1}\frac{\varepsilon}{E_{QCX}}}{E_{QCX}} - \frac{\tan^{-1}\frac{\varepsilon}{E_X}}{E_X}\right]\right). \tag{15}$$

This is the other central result of this paper. At low energies, $\varepsilon \ll E_{QCX}$, ρ, $|M|^2$, and B are constant, leading to a decay rate which goes as $\Gamma(\varepsilon) \simeq \frac{16 J^2 \mathcal{S}}{E_X E_{QCX}}\varepsilon$, *which can exceed* ε *for* $\tilde{J} \to 1$, *leading to ill-defined quasiparticles*. At high energies $\varepsilon \gg E_X$ (but $\varepsilon \ll E_T$), the decay rate due to spin excitations goes to a constant.[17] There will be an additional Fermi liquid broadening,[18] $\Gamma \simeq \varepsilon^2/E_T$ due to neglected interactions. Thus, we have an unusual situation in which the quasiparticles are better defined at high energies than at low energies because the high-energy physics is controlled by the weakly interacting spin-rotation-invariant H_U.[8,9]

In summary, we have presented a *universal interacting crossover regime* in two-dimensional (2D) semiconductor quantum dots, in which a RMT crossover[5] induces strong electronic correlations and, for nonzero ground state S_z, a quasiparticle decay width which can exceed the energy at low energies. Such regimes are the offspring of RMT crossovers and quantum critical phenomena and are quite distinct from new *single-particle* RMT ensembles.[3–5] The regime described here is universal in both the RMT[3] and quantum critical[12] senses. It is universal in the RMT sense because it applies for both ballistic, chaotic, and diffusive dots when all relevant energies are far below E_T. For energies $\delta \ll \varepsilon \ll E_{QCX}$ and $\tilde{J} \to 1$, the regime is universal in the quantum critical phenomena[12] sense because for a given J, E_X physical observables at energy ε [such as $\Gamma(\varepsilon)$] can be expressed as scaling functions (the interacting descendants of ensemble-averaged RMT averages in the RMT crossover[7]) of the scaling variable ε/E_{QCX}. This is a key property of observables in quantum critical phenomena.[12]

It is important to distinguish the quantum critical regimes and zero-dimensional quantum critical theory (0DQCT) from those found earlier. The Pomeranchuk transition[10] leads (for an odd Landau channel) to a 0DQCT in the Caldeira-Leggett class,[13] governed by a dissipative XY action with a twofold anisotropy. The $J_z=0$ model analyzed earlier,[16] besides being unphysical, leads to a dissipative XY action with no anisotropy and no conserved order parameter. The present model is physical and leads to a *different* 0DQCT with a conserved $S_z \neq 0$ and a Berry phase for the XY angle, which changes the low-energy physics. The crucial differences from Ref. 16 are the constant density of low-energy spin excitations and broad fermionic quasiparticles, both potentially measurable and exclusive to the model presented here.

Let us turn to practical issues. One issue is to tune the spin-orbit crossover scale *in situ*. This can be acheived by a vertically coupled geometry of two 2D quantum dots with tunable hopping between them,[20] with one of them fabricated of a material (such as InSb) with a large spin-orbit coupling. Charging effects can be ignored in this geometry.[20] The bigger issue is the small size of $\tilde{J} \approx 0.3$ (Ref. 19) in GaAs at current densities ($r_s \approx 1$). One option is to fabricate samples with $r_s \approx 5$, which would lead to $\tilde{J} \approx 0.5$,[19] and to take advantage of mesoscopic fluctuations,[8,9] which permit a large ground state S_z to occur occasionally. One might also gate diluted magnetic semiconductor films[21] with variable levels of magnetic metal doping to create quantum dots with variable $\tilde{J}$.

Many open questions remain, such as how to incorporate mesoscopic fluctuations,[8,9,14,15] how to characterize the state at and near the transition between S_z steps, how the state responds to an in-plane B field, whether the strong electronic correlation has any signatures in the zero-bias conductance, the question of finite temperature, and the classification of universal interacting crossover regimes into universality classes. The author hopes to explore these and other questions in future work.

It is a pleasure to thank R. Shankar, S. Sachdev, and especially Yoram Alhassid for comments on the manusript.

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