# Surface effects in magnetic superconductors with a spiral magnetic structure

Anatoly Fjodorovich Volkov
*Institute für Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
*and Institute for Radioengineering and Electronics of Russian Academy of Sciences, 11-7 Mokhovaya strasse, Moscow 125009, Russia*
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We consider a magnetic superconductor (MS) with a spiral magnetic structure. On the basis of generalized Eilenberger and Usadel equations, we show that near the boundary of the MS with an insulator (I) or vacuum, the condensate (Gor'kov's) Green's functions are disturbed by boundary conditions and differ essentially from their values in the bulk. Corrections to the bulk quasiclassical Green's functions oscillate with the period of the magnetic spiral, $2\pi/Q$, and decay inside the superconductor over a length of the order $v_F/2\pi T$ (ballistic limit) or $\sqrt{D/\pi T}$ (diffusive limit), where $Q$ is the wave vector of the magnetic spiral, $D=v_F l/3$ is the diffusion coefficient, $v_F$ is the Fermi velocity, and $l$ is the mean free path. We calculate the dc Josephson current in a MS/I/MS tunnel junction and show that the critical Josephson current differs substantially from that obtained with the help of the tunnel Hamiltonian method and bulk Green's functions.

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## I. INTRODUCTION

It is known that in some compounds, the superconducting order can coexist with a magnetic order of the ferromagnetic or antiferromagnetic type. For example, in ternary rare-earth (RE) compounds such as (RE)$Rh_4B_4$ and (RE)$Mo_6X_8$ ($X$ =S,Se), the superconducting and magnetic orderings coexist in a narrow temperature range (see Ref. 2, a review,[1] and a more recent paper[3] and references therein). In $ErRh_4B_4$, superconductivity takes place in the interval $0.7 \leqslant T \leqslant 0.8$ K, and the magnetic ordering arises below $T_m=0.8$ K. In $HoMo_6S_8$, the magnetic ordering occurs below $T_m=0.74$ K, whereas superconductivity exists in the temperature range $0.7 \leqslant T \leqslant 1.8$ K. Besides, the superconducting and magnetic orders are realized in the layered perovskite ruthenocuprate compound $RuSr_2GdCu_2O_8$.[3–7] In this compound, an antiferromagnetic order and, perhaps, a weak ferromagnetism take place.

A uniform magnetization is impossible in a bulk superconductor as the magnetic field destroys superconductivity. In order to explain the coexistence of ferromagnetism and superconductivity, Ginsburg[8] and later Anderson and Suhl[9] supposed that this coexistence is possible in the case of a domain or spiral magnetic structure. The period of the magnetic structure has been calculated in Ref. 9 (see also Ref. 12), and it is equal to $l_m \approx 2\pi(\xi_0 k_F)^{1/3}/k_F$, where $k_F$ is the Fermi momentum and $\xi_0=v_F/\pi\Delta_0$ is the correlation length in a clean superconductor. For example, in $HoMo_6S_8$, the wave vector of the periodic magnetic structure $Q \approx 0.03$ A$^{-1}$ (see Refs. 10–12). This means that the energy $v_F Q/2 \approx 500$ K (for $v_F \approx 5\times 10^7$ cm/s) related to the rotation of the magnetization vector is much larger than the energy related to the superconducting and magnetic transitions ($T_c \approx T_m \approx 1$ K).

As is well known, many characteristics of a superconductor (the critical temperature, the density of states, etc.) can be calculated if the Green's functions of the system, including the anomalous ones (or Gor'kov's functions), $\hat{F}$, are found.[13] These functions for a magnetic superconductor (MS) with a spiral structure have been obtained in Ref. 12. In this case, the functions $\hat{F}$ depend on the center-of-mass coordinate and momentum direction so that the system is anisotropic. Long ago, it was established that surface effects are essential for finite anisotropic samples such as anisotropic superconductors and high $T_c$ superconductors with $d$ wave pairing (see, for example, Ref. 14 and also a review[15] and references therein). In particular, the order parameter may be suppressed near the superconductor (S)/vacuum (V) or superconductor/insulator (I) interface. A high impurity concentration leads to averaging the Green's functions in the momentum space so that in the diffusive limit, the characteristics of the system do not depend on the sample size.

In this paper, we show that the surface effects are important in MSs with a spiral magnetic structure. In particular, the Green's functions of the system are disturbed by boundary conditions at the S/V or S/I interface in samples with any impurity concentration. Corrections to the bulk Green's functions due to boundary conditions oscillate in space with the period $2\pi/Q$ and decay from the interface over a length of the order $\xi_T \approx v/2\pi T$ in the ballistic limit and of the order $\xi_T=\sqrt{D/\pi T}$ in the diffusive limit, where $D=v_F^2\tau/3$ is the diffusion coefficient; $\tau$ is the elastic scattering time.

The surface effects become very important in the cases when one needs to know the Green's functions near the interfaces. For instance, the Josephson current $I_J$ in a MS/I/MS tunnel junction is determined by the values of the Green's functions near the MS/I interface. The Josephson current $I_J$ in the MS/I/MS junction with a spiral magnetic structure was calculated in Ref. 16 on the basis of the tunnel Hamiltonian method. The authors used the Gor'kov's functions calculated in Ref. 12 for an infinite MS with a spiral magnetic structure in the ballistic limit. They have obtained that the Josephson critical current $I_c$ depends on the angle $\theta$ between the magnetization directions in both MSs near the interface and calculated the dependence of $I_c$ on different parameters of the junction (the exchange field, the wave vector of the spiral, $Q$, etc.). It has been established that at some values of parameters, the critical current becomes negative ($\pi$ state). We will show here that, although the current $I_c$ indeed depends on $\theta$ in a way similar to that in Ref. 16, the dependence of $I_c$ on

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various parameters is completely different. The point is that the tunnel Hamiltonian method is not applicable to inhomogeneous superconductors and, in particular, to MSs with a spiral magnetization. Even in the lowest order in the barrier transparency, the results obtained by using this method are incorrect. In order to calculate $I_c$, one has to solve the Eilenberger or Usadel equation with boundary conditions at the MS/I interface. It turns out that the Green's functions at the MS/I interface differ essentially from their values in the bulk and, correspondingly, the Josephson current also differs substantially from its value obtained on the basis of the bulk Green's functions.

The paper is organized as follows. In Sec. II, we analyze the ballistic case. Using the Eilenberger equation generalized for the case of the MS with a magnetic spiral, we find the spatial dependence of corrections to the bulk Green's functions. In Sec. III, the diffusive case will be considered. Using a generalized Usadel equation complemented by boundary conditions at the MS/I interface, we calculate the Josephson current in MS/I/MS tunnel junction and compare the obtained critical Josephson current $I_c$ with that obtained on the basis of the tunnel Hamiltonian method. In Sec. IV, we discuss the obtained results.

## II. BALLISTIC CASE

We consider a MS with a spiral magnetic structure. The exchange field acting on free electrons is assumed to lie in the $(y,z)$ plane and to rotate in space with the wave vector $Q$; that is, the vector of the exchange field is $\mathbf{h}=h[0,\sin\alpha(x),\cos\alpha(x)]$ with $\alpha=Qx+\theta$, $x\geqslant 0$, ($\theta$ is the angle between the magnetization and $z$ axis at $x=0$). The superconducting order parameter $\Delta$ is taken into account in the mean field approximation: $\Delta=\lambda_S\Sigma_p\langle\psi_{\uparrow,p}\psi_{\downarrow,-p}\rangle$, i.e., the singlet pairing is assumed. The Eilenberger equation is derived in a standard way (see, for example, Refs. 17 and 19–22). The main difference between the cases of an ordinary, nonmagnetic superconductor and MS with a spiral structure is that the quasiclassical Green's function $\check{g}$ in the latter case is a $4\times 4$ matrix in the Gor'kov-Nambu and spin spaces. This equation has the form

$$i\mathbf{v}\nabla\check{g}+\omega[\hat{\tau}_3\otimes\hat{\sigma}_0,\check{g}]+i[\mathbf{h}(x)\mathbf{S},\check{g}]+[\hat{\Delta}\otimes\hat{\sigma}_3,\check{g}]+(i/2\tau)\times[\langle\check{g}\rangle,\check{g}]=0, \tag{1}$$

where $v$ is the Fermi velocity, $\mathbf{S}=(\hat{\sigma}_1,\hat{\sigma}_2,\hat{\tau}_3\otimes\hat{\sigma}_3)$, $\hat{\sigma}_k,\hat{\tau}_k$ are the Pauli matrices in the spin and Gor'kov-Nambu spaces, and $\hat{\sigma}_0,\hat{\tau}_0$ are the unit matrices. The square and angular brackets mean the commutator and averaging over angles, respectively, and $\tau$ is an elastic scattering time. The order parameter $\hat{\Delta}$ has the form $\hat{\Delta}=\hat{\tau}_2\Delta$. As in Refs. 12 and 16, we assume the simplest BCS pairing mechanism and isotropic Fermi surface. Therefore, the Green's function $\check{g}$ is angle dependent only due to the presence of the magnetic spiral. In order to exclude the coordinate dependence of the third term in Eq. (1), we perform the transformation (see Ref. 20)

$$\check{g}=\check{U}\otimes\check{g}_n\otimes\check{U}^+, \tag{2}$$

where $\check{U}=\hat{\tau}_0\otimes\hat{\sigma}_0\cos(\alpha/2)+i\sin(\alpha/2)\hat{\tau}_3\otimes\hat{\sigma}_1$ is an operator corresponding to a rotation in the spin and particle-hole spaces, and $\check{g}_n$ is a new matrix. Then, Eq. (1) acquires the form

$$v_F\mu\partial_x\check{g}+[\hat{\tau}_3\otimes(\omega\hat{\sigma}_0+ih\hat{\sigma}_3,\check{g}\ ]+iv_F\mu(Q/2)[\hat{\tau}_3\otimes\hat{\sigma}_1,\check{g}]+[\hat{\Delta}\otimes\hat{\sigma}_3,\check{g}]+(i/2\tau)[\langle\check{g}\rangle,\check{g}]=0, \tag{3}$$

where $\mu=p_x/p$. The subindex $n$ is omitted. From the physical point of view, the transformation given by Eq. (2) means the transition to a rotating coordinate system, in which the magnetization vector is directed along the $z$ axis. That is why the exchange field $h$ in Eq. (3) contains only the $z$ component.

For simplicity, we restrict the consideration with the case of temperatures close to the critical one of the superconducting transition, $T_c$. In this case, the matrix Green's function $\check{g}$ may be represented in the form

$$\check{g}=\mathrm{sgn}\ \omega\cdot\hat{\tau}_3\otimes\hat{\sigma}_0+\check{f}, \tag{4}$$

where the anomalous (Gor'kov's) matrix function $\check{f}$ is assumed to be small, that is, all elements of this matrix are small. The first term is the normal, matrix Green's function in the Matsubara representation.

In this section, we consider the ballistic case, i.e., we suppose that $\tau\to\infty$. By substituting the matrix $\check{g}$ from Eq. (4) into Eq. (1), we come to the equation for the anomalous function $\check{f}$:

$$v_F\mu\hat{\tau}_3\otimes\partial_x\check{f}+iv_F\mu(Q/2)[\hat{\sigma}_1,\check{f}]_++2\omega\check{f}+ih[\hat{\sigma}_3,\check{f}]_+=\hat{\tau}_2\otimes\hat{\sigma}_3\Delta\ \mathrm{sgn}\ \omega. \tag{5}$$

We represent the matrix $\check{f}$ in the form

$$\check{f}=\hat{f}\otimes\hat{\tau}_2+\hat{F}\otimes\hat{\tau}_1, \tag{6}$$

where $\hat{F}$ and $\hat{F}$ are the matrices in the spin space that can be represented as a sum of Pauli matrices:

$$\hat{f}=\sum_k f_k\hat{\sigma}_k,\quad \hat{F}=\sum_k F_k\hat{\sigma}_k, \tag{7}$$

where $k=0,1,3$.

Equation (5) is a system of linear equations with respect to coefficients $f_k$ and $F_k$. The solution of these equations consists of a part, $\bar{f}_k$ and $\bar{F}_k$, constant in space and a nonhomogeneous part, $\delta f_k(x)$ and $\delta F_k(x)$. The latter part arises if there are nontrivial boundary conditions in the problem. The homogeneous part is a solution for an infinite sample when boundary conditions can be ignored. The homogeneous solution can be easily found. It has the form

$$\bar{f}_3=\frac{\Delta(\epsilon_Q^2+\omega^2)}{|\omega|(\epsilon_Q^2+h^2+\omega^2)},\quad \bar{f}_0=\frac{ih\Delta\ \mathrm{sgn}\ \omega}{|\omega|(\epsilon_Q^2+h^2+\omega^2)}, \tag{8}$$

where $\epsilon_Q=\mu v_F Q/2$. All other coefficients (i.e., $f_1,F_k$) are equal to zero. The coefficient $\bar{f}_3$ is the amplitude of the sin-

glet component, and the coefficient $\bar{f}_0$ is the amplitude of the triplet component with zero projection of the total spin of a Cooper pair on the $z$ axis (in the rotating coordinate system), $S_z=0$. The singlet component is an even function of $\omega$, while the triplet component, $\bar{f}_0$, is an odd function of $\omega$ (see Ref. 20). One can see that the exchange field $h$ suppresses the amplitude $\bar{f}_3$, whereas at a sufficiently large wave vector of the spiral $Q$, the amplitude $\bar{f}_3$ is restored to the value $\Delta/|\omega|$, which is the amplitude of the condensate function in a nonmagnetic superconductor. Note that the authors of Ref. 16 used only bulk solutions in the laboratory coordinate frame. These functions may be reduced to the quasiclassical Green's functions in Eq. (8).

The function $\bar{f}_3$ determines a change of the critical temperature of the superconducting transition, $T_c$, due to the exchange field $h$ and wave vector of the magnetic spiral, $Q$ (see, for example, the review articles[19,20]):

$$\frac{T_{c0}-T_c}{T_c}=2\pi T\sum_{\omega}\int_0^1 d\mu\left[\frac{1}{|\omega|}-\frac{\bar{f}_3}{\Delta}\right]$$
$$=2\pi T\sum_{\omega}\int_0^1 d\mu\frac{h^2}{|\omega|(\epsilon_Q^2+h^2+\omega^2)}, \tag{9}$$

where $T_{c0}$ is the critical temperature in the absence of the exchange field $h$. It is seen that with decreasing the spiral period, $2\pi/Q$, the suppression of the critical temperature is reduced and at $vQ\gg h$, the critical temperature is the same as in a nonmagnetic superconductor, i.e., $T_c\to T_{c0}$.

Now, we turn to the calculation of corrections $\delta f_k$ and $\delta F_k$ that arise due to boundary conditions and depend on $x$. Note that if the correction $\delta f_3$ is not small compared to $\bar{f}_3$, a correction to the order parameter $\delta\Delta(x)$ will not be small as well. This circumstance makes the problem rather complicated because Eq. (5) becomes a system of six equations with the right-hand side which depends on $x$. In order to simplify the problem, we assume that the correction $\delta f_3$ is small and we can neglect variation of $\Delta$ in space. We will see below that in a general case, $\delta f_3$ may be comparable with $\bar{f}_3$. In this case, our results are correct up to a numerical factor of the order unity. In the next section, we discuss the validity of the obtained results in more detail.

Thus, in order to find the corrections $\delta f_k$ and $\delta F_k$, we have to solve the system of homogeneous linear equations in Eq. (5) without the right-hand side. Substituting the expansions in Eq. (7) with $\delta f_k$ and $\delta F_k$ as the coefficients of these expansions into Eq. (5) with $\delta\Delta=0$ and representing the coordinate dependence of these coefficients in the form $\{\delta f_k,\delta F_k\}\sim\exp(\kappa x)$, we obtain a system of six linear equations. One can see from these equations that the coefficients $f_1$ and $F_{0,3}$ are antisymmetric functions of $\mu$, whereas the coefficients $f_{0,3}$ and $F_1$ are symmetric functions of $\mu$. We do not write down these equations as they are rather cumbersome. Instead of this, we write the determinant of the system which determines the eigenvalues $\kappa_i$. It is reduced to the cubic algebraic equation

$$\lambda_\omega(1-z)[\lambda_\omega-z(1+\lambda_\omega)+z^2]+(1-\lambda_\omega)(1+z)^2(\lambda_\omega-z)=0, \tag{10}$$

where $z=\epsilon_\kappa^2/\Omega^2$, $\lambda_h=h^2/\Omega^2$, $\lambda_Q=\epsilon_Q^2/\Omega^2$, $\lambda_\omega=\omega^2/\Omega^2$, $\Omega^2=\omega^2+h^2+\epsilon_Q^2$, $\epsilon_\kappa=\mu v\kappa/2$, and $\epsilon_Q=\mu vQ/2$.

In order to find the eigenvalues, one has to solve this equation. We consider the most interesting case of large energy $\epsilon_Q$: $\epsilon_Q\gg T,h$. In this case, the critical temperature $T_c$ is close to $T_{c0}$. The solutions of Eq. (10) are

$$z_1\cong\lambda_\omega,\quad \kappa_1\cong 2|\omega|/(v_F|\mu|) \tag{11}$$

and

$$z_{2,3}\cong-1\pm 2i\sqrt{\lambda_\omega},\quad \kappa_{2,3}\cong\pm iQ-2|\omega|/(v_F|\mu|). \tag{12}$$

Therefore, the eigenfunctions corresponding to $\kappa_{2,3}$ oscillate in space with the period of the spiral and decay over the distance of the order $\xi_T=v_F/2\pi T$. The eigenfunction, which corresponds to $\kappa_1$, decreases monotonously from the interface over the correlation length $\xi_T$.

The amplitudes $f_k$ and $F_k$ may be found from boundary conditions at the MS/V or MS/I interfaces:[34]

$$\check{f}(\mu)-\check{f}(-\mu)=0, \tag{13}$$

which read that the antisymmetric part of the Green's function should turn to zero at the MS/I interface. One can solve the corresponding equations and find the amplitudes $f_k$ and $F_k$. However, we will not do that for two reasons. First, the corresponding expressions are cumbersome. The second and more important reason is that the surface effects are displayed near the interface at which a random (diffusive) scattering takes place. Therefore, the ballistic case considered in this section is not relevant to this situation. In the next section, we consider a more realistic case of a sample with a high impurity concentration (dirty case). We will find the eigenvalues $\kappa_i$ and the amplitudes of eigenfunctions. One can show that the structure and form of the dependencies of the functions $f_k$ and $F_k$ on $\mu$ and $\omega$ are qualitatively the same in both cases, ballistic and diffusive. The only difference is that, whereas in the diffusive case only the zero and first terms in the expansion in spherical harmonics are important, in the ballistic case, the dependence on $\mu$ is more complicated.

## III. DIFFUSIVE CASE

In this section, we consider the influence of the boundary on the condensate functions assuming that the impurity concentration is high and the condition $l\ll 2\pi/Q,\xi_T$ is satisfied, where $l=v_F\tau$ is the mean free path. In this case, the part of the condensate function $\check{f}_{asm}$ antisymmetric in the momentum space is expressed through the symmetric part via the well known expression[18–22]

$$\check{f}_{asm}=(p_x/|p|)\hat{\tau}_3\,\mathrm{sgn}\,\omega\{\partial_x\check{f}+i(Q/2)\hat{\tau}_3[\hat{\sigma}_1,\check{f}]_+\}, \tag{14}$$

where $\hat{\tau}_3\,\mathrm{sgn}\,\omega$ is the ordinary quasiclassical Green's function in the normal state [see Eq. (4)]. The second term on the left-hand side arises as a result of the transformation in Eq.

(2); the term $[\hat{\sigma}_1,\check{f}]_+$ means anticommutator. One can see that the asymmetric part has the opposite parity in $\omega$ compared to the symmetric part $\check{f}$; if $\check{f}$ is an odd function of $\omega$, then $\check{f}_{asm}$ is an even function of $\omega$ and vice versa.[20] In the simplest case of ordinary BCS superconductors, the symmetric function near $T_c$ is equal to $\check{f}=\hat{\tau}_3\otimes\hat{\sigma}_0\Delta/|\omega|$, i.e., is an even function of $\omega$. Obviously, the antisymmetric part $\check{f}_{asm}$ is an odd function of $\omega$. This issue is discussed in detail in Ref. 23.

We assume again that the temperature is close to $T_c$. The symmetric part of the condensate function $\check{f}$ after the transformation in Eq. (2) obeys the equation[20]

$$D\left\{-\partial_{xx}^2\check{f}+\frac{Q^2}{2}(\check{f}+\hat{\sigma}_1\otimes\check{f}\otimes\hat{\sigma}_1)+i\frac{Q}{2}\hat{\tau}_3[\hat{\sigma}_1,\partial_x\check{f}]_+\right\}+2|\omega|\check{f}+ih_\omega[\hat{\sigma}_3,\check{f}]_+=2\hat{\tau}_2\otimes\hat{\sigma}_3\Delta. \tag{15}$$

Here, $h_\omega=\mathrm{sgn}\,\omega h$. As follows from Eqs. (13) and (14), the boundary condition has the form

$$\partial_x\check{f}+i(Q/2)\hat{\tau}_3[\hat{\sigma}_1,\check{f}]_+=0. \tag{16}$$

This means that the spiral axis is assumed to be perpendicular to the MS/V or MS/I interface.

One can see that a coordinate-independent solution for Eq. (15) satisfies the boundary condition only if $Q=0$. If $Q$ is not zero, the anticommutator $[\hat{\sigma}_3,\bar{f}_0\hat{\sigma}_0]_+\neq 0$, and, therefore, $\partial_x\check{f}$ also differs from zero at the boundary.

We have to solve Eq. (15) with the boundary condition in Eq. (16). The uniform solution again has a form similar to Eq. (8):

$$\bar{f}_3=\frac{\Delta(\epsilon_Q+|\omega|)}{|\omega|(\epsilon_Q+|\omega|)+h^2},\quad \bar{f}_0=-\frac{ih_\omega\Delta}{|\omega|(\epsilon_Q^2+|\omega|)+h^2}, \tag{17}$$

with $\epsilon_Q=DQ^2/2$. The correction $\delta\check{f}=\check{f}-\hat{\tau}_2(\bar{f}_3\hat{\sigma}_3+\bar{f}_0\hat{\sigma}_0)$ satisfies the uniform equation in Eq. (15) and may be represented in the form of Eqs. (6) and (7), where only the coefficients $f_{0,3}$ and $F_1$ differ from zero, that is,

$$\check{f}=(f_3\hat{\sigma}_3+f_0\hat{\sigma}_0)\otimes\hat{\tau}_2+F_1\hat{\sigma}_1\otimes\hat{\tau}_1. \tag{18}$$

We look for a solution in the form of exponentially decaying functions: $\delta\check{f}\sim\exp(\kappa x)$ with $\mathrm{Re}\,\kappa<0$. The determinant of the system of Eq. (15) has the form

$$[(1+z)^2+2\lambda_\omega(1-z)+\lambda_\omega^2](\lambda_\omega-z)+\lambda_h^2(1+\lambda_\omega-z)=0, \tag{19}$$

where $z=(\kappa/Q)^2$, $\lambda_\omega=2|\omega|/DQ^2$, $\lambda_h=2h_\omega/DQ^2$.

Again, we consider the most interesting case of small $\lambda_{\omega,h}$ which seems to be relevant to the experiment:[10] $\{\lambda_\omega,\lambda_h\}\ll 1$, i.e., $\{T,h\}\ll DQ^2$ (for example, in $HoMo_6S_8$, the parameter $\lambda_\omega$ is very small: $\lambda_\omega\approx 10^{-4}-10^{-5}$, see Ref. 24). In this limit, the eigenvalues are

$$z_1=\lambda_\omega+\lambda_h^2,\quad \kappa_1=-Q\sqrt{\lambda_\omega+\lambda_h^2} \tag{20}$$

and

$$z_{2,3}=-1\pm i\sqrt{2(2\lambda_\omega+\lambda_h^2)},\quad \kappa_{2,3}=\pm iQ-Q\sqrt{(2\lambda_\omega+\lambda_h^2)/2}. \tag{21}$$

Thus, the correction $\delta f_3$ may be written as

$$\delta f_3(x)=a_1\exp(\kappa_1 x)+a_+\exp(\kappa_+ x)+a_-\exp(\kappa_- x), \tag{22}$$

where $\kappa_+=\kappa_2=+iQ-Q\sqrt{(2\lambda_\omega+\lambda_h^2)/2}$ and $\kappa_-=\kappa_3=\kappa_+^*$. The first term decreases monotonously inside the superconductor, whereas the second and third terms oscillate with the period $2\pi/Q$ and decay over the length of the order of $\min\{\xi_T,\sqrt{(DQ^2/h)(D/h)}\}$. The corrections, $\delta f_0$ and $F_1$, have the form

$$\delta f_0(x)=-i\lambda_h a_1\exp(\kappa_1 x)-(i\lambda_h)^{-1}[(1+i\alpha)a_+\exp(\kappa_+ x)+(1-i\alpha)a_-\exp(\kappa_- x)], \tag{23}$$

$$F_1(x)=2i\lambda_h\sqrt{z_1}a_1\exp(\kappa_1 x)-\lambda_h^{-1}[(1+i\alpha)a_+\exp(\kappa_+ x)-(1-i\alpha)a_-\exp(\kappa_- x)], \tag{24}$$

where $\alpha=\sqrt{2(2\lambda_\omega+\lambda_h^2)}$. The coefficients $a_1$ and $a_\pm$ are found from the boundary condition in Eq. (16):

$$a_1=-\frac{i\lambda_h}{\lambda_h^2+\alpha\sqrt{z_1}/2}\bar{f}_0,\quad a_+=a_-^*=-ia_1\sqrt{z_1}\left[\frac{1}{2}-i\frac{\lambda_h^2+\alpha^2/2}{\alpha}\right]. \tag{25}$$

Making use of Eqs. (22)–(24), one can obtain the values of the condensate function at the interface $\check{f}(0)$ that determine, for example, the Josephson current in MS/I/MS junction. We find

$$f_3(0)\cong-\frac{(\alpha/2)\sqrt{z_1}}{(\lambda_h^2+\alpha\sqrt{z_1}/2)}\bar{f}_3,\quad f_0(0)\cong-i\lambda_h\left(1-\frac{2\sqrt{z_1}}{\alpha}\right)f_3(0) \tag{26}$$

and

$$F_1(0)=-i\frac{2\lambda_h}{\alpha}f_3(0), \tag{27}$$

where the amplitude of the bulk singlet $\bar{f}_3$ component can be expressed in terms of the parameters $\lambda_{\omega,h}$:

$$\bar{f}_3=\frac{(\lambda_\omega+1)}{\lambda_\omega(\lambda_\omega+1)+\lambda_h^2}\frac{2\Delta}{DQ^2}, \tag{28}$$

In the considered limit, $\lambda_{\omega,h}\ll 1$, the function $\bar{f}_3$ is close to the value of the singlet component in an ordinary (nonmagnetic) superconductor. The exchange field, which tries to destroy Cooper pairs, is effectively averaged due to rotation of the magnetization vector.

Now, we discuss the conditions under which the obtained results are valid. Let us first consider the case of a thick sample $[d\gg\xi_{GL}\cong 1.2\sqrt{D/T}(T/\Delta)$, where $d$ is the thickness of the sample and $\xi_{GL}$ is the Ginsburg-Landau correlation

length]. One can see that if $\lambda_h^2 \gg \lambda_\omega(n=0)$, i.e., $h^2 \gg (\pi T)DQ^2$, the value of $f_3(0)$ is $f_3(0)=\bar{f}_3\sqrt{2}/(2+\sqrt{2}) \approx 0.41\bar{f}_3$, i.e., the singlet condensate function at the interface differs from the bulk value by a numerical factor of the order 1. In the limit $\lambda_h^2 \ll \lambda_\omega$, the singlet component is almost constant in space so that $f_3(0) \approx \bar{f}_3$. Therefore, our results are valid in this limit. However, our results are also correct if the thickness of the sample $d$ is less than the Ginsburg-Landau correlation length. In this case, the order parameter $\Delta$ is constant in space[25] so that our assumption about the coordinate-independent $\Delta$ is fulfilled and the obtained results are exact.

Let us discuss the meaning of the component of the condensate function $\check{f}$. As we said above, the function $f_3(0)$ is the amplitude of the singlet component at the interface. The function $f_0(0)$ is the amplitude of the triplet $S_z=0$ component. One can see that both functions, $\bar{f}_0$ and $\delta f_0$ (the bulk value and the correction due to the surface effects), are small compared to the singlet value in the considered limits, $\lambda_{\omega,h} \ll 1$. The function $F_1$ is the amplitude of the triplet component with $|S_z|=1$ in the rotating coordinate system. In the bulk, it is equal to zero. Just this component penetrates the ferromagnet over a long distance in S/feromagnet (F) structures with a rotating magnetization.[20,26–29] This triplet component $F_1(0)$ is of the order of the singlet component in the bulk, $\bar{f}_3$, at $\lambda_\omega \ll \lambda_h^2$ and less than $\bar{f}_3$ at $\lambda_\omega \gg \lambda_h^2$.

By knowing the quasiclassical Green's functions at the MS/I interface, we can calculate the dc Josephson current $I_J$ in a MS/I/MS tunnel junction consisting of two MSs. The Josephson current in this junction is expressed in terms of the components $f_{0,3}$ and $F_1$ at the MS/I, interfaces i.e., at $x=0$ (see the Appendix):

$$I_J = I_c \sin\varphi, \tag{29}$$

$$I_c = (eR_B)^{-1}(2\pi T)\sum_{\omega=0}^{\infty}\{f_3^2(0) + \cos\theta[f_0^2(0) + F_1^2(0)]\},$$

where $R_B$ is the resistance of the junction in the normal state, $\varphi$ is the phase difference, $\omega=\pi T(2n+1)$ is the Matsubara frequency, and $\theta$ is the angle between the magnetization vectors in the right and left magnetic superconductors at the interfaces. Since we are interested in the Josephson current in the lowest order in the parameter $R_B^{-1}$, the functions $f_{0,3}(x)$ and $F_1(x)$ should obey the boundary conditions in Eq. (16) that correspond to the limit $R_B \to \infty$. These functions are given by Eqs. (26) and (27).

A formula, which resembles Eq. (29), was obtained in Ref. 16 on the basis of the tunnel Hamiltonian method. What is the difference between these two formulas? First, the term $F_1^2(0)$ is absent in Ref. 16. Second, instead of terms $f_{0,3}^2(0)$, in Ref. 16, there are terms $\bar{f}_{0,3}^2$ corresponding to the bulk solutions. This difference leads to essential consequences. In particular, the conclusion made in Ref. 16 about the possibility to realize a $\pi$ junction for some values of parameters such as $h, Q$, etc., is not justified.

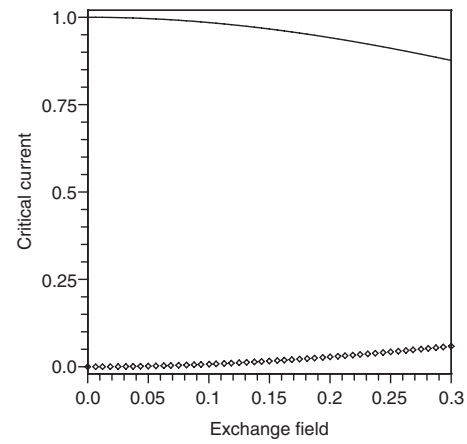


FIG. 1. Contributions of the singlet (solid line) and triplet $S_z=0$ (dotted line) components to the Josephson critical current as a function of the exchange field. This dependence has been obtained on the basis of bulk quasiclassical Green's functions and corresponds to the tunnel Hamiltonian method. The critical current $I_c(h)$ and the exchange field $h$ are plotted in units of $I_c(0)$ and $DQ^2/2$, respectively. The temperature is chosen to be equal to $T=0.1DQ^2/2\pi$.

Figure 1 shows the contributions of the bulk singlet ($\bar{f}_3$) and $S_z=0$ triplet ($\bar{f}_0$) components to the critical current and corresponds to the tunnel Hamiltonian method. Figure 2 displays the contributions of the singlet ($f_3$), $S_z=0$ triplet ($f_0$), and $|S_z|=1$ triplet ($F_1$) components to the critical current. The solid and dotted curves in Fig. 2 are normalized partial critical currents defined as

$$i_3 = \frac{8T^2}{\pi^2\Delta^2}\sum_\omega f_3^2(0), \quad i_0 = \frac{8T^2}{\pi^2\Delta^2}\sum_\omega f_0^2(0), \tag{30}$$

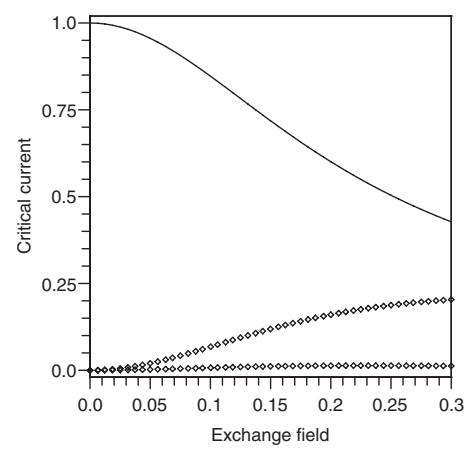


FIG. 2. Contributions of the singlet (solid line) and triplet (dotted lines) components to the Josephson critical current as a function of the exchange field $h$. The upper and lower dotted curves correspond to the triplet $|S_z|=1$ and $S_z=0$ condensate components. These curves are plotted on the basis of Eq. (29). The normalization units are the same as in Fig. 1.

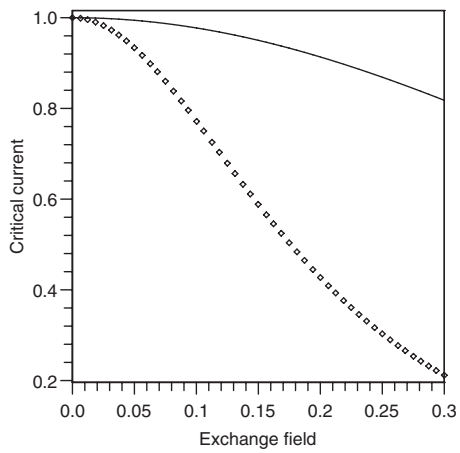


FIG. 3. The total critical current versus the exchange field obtained on the basis of the bulk Green's functions (solid line) and of the Green's functions taken at the MS/I interface (dotted line). The solid line corresponds to the tunnel Hamiltonian method. The normalization units are the same as in Fig. 1.

i1=8T2$\pi$2$\Delta$2$\Sigma\omega$F12(0),where $\pi^2/8=\Sigma_\omega(2n+1)^{-2}$ is the normalization factor. Different contributions to the critical current are normalized to the critical current in an SNS junction, i.e., to the current at $h=0$. The functions $f_{3,0}^2(0)$ and $F_1(0)$ are given by Eqs. (26) and (27). The lower (upper) dotted lines are due to the $S_z=0$ and $|S_z|=1$ triplet components. It is seen that the current $i_3$ due to the singlet component decreases with increasing $\lambda_h$, and the currents $i_{0,1}$ due to the triplet components increase with increasing $\lambda_h$. Interestingly, the current $i_1$ caused by the triplet component with nonzero projection of the total spin on the local $z$ axis is much larger than the current $i_0$ caused by the $S_z=0$ triplet component. Meanwhile, the current $i_1$ is absent in the tunnel Hamiltonian method (compare Figs. 1 and 2).

In Fig. 3, we show the dependence of the total normalized critical current $i_c=i_3+i_0+i_1$ on $\lambda_h$ for $\theta=0$ on the normalized exchange field $\lambda_h$ (dotted line). We compare this dependence with the dependence $\bar{\imath}_c=\bar{\imath}_3+\bar{\imath}_0$ (solid line), i.e., with the critical current given by the tunnel Hamiltonian method, where $\bar{\imath}_{3,0}$ are determined by Eq. (30) with $f_{3,0}^2(0)$ replaced by $\bar{f}_{3,0}^2$. One can see a significant difference between these dependencies.

Note that we considered the case of small $h$ in comparison with the energy $DQ^2$. In this case, one can neglect a change in $T_c$ and $\Delta$ due to the presence of the magnetic spiral. Otherwise, this change should be taken into account. If we ignore the dependence of $\Delta$ on $h$ and $DQ^2$, as it was done in Ref. 16, we would obtain the sign reversal of the critical Josephson current $I_c$ at $\lambda_h\approx 0.6$, whereas at this value of $\lambda_h$, the critical current obtained with the tunnel Hamiltonian method is positive and equals $I_c\approx 0.47I_{c0}$.

## IV. CONCLUSIONS

We have studied the influence of boundary effects on the properties of magnetic superconductors with a spiral magnetic structure. We used the well developed method of quasiclassical Green's functions. These functions obey the Eilenberger (or Usadel) equations generalized to the case of an exchange field **h** acting on spins of free electrons and varying in space. For simplicity, we considered the case of temperatures close to the critical one, $T_c$. Then, one can linearize equations for the condensate matrix Green's functions $\check{f}$. Due to a spatial dependence of the exchange field **h**, coefficients in the Eilenberger (Usadel) equations depend on the coordinate $x$. We excluded this dependence via a transformation which is equivalent to introducing a rotating coordinate system. In this local coordinate system, the field **h** has only the $z$ component and does not depend on $x$. By solving these equations with the corresponding boundary conditions, we have shown that near the boundary of MS with vacuum or an insulator, the condensate functions $\check{f}$ differ essentially from their bulk values.

In the rotating coordinate system, there are two components of the matrix $\check{f}$, $\bar{f}_3$ and $\bar{f}_0$, in the bulk. These correspond to the singlet component and the triplet component with zero projection of the total spin on the $z$ axis. Due to boundary conditions, the corrections $\delta f_{0,3}$ to the bulk functions, $\bar{f}_{0,3}$, arise near the boundary, which are not small in comparison with $\bar{f}_{0,3}$. Besides, the triplet component $F_1$ with nonzero projection of the total spin of Cooper pairs appears in the vicinity of the surface on the scale of the coherence length. The corrections $\delta f_{0,3}$ and function $F_1$ oscillate with the period $2\pi/Q$ in space and decay inside the bulk over a length of the order of $\xi_T=v/2\pi T$ (ballistic case) or $\xi_T=\sqrt{D/2\pi T}$ (diffusive case). The amplitude of the singlet component $f_3$ decreases at the surface, resulting in the suppression of the order parameter $\Delta$ near the surface.

As an example of the importance of the surface effects in MSs, we considered the dc Josephson effect in a MS/I/MS tunnel junction. The critical Josephson current $I_c$ can be expressed in terms of components $f_{0,3}(0)$ and $F_1(0)$ at the MS/I interface. The results are compared with the ones which are obtained on the basis of the tunnel Hamiltonian method and expressed in terms of the bulk condensate functions $\bar{f}_{0,3}$. This method was used in Ref. 16. Although the formulas for $I_c$ in Ref. 16 and in this paper are similar, there is an essential difference between them. In the tunnel Hamiltonian method, the coefficient in front of cos $\theta$ is the squared amplitude of the triplet $S_z=0$ component, $\bar{f}_0^2$. In fact, this coefficient is equal to $f_0^2(0)+F_1^2(0)$ [see Eq. (29)], where $f_0(0)$ is the amplitude of the triplet component with zero projection of the spin on the local $z$ axis and $F_1(0)$ is the amplitude of the $|S_z|=1$ triplet component at the interface. It turns out that, at least near $T_c$, the amplitude $F_1(0)$ is much larger than $f_0(0)$. The tunnel Hamiltonian method can be applied to MSs only if the wave vector of the spiral, $Q$, is small enough: $vQ\ll h$ (ballistic case) or $DQ^2\ll h$ (diffusive case). However, in this case, the exchange field $h$ should be small: $h<\Delta$ ($T\ll\Delta$) or $h\ll(T_c-T)/T$ ($\Delta\ll T$). Otherwise, superconductivity will be destroyed. In this limit of small $Q$, the MS/I/MS junction is equivalent to the FS/I/FS junction. The Josephson current in FS/I/FS junctions was calculated in Refs. 30–33.

The surface effects may also change the other characteristics of MSs such as the density of states (DOS), etc. Our

consideration is restricted with temperatures $T$ near $T_c$, where the DOS is close to that in the normal state and the variation of the DOS due to the surface effects is small. The calculation of the Green's functions in a finite system at low $T$ is a more complicated task because the corresponding equations, strictly speaking, cannot be linearized. This problem is beyond the scope of this paper.

Note also an interesting fact—the appearance of the odd triplet component $F_1(x)$. This means that in a MS/F system with a strong ferromagnet, the component $F_1(x)$ will penetrate the ferromagnet over a long distance.[20]

## ACKNOWLEDGMENT

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## APPENDIX

Here, we obtain a formula for the Josephson current $I_J$ in a MS/I/MS tunnel junction. We consider MSs with a spiral magnetization described by the angle $\alpha(x)=Qx+\theta$ (right superconductor) and $\alpha(x)=Qx$ (left superconductor) so that $\theta$ is the angle between the magnetization vectors at the MS/I interface. In order to obtain the expression for $I_J$, we employ the boundary conditions[20,34,35]

$$\check{f}_l\partial_x\check{f}_l=(2\sigma R_B)^{-1}[\check{f}_l,\check{f}_r], \tag{A1}$$

where $\check{f}_{l,r}$ are the condensate functions in the left (right) superconductor, $\sigma$ is the conductivity of the superconductors in the normal state, and $R_B$ is the junction resistance per unit area. The superconductors are assumed to be identical. The current is equal to[20]

$$\begin{aligned} I &= (\mathcal{S}\sigma/8)i(2\pi T)\sum_{\omega}\ \mathrm{Tr}\{\hat{\tau}_3\otimes\hat{\sigma}_0\otimes\check{f}_l\otimes\partial_x\check{f}_l\} \\ &= \frac{\mathcal{S}}{16R_B}i(2\pi T)\sum_{\omega}\ \mathrm{Tr}\{\hat{\tau}_3\otimes\hat{\sigma}_0\otimes[\check{f}_l,\check{f}_r]\}, \end{aligned} \tag{A2}$$

where $\omega=\pi T(2n+1)$ is the Matsubara frequency and all the functions are taken at the interface ($x=0$).

We assume that the phase of the left superconductor is $\varphi$ and the phase of the right superconductor is zero. Then, we can express the functions $\check{f}_{l,r}$ in terms of the functions $\check{f}$ found above with the help of transformations

$$\check{f}_l\Rightarrow\check{U}_\varphi\otimes\check{U}_l\otimes\check{f}_l\otimes\check{U}_\varphi^+\otimes\check{U}_l^+,\quad \check{f}_r\Rightarrow\check{U}_r\otimes\check{f}_r\otimes\check{U}_r^+. \tag{A3}$$

Here, $\check{U}_\varphi=\cos(\varphi/2)+i\hat{\tau}_3\otimes\hat{\sigma}_0\sin(\varphi/2)$ is the transformation matrix which relates a state with phase equal to zero and a state with a finite phase $\varphi$;[20] $\check{U}_{l,r}=\cos(\alpha_{l,r}/2)+i\hat{\tau}_3\otimes\hat{\sigma}_1\sin(\alpha_{l,r}/2)$ with $\alpha_l=Qx$ and $\alpha_r=Qx+\theta$. Then, we substitute the expressions in Eq. (A3) together with Eq. (18) into Eq. (A2). By calculating the commutator in Eq. (A2), we come to Eq. (29).

It is worth noting that the tunnel Hamiltonian leads to the same formula as Eq. (A2) if the functions $\check{f}_{r,l}$ are replaced by the bulk solutions, $\bar{f}_{0,3}$.


[1] A. I. Buzdin, L. N. Bulaevskii, M. L. Kulic, and S. V. Panyukov, Adv. Phys. **34**, 176 (1985).
[2] L. N. Bulaevskii, A. I. Buzdin, S. V. Panjukov, and M. L. Kulic, Phys. Rev. B **28**, 1370 (1983).
[3] M. Houzet, A. Buzdin, and M. L. Kulic, Phys. Rev. B **64**, 184501 (2001).
[4] L. Bauernfeind, W. Widder, and H. F. Braun, Physica C **254**, 151 (1995); J. Low Temp. Phys. **105**, 1605 (1996).
[5] J. W. Lynn, B. Keimer, C. Ulrich, C. Bernhard, and J. L. Tallon, Phys. Rev. B **61**, R14964 (2000).
[6] G. V. M. Williams and S. Kramer, Phys. Rev. B **62**, 4132 (2000).
[7] C. Bernhard, J. L. Tallon, C. Niedermayer, T. Blasius, A. Golnik, E. Brucher, R. K. Kremer, D. R. Noakes, C. E. Stronach, and E. J. Ansaldo, Phys. Rev. B **59**, 14 099 (1999).
[8] V. L. Ginsburg, Sov. Phys. JETP **4**, 153 (1957).
[9] P. W. Anderson and H. Suhl, Phys. Rev. **116**, 898 (1959).
[10] D. E. Moncton *et al.*, Phys. Rev. Lett. **45**, 2060 (1980).
[11] S. K. Sinha, G. W. Crabtree, D. G. Hinks, and H. Mook, Phys. Rev. Lett. **48**, 950 (1982).
[12] L. N. Bulaevskii, M. L. Kulic, and A. I. Rusinov, Solid State Commun. **30**, 59 (1979); J. Low Temp. Phys. **39**, 255 (1980).
[13] A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1975).
[14] L. J. Buchholtz, M. Palumbo, D. Rainer, and J. A. Sauls, J. Low Temp. Phys. **101**, 1079 (1995); **101**, 1099 (1995).
[15] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
[16] M. L. Kulic and I. M. Kulic, Phys. Rev. B **63**, 104503 (2001).
[17] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[18] K. L. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[19] A. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
[20] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).
[21] A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity*, edited by D. N. Langenberg and A. I. Larkin (Elsevier, Amsterdam, 1984).
[22] N. B. Kopnin, *Theory of Nonequilibrium Superconductivity* (Clarendon, Oxford, 2001).
[23] Y. Tanaka, A. A. Golubov, S. Kashiwaya, and M. Ueda, Phys. Rev. Lett. **99**, 037005 (2007); Y. Tanaka, Y. Tanuma, and A. A. Golubov, Phys. Rev. B **76**, 054522 (2007); T. Yokoyama, Y. Tanaka, and A. A. Golubov, arXiv:cond-mat/0710.2967 (unpublished).
[24] The parameter $\lambda_\omega(n)=(2\pi T/DQ^2)(2n+1)$ depends on $n$ with a characteristic $n_{ch}$ determined by a value at which corresponding sums over the Matsubara frequencies converge. One can see from Eq. (28) that $n_{ch}$ is of the order $(2n_{ch}+1)\gtrsim\lambda_h(h/\pi T)$.
[25] Y. N. Ovchinnikov, Sov. Phys. JETP **29**, 853 (1969).
[26] I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96**, 157002 (2006).

[27] A. F. Volkov, A. Anishchanka, and K. B. Efetov, Phys. Rev. B **73**, 104412 (2006).
[28] A. F. Volkov, Y. V. Fominov, and K. B. Efetov, Phys. Rev. B **72**, 184504 (2005); Y. V. Fominov, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **75**, 104509 (2007).
[29] T. Champel, T. Löfwander, and M. Eschrig, arXiv:cond-mat/0706.3594 (unpublished).
[30] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 3140 (2001).
[31] Ya. V. Fominov, N. M. Chtchelkatchev, and A. A. Golubov, JETP Lett. **74**, 96 (2001).
[32] Yu. S. Barash, I. V. Bobkova, and T. Kopp, Phys. Rev. B **66**, 140503(R) (2002).
[33] J. Linder and A. Sudbo, Phys. Rev. B **76**, 064524 (2007).
[34] A. V. Zaitsev, Sov. Phys. JETP **59**, 1015 (1984).
[35] M. Yu. Kupriyanov and V. F. Lukichev, Sov. Phys. JETP **67**, 1163 (1988).