Heat current in the magnetic field: Nernst-Ettingshausen effect above the superconducting transition

A. Sergeev*
Research Foundation, University at Buffalo, Buffalo, New York 14260, USA

M. Yu. Reizer
5614 Naiche Road, Columbus, Ohio 43213, USA

V. Mitin
Electrical Engineering Department, University at Buffalo, Buffalo, New York 14260, USA
(Received 6 November 2007; published 7 February 2008)

A problem of the definition of the heat transported in thermomagnetic phenomena has been realized well in the late 1960s, but not solved up to date. By ignoring this problem, numerous recent theories grossly overestimate the thermomagnetic coefficients in strongly interacting systems. Here, we develop a gauge-invariant microscopic approach, which shows that the heat transfer should include the energy of the interaction between electrons and a magnetic field. We also demonstrate that the surface currents induced by the magnetic field transfer the charge in the Nernst effect but do not transfer the heat in the Ettingshausen effect. Only with these two modifications of the theory does the physically measurable thermomagnetic coefficients satisfy the Onsager relation. We critically revised the Gaussian-fluctuation theory above the superconducting transition and show that the gauge invariance uniquely relates thermomagnetic phenomena in the Fermi liquid with the particle-hole asymmetry.

DOI: 10.1103/PhysRevB.77.064501 PACS number(s): 74.20.−z, 71.10.−w

I. INTRODUCTION

It is well known that in the electric field, $\mathbf{E}=-\nabla\phi$, and magnetic field $\mathbf{H}$, the energy of a system of charged particles with coordinates $\mathbf{r}_\alpha$ and momentums $\mathbf{p}_\alpha$ is[1]

$$E=\sum_\alpha \frac{p_\alpha^2}{2m}+\sum_\alpha e\phi(\mathbf{r}_\alpha)+\sum_\alpha \frac{e}{2mc}(\mathbf{r}_\alpha\times\mathbf{p}_\alpha)\cdot\mathbf{H}. \quad (1)$$

Considering the heat transfer, it is important to realize which terms in the above equation should be associated with the thermal energy. The uniform electric field accelerates all particles in the same way, while the effect of the magnetic field depends on the particle state. Therefore, it is reasonable to assume that the electric term (the second term) contributes to the potential energy, while the magnetic term (the third term) provides contribution to the thermal energy. Of course, the above consideration cannot be considered as a proof. Moreover, there are no regular methods for the identification of thermal energy. The correctness of our assumption will be verified through the Onsager relation between the Ettingshausen and Nernst coefficients, which describe the heat and electric currents initiated by the electric field and temperature gradient, correspondingly. We will show that the magnetic term overlooked in previous works plays a crucial role and restores the gauge invariance. While in accordance with van Leeuwen's theorem[1] the magnetic contribution to the heat transfer does not allow a classical interpretation, it is critically important for a consistent quantum description of thermomagnetic transport.

The definition of the heat current and the Onsager relation in the magnetic field are long standing problems, which attracted significant attention in the late 1960s in the context of thermomagnetic vortex transport in the type-II superconductors.[2,3] Maki[2] has noted that the phenomenological[4] and microscopic[5] descriptions of thermomagnetic phenomena violate the Onsager relation or the third law of thermodynamics if the heat current operator is artificially corrected to be consistent with the Onsager principle. All attempts to resolve this enigma were based on thermodynamic treatment. Various corrections suggested for the heat current to be expressed in terms of the equilibrium magnetization currents.[2,3] As we will show, the magnetization currents do not transfer the heat and the thermodynamic approach cannot solve this problem.

Recently, much effort has been dedicated to the understanding of thermomagnetic effects in high-T_c superconductors.[6–8] A variety of phenomenological concepts were suggested to explain the anomalously large Nernst signal above the superconducting transition.[9] In many papers,[10–13] the transition is associated with the three-dimensional analog of the Berezinski-Kosterlitz-Thouless (BKT) transition and the Nernst effect is attributed to the motion of vortices created by phase fluctuations above this BKT-like transition but below the mean-field transition. On the contrary, microscopic models[14,15] are based on the Gaussian-fluctuation theory (GFT) above the mean-field transition.[16–20] However, the theories related to vortices are based on the results of Refs. 4 and 5, the inconsistency with basic principles of which was noted in Refs. 2 and 3. GFT and other microscopic models also ignore corrections to the heat current.

While the large Nernst coefficient was obtained in all the above models, how Cooper's interaction could lead to this effect at the microscopic level is still not much understood. In the model of noninteracting electrons, which describes well ordinary metals, the Nernst coefficient is small because it is proportional to the square of the particle-hole asymmetry (PHA).[21] In this case, the Nernst coefficient combines PHAs of the thermoelectric, η, and Hall, σ_{xy}, coefficients: $N_n \simeq \eta \cdot \sigma_{xy}$. The interelectron interaction cannot change PHA of η and σ_{xy}.[22] According to current points of view,[9,14–20] the interaction in the Cooper channel leads to the Nernst coefficient in the zeroth order in PHA. This effect is $(\epsilon_F/T)^2$ times larger than the corresponding correction to $\eta \cdot \sigma_{xy}$ (ϵ_F is the Fermi energy). However, experiments with ordinary superconductors, such as Nb, Al, and Sn, do not support this prediction. This contradiction is a consequence of a number of misconceptions at the microscopic level.

The microscopic formalism based on the Kubo method allows one to calculate the Ettingshausen coefficient Υ, which describes the heat flux induced by crossed electric and magnetic fields, $\mathbf{j}^h = -\Upsilon[\mathbf{E}\times\mathbf{H}]$. The temperature gradient cannot be directly introduced in the Lagrange formalism of the Kubo approach. Therefore, the Nernst coefficient, which relates the electric current with the temperature gradient, $\mathbf{j}^e = N[\nabla T \times \mathbf{H}]$, is found from the Onsager relation,

$$N = \Upsilon / T. \tag{2}$$

The coefficient Υ is calculated for the infinite sample, while the Onsager relation is only valid for a finite sample,[23] where surface currents should be taken into account.

If magnetization depends on temperature, the temperature gradient generates surface magnetization currents, which provide important contribution to the charge transfer in the Nernst effect.[23] Several recent works[18–20,24] state that surface magnetization currents also play an important role in the heat transfer and the Ettingshausen coefficient calculated for the infinite sample, $\delta\Upsilon_{inf}$, should be corrected due to the surface heat currents: $\delta\Upsilon = \delta\Upsilon_{inf} - c\mu$. According to Refs. 16 and 18–20 GFT leads to the bulk coefficient, which is of the same order as the correction due to surface currents: $\delta\Upsilon_{inf} = 3/2c\mu$, where μ is the fluctuation magnetic susceptibility,[20]

$$\mu = -\frac{e^2 T}{6\pi c^2}\begin{cases} 2\alpha/\eta & \text{for two dimensions} \\ \sqrt{\alpha/\eta} & \text{for three dimensions,} \end{cases} \tag{3}$$

$\eta = (T-T_c)/T_c$, and $\alpha = (T-T_c)[\xi(T)]^2$; $\xi(T)$ is the coherence length.

In the current paper, we reconsider basics of the microscopic description of the thermomagnetic effects. This work will address the following questions: (1) Is the heat current operator modified in the presence of a magnetic field? (2) Do surface magnetization currents transfer the heat? (3) How do thermomagnetic coefficients satisfy the Onsager relation? (4) Finally, can the interelectron interaction change the PHA of the Nernst and Ettingshausen coefficients? While our results are quite general, we specify them for GFT, which will be critically revised. In particular, we show that $\delta\Upsilon \sim T\delta N \sim (T_c/\epsilon_F)^2 c\mu$.

II. ETTINGSHAUSEN COEFFICIENT: KUBO METHOD

In the gauge $\mathbf{H} = i[\mathbf{k}_H \times \mathbf{A}_H]$ and $\mathbf{E} = -i\mathbf{k}_E\phi$, contributions of static electric and magnetic fields to the energy of a charged particle are easily separated,

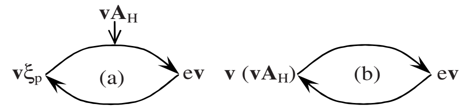

FIG. 1. Diagrams for Ettingshausen coefficient of noninteracting electrons. The straight lines stand for the electron Green functions.

$$\widetilde{E} = \frac{(\mathbf{p} + e\mathbf{A}_H/c)^2}{2m} + e\phi = \xi_p + \frac{e}{c}(\mathbf{v}\cdot\mathbf{A}_H) + e\phi + \mu_0, \tag{4}$$

where $\xi_p = p^2/2m - \mu_0$; μ_0 is the chemical potential. Let us first show the importance of the second (magnetic) term for noninteracting electrons.[25] Thermal energy is counted from the electrochemical potential $e\phi + \mu_0$, so only the first and second terms contribute to the heat current operator,

$$\hat{\mathbf{J}}^h = \sum_{\mathbf{p}} \mathbf{v}\xi_p a^+_{\mathbf{p}} a_{\mathbf{p}} + \sum_{\mathbf{p}} \frac{e\mathbf{v}}{c}(\mathbf{v}\mathbf{A}_H) a^+_{\mathbf{p}} a_{\mathbf{p}}. \tag{5}$$

Here $a^+_{\mathbf{p}}$ and $a_{\mathbf{p}}$ are the electron creation and annihilation operators. For the Ettingshausen coefficient, the corresponding diagrams with two heat current vertices are shown in Fig. 1. Diagram (a) has a form of the Hall diagram,[26] where the electric current operator, $e\mathbf{v}$, is replaced by the heat current operator, $\xi_p\mathbf{v}$. The Hall effect is proportional to PHA, i.e., the corresponding integrant is an odd function of ξ_p and a nonzero Hall coefficient is obtained after expansion of all parameters in ξ_p/ϵ_F near the Fermi surface. With the heat current operator $\xi_p\mathbf{v}$, the integrant becomes an even function and gives a nonzero Ettingshausen coefficient without expansion. However, this large contribution is canceled by diagram (b). The well-known expression for Υ in a system of noninteracting electrons is obtained in the second order in PHA, when both integrants (a) and (b) are expanded in $(\xi_p/\epsilon_F)^2$ (for details, see Ref. 25 and Appendix A).

Now, we consider electrons interacting in the Cooper channel. The heat current operator in this system[27] is easily generalized to include the external magnetic field,

$$\begin{aligned}\hat{\mathbf{J}}^h_C = &\sum_{\mathbf{p}} \mathbf{v}\xi_p a^+_{\mathbf{p}} a_{\mathbf{p}} + \sum_{\mathbf{p}} \frac{e\mathbf{v}}{c}(\mathbf{v}\mathbf{A}_H) a^+_{\mathbf{p}} a_{\mathbf{p}} \\ &- \lambda/2 \sum_{\mathbf{p},\mathbf{p}',\mathbf{p}''} (\mathbf{v}+\mathbf{v}') a^+_{\mathbf{p}+\mathbf{p}'-\mathbf{p}''} a^+_{\mathbf{p}''} a_{\mathbf{p}'} a_{\mathbf{p}} \\ &+ \sum_{\mathbf{p},\mathbf{p}',\mathbf{R}_i} (\mathbf{v}+\mathbf{v}') U_{imp}(\mathbf{R}_i)\exp[i(\mathbf{p}+\mathbf{p}')R_i] a^+_{\mathbf{p}} a_{\mathbf{p}'}. \end{aligned} \tag{6}$$

Here, λ is the interaction constant in the Cooper channel, and U_{imp} is the impurity potential. The first and second terms in Eq. (6) describe the heat flux of noninteracting electrons [Eq. (5)]. The last two terms are due to the electron-electron and electron-impurity interactions. These two terms generate diagram blocks (Aslamazov-Larkin blocks) proportional to PHA, but contributions of all blocks with PHA cancel each other.[27] Thus, when calculating the heat current, we should take into account only heat current vertices of noninteracting electrons, i.e., $\gamma^h_1 = \mathbf{v}\xi_p$ and $\gamma^h_2 = (e\mathbf{v}/c)(\mathbf{v}\mathbf{A}_H) a^+_{\mathbf{p}} a_{\mathbf{p}}$, corre-

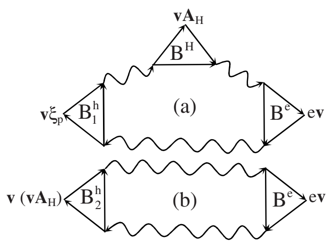

FIG. 2. Fluctuation AL diagrams describing the heat current in crossed electric and magnetic fields. Wavy lines stand for the fluctuation propagators and straight lines stand for the electron Green functions, which form the AL blocks.

sponding to the kinetic and magnetic terms in Eqs. (5) and (6).

The two leading terms in the heat current operator generate two diagrams, which describe Υ in the Aslamazov-Larkin (AL) approximation (see Fig. 2). The wavy lines correspond to the fluctuation propagator,[20,27]

$$L^{R,A}(q,\omega)=(\lambda^{-1}-P^{R,A}(q,\omega))^{-1}, \tag{7}$$

$$P^{R,A}(q,\omega)=-\frac{\nu}{2}\left(\ln\frac{2C_\gamma\omega_D}{\pi T}-\alpha q^2\pm\frac{i\pi\omega}{8T}+\gamma\omega\right), \tag{8}$$

where $P(q,\omega)$ is the polarization operator, ν is the electron density of states, ω_D is the Debye frequency, and C_γ is the Euler constant. The last term in Eq. (8) is proportional to PHA.[27,28]

The AL blocks $\mathbf{B}^{e,h,H}$ presented in Fig. 2 are built from electron Green functions and vertices $\gamma^{e,h,H}$ (see Table I). The left blocks $\mathbf{B}_1^h$ and $\mathbf{B}_2^h$ in Figs. 2(a) and 2(b) are blocks with heat current vertices γ_1^h (kinetic) and γ_2^h (magnetic). The right block $\mathbf{B}^e$ in both diagrams includes the electric current vertex $\gamma^e=e\mathbf{v}\cdot\mathbf{e}_E$, $\mathbf{e}_E=\mathbf{E}/E$. Block $\mathbf{B}^H$ includes the magnetic vertex $\gamma^H=(e/c)\mathbf{v}\cdot\mathbf{A}_H$. The results of calculation are summarized in Table I. AL blocks are obtained by inserting vertices γ into the polarization operator and can be expressed through $\nabla_\mathbf{q}P^R(\mathbf{q},0)$. Blocks $\mathbf{B}^e$, $\mathbf{B}^H$, and $\mathbf{B}_1^h$ are well known.[20] The block $\mathbf{B}_1^h$ describing the heat current in the absence of magnetic field has been calculated in Ref. 27 (see also Refs. 18 and 19). Here, we introduce $\mathbf{B}_2^h$, which is based on the electron vertex γ_2^h and describes the magnetic correction to the heat current.

The first AL diagram [Fig. 2(a)] was investigated in Ref. 19 and its contribution is $\delta\Upsilon_{inf}^{(1)}=3/2c\mu$. The same result has been obtained in the time dependent Ginzburg-Landau (TDGL) formalism.[16,18,20]

The contribution of the second AL diagram [Fig. 2(b)] is

$$\Upsilon_{inr}^{(2)}H=\Im\int\frac{d\mathbf{q}}{(2\pi)^n}\frac{d\omega}{2\pi}\frac{\mathbf{B}_2^h\mathbf{B}^e}{2\Omega}(L_+^CL_-^A+L_+^RL_-^C), \tag{9}$$

where $L^C=\coth(\omega/2T)(L^R-L^A)$, $L_\pm$ is used for $L(\mathbf{q}\pm\mathbf{k}/2,\omega\pm\Omega/2)$, and n is the system dimensionality with respect to the coherence length $\xi(T)$. Expanding the integrant to the linear order in Ω and $\mathbf{k}$ and calculating the integrals over ω and $\mathbf{q}$, we find that the contribution of the second diagram, $\Upsilon_{inr}^{(2)}$, cancels completely the contribution of the first one.

Thus, without PHA, the Ettingshausen effect is absent, $\delta\Upsilon=0$. To get nonzero result, we should expand the fluctuation propagator [Eq. (7)] up to the second order in PHA. Expanding the polarization operator [Eq. (8)] to the second order in $\gamma\omega$, we get

$$\frac{\delta\Upsilon}{T}=\delta N=-\frac{5e^2}{4\pi c}\left(\frac{8T\gamma}{\pi}\right)^2\begin{cases}2\alpha/\eta & \text{for two dimensions}\\ \sqrt{\alpha/\eta} & \text{for three dimensions,}\end{cases} \tag{10}$$

where[28] $\gamma=(1/2\epsilon_F)(\partial\ln\nu/\partial\ln\epsilon_F)\ln(2C_\gamma\omega_D/\pi T_c)$. Thus, thermomagnetic coefficients in the fluctuation region are proportional to $(T/\epsilon_F)^2$.

In summarizing the results of this section, we would like to note that the total heat current operator of the fluctuating pairs in the magnetic field,

$$\mathbf{B}^h=\mathbf{B}_1^h+\mathbf{B}_2^h=\omega\nu\alpha[\mathbf{q}+(2e/c)\mathbf{A}_H], \tag{11}$$

may be considered as the gauge-invariant extension of the operator $\mathbf{B}_1^h$ without $\mathbf{H}$. This is a key point because the further calculations of the diagrams in Fig. 2 are similar to that for noninteracting electrons (Fig. 1): the kinetic and magnetic terms in $\mathbf{B}^h$ generate two diagrams, which cancel each other in the zeroth order in PHA.

In the above calculations, the interaction with the magnetic field has been included in the heat current. Note that Eq. (11) can also be derived in another, more formal approach, where the magnetic field is initially included in electron states. Without the magnetic field, the thermoelectric coefficient is described by the AL diagram with the heat and electric current operators,[20,27] $\mathbf{B}_1^h$ and $\mathbf{B}^e$. In the magnetic field, the momentum of the Cooper pair is given by $\mathbf{q}+2e\mathbf{A}_H/c$, and the polarization operator has the form[20] $P(\mathbf{q}+2e\mathbf{A}_H/c,\omega)$. In calculating the thermomagnetic response, all blocks of the diagram should be expanded in $\mathbf{A}_H$. By expanding polarization operators in the fluctuation propagators, we obtain the first diagram for the thermomagnetic coefficient.[19,20] By expanding the heat current block $\mathbf{B}_1^h=\omega\nabla_\mathbf{q}P^R(\mathbf{q}+2e\mathbf{A}_H/c)$, we immediately obtain the block $\mathbf{B}_2^h$, which forms the second diagram. This magnetic term has been lost in all previous works.[16–19] Using Eq. (11) as the

TABLE I. AL blocks (operators for fluctuating pairs) $\mathbf{B}$ based on electron operators $\boldsymbol{\gamma}$.

$\gamma^e=e\mathbf{v}\cdot\mathbf{e}_E$	$\gamma^H=(e/c)\mathbf{v}\cdot\mathbf{A}$	$\gamma_1^h=\xi\mathbf{v}\cdot\mathbf{e}_{j^h}$	$\gamma_2^h=(\mathbf{v}\cdot\mathbf{A}_H)(\mathbf{v}\cdot\mathbf{e}_{j^h})$
$\mathbf{B}^e=2e\nabla_\mathbf{q}P^R(\mathbf{q},0)\cdot\mathbf{e}_E$	$\mathbf{B}^H=(2e/c)\nabla_\mathbf{q}P^R(\mathbf{q},0)\cdot\mathbf{A}_H$	$\mathbf{B}_1^h=\omega\nabla_\mathbf{q}P^R(\mathbf{q},0)\cdot\mathbf{e}_{j^h}$	$\mathbf{B}_2^h=2\omega\nabla_\mathbf{q}^2P^R(\mathbf{q},0)A_H$
$=2e\nu\alpha\mathbf{q}\cdot\mathbf{e}_E$	$=(2e/c)\nu\alpha\mathbf{q}\cdot\mathbf{A}_H$	$=\omega\nu\alpha\mathbf{q}\cdot\mathbf{e}_{j^h}$	$=(2e/c)\omega\nu\alpha\mathbf{A}_H\cdot\mathbf{e}_{j^h}$

heat current operator for fluctuating pairs, our results can be also obtained in the TDGL formalism.

III. NERNST COEFFICIENT: QUANTUM TRANSPORT EQUATION

To investigate a response of the electron system to ∇T, one can use the quantum transport equation. Previously, we adapted this method to calculations of the themoelectric and Hall coefficients in GFT.[27,29] In this approach, the electric current is given by

$$j^e = e\nu\alpha \int \frac{d\mathbf{q}}{(2\pi)^n}\frac{d\omega}{(2\pi)}\mathbf{q}\ \mathrm{Im}\ \delta L^C(\mathbf{q},\omega), \tag{12}$$

where $\delta L^C(\mathbf{q},\omega)$ is the nonequilibrium correction to the fluctuation propagator.

In the equilibrium, $L^C = L^R P^C L^A$, where the Keldysh component of the polarization operator $P^C = i\pi\nu/4$ at $T-T_c \ll T_c$. The nonequilibrium effects are taken into account by the ∇T and $\mathbf{H}$-Poisson brackets between polarization operators,[25,27,29]

$$\{P_1,P_2\}_T = \nabla T\left(\frac{\partial P_1}{\partial T}\frac{\partial P_2}{\partial \mathbf{q}} - \frac{\partial P_2}{\partial T}\frac{\partial P_1}{\partial \mathbf{q}}\right), \tag{13}$$

$$\{P_1,P_2\}_H = \frac{e}{c}\mathbf{H}\cdot\left(\frac{\partial P_1}{\partial \mathbf{q}} \times \frac{\partial P_2}{\partial \mathbf{q}}\right), \tag{14}$$

where P_1 and P_2 are polarization operators [Eq. (8)], which form the fluctuation propagators [Eq. (7)]. Calculation of $\delta L^C(\mathbf{q},\omega)$ for the Nernst coefficient is analogous to its calculation for the Hall effect.[29] The only difference is that the derivative $\partial P/\partial\omega$ in the electric field $\mathbf{E}$-Poisson bracket in the Hall effect[29] should be replaced by the derivative $\partial P/\partial T$ in the ∇T-Poisson bracket in the Nernst coefficient [Eq. (12)].

To get the vector product $\mathbf{H}\times\nabla T$, the $\mathbf{H}$ bracket should include the same polarization operator as the ∇T bracket. Thus, in the first order in $\mathbf{H}\times\nabla T$, the nonequilibrium fluctuation propagator $\delta L^C(\mathbf{q},\omega)$ is described by the diagrams shown in Fig. 3. Taking into account that $(\nabla_q)^2 P^{R(A)} = \alpha\nu$, we get

$$\delta L^C = \frac{\pi i}{4}\frac{e}{c}\alpha\nu^2[\mathbf{H}\times\nabla T] \times \mathrm{Re}\left[L^A(L^A - 2L^R)\frac{\partial^2 L^R}{\partial T\partial \mathbf{q}} + 2L^A\frac{\partial L^R}{\partial T}\frac{\partial L^R}{\partial \mathbf{q}}\right]. \tag{15}$$

Finally, by calculating the Nernst current in the interior of the sample [Eq. (12)], for a two-dimensional superconductor we find

$$\delta N_{inf} = \delta N + \frac{e^2}{3\pi c}\left(\frac{\alpha}{\eta^2} - \frac{\alpha}{\eta} - \frac{\alpha}{\eta}\frac{\partial\alpha}{\partial T}\right), \tag{16}$$

where δN is equal to the term that was calculated in the previous section from the Onsager relation [see Eq. (10)]. Thus, in the infinite sample or in the interior of the finite sample, the Nernst coefficient consists of two terms. The

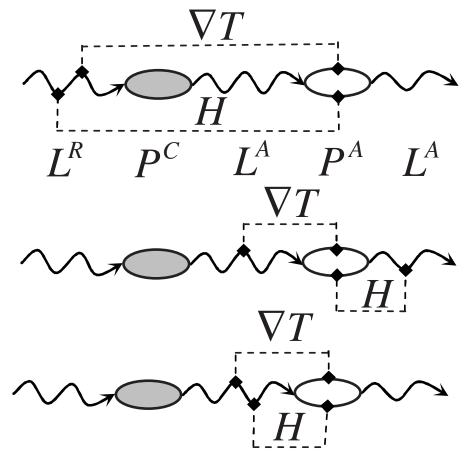

FIG. 3. The nonequilibrium fluctuation propagator δL^c due to $\mathbf{H}$- and ∇T-Poisson brackets.

second term in Eq. (16) has the zeroth order in PHA and violates the Onsager relation. As it will be shown in the next section, in the finite sample, this term is canceled by the contribution of the magnetization currents.

IV. ONSAGER RELATION IN MAGNETIC FIELD

The above results [Eqs. (10) and (16)], have been calculated for the infinite sample. Referring to Ref. 23, recent works[18,19,24] state that for a finite sample, both coefficients should be corrected due to charge and heat transfer by surface magnetization currents. Here, we show that the magnetization currents contribute only to the charge transfer, and the results of Ref. 23 have been misinterpreted.

The electric magnetization current $\mathbf{j}^e_{mag}$ in the potential relief $\phi(\mathbf{r})$ transfers the energy flux $\mathbf{j}^\epsilon_{mag} = \phi\mathbf{j}^e_{mag}$ [Eq. (23) in Ref. 23]. Using $\mathbf{j}^e_{mag} = c\mu k^2\mathbf{A}_H$, we get

$$\mathbf{j}^\epsilon_{mag} = c\mu[\mathbf{H}\times\mathbf{E}]. \tag{17}$$

This term was erroneously attributed to the heat flux.[18–20,24] As we discussed, the electric potential ϕ and the corresponding vertex $\gamma^\phi = e\mathbf{v}\phi$ do not contribute to the heat current because the thermal energy should be counted from the electrochemical potential.

In the interior of the sample, the electric current consists of the transport and magnetization components, $\mathbf{j}^e_{inr} = \mathbf{j}^e_{tr} + \mathbf{j}^e_{mag}$. The magnetization component is[23]

$$\mathbf{j}^e_{mag} = c\frac{\partial\mu}{\partial T}(\nabla T\times\mathbf{H}). \tag{18}$$

The magnetization currents are divergence-free. The total magnetization current through the sample cross section must be zero, i.e., the bulk magnetization currents are canceled by the surface currents. Therefore, the Nernst coefficient measured in the finite sample is determined by the transport currents:[23] $N = \mathbf{j}^e_{tr}/[\nabla T\times\mathbf{H}]$. The Nernst coefficient in the infinite sample [Eq. (16)] is associated with the bulk current in the finite sample, $N_{inf} = \mathbf{j}^e_{inr}/[\nabla T\times\mathbf{H}]$. Using Eq. (18), we get

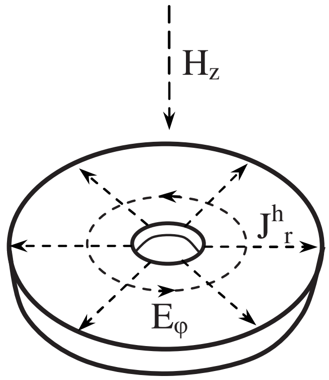

FIG. 4. The interior heat currents and corresponding Ettingshausen coefficient Υ_{inf} can be measured in the Gorbino disk geometry with the circular inductive electric field. The interior electric currents and the Nernst coefficient N_{inf} cannot be measured because the circular temperature gradient does not exist.

$$\delta N = \frac{\mathbf{j}^{e}_{inr} - \mathbf{j}^{e}_{mag}}{[\nabla T \times \mathbf{H}]} = \delta N_{inf} - c\frac{\partial \mu}{\partial T}. \tag{19}$$

Taking into account Eq. (3), we see that the second term in the last equation, $c\partial\mu/\partial T$, cancels completely the second term in δN_{inf} [Eq. (16)]. The rest is equal to δN, which satisfies the Onsager relation $\delta N = \delta\Upsilon/T$.

Note that if in contradiction to our results the surface magnetization currents provide the heat transfer, this effect could be found in transport measurements. In the Gorbino disk geometry with the magnetic field perpendicular to the disk and the circular inductive electric field in the plane (Fig. 4), the heat current in the radial direction does not contain the surface components, which were predicted for the standard parallelepiped geometry in Refs. 18, 19, and 24. According to our results, both experiments will give the same results. The surface electric current generated by ∇T is very significant [see Eq. (18)]. However, it cannot be experimentally separated from the interior current because, contrary to the circular electric field, the circular temperature gradient does not exist. This difference between $\mathbf{E}$ and ∇T is reflected in the asymmetry of the Nernst and Ettingshausen coefficients calculated for the infinite sample.

V. PARTICLE-HOLE ASYMMETRY IN THERMOMAGNETIC EFFECTS

Now, we show that in the general case, the interelectron interaction cannot change PHA requirements for N and Υ, i.e., the thermomagnetic coefficients are always proportional to the square of PHA.

Assuming that electron scattering from impurities is the main mechanism of the momentum relaxation, it is easy to see[25,27,29] that the magnetic field and temperature gradient enter into the transport equation formalism through the distribution functions of noninteracting electrons and the Poisson brackets. In fact, the terms proportional to $\nabla T \times \mathbf{H}$ can appear in three different ways:[25] (a) through the Nernst nonequilibrium distribution function of noninteracting electrons, $(e\tau^2/cm)\mathbf{v}\cdot[\nabla T \times \mathbf{H}](\partial S/\partial T)$, (b) through the $\mathbf{H}$-Poisson bracket that involves the nonequilibrium distribution function under the temperature gradient, $-e\tau(\mathbf{v}\cdot\mathbf{E})(\partial S/\partial T)$, and, finally, (c) due to double, ∇T- and $\mathbf{H}$-, Poisson brackets. It is evident that (a)- and (b)-type terms have already included the Hall PHA, which is proportional to $(\partial v/\partial p) = 1/m$. The (c)-type terms in the form of the double Poisson brackets describe the AL process, which has been investigated above. As we have seen, the AL diagram gives the contribution in the zeroth order in PHA; however, this contribution is canceled by the contribution of the surface magnetization currents. Thus, we conclude that the interelectron interaction can provide many-body thermomagnetic effects only in the second order in PHA.

VI. CONCLUSIONS

We have shown that the magnetic term in the Hamiltonian of charged particles [Eq. (1)] should be associated with the thermal energy. The corresponding term in the heat current operator [Eq. (5)] restores the gauge invariance and gives important contribution to the Ettingshausen coefficient. We also found that the surface magnetization currents do not contribute to the heat current but provide substantial contribution to the charge transfer in the Nernst effect [Eq. (19)]. Our gauge-invariant scheme gives the thermomagnetic coefficients that satisfy the Onsager relation [see Eqs. (19) and (16)]. In the general case of the Fermi liquid with particle-hole excitations, we conclude that the measured thermomagnetic coefficients are always proportional to the square of PHA. Any interaction by itself, i.e., without changing the electron band structure or character of elementary excitations, cannot provide large thermomagnetic effects.

The developed approach has been applied to effects of superconducting fluctuations (GFT) above the mean-field transition temperature. We have shown that the gauge-invariant form of the heat current operator of fluctuating pairs is $\nu\alpha\omega(\mathbf{q}+2e\mathbf{A}_H/c)$. The second (magnetic) term missed in previous publications plays an important role: as in the case of noninteracting electrons (Fig. 1), the corresponding diagram (Fig. 2) cancels completely the large, zeroth order in the PHA term in the Ettingshausen coefficient. We also show that the Nernst coefficient in the infinite superconducting sample consists of an anomalously large, zeroth order in PHA term [Eq. (16)]. However, in the finite sample, this term is canceled by the surface magnetization currents [Eq. (19)] and thermomagnetic coefficients satisfy the Onsager relation. Our results for δN are different by a huge factor of $(\epsilon_F/T)^2$ from the previous works,[16–20] which claim that the attractive interaction in the Cooper channel provides thermomagnetic transport without PHA at all. We can also easily rebuff this claim, if we just change a sign of the interaction constant and consider the repulsive interaction in the Cooper channel. As is known, in ordinary metallic films, this interaction results in corrections to conductivity, which are a factor of $(\epsilon_F\tau)^{-1}$ smaller than the conductivity of noninterating electrons.[30] If the statement of Refs. 16–20 is correct, the thermomagnetic effects in ordinary metals would be $(\epsilon_F\tau)^{-1}(\epsilon_F/T)^2 \sim (\epsilon_F/T)/(T\tau)$ larger than those predicted for noninteracting electrons. Certainly, this huge effect is not known.

No thermomagnetic measurements have shown huge fluctuation effects in ordinary superconductors, such as Nb, Al, and Sn. While claiming the opposite, the very recent work on highly disordered (close to the metal-dielectic transition), superconducting NbSi (Ref. 31) also does demonstrate fluctuation phenomena, because the large Nernst coefficient was observed in the wide temperature range $T-30T_c$, which drastically exceeds the fluctuation region and, therefore, it is not related to fluctuation phenomena.

Returning to the problem of high-T_c superconductors, we should note that in a number of models, including fluctuation exchange[15] and preformed pairs,[14] the large Nernst effect has been calculated above the mean-field transition temperature in zeroth order in PHA due to solely interaction effects. This work shows that any interaction by itself cannot lead to thermomagnetic effects in the Fermi liquid without PHA.

As we have shown here, large thermomagnetic effects require elementary excitations other than particles and holes.[32] While a vortex theory consistent with the Onsager relation should still be developed,[2,3,33,34] there is no doubt that the magnetic vortices above the BKT-like transition but below the mean-field transition can be such excitations.[34] Note that the vortex scenario is also employed for cuprates to explain magnetization data above T_c, which is associated with the BKT transition.[35,36] However, the magnetization data can be also understood in the GFT, where T_c is associated with the mean-field transition.[37] Therefore, the magnetization data by themselves do not provide solid evidence of vortices and related specific phase fluctuations.[37–39] According to our results and in agreement with numerous data in traditional superconductors,[34] strong fluctuation diamagnetism above the mean-field transition temperature does not correlate with the small Nernst effect. Therefore, the thermomagnetic phenomena rather than magnetization can help identify the nature of superconducting transition and provide information about elementary excitations in the electron system.

Using GFT as an example, we have shown that the gauge-invariant form of the heat current is critical for microscopic description of the Ettingshausen effect and that the surface currents are important for the Nernst effect. Calculations related to other models including vortex thermomagnetic transport should be reconsidered taking into account the correct form of the heat current operator as well as the electric current due to magnetization in the presence of ∇T.

ACKNOWLEDGMENTS

We would like to acknowledge useful discussions with A. Larkin, D. Livanov, A. Varlamov, and I. Ussishkin at the initial step of this work. We are grateful to I. Aleiner for very valuable discussion of charge and energy transfers by magnetization currents. In addition, two of us (A.S. and V.M.) acknowledge support provided by NYSTAR.

APPENDIX A: KUBO METHOD FOR NONINTERACTING ELECTRONS

In this appendix, we present detailed calculations of the Ettingshausen coefficient for noninteracting electrons using the Kubo method.

According to Eq. (4), the heat current vertex for noninteracting electrons in the magnetic field is given by

$$\gamma^h = \gamma_1^h + \gamma_2^h = \xi_p \mathbf{v} + (e/c)(\mathbf{v}\cdot\mathbf{A}_H)\mathbf{v}. \tag{A1}$$

Two vortices γ_1^h and γ_2^h create two diagrams, shown in Fig. 1. The solid lines in the diagrams represent the electron Green functions,

$$G_p^R = [G_p^A]^* = (\epsilon - \xi_p + i/2\tau)^{-1}. \tag{A2}$$

To get the Ettingshausen coefficient proportional to $\mathbf{A}(\mathbf{k}\cdot\mathbf{E})$, one should expand the Green function $G(\mathbf{p}+\mathbf{k})$ in powers of $\mathbf{k}\cdot\mathbf{v}$. Then, the contribution of the first diagram is given by

$$\Upsilon_1 = \frac{e^2}{cH}\int \frac{d\mathbf{p}}{(2\pi)^n}\frac{d\epsilon}{2\pi}\frac{\partial S_0}{\partial\epsilon}\xi_p \mathbf{v}(\mathbf{v}\cdot\mathbf{A})(\mathbf{v}\cdot\mathbf{E})(\mathbf{v}\cdot\mathbf{k})I_1, \tag{A3}$$

where $S_0 = -\tanh(\epsilon/2T)$, n is the dimensionality of the system, and the combination of the Green functions is

$$I_1 = 2iG_p^A G_p^R \,\mathrm{Im}(G_p^A)^2 = 2i\tau^2 \,\mathrm{Im}(G_p^A)^2. \tag{A4}$$

The contribution of the second diagram is

$$\Upsilon_2 = \frac{e^2}{cH}\int \frac{d\mathbf{p}}{(2\pi)^n}\frac{d\epsilon}{2\pi}\frac{\partial S_0}{\partial\epsilon}\mathbf{v}(\mathbf{v}\cdot\mathbf{A})(\mathbf{v}\cdot\mathbf{E})(\mathbf{v}\cdot\mathbf{k})I_2, \tag{A5}$$

where the combination of the Green functions I_3 is

$$I_2 = 2iG_p^A \,\mathrm{Im}(G_p^A)^2 = 2i\tau^2 \,\mathrm{Im}\, G_p^A. \tag{A6}$$

Integration over angles of the electron momentum in Eqs. (A3) and (A5) gives

$$\int d\Omega_{\mathbf{p}} \mathbf{v}(\mathbf{v}\cdot\mathbf{A})(\mathbf{v}\cdot\mathbf{k})(\mathbf{v}\cdot\mathbf{E}) = \frac{v^4}{n(n+2)}\mathbf{A}(\mathbf{k}\cdot\mathbf{E}). \tag{A7}$$

Then, the total contribution may be presented as

$$\Upsilon = \frac{ie^2}{n(n+2)cH}\int \frac{d\epsilon}{2\pi}d\xi_p \frac{\partial S_0(\epsilon)}{\partial\epsilon}\mathbf{A}(\mathbf{k}\cdot\mathbf{E})v^4\tau^2\nu(\xi_p \,\mathrm{Im}(G_p^A)^2 + \mathrm{Im}\, G_p^A). \tag{A8}$$

Without taking PHA into account, the total contribution of the two diagrams goes to zero after integration over ξ_p because

$$\int d\xi_p(\xi_p(G_p^A)^2 + \mathrm{Im}\, G_p^A) = 0. \tag{A9}$$

Nonzero contribution arises from terms proportional to ϵ^2; thus we should expand all electron parameters near the Fermi surface. For example, for a three-dimensional conductor,

$$v^4\nu = v_0^4\nu_0\left[1 + \frac{5}{2}\frac{\xi_p}{\epsilon_F} + \frac{15}{8}\left(\frac{\xi_p}{\epsilon_F}\right)^2 + \cdots\right], \tag{A10}$$

$$\tau^2 = \tau_0^2\left[1 - \frac{\epsilon}{\epsilon_F} - \left(\frac{\epsilon}{\epsilon_F}\right)^2 + \cdots\right]. \tag{A11}$$

Taking into account terms proportional to the square of PHA, e.g., ξ^2/ϵ_F^2 or $\xi\epsilon/\epsilon_F^2$, we get

$$\int d\xi_p v^4 \nu \tau^2 (\xi_p \operatorname{Im}[G^A(P)^2 + \operatorname{Im} G^A(P)\ )$$

$$= -\pi v_0^4 \nu_0 \tau_0^2 \frac{5}{4}\frac{\epsilon^2}{\epsilon_F^2} = -\pi \frac{5}{2}\frac{v_0^2 \tau_0^2 \nu_0}{m}\frac{\epsilon^2}{\epsilon_F}. \tag{A12}$$

Substituting this result into Eq. (A8) and performing integration over ϵ, we get the well-known result for the Ettingshausen coefficient of noninteracting electrons,

$$\Upsilon_{3D} = -\frac{\pi^2}{6}\frac{T^2}{\epsilon_F}(\Omega \tau_0)\frac{\sigma_{xx}}{H}, \tag{A13}$$

where $\Omega = eH/mc$ is the cyclotron frequency and σ_{xx} is the Drude conductivity. For two-dimensional conductors, the corresponding relation between Υ_{2D} and two-dimensional conductivity has an additional numeric factor of 2.

Thus, calculating the Ettingshausen coefficient of noninteracting electrons, we demonstrated that the magnetic field should be taken into account in the heat current vertex. The diagram with this magnetic vertex in the heat current cancels the basic diagrams in the zeroth order in PHA. The nonzero Ettingshausen coefficient arises only in the second order in PHA. As it has been shown in the main text, the above conclusions are also relevant to any many-body correction to thermomagnetic coefficients.

APPENDIX B: ASLAMAZOV-LARKIN BLOCKS

For an arbitrary electron momentum relaxation time τ, the AL blocks $\mathbf{B}^{e,h,H}$ built from electron Green functions $G^{R(A)}$ with electron vertices γ (γ^e, γ^H, γ_1^h, and γ_2^h) are given by[20]

$$\mathbf{B}_i^{e,h,H} = \operatorname{Im} \int \frac{d\mathbf{p}}{(2\pi)^n}\frac{d\epsilon}{2\pi}\gamma_i^{e,h,H} S_0(\epsilon)\frac{(G_p^A)^2 G_{q-p}^R}{(1-\zeta)^2}, \tag{B1}$$

$$\zeta = \frac{1}{\pi \nu \tau}\int \frac{d\mathbf{p}}{(2\pi)^3} G_p^A G_{q-p}^R, \tag{B2}$$

where the electron Green functions are given by Eq. (A2).

The block $\mathbf{B}^e$ with the electric current vertex, $\gamma^e = e\mathbf{v}\cdot\mathbf{e}_E$, may be presented as[20]

$$\mathbf{B}^e(\mathbf{q}) = 2e\nabla_{\mathbf{q}} P^R(\mathbf{q},0)\cdot \mathbf{e}_E = 2e\nu\alpha \mathbf{q}\cdot\mathbf{e}_E. \tag{B3}$$

The block $\mathbf{B}^H$ with the vertex $\gamma^H = (e/c)\mathbf{v}\cdot\mathbf{A}_H$ is given by[20]

$$\mathbf{B}^A(\mathbf{q}) = \frac{2e}{c}\nu\alpha \mathbf{q}\cdot\mathbf{A}_H. \tag{B4}$$

The block $\mathbf{B}_1^h$ with the kinetic heat current vertex, $\gamma_1^h = \xi \mathbf{v}\cdot\mathbf{e}_{j^h}$ ($\mathbf{e}_{j^h} = \mathbf{j}^h/j^h \parallel \mathbf{A}$), is given by[27] (see also Refs. 18–20)

$$\mathbf{B}_1^h(\mathbf{q},\omega) = \omega \nabla_{\mathbf{q}} P^R(\mathbf{q},0)\cdot\mathbf{e}_{j^h} = \omega\nu\alpha\mathbf{q}\cdot\mathbf{e}_{j^h}. \tag{B5}$$

Next, we calculate the block $\mathbf{B}_2^h$ with the magnetic heat current vertex $\gamma_2^h = (\mathbf{v}\cdot\mathbf{A}_H)(\mathbf{v}\cdot\mathbf{e}_{j^h})$. The integral over angles of the electron momentum involves only the vertex γ_2^h because the heat current is in the direction of $\mathbf{A}_H$. To obtain an imaginary part in Eq. (B1), the integral

$$\int d\xi (G_p^A)^2 G_{q-p}^R = \frac{2\pi i}{(2\epsilon - \omega - \mathbf{q}\cdot\mathbf{v} - i/\tau)^2} \tag{B6}$$

should be expanded in ω (in calculations of $\mathbf{B}_e$, it is expanded in $\mathbf{q}\cdot\mathbf{v}$). Finally, we get

$$\mathbf{B}_2^h(\mathbf{q},\omega) = 2\omega\nabla_{\mathbf{q}}^2 P^R(\mathbf{q},0)A_H = 2(e/c)\omega\nu\alpha A_H. \tag{B7}$$

APPENDIX C: GAUGE $\mathbf{E}=i\Omega\mathbf{A}_E/c$

As we stressed above, our approach is gauge invariant. Here, we briefly demonstrate how the above results can be directly obtained in the gauge, where $\mathbf{E} = i\Omega\mathbf{A}_E/c$ and $\mathbf{H} = i[\mathbf{k}\times\mathbf{A}_H]$. In the electric and magnetic fields, the kinetic energy has the form $K = (\mathbf{p} + e\mathbf{A}/c)^2/2m$, and the part of the Hamiltonian describing the interaction with external fields is given by

$$H' = \frac{e}{mc}\mathbf{p}(\mathbf{A}_H + \mathbf{A}_E) + \frac{e^2}{2mc^2}(\mathbf{A}_H + \mathbf{A}_E)^2, \tag{C1}$$

Calculating the response to $\mathbf{E}\times\mathbf{H} = (\Omega/c)[\mathbf{A}_H(\mathbf{k}\cdot\mathbf{A}_E) - \mathbf{k}(\mathbf{A}_E\cdot\mathbf{A}_H\ ]$, it is convenient to use the gauge conditions $\mathbf{k}\cdot\mathbf{A} = 0$ and $\mathbf{A}_H\cdot\mathbf{A}_E = 0$.[26] In this gauge, the second term in Eq. (C1) can be neglected. By including the interaction with the magnetic field, we get the heat current operator,

$$\hat{\mathbf{J}}^h = \sum_{\mathbf{p}} \mathbf{v}\xi_p a_{\mathbf{p}}^+ a_{\mathbf{p}} + \sum_{\mathbf{p}} \frac{e\mathbf{v}}{c}(\mathbf{v}\cdot\mathbf{A}_H) a_{\mathbf{p}}^+ a_{\mathbf{p}}. \tag{C2}$$

As it is expected, the term in the heat current describing the interaction with the magnetic field is independent of the presentation of the electric field [see Eqs. (5) and (A1)]. Therefore, all further calculations of thermomagnetic coefficient are the same as in the gauge $E = -\nabla\phi$.

*asergeev@eng.buffalo.edu

[1] R. M. White, *Quantum Theory of Magnetism* (Springer-Verlag, Berlin, 1983).
[2] K. Maki, Phys. Rev. Lett. **21**, 1755 (1968).
[3] C.-R. Hu, Phys. Rev. B **13**, 4780 (1976).
[4] M. J. Stephen, Phys. Rev. Lett. **16**, 801 (1966).
[5] C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967).
[6] Z. A. Xu, N. P. Ong, Y. Wang, T. Kakeshita, and S. Uchida, Nature (London) **406**, 486 (2000).
[7] Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, Science **299**, 86 (2003).
[8] Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B **73**, 024510 (2006).
[9] P. A. Lee, N. Nagaosa, and X. G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
[10] V. J. Emery and S. A. Kivelson, Nature (London) **374**, 434 (1995).

[11] O. Vafek and Z. Tesanovic, Phys. Rev. Lett. **91**, 237001 (2003).
[12] P. W. Anderson, arXiv:cond-mat/0603726 (unpublished).
[13] A. M. Tsvelik and A. V. Chubukov, Phys. Rev. Lett. **98**, 237001 (2007).
[14] S. Tan and K. Levin, Phys. Rev. B **69**, 064510 (2004).
[15] H. Kontani, Phys. Rev. Lett. **89**, 237003 (2002); Phys. Rev. B **67**, 014408 (2003).
[16] S. Ullah and A. T. Dorsey, Phys. Rev. Lett. **65**, 2066 (1990); Phys. Rev. B **44**, 262 (1991).
[17] A. A. Varlamov and D. V. Livanov, Sov. Phys. JETP **72**, 1016 (1991).
[18] I. Ussishkin, S. L. Sondhi, and D. A. Huse, Phys. Rev. Lett. **89**, 287001 (2002).
[19] I. Ussishkin, Phys. Rev. B **68**, 024517 (2003).
[20] A. Larkin and A. Varlamov, *Theory of Fluctuations in Superconductors* (Oxford University Press, New York, 2005).
[21] T. C. Harman and J. M. Honig, *Thermoelectric and Thermomagnetic Effects and Applications* (McGraw-Hill, New York, 1967).
[22] J. J. Krempasky and A. Schmid, J. Low Temp. Phys. **34**, 197 (1979).
[23] N. R. Cooper, B. I. Halperin, and I. M. Ruzin, Phys. Rev. B **55**, 2344 (1997).
[24] S. Mukerjee and D. A. Huse, Phys. Rev. B **70**, 014506 (2004).
[25] M. Reizer and A. Sergeev, Phys. Rev. B **61**, 7340 (2000).
[26] B. L. Altshuler, D. Khmel'nitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
[27] M. Yu. Reizer and A. V. Sergeev, Phys. Rev. B **50**, 9344 (1994).
[28] H. Fukuyama, H. Ebisawa, and T. Tsuzuki, Prog. Theor. Phys. **46**, 1028 (1971).
[29] A. Sergeev, M. Yu. Reizer, and V. Mitin, Phys. Rev. B **66**, 104504 (2002).
[30] B. L. Altshuler and A. G. Aronov, *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Polak (North-Holland, Amsterdam, 1985).
[31] A. Pourret, H. Aubin, J. Lesueur, C. Marrache-Kikuchi, L. Berge, L. Dumoulin, and K. Behnia, Nat. Phys. **2**, 683 (2006).
[32] Note that the two-band model with electron and hole bands also lead to large thermomagnetic effects (see Ref. 21), but this possibility seems to be excluded by band-structure calculations in cuprates.
[33] H. Takayama and H. Ebisawa, Prog. Theor. Phys. **44**, 1450 (1970).
[34] R. P. Huebener, *Magnetic Flux Structures in Superconductors* (Springer-Verlag, Berlin, 1979) and references therein.
[35] Y. Wang, L. Li, M. J. Naughton, G. D. Gu, S. Uchida, and N. P. Ong, Phys. Rev. Lett. **95**, 247002 (2005).
[36] Lu Li, J. G. Checkelsky, Seiki Komiya *et al.*, Nat. Phys. **3**, 311 (2007).
[37] L. Cabo, J. Mosqueira, and F. Vidal, Phys. Rev. Lett. **98**, 119701 (2007).
[38] C. Bergemann, A. W. Tyler, A. P. Mackenzie, J. R. Cooper, S. R. Julian, and D. E. Farrell, Phys. Rev. B **57**, 14387 (1998).
[39] N. P. Ong, Y. Wang, L. Li, and M. J. Naughton, Phys. Rev. Lett. **98**, 119702 (2007).