

Exotic pseudospin Kondo effect in laterally coupled double quantum dots

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We study the pseudospin Kondo effect in laterally coupled double quantum dots using the slave-boson mean-field theory. We find that the exotic pseudospin Kondo effect appears when a coherent indirect coupling with parameter α is present through the common reservoirs. When $0 < |\alpha| < 1$, for finite interdot tunnel coupling, we obtain an asymmetric density of states due to pseudospin-dependent linewidth functions. The differential conductance reveals an additional structure in the split peaks caused by the interdot tunnel coupling because of the asymmetry of the density of states.

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The recent development of semiconductor quantum dots (QDs) whose electronic configurations are well defined¹ has revived interest in the Kondo effect. Using the wide tunability of various physical parameters, e.g., the number of electrons and the dot-lead tunnel coupling strength, various kinds of Kondo effect not seen in conventional metallic systems have been revealed, including enhancement of the Kondo effect by state degeneracy,^{2,3} the unitary limit of the Kondo effect,⁴ and the nonequilibrium Kondo effect.⁵ The Kondo effect for single QDs is usually promoted by coherent singlet coupling between an unpaired electron spin in a QD and the Fermi sea in the tunnel-coupled leads. For coupled double quantum dots (DQDs), another degree of freedom for the Kondo coupling or quasispin can be introduced. The pseudospin (PS) state is defined by an orbital state such that an extra electron is placed in one of the two coupled QDs. The PS Kondo effect has been actively studied for capacitively coupled DQDs as shown in Fig. 1(a).⁶⁻⁸ When the two reservoirs below or above the DQD are common, coherent interdot coupling occurs not only directly through the interdot tunnel barrier but also indirectly through the common reservoir as shown in Fig. 1(b).⁹⁻¹³ The latter is equivalent to the case for an Aharonov-Bohm interferometer,^{14,15} and can also significantly modify the Kondo effect. However, there have been no detailed studies about the effect of the indirect interdot coupling reported to date.

In this paper, we discuss the effect of quantum coherence on the PS Kondo effect in laterally coupled DQDs. The coherent direct tunnel coupling t_c between two QDs leads to level repulsion of the DQDs and is equivalent to the Zeeman effect for the PS. Then what kind of effect will be induced by the coherent indirect coupling between two QDs via the reservoirs? This corresponds to coherent coupling between two levels via a third common level, which is important for electromagnetically induced transparency¹⁶ (EIT) actively discussed in quantum optics. With EIT, this coherent coupling creates a superposition state (dark state) and the coupling between this state and the third level vanishes. As regards our problem, coherent indirect coupling between two QDs via the Fermi level (third level) in the reservoir leads to the quantum mechanical interference effect and this gives interesting results.¹⁷⁻¹⁹ In this paper, we discuss the effect of α on the Kondo problem. Although the modification of the Kondo

effect by coherent indirect coupling was discussed in Ref. 20, the situation was limited to $\alpha=1$. In DQDs, a down PS state becomes a dark state as in EIT when $\alpha=1$. As a result, the PS Kondo effect vanishes. The situations shown in Figs. 1(a) and 1(b) are very special, and various experimental conditions correspond to the intermediate condition shown in Fig. 1(c).^{13-15,21-24} For a highly symmetric configuration, Chudnovskiy investigated the coherent indirect coupling dependence of the Kondo temperature for the spin on PS SU(4) Kondo effect.²⁵ We neglect the spin degree of freedom and discuss the nonlinear transport. We find that there exist two kinds of peaks in the differential conductance because of formation of the dark state by modulation of the coherent indirect coupling. Moreover, we examine the change in the differential conductance as a result of the asymmetry of the system.

We consider DQDs coupled to two reservoirs as shown in Fig. 1(c). We assume just a single energy level for each QD

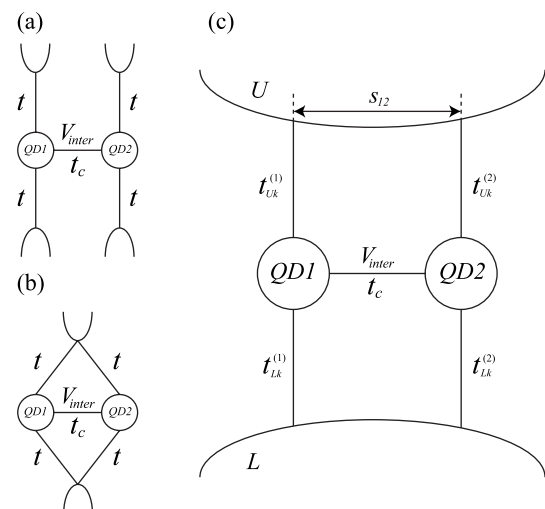


FIG. 1. Schematic diagrams of laterally coupled DQDs. (a) Two reservoirs are completely separated, namely, there is no tunneling process between the two QDs via the reservoirs. Here $\alpha=0$. (b) The electrons can tunnel indirectly only via a point in the reservoirs between the two QDs. Here $\alpha=1$. (c) $0 < |\alpha| < 1$. s_{12} is the minimum distance that electrons propagate in the reservoirs.

and ignore the spin degree of freedom. The Hamiltonian is

$$H = H_R + H_{\text{DQD}} + H_T, \quad (1)$$

where H_R describes the Fermi seas of noninteracting electrons in the two reservoirs

$$H_R = \sum_{\nu \in \{U,L\}} \sum_k \epsilon_{\nu k} c_{\nu k}^\dagger c_{\nu k}. \quad (2)$$

Here $\epsilon_{\nu k}$ is the electron energy with wave number k in the reservoir ν and the operator $c_{\nu k}$ ($c_{\nu k}^\dagger$) annihilates (creates) an electron in the reservoirs. H_{DQD} is the Hamiltonian of the isolated DQDs,

$$H_{\text{DQD}} = \sum_{i=1}^2 \epsilon_i d_i^\dagger d_i + t_c (d_1^\dagger d_2 + \text{H.c.}) + V_{\text{inter}} n_1 n_2, \quad (3)$$

where ϵ_i is the single-particle energy level and d_i (d_i^\dagger) annihilates (creates) an electron in the i th QD. The second term represents direct tunneling between two QDs. The last term describes the interdot Coulomb interaction, and $n_i = d_i^\dagger d_i$ is the number operator of the i th QD. H_T is the tunneling Hamiltonian between the reservoirs and QDs,

$$H_T = \sum_{\nu \in \{U,L\}} \sum_k \sum_{i=1}^2 (t_{\nu k}^{(i)} c_{\nu k}^\dagger d_i + \text{H.c.}), \quad (4)$$

where $t_{\nu k}^{(i)}$ is the tunneling amplitude between the reservoir ν and the i th QD.

We take account of the propagation of electrons in the reservoirs. This propagation process (in other words, the wave number dependence of $t_{\nu k}^{(i)}$) leads to coherent indirect coupling via the reservoir ν between two QDs,^{17,18} which is characterized by the parameter α^ν . Using this parameter α^ν , the linewidth function in the local basis is

$$\Gamma^\nu = \begin{pmatrix} \gamma_{\nu 1} & \alpha^\nu \sqrt{\gamma_{\nu 1} \gamma_{\nu 2}} \\ \alpha^\nu \sqrt{\gamma_{\nu 1} \gamma_{\nu 2}} & \gamma_{\nu 2} \end{pmatrix}, \quad (5)$$

where $\gamma_{\nu i} \equiv 2\pi \sum_k t_{\nu k}^{(i)*} t_{\nu k}^{(i)}$. In general, $|\alpha^\nu| \leq 1$.

We discuss the symmetric coupling between the reservoirs and DQDs, namely, $\gamma_{\nu i} = \gamma_\nu$, and $\alpha^\nu = \alpha$. The effect of asymmetric coupling is discussed in the last part of this paper. Then, the linewidth function is diagonalized:

$$\Gamma^\nu = \gamma_\nu \begin{pmatrix} 1 + \alpha & 0 \\ 0 & 1 - \alpha \end{pmatrix}. \quad (6)$$

Here we transformed the basis by using the symmetric and antisymmetric modes $d_s = (d_1 + d_2)/\sqrt{2}$ and $d_a = (d_1 - d_2)/\sqrt{2}$. With this new basis, the Hamiltonian is

$$H = \sum_{\nu \in \{U,L\}} \sum_k \epsilon_{\nu k} c_{\nu k}^\dagger c_{\nu k} + \sum_{n \in \{s,a\}} \epsilon_n d_n^\dagger d_n + V_{\text{inter}} n_s n_a + \frac{1}{2} \delta \epsilon (d_s^\dagger d_a + \text{H.c.}) + \sum_{\nu \in \{U,L\}} \sum_k \sum_{n \in \{s,a\}} (t_{\nu k}^{(n)} c_{\nu k}^\dagger d_n + \text{H.c.}), \quad (7)$$

where $\epsilon_s = \epsilon_0 + t_c$, $\epsilon_a = \epsilon_0 - t_c$, $\epsilon_0 = (\epsilon_1 + \epsilon_2)/2$ and $\delta \epsilon = \epsilon_1 - \epsilon_2$ are the average dot energy and the energy offset, respec-

tively. $n_{s(a)} = d_{s(a)}^\dagger d_{s(a)}$ is the number operator of the symmetric (antisymmetric) state, $t_{\nu k}^{(s)} = (t_{\nu k}^{(1)} + t_{\nu k}^{(2)})/\sqrt{2}$, and $t_{\nu k}^{(a)} = (t_{\nu k}^{(1)} - t_{\nu k}^{(2)})/\sqrt{2}$. Since H_{DQD} and the linewidth functions are simultaneously diagonalized when $\delta \epsilon = 0$, we focus on this condition. Then, the energy levels ϵ_s and ϵ_a correspond to the energies of the symmetric and antisymmetric states, respectively, where we choose a negative t_c .

It is noted that the symmetric state, antisymmetric state, interdot tunnel coupling, and interdot Coulomb interaction correspond to the spin-up state, spin-down state, Zeeman energy, and on-site Coulomb interaction, respectively, in the conventional spin $S=1/2$ Anderson model under a finite magnetic field. As shown in Eq. (6), when $\alpha \neq 0$, we have PS-dependent linewidth functions.

We assume that $V_{\text{inter}} \rightarrow \infty$, which prevents the occupation of both the symmetric and antisymmetric states. To describe this situation, we adopt the slave-boson (SB) formalism.^{26,27} The SB representation includes the pseudofermion operator f_n^\dagger , which creates a singly occupied state with the PS n , and the SB operator b^\dagger , which creates an empty state. Moreover, the following constraint has to be imposed on these operators: $b^\dagger b + \sum_{n \in \{s,a\}} f_n^\dagger f_n = 1$.

In the physical subspace defined by this constraint, the annihilation (creation) operators of the DQDs are replaced by $d_n = b^\dagger f_n$ ($d_n^\dagger = f_n^\dagger b$). In the mean-field theory (MFT), the SB operator is replaced by a constant real number \bar{b} and the Hamiltonian (7) can be rewritten as

$$H_{\text{SB}} = \sum_{\nu \in \{U,L\}} \sum_k \epsilon_{\nu k} c_{\nu k}^\dagger c_{\nu k} + \sum_{n \in \{s,a\}} \epsilon_n f_n^\dagger f_n + \sum_{\nu \in \{s,a\}} \sum_k \sum_{n \in \{s,a\}} (\bar{b} t_{\nu k}^{(n)} c_{\nu k}^\dagger f_n + \text{H.c.}) + \lambda \left(\bar{b}^2 + \sum_{n \in \{s,a\}} f_n^\dagger f_n - 1 \right), \quad (8)$$

where λ is a Lagrange multiplier. We determine \bar{b} and λ by minimizing the expectation value of the Hamiltonian (8). Then the effective energy level and linewidth function are $\tilde{\epsilon}_n = \epsilon_n + \lambda$, $\tilde{\Gamma}_n^\nu = \bar{b}^2 \Gamma_n^\nu$, and $\tilde{\Gamma}_n = \tilde{\Gamma}_n^U + \tilde{\Gamma}_n^L$, respectively. From the constraint and the equation of motion of the SB operator, the self-consistent equations with unknown parameters \bar{b} and λ are

$$\sum_{n \in \{s,a\}} \int \frac{d\epsilon}{2\pi\hbar i} G_n^<(\epsilon) + \bar{b}^2 - 1 = 0, \quad (9)$$

$$\sum_{n \in \{s,a\}} \int \frac{d\epsilon}{2\pi\hbar i} G_n^<(\epsilon) (\epsilon - \tilde{\epsilon}_n) + 2\lambda \bar{b}^2 = 0, \quad (10)$$

in terms of the Fourier transform of the lesser Green's function of the DQDs, $G_n^<(t, t') \equiv i \langle f_n^\dagger(t') f_n(t) \rangle$.

The current through the DQDs is written as²⁸

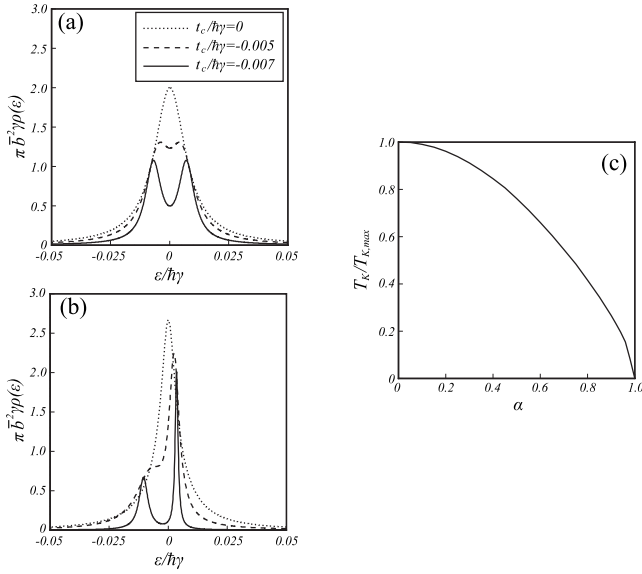


FIG. 2. Density of states when $\epsilon_0/\hbar\gamma=-3$ and $V_{SD}=0$. $\alpha=(a)$ 0 and (b) 0.5. (c) The α dependence of the Kondo temperature when $t_c=0$.

$$I = \frac{e}{h} \sum_{n \in \{s,a\}} \frac{\tilde{\Gamma}_n^U \tilde{\Gamma}_n^L}{\tilde{\Gamma}_n^U + \tilde{\Gamma}_n^L} \int d\epsilon [f_U(\epsilon) - f_L(\epsilon)] \rho_n(\epsilon, eV_{SD}), \quad (11)$$

where $f_\nu(\epsilon) = 1/(1 + e^{(\epsilon - \mu_\nu)/k_B T})$ with $\nu = U/L$ is the Fermi-Dirac distribution function. The upper and lower reservoirs have chemical potentials $\mu_U = \mu + eV_{SD}/2$ and $\mu_L = \mu - eV_{SD}/2$ with the source-drain bias voltage V_{SD} , and $\mu = 0$ is the origin of the energy. $\rho_n(\epsilon, eV_{SD})$ is the density of states (DOS) for the PS n state given by $\rho_n(\epsilon, eV_{SD}) = \frac{\tilde{\Gamma}_n}{[(\epsilon - \tilde{\epsilon}_n)/\hbar]^2 + (\tilde{\Gamma}_n)^2}$.

In the following discussions, we examine only $\epsilon_0/\hbar\gamma = -3$ (Kondo regime) and $\gamma_U = \gamma_L$. We numerically solve the set of nonlinear equations (9) and (10) at zero temperature for $D/\hbar\gamma = 100$, where D is the width of the conduction band in the reservoirs. First, we discuss the linear response regime. The total DOS $\rho(\epsilon) = \rho_s(\epsilon) + \rho_a(\epsilon)$ is shown in Figs. 2(a) and 2(b). When $\alpha = 1$, we have only a single conduction mode.¹⁸ Therefore, we do not expect the PS Kondo effect. In the following, we discuss the situation when $|\alpha| < 1$, where there are two conduction modes. Then, the PS of the DQDs is screened by the PS modes in the reservoirs and the PS Kondo effect occurs. As shown in Figs. 2(a) and 2(b) by the dotted lines, the full width at half maximum (FWHM) of the Kondo resonant peak, which characterizes the Kondo temperature,^{29,30} decreases as α increases. The α dependence of the Kondo temperature, shown in Fig. 2(c), is nearly proportional to $\sqrt{1 - \alpha^2}$. The Kondo effect is suppressed since $\alpha \neq 0$ leads to the polarization of the PS.

The interdot tunnel coupling causes the Kondo resonant peak at the Fermi level to split symmetrically with respect to the Fermi level ($\epsilon = 0$) when $\alpha = 0$ [Fig. 2(a)]. This corresponds to the conventional spin $S = 1/2$ Anderson model with

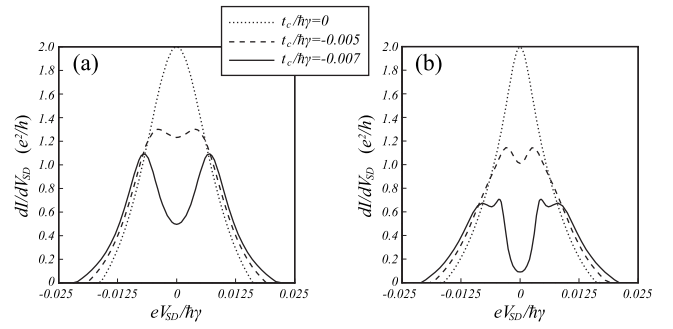


FIG. 3. dI/dV_{SD} curves when $\epsilon_0/\hbar\gamma = -3$. $\alpha=(a)$ 0 and (b) 0.5.

a finite Zeeman splitting. For $0 < |\alpha| < 1$ [Fig. 2(b)], however, the Kondo resonant peak splits asymmetrically due to the interdot tunnel coupling. The FWHMs of the two Kondo resonant peaks shown in Fig. 2 are proportional to $1 + \alpha$ and $1 - \alpha$.

In Fig. 3, we plot the differential conductance dI/dV_{SD} . As seen from Eq. (11), dI/dV_{SD} is given by $dI/dV_{SD} \propto \tilde{b}^2 [\rho(eV_{SD}/2, eV_{SD}) + \rho(-eV_{SD}/2, eV_{SD})]$. It is noted that the non-equilibrium calculation using the SB MFT is restricted to low bias voltages $|eV_{SD}| \ll |\epsilon_0|$. When there is no interdot tunnel coupling (dotted line in Fig. 3), we find a zero-bias peak at the Fermi level. It is well known that interdot tunnel coupling causes the zero-bias peak to split and the linear conductance (dI/dV_{SD} at zero bias) is suppressed. When $\alpha = 0.5$, there is an additional structure in each split peak for larger interdot tunnel coupling [solid line in Fig. 3(b)]. The subpeak at the smaller (larger) $|V_{SD}|$ in the additional structure arises from the sharp peak at lower energy (broad peak at higher energy) in the asymmetric DOS shown by the solid line in Fig. 2(b).

Finally, we discuss the effect of the asymmetry in the tunnel coupling to the reservoirs. In general, since the proportionate coupling condition ($\Gamma^U \propto \Gamma^L$) may not always be satisfied in laterally coupled DQDs, we need to use the general current formula²⁸ instead of Eq. (11),

$$I = \frac{e}{h} \int d\epsilon [f_U(\epsilon) - f_L(\epsilon)] \text{Tr}\{\mathbf{G}^r(\epsilon) \mathbf{\Gamma}^U \mathbf{G}^a(\epsilon) \mathbf{\Gamma}^L\}. \quad (12)$$

In the following, we consider $\delta\epsilon = 0$ and $t_c = 0$. In the linear response regime, the DOS and the Kondo temperature depend only on the total linewidth function $\tilde{\Gamma}_n$. From Eq. (12), with asymmetric coupling, the differential conductance dI/dV_{SD} is no longer directly related to the DOS and depends on the individual linewidth functions Γ^U and Γ^L . For a fixed total linewidth function (the Kondo temperature is the same in the linear response regime)

$$\mathbf{\Gamma} = \gamma \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad (13)$$

we calculate dI/dV_{SD} for the following cases (Fig. 4). Case (i): $\Gamma^U = \Gamma^L = \frac{1}{2}\mathbf{\Gamma}$. This is equivalent to the case shown by the dotted line in Fig. 3(b). Case (ii): $\Gamma^U = \frac{3}{4}\mathbf{\Gamma}$ and $\Gamma^L = \frac{1}{4}\mathbf{\Gamma}$,

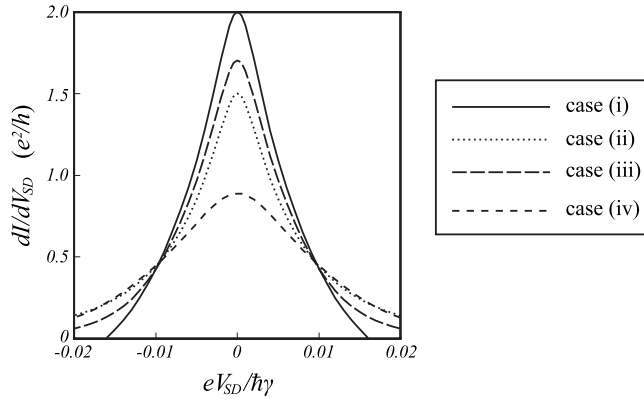


FIG. 4. dI/dV_{SD} curves when $\epsilon_0/\hbar\gamma = -3$, which is the same total linewidth function as cases (i), (ii), (iii), and (iv) (see text).

where dI/dV_{SD} is suppressed due to the prefactor in Eq. (11). The FWHM of case (ii) is the same ($\approx 0.01\hbar\gamma$) as that of case (i). In both cases (i) and (ii), proportionate coupling is satisfied, where dI/dV_{SD} is expressed by the DOS. In contrast, when proportionate coupling is not satisfied, as in case (iii), we consider the situation

$$\Gamma^U = \gamma \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 4 \\ 2 & 3 \end{pmatrix}, \quad \Gamma^L = \gamma \begin{pmatrix} 4 & 1 \\ 3 & 2 \\ 1 & 2 \\ 2 & 3 \end{pmatrix}, \quad (14)$$

where $\alpha_U = \alpha_L$. The FWHM ($\approx 0.011\hbar\gamma$) is larger than in cases (i) and (ii). Case (iv):

$$\Gamma^U = \gamma \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \Gamma^L = \gamma \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (15)$$

which corresponds to $\alpha^U = 1$ and $\alpha^L = 0$, and we have only a single conduction mode. However, there is Kondo screening due to the higher-order tunneling processes between the DQDs and one reservoir L . Then we have a zero-bias peak in dI/dV_{SD} . The maximum value of dI/dV_{SD} is less than 1 since there is one conduction mode (FWHM $\approx 0.02\hbar\gamma$).

In conclusion, we investigated the exotic PS Kondo effect in laterally coupled DQDs. We introduced the coherent indirect coupling parameter α . We have shown that there is no PS Kondo effect when $|\alpha| = 1$. When $0 < |\alpha| < 1$, the PS-dependent linewidth functions lead to an asymmetric DOS for finite interdot tunnel coupling. As a result, we expect two kinds of peak in the dI/dV_{SD} curve when the proportionate coupling for the linewidth functions is satisfied. From the sharpness of peaks in the dI/dV_{SD} curve, we can observe the formation of the dark state. In QQDs, however, the proportional coupling may not be satisfied and we cannot simply deduce dI/dV_{SD} from the DOS. We can no longer observe two kinds of peaks in the dI/dV_{SD} curve when proportionate coupling is not satisfied. Therefore, it should be noted that the symmetry of the system is important in order to observe the formation of the dark state clearly.

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