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Persistent spin current in nanodevices and definition of the spin current

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We investigate two closely related subjects: (i) the existence of a pure persistent spin current without an accompanying charge current in a semiconducting mesoscopic device with a spin-orbit interaction (SOI) and (ii) the definition of the spin current in the presence of SOI. Through physical argument from four physical pictures in different aspects, we provide strong evidences that the equilibrium persistent spin current does exist in a device with SOI in the absence of any magnetic field or magnetic materials. This persistent spin current is an analog of the persistent charge current in a mesoscopic ring threaded by a magnetic flux, and it describes the real spin motion and can be measured experimentally. We then investigate the definition of the spin current. We point out that (i) the nonzero spin current in the equilibrium SOI device is the persistent spin current, (ii) the spin current is, in general, not conserved, and (iii) the Onsager relation is violated for the spin transport no matter what definition of the spin current is used. These issues, the nonzero spin current in the equilibrium case, the nonconserved spin current, and the violation of the Onsager relation, are intrinsic properties of spin transport. We note that the conventional definition of the spin current has very clear physical intuition and describes the spin motion very well. Therefore, we feel that the conventional definition of the spin current makes physical sense, and there is no need to modify it. (Note that this conclusion is not in contradiction with the opinions in our previous papers). In addition, the relationship between the persistent spin current and transport spin current, the persistent linear and angular spin currents in the SOI region of the hybrid ring, and the measurement of the persistent spin current are discussed. Finally, we show that if the spin-spin interaction is included into the Hamiltonian, the persistent spin current is automatically conserved using the conventional definition.

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I. INTRODUCTION

In traditional charge-based electronics, the spin degree of freedom has not been fully explored. Until recently, the scientists found that spin control and manipulation in the nanoscale can enhance operational speed and integration density of conventional charge-based electronic devices.[1,2] In order to successfully use the spin degree of freedom of electrons in the conventional semiconductor devices, one has to have a good control, manipulation, and detection of the spin and its flow in nanodevices. This emerging field called spintronics is a new subdiscipline of condensed matter physics, and it is growing rapidly and generating great interests in recent years.[1,2]

It is well known that the spin-orbit interaction (SOI) plays an important role in the emerging field of semiconductor spintronics. SOI couples the spin degree of freedom of electrons to their orbital motions, thereby giving rise to a useful way to manipulate and control the spin of electron by an external electric field or a gate voltage. SOI is an intrinsic interaction having its origin from relativistic effects that can be quite significant in some semiconductors.[3] For instance, experiments show that the significant SOI indeed exists in some semiconductors;[4–8] e.g., Rashba SOI was found in the InGaAs/InAlAs heterostructure or InAs quantum wells,[4,5] and both the Rashba and Dresselhaus SOI were detected in various III-V material based two-dimensional (2D) structures at room temperature,[6] to name just a few. Moreover, the strengths of these SOI have been well modulated by the gate voltage experimentally.[7]

Many interesting effects resulting from SOI have been predicted. For example, using the effect of spin precessions due to the Rashba SOI, Datta and Das proposed a spin transistor more than ten years ago.[9] By using the Rashba SOI, a method to efficiently control and manipulate the spin of the electron in the quantum dot has been proposed.[10] Very recently, a very interesting effect, the intrinsic spin Hall effect, was theoretically predicted by Murakami *et al.*[11] and Sinova *et al.*[12] in a Luttinger SOI three-dimensional *p*-doped semiconductor and a Rashba SOI 2D electron gas, respectively, stating that a substantial amount of dissipationless spin current can be generated from the interplay between the electric field and the SOI. Since then, the spin Hall effect has generated tremendous interests with a great amount of works focusing in the field of spintronics.[13–18] On the experimental side, Kato *et al.*[19] and Wunderlich *et al.*[20] observed the transverse opposite spin accumulations near two edges of their devices when the longitudinal voltage bias was added. In addition, a third group by Valenzuela and Tinkham took the electric measurement of the reciprocal spin Hall effect,[21] and they observed an induced transverse voltage in a diffusive metallic conductor when a longitudinal net spin current flowed through it.

In this paper, we study two closely related subjects: (i) We predict another interesting effect that a persistent spin current without accompanying charge current exists in a coherent mesoscopic semiconductor ring with symplectic symmetry, i.e., with SOI but maintaining the time-reversal symmetry; (ii) we examine the issue of whether it is needed to redefine the conventional spin current or, in other words, whether the conventional definition of the spin current, $\mathbf{I}_S = \mathrm{Re}\{\Psi^\dagger \hat{v}\hat{s}\Psi\}$,[22] is reasonable in the presence of SOI. Since these two subjects are closely related, we investigate them together here. In fact, a few years ago, by using the conventional definition of the spin current, Rashba found a nonzero spin current I_S in an infinite two-dimensional system with Rashba SOI in the thermodynamic equilibrium.[23] In his opinion, this spin current I_S is not associated with real spin transport and, therefore, should be eliminated in calculating the transport current by modifying the conventional definition of the spin current. After his work, many subsequent works have discussed the definition of the spin current.[24–27] So, to uncover the physical meaning of the nonzero spin current in the equilibrium system, we must face the question of whether one needs to redefine the spin current.

More than two decades ago, the persistent (charge) current in a mesoscopic ring threaded by a magnetic flux has been predicted theoretically,[28] and later observed experimentally in the early 1990s.[29] It is now well known that the persistent charge current is a pure quantum effect and can sustain without dissipation in the equilibrium case. There has also been many investigations on the persistent spin current.[30–34] For example, in a mesoscopic ring with a crown-shape inhomogeneous magnetic field[30] or threaded by a magnetic flux,[31] the persistent spin current has been predicted and is related to Berry's phase. Recently, the persistent spin current carried by Bosonic excitations has also been predicted in a Heisenberg ring with the magnetic field or in the ferromagnetic material.[32] The reason that the persistent spin current exists may be explained as follows. Due to the magnetic field or the magnetic flux, there are persistent flows of both spin up and down electrons. In the absence of SOI, this gives rise to the well known persistent charge current. In the presence of SOI or magnetic field, the persistent charge current is spin polarized, resulting in a nonzero persistent spin current. Hence, the origin of this persistent spin current is the same as that of persistent charge current, so that the persistent spin current is always accompanied by a persistent charge current. Recently,[35,36] we have reported that a persistent spin current without accompanying charge current (a pure persistent spin current) can sustain in a mesoscopic semiconducting ring with SOI in the absence of the external magnetic field, magnetic flux, and magnetic material. This pure persistent spin current is induced solely by SOI, which is different from the previous case. In the main part of this paper, we will give detailed discussions on the existence of the persistent spin current and the behavior of the spin current.

Another motivation of the present paper is to study the definition of the spin current in the presence of SOI. In fact, the spin current is the most important physical quantity in the field of spintronics. So, it is important to give an appropriate definition of the spin current. This problem, the discussion of the definition of the spin current, was first tackled by Rashba. In a recent work by Rashba,[23] he found a nonzero spin current I_S in an infinite two-dimensional system with Rashba SOI even in the equilibrium case by using the conventional definition of the spin current,[22] $\mathbf{I}_S = \mathrm{Re}\,\Psi^\dagger \hat{v}\hat{s}\Psi$. So, he questioned the correctness of the conventional definition of the spin current and suggested that the conventional definition should be modified to eliminate the equilibrium nonzero spin current. Besides the nonzero spin current in the equilibrium, there are other problems with the conventional definition: (1) The spin current is not conserved using the conventional definition in the presence of SOI and/or the magnetic field. This is because the operator $\hat{s}$ does not commute with the Hamiltonian H when the system has SOI and/or the magnetic field. (2) The Onsager relation is violated using the conventional definition. After Rashba's work, many subsequent papers have discussed the definition of the spin current.[24–27] For instance, Sun and Xie suggested that in addition to the conventional (linear) spin current $\mathbf{I}_S = \mathrm{Re}\,\Psi^\dagger \hat{v}\hat{s}\Psi$, one needs to introduce the angular spin current $\mathbf{I}_\omega = \mathrm{Re}\,\Psi^\dagger (d\vec{s}/dt)\Psi = \mathrm{Re}\,\Psi^\dagger \hat{\omega}\times\hat{s}\Psi$ to describe the rotational motion (precession) of a spin[24] because the spin operator is a vector. Similar to the conventional linear spin current, the angular spin current can also induce an electric field. Shi *et al.*[26] gave a new definition of the spin current with $\mathbf{I}_S = \mathrm{Re}\,\Psi^\dagger d(\mathbf{r}\hat{s})/dt\Psi$, in which the operator of the spin current $\hat{I}_S = d(\mathbf{r}\hat{s})/dt$ is the derivative of the whole $\mathbf{r}\hat{s}$. Compared with the conventional definition of the spin current, it has an extra term $\mathbf{r}(d\hat{s}/dt)$ in this new definition. Wang *et al.*[27] pointed out that the spin current is automatically conserved using the conventional definition of the spin current if the spin-spin interaction is included. In Sec. VI in the present paper, we will explore the definition of the spin current.

In this paper, we will first give a physical argument as to why the persistent spin current can exist in equilibrium. In order to show that the persistent spin current should exist in the mesoscopic system with solely SOI, four physical pictures or physical arguments from the different aspects will be discussed: (i) from the picture of electric and magnetic correspondence to analyze the driving force of persistent spin current [in Figs. 1(a) and 1(b)]; (ii) from the point of view of the spin Berry phase; (iii) from the comparison among four effect (the Hall effect, the spin Hall effect, the persistent (charge) current, and the persistent spin current) (see Fig. 2); and (iv) from the point of view of the motion of the spin. As an example, we then consider a semiconducting SOI-normal hybrid mesoscopic ring and show that, indeed, a pure persistent spin current can emerge with solely SOI. We note that currently there is no consensus on the definition for the spin current in the presence of SOI. In order to avoid the problem of the definition of the spin current, here, we first use the following approach: We consider a mesoscopic hybrid ring

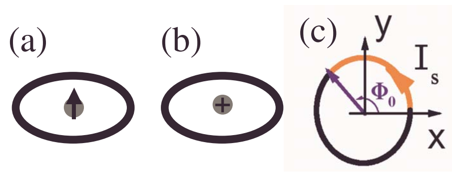

FIG. 1. (Color online) (a) and (b) are the schematic diagrams for a mesoscopic ring with a magnetic atom or an ion at its center. (c) Schematic diagram for a hybrid mesoscopic ring having the Rashba SOI in one part of the ring while the other part being normal.

that consists of a Rashba SOI's region and a normal region without SOI, as shown in Fig. 1(c). Since there is no spin flip in this normal region, the definition of the spin current in that region is without controversy. So, we can calculate and study the spin current in the normal region, and make sure that the persistent spin current can indeed be induced solely by SOI. After ensuring the existence of the persistent spin current, we then investigate the definition of the spin current. (i) We point out that the nonzero spin current in the equilibrium system in the presence of the SOI is the persistent spin current. It describes a real spin motion and has a physical meaning, so this spin current should be kept as it is. (ii) Since spin operator is a vector, it has a rotational motion (precession) due to the SOI accompanying the particle translational motion, so, in general, spin current is not a conserved quantity. In fact, there are experimental indications that the spin current is not conserved.[21] On the other hand, in certain cases, if one includes the strong spin-spin interaction into the Hamiltonian, the spin current obtained from the conventional definition will be conserved automatically. (iii) It is well known that the Onsager relation holds under certain conditions. We will show that for the spin system, this condition does not satisfy. We think that the above three points are actually the intrinsic properties of spin transport, so we feel that the conventional definition of the spin current need not be modified.[37]

In addition, we also address the following issues: (1) We note that persistent spin current and transport spin current cannot be distinguished from each other in the coherent part

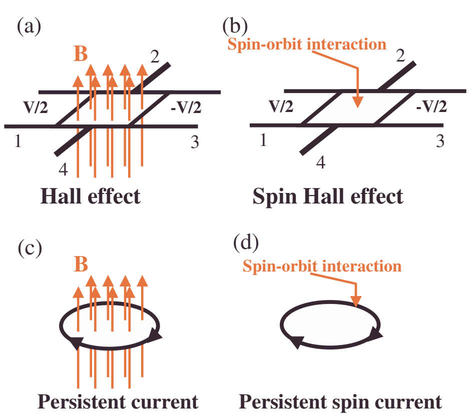

FIG. 2. (Color online) (a), (b), (c), and (d) are schematic diagrams for the devices of the Hall effect, the spin Hall effect, the persistent (charge) current, and the persistent spin current, respectively.

of the device. (2) In calculating the persistent linear and angular spin currents in the SOI's region of the SOI-normal hybrid ring, we find that the persistent spin current still exists in the SOI's region even if the whole ring has the SOI. So, the normal part is not necessary for generating the persistent spin current. (3) The measurement of the persistent spin current is discussed; we suggest that this persistent spin current can be observed by detecting its induced electric fields. (4) Including the spin-spin interaction in the ring, we find that the persistent spin current calculated using the conventional definition is a conserved quantity.

The rest of the paper is organized as follows. In Sec. II, we provide physical arguments and physical pictures from four different aspects to show the existence of the persistent spin current. In Sec. III, we consider a SOI-normal hybrid mesoscopic semiconducting ring device to show that, indeed, a pure persistent spin current can emerge in the normal region where the definition of the spin current is without controversy. Then, the effect of a sharp interface between the SOI's region and the normal region, and the relation of the persistent spin current and the transport spin current, are discussed in Sec. IV and V, respectively. In Sec. VI, we study the definition of the spin current. In Sec. VII, the persistent linear and angular spin currents in the SOI's region as well some conserved quantities in the hybrid ring device are explored. In Sec. VIII, we investigate electric fields induced by the persistent spin current. This provides a way to detect the persistent spin current. In Sec. IX, we discuss the effect of spin-spin interaction in the ring, which makes the persistent spin current conserved. Finally, Sec. X summarizes the results of our work.

II. PHYSICAL ARGUMENTS FOR THE EXISTENCE OF THE PERSISTENT SPIN CURRENT

In this section, we argue that the pure persistent spin current (without an accompanying charge current) should exist in the mesoscopic semiconducting ring device with the SOI. In particular, this pure persistent spin current can be induced solely by SOI even at zero magnetic flux or magnetic field. We examine this effect from the following four different aspects.

A. Analysis of the driving force

For the persistent charge current in the mesoscopic ring, the magnetic flux or magnetic field acts like a "driving force," so one naturally looks for the analogous driving force in the spin case. To discuss this question, let us consider two devices. The first device consists of a mesoscopic ring (without SOI) where a magnetic atom with a magnetic dipole moment is placed at the center of the ring [see Fig. 1(a)]. In the second device, the magnetic atom is replaced by a charged atom, e.g., an ion [see Fig. 1(b)]. The magnetic atom produces a vector potential **A** on the perimeter of the ring, which drives the persistent charge current. By analogy, a charged atom, which produces a scalar potential ϕ on the perimeter of the same ring, should drive a persistent spin current.[38] Since the presence of this ionic center generates a

SOI in the relativistic limit, we expect that this SOI, which plays the role of the spin driving force, will induce a pure persistent spin current. In addition, from the picture of electric and magnetic correspondence, the persistent charge current in the ring should also change into the persistent spin current when the center magnetic atom [in Fig. 1(a)] is substituted by a charged atom [in Fig. 1(b)].

B. From the point of view of the spin Berry phase

The existence of the pure persistent spin current can be examined from another point of view using the spin Berry phase.[39,40] This physical arguments has been provided in detail in our previous letter (see the fourth paragraph in Ref. 35), so we omit the discussion here.

C. Comparison among four effects: The Hall effect, the spin Hall effect, the persistent (charge) current, and the persistent spin current

In the following, let us compare the four effects: the Hall effect, the spin Hall effect, the persistent charge current, and the persistent spin current, from which one expect that the persistent spin current should exist in the mesoscopic ring with solely SOI.

(i) We consider a four-terminal device with a bias V added between terminals 1 and 3 and a perpendicular magnetic field $\mathbf{B}$ [as shown in Fig. 2(a)]. For this system, there exists the Hall effect, and a charge current is induced in the transverse terminals 2 and 4. (ii) Replacing the perpendicular magnetic field by the SOI [see Fig. 2(b)] a spin current emerges (instead of the charge current) in the transversal terminals 2 and 4. This is the spin Hall effect that was predicted recently and that generated tremendous interests.[11–17] (iii) Consider a mesoscopic ring with a perpendicular magnetic field [see Fig. 2(c)]; a persistent charge current is induced in the ring. This is the persistent current, which is well known now.[28,29] (iv) By analogy, a persistent spin current instead of the persistent charge current should be induced when the SOI replaces the perpendicular magnetic field [see Fig. 2(d)].

Let us discuss the Hamiltonian in four devices in Fig. 2. (i) In Fig. 2(a), the Hamiltonian is $H=\frac{(\mathbf{p}+e\mathbf{A}/c)^2}{2m}+V(\mathbf{r})$, and there exists the Hall effect because of the vector potential $\mathbf{A}$. (ii) To replace $\mathbf{A}$ by the Rashba SOI with $H=\frac{(\mathbf{p}+\alpha\hat{\sigma}\times\hat{z})^2}{2m}+V(\mathbf{r})$, the Hall effect change into the spin Hall effect, and a spin current instead of the charge current emerges in the terminals 2 and 4 [see Fig. 2(b)]. (iii) In Fig. 2(c), the Hamiltonian is $H=\frac{(\mathbf{p}+e\mathbf{A}/c)^2}{2m}+V(\mathbf{r})$, and there exists the persistent charge current in the ring because of the vector potential $\mathbf{A}$. (iv) By analogy, when $\mathbf{A}$ is replaced by the Rashba SOI, a persistent spin current should emerge instead of the persistent charge current.

D. From the point of view of the motion of the spin

By analyzing the motion of the spin, including the translational and rotational motions (precession) of a spin, one can also show the existence of the persistent spin current in the mesoscopic device with the SOI.[36] In order to analyze the motion of the spin, we need to solve the wave functions of the electron, so we put this analysis in the Appendix, where the wave functions have been solved.

III. PERSISTENT SPIN CURRENT IN THE NORMAL REGION OF THE SOI-NORMAL HYBRID RING

In this section, we present an example to show that, indeed, a pure persistent spin current can exist for a mesoscopic semiconducting ring with SOI. In the presence of SOI, the spin of an electron experiences a torque, and hence σ_i $(i=x,y,z)$ is not a good quantum number anymore. Because of this, the spin current is not conserved using the conventional definition. At present, there are controversies on whether one should define a conserved spin current or whether there exists a conserved spin current.[24–27] In other words, so far, there is no consensus on the definition for the spin current in the presence of SOI. In this section, we use the following approach. We first discuss the persistent spin current for a one-dimensional mesoscopic semiconducting ring that consists of a Rashba SOI's region and a normal region without SOI, as shown in Fig. 1(c). Since there is no spin flip in the normal region, the spin current can be calculated using a conventional definition without controversy, so that we can ensure the existence of the persistent spin current in the equilibrium case with solely SOI. After we ensure the existence of the persistent spin current, we then go back to examine the definition of the spin current, which is given in Sec. VI.

The Hamiltonian of our system is given by[31,41]

$$H=-E_a\frac{\partial^2}{\partial\varphi^2}-\frac{i\sigma_r}{2a}\left[\alpha_R(\varphi)\frac{\partial}{\partial\varphi}+\frac{\partial}{\partial\varphi}\alpha_R(\varphi)\right]-i\frac{\alpha_R(\varphi)}{2a}\sigma_\varphi, \tag{1}$$

where $E_a=\hbar^2/2ma^2$, a is the radius of the ring, m is the effective mass of the electron, $\sigma_r=\sigma_x\cos\varphi+\sigma_y\sin\varphi$, and $\sigma_\varphi=-\sigma_x\sin\varphi+\sigma_y\cos\varphi$. $\alpha_R(\varphi)$ is the strength of the Rashba SOI, $\alpha_R(\varphi)=0$ while $0<\varphi<\Phi_0$, i.e., in the normal region, and $\alpha_R(\varphi)$ is a constant α_R in the SOI's region with $\Phi_0<\varphi<2\pi$.

The eigenstates of Hamiltonian (1) can be solved numerically in the following way. First, in the Rashba SOI's region $(\alpha_R\neq 0)$, the equation $H\Psi(\varphi)=E\Psi(\varphi)$ has four independent solutions $\Psi_i^{SO}(\varphi)$ $(i=1,2,3,4)$:[31]

$$\Psi_{1/2}^{SO}(\varphi)=\begin{pmatrix}\cos(\theta/2)e^{ik_{1/2}\varphi}\\ -\sin(\theta/2)e^{i(k_{1/2}+1)\varphi}\end{pmatrix}, \tag{2}$$

and $\Psi_{3/4}^{SO}=\hat{T}\Psi_{1/2}^{SO}$, with $\hat{T}$ being the time-reversal operator. In Eq. (2), the wave vectors $k_{1/2}=-1/2+1/(2\cos\theta)\pm(1/2)\sqrt{(1/\cos^2\theta)-1+4E/E_a}$, and the angle θ is given by $\tan(\theta)=\alpha_R/(aE_a)$. Similarly, in the normal region $(0<\varphi<\Phi_0)$, the Schrödinger equation has four independent solutions: $\Psi_1^N(\varphi)=(1,0)^\dagger e^{ik\varphi}$, $\Psi_2^N(\varphi)=(1,0)^\dagger e^{-ik\varphi}$, and $\Psi_{3/4}^N=\hat{T}\Psi_{1/2}^N$, with $k=\sqrt{E/E_a}$. Second, the eigen-wave-function $\Psi(\varphi)$ with the eigenenergy E can be represented as

$$\Psi(\varphi)=\begin{cases}\sum_i a_i\Psi_i^N(\varphi) & \text{while } 0<\varphi<\Phi_0\\ \sum_i b_i\Psi_i^{SO}(\varphi) & \text{while } \Phi_0<\varphi<2\pi,\end{cases} \quad (3)$$

where a_i and b_i $(i=1,2,3,4)$ are constants to be determined by the boundary conditions at the interfaces $\varphi=0$ and Φ_0. Here, the boundary conditions are the continuity of the wave function $\Psi(\varphi)|_{\varphi=0^+/\Phi_0^+}=\Psi(\varphi)|_{\varphi=2\pi^-/\Phi_0^-}$ and the continuity of its flux[42] $\hat{v}_\varphi\Psi|_{\varphi=0^+/\Phi_0^+}=\hat{v}_\varphi\Psi|_{\varphi=2\pi^-/\Phi_0^-}$, where $\hat{v}_\varphi=(2aE_a/i\hbar)\times[\partial/\partial\varphi+(i/2)\sigma_r\tan(\theta)]$ is the velocity operator. By using the boundary conditions, we obtain eight series of linear equations:

$$a_1+a_2+\cos(\theta/2)e^{i2\pi k_1}b_1+\cos(\theta/2)e^{i2\pi k_2}b_2 + \sin(\theta/2)e^{-i2\pi k_1}b_3+\sin(\theta/2)e^{-i2\pi k_2}b_4=0, \quad (4)$$

$$a_3+a_4-\sin(\theta/2)e^{i2\pi k_1}b_1-\sin(\theta/2)e^{i2\pi k_2}b_2 + \cos(\theta/2)e^{-i2\pi k_1}b_3+\cos(\theta/2)e^{-i2\pi k_2}b_4=0, \quad (5)$$

$$e^{ik\Phi_0}a_1+e^{-ik\Phi_0}a_2+\cos(\theta/2)e^{ik_1\Phi_0}b_1+\cos(\theta/2)e^{ik_2\Phi_0}b_2 + \sin(\theta/2)e^{-i(k_1+1)\Phi_0}b_3+\sin(\theta/2)e^{-i(k_2+1)\Phi_0}b_4=0, \quad (6)$$

$$e^{-ik\Phi_0}a_3+e^{ik\Phi_0}a_4-\sin(\theta/2)e^{i(k_1+1)\Phi_0}b_1-\sin(\theta/2)e^{i(k_2+1)\Phi_0}b_2 + \cos(\theta/2)e^{-ik_1\Phi_0}b_3+\cos(\theta/2)e^{-ik_2\Phi_0}b_4=0, \quad (7)$$

$$ka_1-ka_2+A_1(k_1)e^{i2\pi k_1}b_1+A_1(k_2)e^{i2\pi k_2}b_2-A_2(k_1)e^{-i2\pi k_1}b_3 - A_2(k_2)e^{-i2\pi k_2}b_4=0, \quad (8)$$

$$-ka_3+ka_4-A_2(k_1)e^{i2\pi k_1}b_1-A_2(k_2)e^{i2\pi k_2}b_2 - A_1(k_1)e^{-i2\pi k_1}b_3-A_1(k_2)e^{-i2\pi k_2}b_4=0, \quad (9)$$

$$ke^{ik\Phi_0}a_1-ke^{-ik\Phi_0}a_2+A_1(k_1)e^{ik_1\Phi_0}b_1+A_1(k_2)e^{ik_2\Phi_0}b_2 - A_2(k_1)e^{-i(k_1+1)\Phi_0}b_3-A_2(k_2)e^{-i(k_2+1)\Phi_0}b_4=0, \quad (10)$$

$$-ke^{-ik\Phi_0}a_3+ke^{ik\Phi_0}a_4-A_2(k_1)e^{i(k_1+1)\Phi_0}b_1-A_2(k_2)e^{i(k_2+1)\Phi_0}b_2 - A_1(k_1)e^{-ik_1\Phi_0}b_3-A_1(k_2)e^{-ik_2\Phi_0}b_4=0, \quad (11)$$

where $A_1(x)\equiv x\cos(\theta/2)-\frac{1}{2}\tan(\theta)\sin(\theta/2)$ and $A_2(x)\equiv(x+1)\sin(\theta/2)-\frac{1}{2}\tan(\theta)\cos(\theta/2)$. The eigenvalue E_n can be solved numerically by setting the determinant of the coefficient of the variables a_i and b_i in the above eight series of linear equations to zero.

Now, we present the numerical results. Figure 3(a) shows the eigenvalues E_n versus the Rashba SOI's strength α_R. For the normal ring ($\alpha_R=0$), the eigenvalues are n^2E_a with fourfold degeneracy, and the corresponding eigenstates are $(1,0)^\dagger e^{\pm in\varphi}$ and $(0,1)^\dagger e^{\pm in\varphi}$.[43] As the SOI is turned on, the degenerate energy levels split while maintaining twofold Kramers degeneracy. The higher the energy level, the larger this energy split. Typically, the splits are on the order of E_a at $\alpha_R=10^{-11}$ eV m, with $E_a\approx 0.42$ meV for the ring's radius $a=50$ nm and the effective mass $m=0.036m_e$. The eigenvalues E_n versus the normal region's angle Φ_0 are also shown [see Fig. 3(b)]. For $\Phi_0=2\pi$, the whole ring is normal and E_n are fourfold degenerate.[43] When Φ_0 is away from 2π, the degenerated levels are split into two, and the splits are larger with the smaller Φ_0. When $\Phi_0=0$, the whole ring has the Rashba SOI, and the split reaches the maximum.

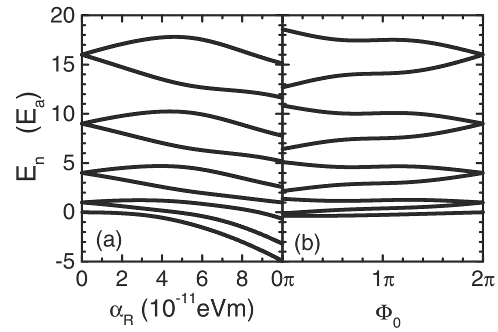

FIG. 3. (a) and (b) show the eigenenergies E_n vs α_R for $\Phi_0=\pi$ and vs Φ_0 for $\alpha_R=3\times10^{-11}$ eV m, respectively. The radius of the ring $a=50$ nm.

Since E_n is twofold degenerate, we obtain two eigenstates for each E_n, which are labeled $\Psi_n(\varphi)$ and $\hat{T}\Psi_n(\varphi)$.[44] With the wave functions, the spin current contributed from the level n can be calculated straightforwardly using the conventional definition $I^n_{S\varphi i}(\varphi)=\text{Re}\,\Psi_n^\dagger\hat{v}_\varphi\hat{\sigma}_i\Psi_n$ $(i=x,y,z)$. Notice that the spin current I_{Sji} is a tensor, where $i,j=r,\varphi,z$ in the cylinder coordinates and $i,j=x,y,z$ in the orthogonal coordinates. The first index j describes the direction of the motion of the electron, and the second index i represents the direction of the spin. Because the device in the present paper is a ring, the motion of the electron has no components along radial (r) and z axes. Hence, the spin currents I^n_{Sri} and I^n_{Szi} $[I^n_{S(r/z)i}(\varphi)=\text{Re}\,\Psi_n^\dagger\hat{v}_{r/z}\hat{\sigma}_i\Psi_n]$ are zero. Only spin current $I^n_{S\varphi i}$, with the electron moving along the φ direction, is nonzero. To simplify the notation, hereafter, we use the symbol I^n_{Si} to replace $I^n_{S\varphi i}$. Since there is a controversy about the definition of spin current in the SOI's region, we will calculate the spin current only in the normal region in this section. The spin current in the SOI's region will be studied in Sec. VII after the definition of the spin current is investigated in Sec. VI. In the normal region, the spin current is conserved, so $I^n_{Si}(\varphi)$ is independent of the angle coordinate φ.

Figure 4 shows the spin current I^n_{Si} versus the Rashba SOI's strength α_R for $\Phi_0=\pi$.[45] Since E_n is twofold degenerate, the wave functions can be an arbitrary combination of $\Psi_n(\varphi)$ and $\hat{T}\Psi_n(\varphi)$. However, the spin current I^n_{Si} remains the same. Our results in Fig. 4 show that I^n_{Sx} is exactly zero for all level n, while $\Phi_0=\pi$, and I^n_{Sy} and I^n_{Sz} exhibit the oscillatory pattern with α_R. A $\pi/2$-phase shift between I^n_{Sy} and I^n_{Sz} is observed, with $\sqrt{(I^n_{Sy})^2+(I^n_{Sz})^2}$ approximately constant. For two adjacent levels $2n-1$ and $2n$, their spin current have opposite signs, and $I^{2n-1}_{Si}+I^{2n}_{Si}=0$ if $\alpha_R=0$. We note that the spin current I^n_{Si} is quite large. For example, the value E_a is equivalent to the spin current of a moving electron in the ring with its speed of 4×10^5 m/s.

The spin current I^n_{Si} versus the angle Φ_0 that describes the normal region at a fixed $\alpha_R=3\times10^{-11}$ eV m is shown in Fig. 5. When $\Phi_0=2\pi$, the whole ring is normal, I^n_{Sy} and I^n_{Sz} are

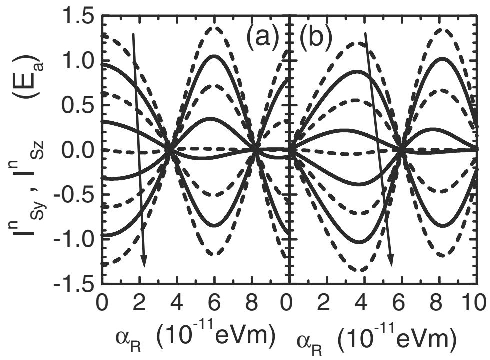

FIG. 4. I^n_{Sy} (a) and I^n_{Sz} (b) vs α_R for $\Phi_0=\pi$ and $a=50$ nm. Along the arrow direction, $n=7,5,3,1,0,2,4,6$, and 8. Here, the level indices $n=0,1,2,\ldots$, represent the ground state, the first excited state, the second excited state,…, respectively.

exactly zero. However, I^n_{Sx} is nonzero [see Fig. 5(a)]. Note that $I^{2n-1}_{Sx}+I^{2n}_{Sx}=0$ $(n=1,2,3,\ldots)$ and $I^0_{Sx}=0$ at $\Phi_0=2\pi$, so the total spin current I_{Sx} is still identically zero because the $(2n-1)$-th and the $2n$th states are degenerate and have the same occupied probability at $\Phi_0=2\pi$. When $\Phi_0\neq 2\pi$, where part of the ring has the SOI, three components $I^n_{Sx/y/z}$ of the spin current can be nonzero. For larger n, the absolute value of the spin current $|I^n_{Si}|$ is larger. For two adjacent levels $2n-1$ and $2n$, their spin current have opposite signs, which is similar to the result of Fig. 4.

Now, we calculate the equilibrium total spin current I_{Si} contributed from all occupied energy levels: $I_{Si}=2\sum_n I^n_{Si} f(E_n)$, where $f(E)=1/\{\exp[(E-E_F)/k_BT]+1\}$ is the Fermi distribution with the Fermi energy E_F and the temperature T, and the factor 2 is due to the Kramers degeneracy. The persistent charge current and the equilibrium spin accumulation are found to be zero because the system has a time-reversal symmetry. Figures 6(a) and 6(b) show the total spin currents I_{Si} versus the Rashba SOI's strength α_R for different Fermi energies E_F. One of the main results is that the spin current is indeed nonzero when $\alpha_R\neq 0$. The persistent spin currents I_{Si} in Fig. 6 have the following features. At $\alpha_R=0$, the whole ring is normal, so I_{Si} is exactly zero. With increasing α_R, I_{Si} increases initially and then oscillates for the large α_R. In Figs. 6(a) and 6(b), the parameter Φ_0 is π; i.e., half of the ring is normal and the other half has SOI; then, I_{Sx} is zero, and only I_{Sy} and I_{Sz} are nonzero. If $\Phi_0\neq\pi$, the components $I_{Sx/y/z}$ can be nonzero. At certain α_R, there is a jump in the curve of I_{Si} versus α_R. This is because for this α_R the Fermi energy E_F is in line with level E_n, leading to a change of its occupation. At zero temperature, the jump is abrupt, as shown in Figs. 6(a) and 6(b). However, at finite temperature, this jump will be washed out. In fact, these results are similar to the persistent (charge) current in the mesoscopic ring.[28]

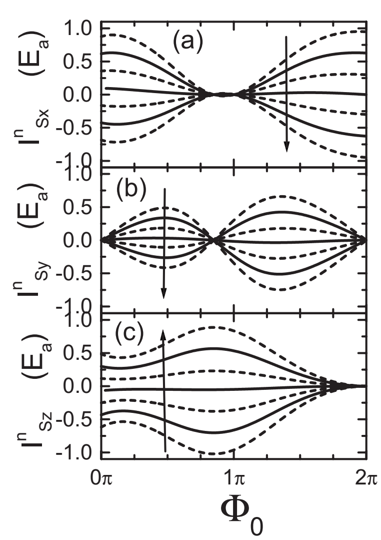

FIG. 5. (a) I^n_{Sx}, (b) I^n_{Sy}, and (c) I^n_{Sx} vs Φ_0 for $\alpha_R=3\times10^{-11}$ eV m and $a=50$ nm. Along the arrow direction, $n=6$, 4, 2, 0, 1, 3, and 5. Here, the level indices $n=0,1,2,\ldots$, represent the ground state, the first excited state, the second excited state, …, respectively.

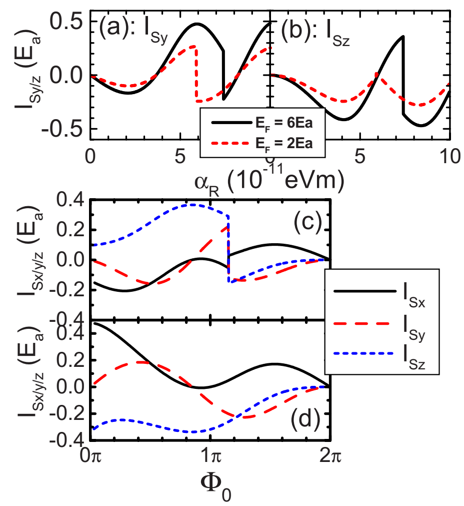

FIG. 6. (Color online) (a) and (b) show I_{Sy} and I_{Sz} vs α_R for $\Phi_0=\pi$. (c) and (d) show $I_{Sx/y/z}$ vs the angle of the normal region Φ_0 for $\alpha_R=3\times10^{-11}$ eV m and (c) $E_f=3E_a$ and (d) $E_f=6E_a$. The radius of the ring $a=50$ nm and temperature $T=0$.

The spin current I_{Si} versus the angle Φ_0 of normal region at a fixed $\alpha_R=3\times10^{-11}$ eV m is shown in Figs. 6(c) and 6(d). When $\Phi_0=2\pi$, the whole ring is normal, and we have $I_{Sx/y/z}=0$. When Φ_0 is away from 2π, the spin current $I_{Sx/y/z}$ emerges. For some Fermi energy, a jump appears in the curve I_{Si}-Φ_0 [as shown in Fig. 6(c)], whose behaviors are similar as the jump in the curve I_{Si}-α_R. For other Fermi energies, however, the jump in the curve I_{Si} versus Φ_0 [see Fig. 6(d)] disappears when the Fermi energy E_F is not in line with level E_n at all values of Φ_0. In particular, in the limit when Φ_0 goes to zero, i.e., when there is no normal region in the ring, the spin currents I_{Sx} and I_{Sz} still exist. This indicates that the normal region is not necessary for generating I_{Si}.

In the above numerical calculation, the temperature T is set to zero. Now, we consider the effect of the finite temperature T. Figure 7 shows the persistent spin currents $I_{Sy/z}$ versus the strength of SOI α_R at $\Phi_0=\pi$ with the different temperatures k_BT. When the temperature $k_BT=0$, the spin currents $|I_{Si}|$ are the largest and I_{Si} shows a jump if the Fermi energy

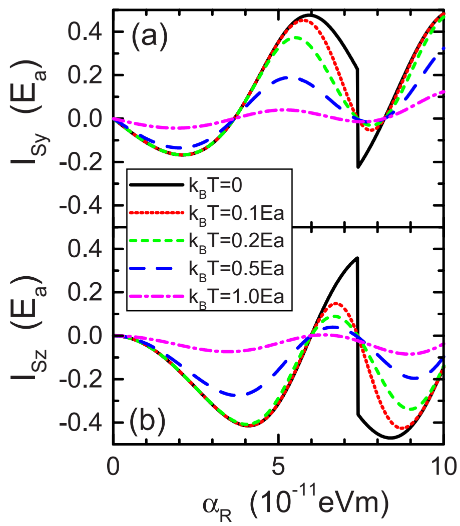

FIG. 7. (Color online) (a) and (b) show I_{Sy} and I_{Sz} vs α_R at different temperatures T. The radius of the ring $a=50$ nm and $\Phi_0=\pi$.

E_F is in line with level E_n. With the increase of the temperature from zero, this jump is smoothed, and the spin currents $|I_{Si}|$ at these α_R near the point of the jump (e.g., 7.4×10^{-11} eV m in Fig. 7) decreases sharply even at the very low temperature. However, the spin currents $|I_{Si}|$ at these α_R that are far away from this point of jump (e.g., $\alpha_R<5\times10^{-11}$ eV m in Fig. 7) are not effected very much by the temperature k_BT even for k_BT reaching $0.2E_a$. Upon further increase of the temperature k_BT and on the same order of the energy-level interval, the spin currents I_{Si} is reduced for all values of α_R. This is because the probability of occupation, i.e., $f(E_n)$, of the energy level varies smoothly versus the level index n, and the spin current due to the adjacent levels is opposite in sign. However, even when k_BT reaches $k_BT=0.5E_a$, the persistent spin current I_{Si} is still quite large. For a ring's radius $a=50$ nm and an effective mass $m=0.036m_e$, E_a is about 0.42 meV. Then, the temperature T is approximately 2.1 K at $k_BT=0.5E_a$. This temperature T is easily reached at the present technology.[7,46]

IV. DISCUSSION OF THE EFFECT OF SHARP INTERFACE BETWEEN THE SOI REGION AND THE NORMAL REGION

In the above section, the coefficient Rashba SOI $\alpha_R(\varphi)$ varies sharply in the interface of the normal and of SOI's part. Now, we examine the effect of sharp interface. Let us consider a hybrid ring device with the SOI coefficient $\alpha_R(\varphi)$ varying continuously along the ring. The Hamiltonian is the same as Eq. (1), with $\alpha_R(\varphi)=0$ for $0<\varphi<\pi$ and $\alpha_R(\varphi)=\alpha_R\sin^2(\varphi)$ for $\pi<\varphi<2\pi$. In this case, both $\alpha_R(\varphi)$ and $d\alpha_R(\varphi)/d\varphi$ are continuous at the interfaces $\varphi=0$ and π. In this section, we show that a persistent spin current still exists for a hybrid ring device with SOI varying continuously along the ring.

For this continuous varying Rashba SOI coefficient $\alpha_R(\varphi)$, the Hamiltonian can not be solved analytically. Here we numerically solve this Hamiltonian by using the discrete tight-binding model. Notice that $-\frac{i\sigma_r}{2a}[\alpha_R(\varphi)\frac{\partial}{\partial\varphi}+\frac{\partial}{\partial\varphi}\alpha_R(\varphi)]-i\alpha_R(\varphi)\sigma_\varphi/(2a)=-\frac{i}{2a}[\alpha_R(\varphi)\sigma_r\frac{\partial}{\partial\varphi}+\frac{\partial}{\partial\varphi}\sigma_r\alpha_R(\varphi)]$, and the discretized Hamiltonian discrete becomes[47]

$$H=\sum_{j=1}^{N}(2ta_j^\dagger a_j-ta_{j+1}^\dagger a_j-ta_j^\dagger a_{j+1})+\sum_{j=1}^{N}\left(a_{j+1}^\dagger\frac{i}{2a}\frac{\sigma_{r,j}\alpha_{R,j}+\sigma_{r,j+1}\alpha_{R,j+1}}{2\Delta\varphi}a_j+\text{H.c.}\right), \quad (12)$$

where a_j and $a_j^\dagger$ are annihilation and creation operators at point j, N is the number of the points in the ring, $\Delta\varphi=2\pi/N$ is the angle between two neighboring points, $t=E_a/(\Delta\varphi)^2$, $\sigma_{r,j}=\sigma_r(j\Delta\varphi)=\sigma_x\cos(j\Delta\varphi)+\sigma_y\sin(j\Delta\varphi)$, and $\alpha_{R,j}=\alpha_R(j\Delta\varphi)$. In the above Hamiltonian, the point index $N+1$ is the same with point index 1. Then, by calculating the eigenvalues and eigenvectors of the Hamiltonian matrix with the dimension N, the eigenvalues E_n and the eigen-wave-functions $\Psi_{n,j}$ $[\Psi_{n,j}=\Psi_n(j\Delta\varphi)]$ of the ring device can be easily solved.[44] After solving the eigen-wave-functions $\Psi_{n,j}$, the spin current $I_{Sk}^n(\varphi)=\text{Re}\,\Psi_n^\dagger\hat{v}_\varphi\hat{\sigma}_k\Psi_n=\frac{\hbar^2}{2ma}\text{Re}(-i)\Psi_n^\dagger\hat{\sigma}_k\frac{\partial}{\partial\varphi}\Psi_n$ $(k=x,y,z)$ in the normal region can be obtained straightforwardly from

$$I_{Sk,j}^n=\frac{\hbar^2}{2ma}\,\text{Im}\,\Psi_{n,j}^\dagger\hat{\sigma}_k\frac{\Psi_{n,j+1}-\Psi_{n,j-1}}{2\Delta\varphi}. \quad (13)$$

In order to show that the above method is correct and reliable, we first solve the model of Sec. III again, with the sharp varying SOI's coefficient $\alpha_R(\varphi)$: $\alpha_R(\varphi)=0$ while $0<\varphi<\Phi_0$, and $\alpha_R(\varphi)=\alpha_R$ while $\Phi_0<\varphi<2\pi$. The results of the eigenenergies are shown in Fig. 8. When the number of points $N=20$, we can see that the eigenenergies from the above discrete method are quite different from the exact values obtained from the method in Sec. III [see Fig. 8(a)]. However, with increasing N (e.g., $N=50$), this difference becomes very small [see Fig. 8(b)]. When $N=150$, the eigenenergies from the discrete method are in excellent agreement with the exact results [see Fig. 8(c)]. This means that our results using the above discrete method converges for large N.

Since E_n is twofold degenerate, the arbitrary combination $c_1\Psi_n(\varphi)+c_2\hat{T}\Psi_n(\varphi)$ is still the eigen-wave-functions, so the wave function is uncertain. In the following, we examine the correctness of our spin current I_{Si}^n $(i=x,y,z)$, which is calculated from the wave functions. Figure 9 shows the spin currents I_{Sy}^n contributed from the level n for the case of the sharp varying SOI, and it shows that the results of the spin current of the above discrete method are also in excellent agreement with the exact result at large N. In particular, it is surprising that the number of points N need not be very large. For $N=50$, the difference between numerical and exact results is already very small, and for $N=150$, there is almost no difference.

Now, we are ready to examine the effect of sharp interface. Figure 10 shows the results of the hybrid ring with SOI varying continuously along the ring, with $\alpha_R(\varphi)=0$ for 0

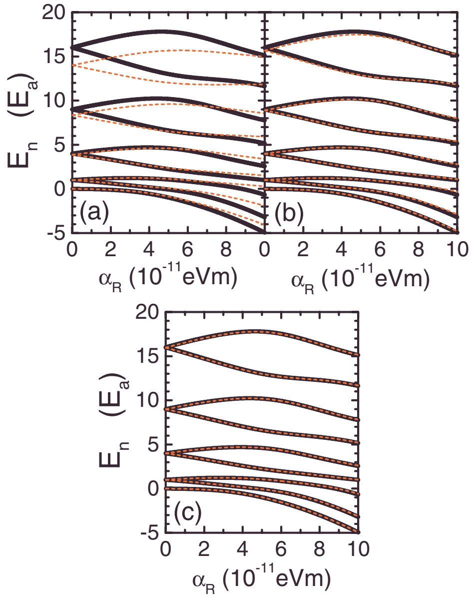

FIG. 8. (Color online) The eigenenergies E_n vs α_R. The dotted curves are for the tight-binding model of the discrete Hamiltonian with the lattice points (a) $N=20$, (b) $N=50$, and (c) $N=150$. The black solid curves are from the exact method used in Sec. III; i.e., these black solid curves are completely the same as the curves in Fig. 3(a). All parameters are the same as in Fig. 3(a).

$<\varphi<\pi$ and $\alpha_R(\varphi)=\alpha_R \sin^2(\varphi)$ for $\pi<\varphi<2\pi$, otherwise. Figures 10(a), 10(b), and 10(c) are for the eigenvalues, the spin current I^n_{Sy} contributed from the level n, and the equilibrium total spin current I_{Si} in the normal region versus α_R, respectively. The eigenvalues are fourfold degenerate at $\alpha_R=0$, and these degeneracies are split into two twofold Kramers degenerated states. The spin current I^n_{Sy} contributed from the level n oscillates with α_R. These results of the eigenvalues and I^n_{Sy} are similar to the case of the sharp interface. In particular, the results show that the (equilibrium) persistent spin current I_{Si} is still nonzero and has a quite large value [see Figs. 10(c) and 10(d)]. This indicates that the persistent spin current I_{Si} indeed originates from the SOI, and it is not an artifact of sharp interface.

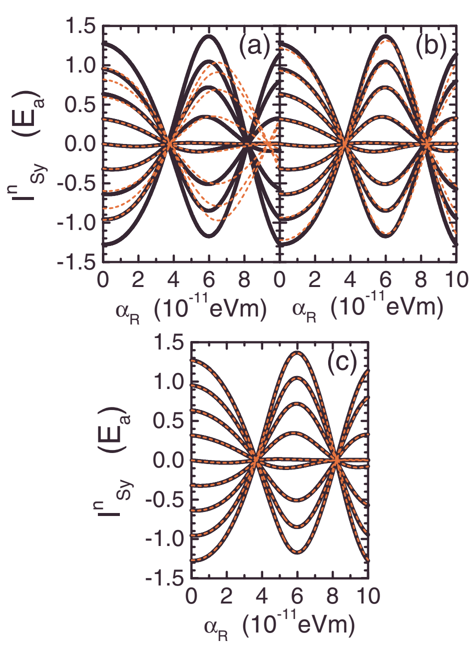

FIG. 9. (Color online) I^n_{Sy} vs α_R. The dotted curves are for the tight-binding model of the discrete Hamiltonian with the lattice points (a) $N=20$, (b) $N=50$, and (c) $N=150$. The black solid curves are for the exact method used in Sec. III; i.e., these black solid curves are completely the same as the curves in Fig. 4(a). All parameters are the same as in Fig. 4(a).

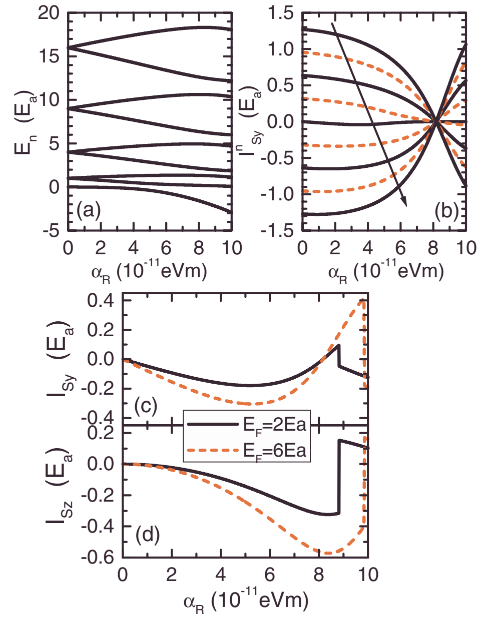

FIG. 10. (Color online) (a) The eigenenergies E_n vs α_R, (b) I^n_{Sy} vs α_R, (c) the persistent spin current I_{Sy} vs α_R, and (d) the persistent spin current I_{Sz} vs α_R for the ring with a nonsharp interface (see the text). The ring radius $a=50$ nm and the number of lattice points $N=200$. In (b), along the arrow direction, $n=7$, 5, 3, 1, 0, 2, 4, 6, and 8. In (c) and (d), the temperature $T=0$.

V. RELATION BETWEEN THE PERSISTENT SPIN CURRENT AND THE TRANSPORT SPIN CURRENT

Through the physical arguments and physical pictures from four different aspects in Sec. II and the analytic results of an example of a SOI-normal ring (in which the definition of the spin current is without controversy in the normal region) in Sec. III, as well as the discussion concerning the interface in Sec. IV, so far we have plenty evidence to demonstrate that the persistent spin current indeed exists in a mesoscopic semiconducting ring device with an intrinsic SOI. This persistent spin current can be induced solely by a SOI, and it exists in an equilibrium mesoscopic device without a magnetic field, a magnetic flux, and any magnetic materials. Besides the above mentioned ring geometry, we find that the device can also be of other shapes,[36,48] e.g., a disk device, a quantum wire, and a two-dimensional system. Thus, it is a generic feature that a pure persistent spin current appears in a system with SOI. In this section, we will discuss

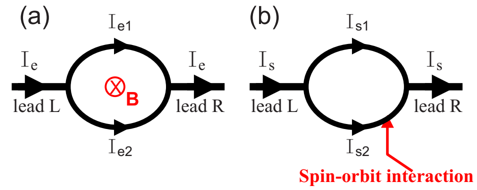

FIG. 11. (Color online) (a) is a schematic diagram for the ring device threaded by a magnetic flux and coupled to left and right leads. (b) is a schematic diagram for the ring device with the SOI and coupled to left and right leads.

the relation between the persistent spin current and the normal transport spin current.

First, let us review and discuss the relation of the persistent (charge) current and the transport (charge) current in a mesoscopic ring device. To consider a mesoscopic ring threaded by a magnetic flux coupled to two leads (the lead L and the lead R), which act as the electron reservoirs [see Fig. 11(a)]. There exists dissipation in the two leads, and they are always in equilibrium for an isolated lead L (R). The size of the ring is assumed within the coherent length and no dissipation in the ring. At zero bias (i.e., the equilibrium case), the transport charge current is zero everywhere, including the two leads and the ring, but the persistent charge current exists only in the ring, but not the leads because of the presence of dissipation in the leads. If a nonzero bias is added between two leads, a transport charge current flows from one lead through the ring to the other lead, so there exists both transport and persistent charge currents in the ring but only the transport current in the leads because of dissipation. In other words, at a finite bias, the charge currents $I_{e1(e2)}$ in the two arms of the ring are the sum of the transport currents $I_{e1(e2),t}$ and the persistent current $I_{e,p}$: $I_{e1}=I_{e1,t}+I_{e,p}$ and $I_{e2}=I_{e2,t}-I_{e,p}$. Can one distinguish the transport charge currents $I_{e1(e2),t}$ and the persistent charge current $I_{e,p}$? In fact, they ($I_{e1(e2),t}$ and $I_{e,p}$) cannot be distinguished either in theory or in experiment. The transport charge currents $I_{e1(e2),t}$ and the persistent charge current $I_{e,p}$ in the ring have identical behaviors; both of them are dissipationless,[49] capable of inducing a magnetic field, etc. So, in principle, only the total charge currents $I_{e1(e2)}$ in the arms are observable physical quantities. If some dissipative impurities are introduced in the ring, then the transport charge currents $I_{e1(e2),t}$ show dissipation while the persistent charge current $I_{e,p}$ does not. Under this circumstance, can one distinguish the transport currents $I_{e1(e2),t}$ and the persistent currents $I_{e,p}$? It turns out that one still cannot distinguish these types of currents. When dissipative impurities are introduced, the original dissipationless ring is changed into a new different ring, in which the persistent charge current $I_{e,p}$ is quenched while the transport charge currents $I_{e1(e2),t}$ are normally reduced due to dissipation. Thus, it is impossible to obtain the $I_{e1(e2),t}$ and $I_{e,p}$ of the original dissipationless ring.

The relation between the persistent spin current and the transport spin current is identical to the relation between the two charge currents discussed above. Consider that part of a mesoscopic ring contains a SOI, but without a magnetic flux and with two leads coupled to this ring [see Fig. 11(b)]. In the equilibrium case, a persistent spin current emerges in the ring; neither the transport spin current nor the persistent spin current is present in the leads. Under a spin-motive force,[18,50,51] a transport spin current flows from one lead through the ring to the other lead. The persistent spin current in the leads is always absent with or without a spin-motive force due to dissipation. On the other hand, with a spin-motive force, both the persistent spin current and the transport spin current exist in the ring. Similar to the charge currents, both persistent spin current $I_{s,p}$ and transport spin currents $I_{s1(s2),t}$ in the two arms of the ring are indistinguishable since they behave identically in all physically measurable properties. Both of them are dissipationless,[49] describing the real spin motion, capable of inducing an electric field, and so on.

VI. DEFINITION OF THE SPIN CURRENT

From Secs. II–IV, we have made sure that the persistent spin current exists in the equilibrium mesoscopic semiconducting device in the presence of SOI. In this section, we examine the definition of the spin current. The first work to question the conventional definition of the spin current is by Rashba.[23] After that, many subsequent papers have discussed the definition of the spin current as mentioned in the Introduction.[24–27] In summary,[26] in the presence of SOI, one faces three problems when using the conventional definition $\mathbf{I}_S=\text{Re}\,\Psi^\dagger\hat{v}\hat{s}\Psi$:[22] (i) There exists a nonzero spin current even in the equilibrium system, (ii) the spin current is usually nonconservative, and (iii) the Onsager relation is violated. Therefore, suggestions have been made in previous papers that one needs to modify the conventional definition of the spin current. In the following, we examine these three problems in detail, and we argue that there is no need to modify this conventional definition[22] $\mathbf{I}_S=\text{Re}\,\Psi^\dagger\hat{v}\hat{s}\Psi$.

A. Nonzero spin current in the equilibrium system

From the investigation and the discussion in Secs. II–IV, we have clearly shown that this nonzero spin current is the persistent spin current. It describes the real motion of spins, has the physical meaning, and can be observed in the experiment, in principle (see Sec. VIII). So, this nonzero spin current should be kept in the calculation of the total spin current.

B. Nonconservation of the spin current

In this subsection, we argue that, in general, the spin current is not conserved in the presence of the SOI and/or the magnetic field. However, in certain cases, the spin current can be conserved by including the strong spin-spin interaction. Let us analyze this problem from both aspects of theory and experiment. In the aspect of theory, we give two examples of the nonconserved spin current (for intuition, the readers can also consider the spin as a classic vector).

First, we want to show that, in general, if the system has a spin flip mechanism, the spin current will not be conserved. For instance, in the presence of a rotating magnetic field or a

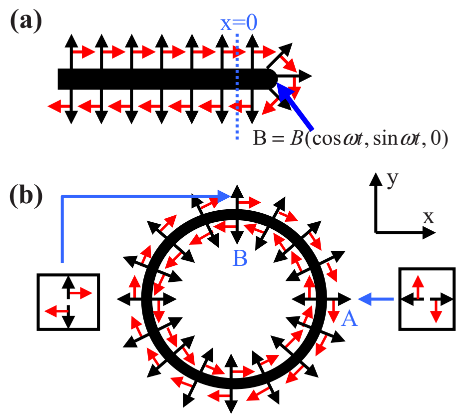

FIG. 12. (Color online) (a) Schematic diagram for a spin movement in a terminal device, in which the spin moves along the x axis and flips about $x=0$. (b) Schematic diagram for the spin movement and precession in a ring.

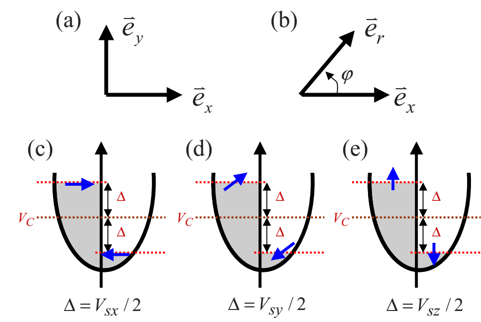

FIG. 13. (Color online) (a) and (b) are schematic diagrams for the base vectors $\vec{e}_x$ and $\vec{e}_y$, and $\vec{e}_x$ and $\vec{e}_r$. (c), (d), and (e) are schematic diagrams for the spin potential V_{sx}, V_{sy}, and V_{sz}, respectively.

circular polarized light,[52] the spin current exists in a device connected to only one terminal. This one-terminal device and its spin translational motion and precession are as shown in Fig. 12(a). Here, at $x<0$, the spin is without precession, and the spin pointing to the y direction moves along the $+x$ direction, while the spin pointing to the $-y$ direction moves along the opposite direction. So, it has a nonzero spin current $I_{s,xy}$ at $x<0$. Near the point $x=0$ where the quantum dot is located, the spin precesses and is flipped due to the presence of a rotating magnetic field or a circularly polarized light.[52] Then, the spin accumulation does not vary with time, and the system maintains the steady state. It is obvious that the spin current is not conserved because of only one terminal in this device.

In the second theoretical example, we consider the spin translational motion and precession on a ring, as shown in Fig. 12(b). At point A with its angular coordinate $\varphi=0$, the spin pointing to the $+x$ direction moves down while the spin pointing to the $-x$ direction moves up, then the nonzero element of the spin current is $I_{S,yx}$. At point B, with its angular coordinate $\varphi=\pi/2$, the spin pointing to the $+y$ direction moves along the $+x$ axis and the spin pointing to the $-y$ direction moves along the $-x$ axis, then the nonzero element of the spin current is $I_{S,xy}$. So, it is obvious that the spin current is not conserved in this ring device, but the spin accumulation still remains invariant. In fact, this example is similar to the persistent spin current in the mesoscopic ring while the whole ring has a SOI (see Sec. VII C and the Appendix), in which the motion of the x-y plane elements of the spin is as shown in Fig. 12(b) and the spin accumulation is zero everywhere.

In addition, there are experimental indications that the spin current is not conserved. For example, let us consider the experimental result by Valenzuela and Tinkham.[21] In this experiment, they have clearly shown that a pure spin current injects from the FM1 electrode into the Al strip, reduces with its flowing forward because of the spin flip, and finally disappears while the distance is much longer than the spin diffusion length [see Figs. 1(a)–1(c), and Fig. 4 in Ref. 21]. This experimental result gives a strong proof of the fact that the spin current should be nonconserved in the presence of the SOI, the magnetic field, the magnetic impurity, and others.

C. Violation of the Onsager relation

The Onsager reciprocal relation is an important theorem of the near-equilibrium transport theory. Up to now, the Onsager reciprocal relation is always satisfied for the transport of any physical quantity (e.g., charge transport and thermal transport) by suitably defining a corresponding current. However, in the following, we point out that the Onsager relation for the spin transport is, in general, violated. In particular, it is impossible to restore the Onsager relation regardless of how to modify the definition of the spin current. This is very different from all previous cases. In this subsection, we first recall the tenable condition of the Onsager reciprocal relation and then examine the case of the spin current. We find that the spin transport does not always meet this condition no matter how we define the spin current, although this condition is met for all previous transport phenomena people have studied. So, the Onsager relation is, in general, violated for the spin transport.

First, let us recall the tenable condition of the Onsager relation. Considering the currents $\vec{I}=\{I_i\}$ and its corresponding forces $\vec{F}=\{F_i\}$, they have the following relationship:

$$I_i = \sum_j G_{ij} F_j, \tag{14}$$

where G_{ij} is the conductivity. *If* the local entropy production dS/dt per unit time can be expressed as $dS/dt=\Sigma_i I_i F_i$, there will exist an Onsager relation $G_{ij}=G_{ji}$ (assuming that the system has a time-reversal symmetry).

Next, we give an intuitive example to show the tenable condition of the Onsager relation. Let us consider the charge conductivity in the two-dimensional system. If we take the vectors $\vec{e}_x$ and $\vec{e}_y$ [as shown in Fig. 13(a)] as the base vectors of the charge current and its force [i.e., the gradient of the potential, $\vec{\nabla} V(\mathbf{r})$], Eq. (14) becomes

$$\begin{pmatrix} I_x \\ I_y \end{pmatrix} = \begin{pmatrix} G_{xx} & G_{xy} \\ G_{yx} & G_{yy} \end{pmatrix} \begin{pmatrix} \nabla_x V \\ \nabla_y V \end{pmatrix}. \tag{15}$$

In this case, the Onsager relation is tenable, and $G_{xy}=G_{yx}\equiv G_o$. However, if we take the nonorthonormal vectors $\vec{e}_x$ and $\vec{e}_r$ [as shown in Fig. 13(b)] as the base vectors of the current and the force, the Onsager relation will be violated,as shown in the following. For the base vectors $\vec{e}_x$ and $\vec{e}_r$, any current vector $\vec{I}$ and force vector $\vec{\nabla}V$ can still be expressed as $\vec{I}=I_x\vec{e}_x+I_r\vec{e}_r$ and $\vec{\nabla}V=\nabla_x V\vec{e}_x+\nabla_r V\vec{e}_r$. Then, it is easy to obtain the relation between $(\nabla_x V,\nabla_y V)$ and $(\nabla_x V,\nabla_r V)$:

$$\begin{pmatrix}\nabla_x V\\ \nabla_y V\end{pmatrix}=\mathbf{U}\begin{pmatrix}\nabla_x V\\ \nabla_r V\end{pmatrix}\quad\text{and}\quad\begin{pmatrix}\nabla_x V\\ \nabla_r V\end{pmatrix}=\mathbf{U}^{-1}\begin{pmatrix}\nabla_x V\\ \nabla_y V\end{pmatrix},$$

where

$$\mathbf{U}=\begin{pmatrix}1 & \cos\varphi\\ 0 & \sin\varphi\end{pmatrix}. \tag{16}$$

So, under the base vectors $\vec{e}_x$ and $\vec{e}_r$, Eq. (15) changes to

$$\begin{pmatrix}I_x\\ I_r\end{pmatrix}=\mathbf{U}^{-1}\begin{pmatrix}G_{xx} & G_o\\ G_o & G_{yy}\end{pmatrix}\mathbf{U}\begin{pmatrix}\nabla_x V\\ \nabla_r V\end{pmatrix}=\begin{pmatrix}G_{xx}-G_o\cos\varphi/\sin\varphi & (G_{xx}-G_{yy})\cos\varphi-G_o\cos(2\varphi)/\sin\varphi\\ G_o/\sin\varphi & G_{yy}+G_o\cos\varphi/\sin\varphi\end{pmatrix}\begin{pmatrix}\nabla_x V\\ \nabla_r V\end{pmatrix}. \tag{17}$$

It is obvious that the two off-diagonal elements of the conductivity in the above equation are not equal, so the Onsager relation is violated in the nonorthonormal base vectors $\vec{e}_x$ and $\vec{e}_r$. In fact, for the charge current, the Onsager relation is only tenable under the orthonormal and linear independent base vectors.

Now, let us discuss the spin current. The spin current has $3\times 3=9$ elements, and the charge current has three elements. So, here, the current $\vec{I}$ (including spin and charge) and the corresponding force $\vec{F}$ has a total of 12 elements and the conductivity has $12\times 12=144$ elements. For simplicity and clarity, we consider a one-dimensional system, and the electron can only move along the x axis. In this case, the current $\vec{I}$ and the force $\vec{F}$ have only four nonzero elements, and they are

$$\vec{I}=(I_{sxx},I_{sxy},I_{sxz},I_{cx}), \tag{18}$$

$$\vec{F}=(\nabla_x V_{sx},\nabla_x V_{sy},\nabla_x V_{sz},\nabla_x V_c), \tag{19}$$

where V_{si} $(i=x,y,z)$ is the spin chemical potential and V_c is the (charge) chemical potential.[18,50,51] The spin chemical potential V_{si} means that the electrons of the spin pointing to $+i$ and $-i$ directions occupy up to $V_c+V_{si}/2$ and $V_c-V_{si}/2$ [shown in Figs. 13(c)–13(e)], respectively. Then, Eq. (14) becomes

$$\begin{pmatrix}I_{sxx}\\ I_{sxy}\\ I_{sxz}\\ I_{cx}\end{pmatrix}=\begin{pmatrix}G_{xx} & G_{xy} & G_{xz} & G_{xc}\\ G_{yx} & G_{yy} & G_{yz} & G_{yc}\\ G_{zx} & G_{zy} & G_{zz} & G_{zc}\\ G_{cx} & G_{cy} & G_{cz} & G_{cc}\end{pmatrix}\begin{pmatrix}\nabla_x V_{sx}\\ \nabla_x V_{sy}\\ \nabla_x V_{sz}\\ \nabla_x V_c\end{pmatrix}, \tag{20}$$

and the conductivity has $4\times 4=16$ elements.

Now, we show that the spin transport does not meet the tenable condition of the Onsager relation. First, the three base vectors σ_i $(i=x,y,z)$ are not orthonormal, and it is impossible to find a series of the orthonormal base vectors regardless of the combination of the three σ_i.

In addition, once the spin chemical potential of one component is fixed, the other two spin potentials V_{si} cannot exist. For example, if we give the value for the spin potential V_{sz}, this means that the electron of the spin along the $+z$ direction occupies up to $V_{sz}/2$ and the electron of the spin at the $-z$ direction occupies up to $-V_{sz}/2$ if $V_c=0$ [see Fig. 13(e)],[18,50] and the electron occupational state has completely been determined. Then, one cannot further specify the spin states and the corresponding occupation number along $+x$ and $-x$ (or $+y$ and $-y$). Therefore, while giving the value for one component of the spin potential, e.g., V_{sz}, the others V_{sx} and V_{sy} do not exist. This conclusion holds even for spin-free and conserved systems with $[\sigma_i,H]=0$ for all i $(i=x,y,z)$. Due to the fact that three spin potentials V_{si} cannot be simultaneously evaluated, Eq. (20) [i.e., Eq. (14)] does not exist for the spin transport, regardless of how to define the spin current. Therefore, there is no Onsager relation in spin transport, and it is impossible to restore the Onsager relation by modifying the definition of the spin current. In fact, the Onsager relation is not satisfied for any existing definitions of the spin current.

Furthermore, we can also use the four normal terminal device (as the device in Ref. 18) to examine the Onsager relation. Because the four terminal leads are normal metals without the SOI, there is no spin flip. For this system, there is no controversy for the definition of the spin current in the terminal leads. Therefore, we can avoid the definition of the spin current and examine the Onsager relation. The results also show that the Onsager relation does not exist for the spin transport. This gives an additional proof.

We wish to mention that because V_{sz} and V_c (or V_{sx} and V_c, or V_{sy} and V_c) can be determined simultaneously, the Onsager relation for $G_{zc}=G_{cz}$ (or $G_{xc}=G_{cx}$, or $G_{yc}=G_{cy}$) might exist,[18,26] e.g., in the suitable boundary condition in the four-terminal device.[18] However, it is impossible that six relations ($G_{zc}=G_{cz}$, $G_{xc}=G_{cx}$, $G_{yc}=G_{cy}$, $G_{xy}=G_{yx}$, $G_{xz}=G_{zx}$, and $G_{yz}=G_{zy}$) are satisfied simultaneously.[18] For three-dimensional systems, the conductivity has $12\times 12=144$ elements G_{ij}; a similar conclusion applies, i.e., it is impossible that $G_{ij}=G_{ji}$ is satisfied for all off-diagonal matrix elements simultaneously.

D. Discussions

From the discussion in the last three subsections, we have clearly shown that (i) the nonzero spin current in the equilibrium SOI's device is the persistent spin current, (ii) in general, the spin current is not conserved, and (iii) the Onsager relation is violated for the spin transport. In particular, it cannot be restored regardless of how to modify the definition of the spin current. Therefore, the three "flaws" of the conventional definition of the spin current, which has been mentioned and commented in some previous papers, are intrinsic properties of spin transport. In addition, the conventional definition has very clear physical intuition and has described the spin motion very well. Using this conventional definition, one can account for many effects that relate the spin transport,[24] e.g., the heat produced by the spin current, the spin current induced electric field, and the force and the torque acting on the spin current in the presence of electric field. Therefore, we make the conclusion that there is no need to modify the conventional definition $\mathbf{I}_s=\mathrm{Re}\,\Psi^\dagger\hat{v}\hat{s}\Psi$.[22]

Before we end this section, we wish to mention that if the spin-spin interaction is included into the Hamiltonian, the spin current calculated using the conventional definition will be conserved automatically.[27] We will discuss this in detail in Sec. IX. We also note that our discussion above does not contradict with the angular spin current in our previous paper by Sun and Xie.[24] To see this, let us first recall the (linear) velocity $\vec{v}$ and the angular velocity $\vec{\omega}$. (a) The (linear) velocity $\vec{v}=d\mathbf{r}/dt$ (or the velocity operator $\hat{v}=d\hat{\mathbf{r}}/dt$) can describe the translational motion of the vector (or the rigid body), and there is no need to modify this velocity definition $\vec{v}=d\mathbf{r}/dt$. (b) On the other hand, the vector has the rotational degrees of freedom except for its translational motion, so we need to introduce the angular velocity $\vec{\omega}$ to describe its rotational motion. We emphasize that two statements (a) and (b) do not contradict each other and can be rephrased as follows: (a) The (linear) spin current $\mathbf{I}_s=\mathrm{Re}\,\Psi^\dagger\hat{v}\hat{s}\Psi$ can describe the translational motion of the spin and there is no need to modify its definition. (b) On the other hand, the spin has the rotational degrees of freedom (precession) except for its translational motion, so we need to introduce the angular spin current $\mathbf{I}_\omega=\mathrm{Re}\,\Psi^\dagger(d\hat{s}/dt)\Psi=\mathrm{Re}\,\Psi^\dagger\hat{\omega}\times\hat{s}\Psi$ to describe its rotational motion.[53]

VII. PERSISTENT SPIN CURRENT IN THE REGION WITH SPIN-ORBIT INTERACTION

After clarifying the definition of the spin current, we return to discuss the persistent spin current in the SOI's region of the normal-SOI ring in this section. In Sec. VII A, we investigate the linear and angular persistent spin current. In Sec. VII B, we show that the persistent spin current still exists even when using the new definition of the spin current as in Ref. 26. The case of the entire ring with SOI is studied in Sec. VII C. At Sec. VII D, we present some conserved quantities that are related to the persistent spin current.

A. Linear and angular persistent spin currents in the spin-orbit interaction region

From the definition of the (linear) spin current $\mathbf{I}_{Si}=\mathrm{Re}\,\Psi^\dagger(1/2)(\hat{v}_\varphi\hat{s}_i+\hat{s}_i\hat{v}_\varphi)\Psi$, for a ring device we have

$$I_{Sx}(\varphi)=\mathrm{Im}\left\{\Psi^\dagger\left[\sigma_x aE_a\frac{\partial}{\partial\varphi}+\frac{i\alpha_R(\varphi)}{2}\cos\varphi\right]\Psi\right\},$$

$$I_{Sy}(\varphi)=\mathrm{Im}\left\{\Psi^\dagger\left[\sigma_y aE_a\frac{\partial}{\partial\varphi}+\frac{i\alpha_R(\varphi)}{2}\sin\varphi\right]\Psi\right\},$$

$$I_{Sz}(\varphi)=\mathrm{Im}\left\{\Psi^\dagger\left[\sigma_z aE_a\frac{\partial}{\partial\varphi}\right]\Psi\right\}. \quad (21)$$

To make the above equations discrete, $I_{Si}(\varphi)$ changes into $I_{Si,j}=I_{Si}(j\Delta\varphi)$:

$$I_{Sx,j}=aE_a\,\mathrm{Im}\left\{\Psi_j^\dagger\left[\sigma_x\frac{\Psi_{j+1}-\Psi_{j-1}}{2\Delta\varphi}+\frac{i\alpha_{R,j}}{2E_a a}\cos(j\Delta\varphi)\Psi_j\right]\right\},$$

$$I_{Sy,j}=aE_a\,\mathrm{Im}\left\{\Psi_j^\dagger\left[\sigma_y\frac{\Psi_{j+1}-\Psi_{j-1}}{2\Delta\varphi}+\frac{i\alpha_{R,j}}{2E_a a}\sin(j\Delta\varphi)\Psi_j\right]\right\},$$

$$I_{Sz,j}=aE_a\,\mathrm{Im}\left\{\Psi_j^\dagger\sigma_z\frac{\Psi_{j+1}-\Psi_{j-1}}{2\Delta\varphi}\right\}. \quad (22)$$

Let us calculate the angular spin current $\mathbf{I}_\omega$,[24] which describes the rotational motion (precession) of the spin. For a ring device with the Hamiltonian of Eq. (1), one has

$$d\hat{s}_x/dt=(-i\sigma_z/2a)\{\alpha_R(\varphi)\sin\varphi,\partial/\partial\varphi\},$$

$$d\hat{s}_y/dt=(i\sigma_z/2a)\{\alpha_R(\varphi)\cos\varphi,\partial/\partial\varphi\},$$

$$d\hat{s}_z/dt=(-i/2a)\{\alpha_R(\varphi)\sigma_\varphi,\partial/\partial\varphi\}, \quad (23)$$

where $\{A,B\}\equiv AB+BA$. Using the cylindrical coordinates, $d\hat{\vec{s}}/dt=(d\hat{s}_r/dt,d\hat{s}_\varphi/dt,d\hat{s}_z/dt)=\frac{-i}{2a}(0,$ $-\sigma_z\{\alpha_R(\varphi),\partial/\partial\varphi\},\{\alpha_R(\varphi)\sigma_\varphi,\partial/\partial\varphi\})$. Then, the angular spin current can be calculated from its definition,[24] $\mathbf{I}_\omega=\mathrm{Re}\,\Psi^\dagger(d\hat{\vec{s}}/dt)\Psi$ straightforwardly, and so does its discrete version:

$$I_{\omega r,j}=0,$$

$$I_{\omega\varphi,j}=\mathrm{Re}\left\{\Psi_j^\dagger\frac{i\sigma_z}{2a}\left[\alpha_{R,j}\frac{\Psi_{j+1}-\Psi_{j-1}}{2\Delta\varphi}+\frac{\alpha_{R,j+1}\Psi_{j+1}-\alpha_{R,j-1}\Psi_{j-1}}{2\Delta\varphi}\right]\right\},$$

$$I_{\omega z,j}=\mathrm{Re}\left\{\Psi_j^\dagger\frac{-i}{2a}\left[\sigma_{\varphi,j}\alpha_{R,j}\frac{\Psi_{j+1}-\Psi_{j-1}}{2\Delta\varphi}+\frac{\sigma_{\varphi,j+1}\alpha_{R,j+1}\Psi_{j+1}-\sigma_{\varphi,j-1}\alpha_{R,j-1}\Psi_{j-1}}{2\Delta\varphi}\right]\right\}. \quad (24)$$

Since the wave function $\Psi_n(\varphi)$ for each eigenstate is known from Sec. III or IV, the (linear) spin current I_{Si}^n and

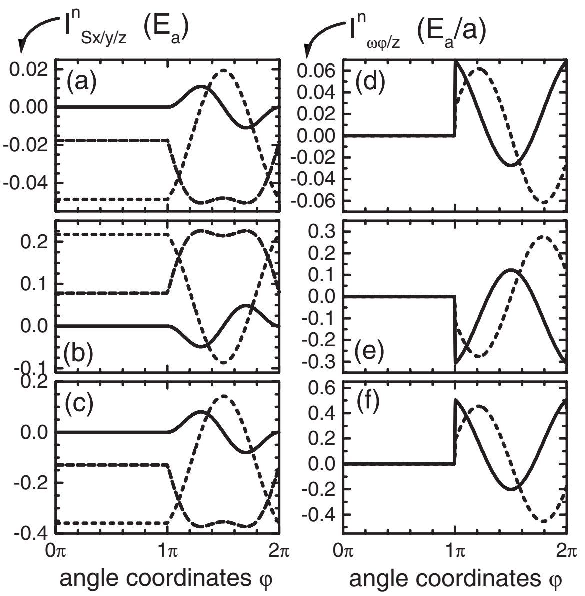

FIG. 14. The linear spin current $I^n_{Sx/y/z}$ and the angular spin current $I^n_{\omega\varphi/z}$ vs the angle coordinates φ for the level [(a) and (d)] $n=0$ [(b) and (e)] $n=1$, and [(c) and (f)] $n=2$. The parameters are $\alpha_R=3\times10^{-11}$ eV m, $\Phi_0=\pi$, and the ring radius $a=50$ nm. The solid curve, dashed curve, and dotted curve in panels (a), (b), and (c) correspond to I^n_{Sx}, I^n_{Sy}, and I^n_{Sz}, respectively. The solid curve and dotted curve in panels (d), (f), and (e) correspond to $I^n_{\omega\varphi}$ and $I^n_{\omega z}$, respectively.

the angular spin current $I^n_{\omega i}$ from the nth state can be easily obtained. Using the exact method in Sec. III or the discrete method in Sec. IV, we find that the results for I^n_{Si} and $I^n_{\omega i}$ match perfectly. Figure 14 shows the linear spin current $I^n_{Sx/y/z}$ (i.e., the element in the orthogonal coordinates) and the angular spin current $I^n_{\omega\varphi/z}$ (i.e., the element in the cylindrical coordinates) versus the angular coordinates φ. In the normal region, $I^n_{Sx/y/z}$ is conserved and is independent of the coordinates φ, and $I^n_{\omega\varphi/z}$ is zero, since the spin has only the translational motion without the precession there. On the other hand, in the SOI region, except for $I^n_{\omega r}=0$, the (linear) spin current $I^n_{sx/y/z}$ and the angular spin current $I^n_{\omega\varphi/z}$ are all nonzero and nonconserved because of the spin precession in the presence of the SOI. The linear spin current $I^n_{Sx/y/z}(\varphi)$ versus φ is always continuous, even at the interface between the normal region and the SOI region where the strength of SOI $\alpha_R(\varphi)$ changes abruptly. However, the angular spin current $I^n_{\omega\varphi/z}(\varphi)$ versus φ shows a jump whenever $\alpha_R(\varphi)$ versus φ has an abrupt change, as shown in Figs. 14(d)–14(f). The jump position of $I^n_{\omega\varphi/z}$ is located at the abrupt point of $\alpha_R(\varphi)$. The spin currents I^n_{Si} (and I^n_{ω}) versus φ for the states with the same parity, e.g., $n=1,3,5,\ldots$ or $n=0,2,4,6,\ldots$, are similar in shape [see Figs. 14(a), 14(c), 14(d), and 14(f)], but the value of $|I^n_{Si}|$ (or $|I^n_{\omega}|$) is much larger for a larger n. Thus, the spin current from the highest occupied level dominates in the (total) persistent spin current.

Figure 15 shows the total persistent spin current $I_{Sx/y/z}$ versus the angular coordinates φ for different Fermi energies E_F. The persistent spin current is conserved in the normal region, but not so in the SOI region because of the spin precession. Due to the fact that the spin current $I^{max\ n}_{Sx/y/z}$ from the highest occupied level $max\ n$ dominates in $I_{Sx/y/z}$, $I_{Sx/y/z}$ behaves similarly as $I^{max\ n}_{Sx/y/z}$ (see Figs. 14 and 15).

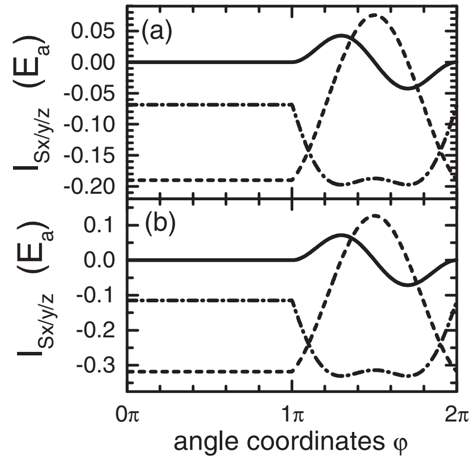

FIG. 15. The persistent spin currents $I_{Sx/y/z}$ vs the angle coordinates φ for the Fermi energy (a) $E_F=2E_a$ and (b) $E_F=6E_a$. The temperature $T=0$ and the other parameters are the same with those Fig. 14. The solid curve, dashed-dotted curve, and dotted curve correspond to I_{Sx}, I_{Sy}, and I_{Sz}, respectively.

B. Calculating the spin current using the definition in Reference 26

We have clarified and demonstrated that the conventional definition of the spin current makes sense in Sec. VI. However, in this subsection, we present our calculated results of the persistent spin current by using the new definition of the spin current (Ref. 26), and discuss its consequences. In the normal region, the persistent spin current $\mathbf{I}_S$ is the same regardless of which definition is used. However, in the SOI region, $\mathbf{I}_S$ depends on the definition. In the following, let us discuss $\mathbf{I}_S$ in the SOI region. By using the new definition in Ref. 26, $\mathbf{I}_S\equiv\text{Re}\,\Psi^\dagger\frac{d\mathbf{r}\hat{s}}{dt}\Psi=\text{Re}\,\Psi^\dagger[\hat{v}\hat{s}+\mathbf{r}d\hat{s}/dt]\Psi$, the spin current from level n and then the total persistent spin current depend on the choice of the coordinate origin. First, if setting the origin at the center of the ring, the element $I_{S\varphi i}$ $(i=x,y,z)$ with the spin motion along the φ direction is $\text{Re}\,\Psi^\dagger\hat{v}_\varphi\hat{s}_i\Psi$. So, $I_{S\varphi i}$ is completely the same with the result using the conventional definition, and it is still nonzero in the equilibrium case and nonconversed in the presence of a SOI. On the other hand, the element I_{Sri} $(i=x,y,z)$ with the spin motion along the radial direction is $a\,\text{Re}\,\Psi^\dagger(d\hat{s}_i/dt)\Psi=aI_{\omega i}$, so it is also nonzero, but the same element is exactly zero using the conventional definition. The element I_{Szi} $(i=x,y,z)$ with the spin motion along the z direction is zero, same as with the conventional definition. Second, if the coordinate origin is not located at the center of the ring, all nine elements of the spin current are, in general, different from those by using the conventional definition. In particular, they are all nonzero and nonconserved in equilibrium.

C. Case when the entire ring is with a spin-orbit interaction

Let us consider the case of $\Phi_0\to0$; i.e., the normal region is gradually getting smaller and at the end the whole ring

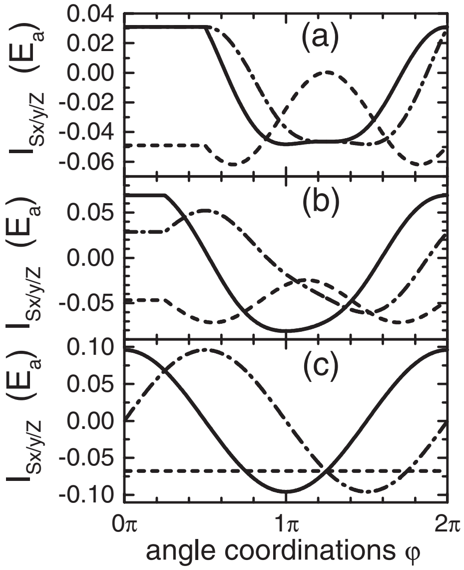

FIG. 16. The persistent spin currents $I_{Sx/y/z}$ vs the angle coordinates φ for (a) $\Phi_0=\pi/2$, (b) $\Phi_0=\pi/4$, and (c) $\Phi_0=0$. The temperature $T=0$ and the other parameters are same with those Fig. 14. The solid curve, dashed-dotted curve, and dotted curve correspond to I_{Sx}, I_{Sy}, and I_{Sz}, respectively.

contains the SOI. Figure 16 shows the persistent spin current $I_{Sx/y/z}$ versus the angle coordinate φ for $\Phi_0=\pi/2$, $\pi/4$, and 0. The results clearly show that the persistent spin current $I_{Sx/y/z}$ does exist, and its value $|I_{Sx/y/z}|$ is even larger with the decrease of the normal region, i.e., Φ_0. Eventually, when the entire ring contains the SOI, $|I_{Sx/y/z}|$ reaches its maximum value. This means that the normal region is not necessary for the existence of the persistent spin current.

In fact, if the whole ring has a constant SOI with $\alpha_R(\varphi)=\alpha_R$, the persistent spin current can be analytically obtained. In this case, the eigen-wave-function is[31]

$$\Psi_n(\varphi)=\begin{pmatrix}\cos(\theta/2)e^{in\varphi}\\ -\sin(\theta/2)e^{i(n+1)\varphi}\end{pmatrix}, \tag{25}$$

where $n=0,\pm 1,\pm 2,\ldots$ and the eigenvalue E_n is given by

$$E_n=E_a[n^2+(n+1/2)(1-1/\cos\theta)]. \tag{26}$$

Then, the linear spin current $\mathbf{I}_S^n$ and the angular spin current $\mathbf{I}_\omega^n$ from the state n are as follows:

$$\mathbf{I}_S^n=-E_aF(\theta)[\vec{e}_x\sin\theta\cos\varphi+\vec{e}_y\sin\theta\sin\varphi-\vec{e}_z\cos\theta], \tag{27}$$

$$\begin{aligned}\mathbf{I}_\omega^n&=(E_a/a)F(\theta)[\vec{e}_x\sin\theta\sin\varphi-\vec{e}_y\sin\theta\cos\varphi]\\&=-(E_a/a)F(\theta)\vec{e}_\varphi\sin\theta,\end{aligned} \tag{28}$$

where $F(\theta)=[n+1/2-1/(2\cos\theta)]/2\pi$. The persistent linear and angular spin currents $\mathbf{I}_S$ and $\mathbf{I}_\omega$ are obtained by summing $\mathbf{I}_S^n$ and $\mathbf{I}_\omega^n$ over the occupied states. In addition, from the wave function [Eq. (25)] and the spin currents [Eqs. (27) and (28)], the spin motion in the ring can be obtained (see the discussion in the Appendix).

D. Three conserved quantities in a ring device

From the results of Eqs. (27) and (28), three quantities characterizing the spin current $\mathbf{I}_S^n$ are found to be conserved, although the spin current itself is not conserved in the presence of a SOI. (a) The spin current $\mathbf{I}_{Sz}^n$ with spin polarization along the z direction is a conserved quantity for the ring geometry. (b) The magnitude of the spin current $I_S^n=\sqrt{(I_{Sx}^n)^2+(I_{Sy}^n)^2+(I_{Sz}^n)^2}=E_aF(\theta)$ is a constant of motion. (c) For a given cross section of the ring, the vector of spin polarization for the spin current $\mathbf{I}_S^n$ makes a fixed angle with the normal direction of that cross section. This angle is a constant for any cross section of the ring. In this sense, the spin current $\mathbf{I}_S^n$ is "conserved," although the direction of spin polarization for $\mathbf{I}_S^n$ is not a constant of motion due to a SOI. So the nonconservation of spin current in the ring device means that while moving along the ring the direction of spin polarization is precessing due to the torque from the SOI. For the hybrid ring, our numerical results also show that the magnitude of spin current I_S^n is again a constant of motion across the hybrid ring, but the element of the z-direction $\mathbf{I}_{Sz}^n$ and the angle in (c) are not.

VIII. INDUCED ELECTRIC FIELD BY A PERSISTENT SPIN CURRENT

There are a number of experiments that have been carried out to confirm the existence of spin current,[19–21] e.g., to observe the spin current induced spin accumulations by the Kerr effect[19] or to make the electric measurement through the reciprocal spin Hall effect.[21] Since the persistent spin current is an equilibrium property, the above mentioned methods are not suitable. There is also a proposal that a spin current may cause a spin torque that can be measured experimentally.[54,55] Very recently, Sonin pointed out that this method can be employed to detect the persistent spin current.[55,56] On the other hand, we note that the persistent charge current can be detected by measuring its induced magnetic field.[29] It has been shown that the persistent spin current can also generate an electric field.[24,32,57,58] So, this offers another way to detect the persistent spin current by measuring its induced electric field. In the following, we calculate the persistent spin current induced electric field and electric potential, and show that this electric field or the electric potential can be observed in the present technology.

The induced electric fields $\vec{E}_S(\mathbf{r})$ and $\vec{E}_\omega(\mathbf{r})$ at space point $\mathbf{r}=(x,y,z)$ by the linear and angular spin currents $\mathbf{I}_S$ and $\mathbf{I}_\omega$ in the ring device are[24]

$$\vec{E}_S=-\frac{\mu_0 g\mu_B}{h}\nabla\times\int_0^{2\pi}\mathbf{I}_S(\varphi)\cdot\frac{\mathbf{r}-\mathbf{r}'(\varphi)}{|\mathbf{r}-\mathbf{r}'(\varphi)|^3}a\,d\varphi, \tag{29}$$

$$\vec{E}_\omega=-\frac{\mu_0 g\mu_B}{h}\int_0^{2\pi}\mathbf{I}_\omega(\varphi)\times\frac{\mathbf{r}-\mathbf{r}'(\varphi)}{|\mathbf{r}-\mathbf{r}'(\varphi)|^3}a\,d\varphi, \tag{30}$$

where μ_B is the Bohr magneton and $\mathbf{r}'(\varphi)=a(\cos\varphi,\sin\varphi,0)$ is the position vector in the ring. Considering the whole ring having a constant SOI, the persistent linear and angular spin currents have been solved in Sec.

VII C [see Eqs. (27) and (28)]. Substituting them into the above formulas [Eqs. (29) and (30)] (note that I_{Si} is the element $I_{S\varphi i}$ of the linear spin current) and with the help of the first and second kind complete elliptic integral functions $K(x)$ and $E(x)$,

$$K(x) = \int_0^{\pi/2} 1/\sqrt{1 - x(\sin\varphi)^2}\, d\varphi, \tag{31}$$

$$E(x) = \int_0^{\pi/2} \sqrt{1 - x(\sin\varphi)^2}\, d\varphi, \tag{32}$$

the induced electric fields $\vec{E}_S(\mathbf{r})$ and $\vec{E}_\omega(\mathbf{r})$ can be obtained straightforwardly. The electric fields $\vec{E}_S(\mathbf{r})$ and $\vec{E}_\omega(\mathbf{r})$ are rotational invariant about the z axis, and in the plane $\vec{e}_z$-$\vec{e}_r$, the elements $\vec{E}_{S\varphi}$ and $\vec{E}_{\omega\varphi}$ are zero. So, here, we only show $\vec{E}_S(\mathbf{r})$ and $\vec{E}_\omega(\mathbf{r})$ in the x-z plane with $\mathbf{r}=(x,y,z)=(x,0,z)$:

$$E_{Sx} = \frac{2c}{axR_-^3R_+^4}\{K(A)R_+^2[a(R_+^2R_-^2 - z^2(a^2+x^2+z^2))\cos\theta - z(R_+^2R_-^2 + a^2(a^2-x^2+z^2))\sin\theta] - E(A)[a(R_+^2R_-^2(a^2 + x^2) - 16a^2x^2z^2)\cos\theta - z(R_+^2R_-^2(2a^2+x^2+z^2) + 8a^2x^2(a^2-x^2-z^2))\sin\theta]\}, \tag{33}$$

$$E_{Sz} = \frac{2c}{aR_-^3R_+^4}\{K(A)R_+^2[az(-a^2+x^2+z^2)\cos\theta + (R_+^2R_-^2 + 2a^2z^2)\sin\theta] + E(A)[az(-R_+^2R_-^2 + 8a^2(a^2-x^2+z^2))\cos\theta + (R_+^2R_-^2(3a^2-x^2-z^2) - 8a^2z^2(a^2+x^2+z^2))\sin\theta]\}, \tag{34}$$

$$E_{\omega x} = \frac{-2cz\sin\theta}{axR_-R_+^2}[(a^2+x^2+z^2)E(A) - R_+^2K(A)], \tag{35}$$

$$E_{\omega z} = \frac{-2c\sin\theta}{aR_-R_+^2}[(a^2-x^2-z^2)E(A) + R_+^2K(A)], \tag{36}$$

and $E_{Sy}=E_{\omega y}=0$. Here, $R_\pm^2=(a\pm x)^2+z^2$, $A=-4ax/R_-^2$, and $c=-\mu_0 g\mu_B E_a F(\theta)/h$. Then, the total electric field $\mathbf{E}_T=\mathbf{E}_S+\mathbf{E}_\omega$ is also easily obtained, and $\mathbf{E}_T$ can be expressed as a gradient of a potential $V(\mathbf{r})$, $\mathbf{E}_T(\mathbf{r})=-\nabla V(\mathbf{r})$, where

$$V(\mathbf{r}) = \frac{2c}{R_-R_+^2}[R_+^2K(A)\cos\theta + ((a^2-x^2-z^2)\cos\theta - 2az\sin\theta)E(A)]. \tag{37}$$

In fact, this total electric field $\mathbf{E}_T$ can also be expressed as

$$\mathbf{E}_T = -\nabla V = -c\nabla\int \vec{P}_e(\varphi)\cdot\frac{\mathbf{r}-\mathbf{r}'(\varphi)}{|\mathbf{r}-\mathbf{r}'(\varphi)|^3}\, a d\varphi, \tag{38}$$

i.e., $\mathbf{E}_T$ is equivalent to the electric field generated by a one-dimensional electric dipole moment $\vec{P}_e(\varphi)=-(\cos\theta\cos\varphi, \cos\theta\sin\varphi, \sin\theta)=-\vec{e}_\varphi\times(-\sin\theta\vec{e}_r+\cos\theta\vec{e}_z)\propto\vec{v}\times\vec{S}$ in the ring [see Fig. 17(c)].

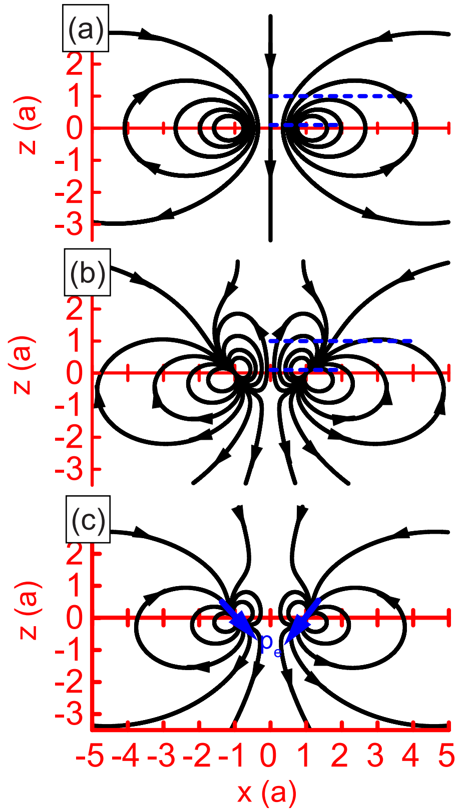

FIG. 17. (Color online) Schematic plots of electric-field lines of the electric fields (a) $\vec{E}_\omega$, (b) $\vec{E}_S$, and (c) $\vec{E}_T$ for the case when the entire ring has a constant α_R.

Figure 17 shows the electric-field lines of $\mathbf{E}_S$, $\mathbf{E}_\omega$, and $\mathbf{E}_T$ in the x-z plane. The across points of the ring and the x-z plane are at $(a,0,0)$ and $(-a,0,0)$. The electric-field lines have the following characteristics: The field lines are in the x-z plane and $E_{Sy}=E_{\omega y}=E_{Ty}=0$. $\nabla\times\mathbf{E}_S$ and $\nabla\times\mathbf{E}_\omega$ are usually nonzero, but the total electric field $\mathbf{E}_T$ has the behavior $\nabla\times\mathbf{E}_T=0$, i.e., $\oint\mathbf{E}_T\cdot d\mathbf{l}=0$. The electric fields in the x-z plane are mirror symmetry around the z axis with $E_{S/\omega/T,x}(x,z)=-E_{S/\omega/T,x}(-x,z)$ and $E_{S/\omega/T,z}(x,z)=E_{S/\omega/T,z}(-x,z)$.

Figure 18 shows the electric-field strengths $E_{Sx/z}$, $E_{\omega x/z}$, and $E_{Tx/z}$ along the two horizontal dashed lines [from the point $(0,0,a)$ to the point $(4a,0,a)$ or from the point $(0,0,0.1a)$ to the point $(2a,0,0.1a)$] in Fig. 17. In this calculation, we consider that only the lowest level $n=0$ (the ground state) in the ring device is occupied by the electron, i.e., taking the parameter $E_0<E_F<E_1$ and, in this case, $\mathbf{I}_S=\mathbf{I}_S^0$ and $\mathbf{I}_\omega=\mathbf{I}_\omega^0$. We also take the parameter g factor, $g=2$, and the efficient electron mass $m=0.036m_e$. At the point $x=0$, the x-direction element $E_{S/\omega/T,x}=0$ due to the mirror symmetry, but the z-direction element $E_{S/\omega/T,z}$ remains a quite large value still. Slightly far away from the ring device [e.g., $z=a$ in Figs. 18(a), 18(c), and 18(e)] the electric field $\mathbf{E}_\omega$ induced from the persistent angular spin current is in the same order with the electric field $\mathbf{E}_S$ induced from the persistent linear spin current. So, $\mathbf{E}_\omega$ is important for contributing to the total electric field $\mathbf{E}_T$. On the other hand, while very near the ring device [e.g., $z=0.1a$ in Figs. 18(b), 18(c),

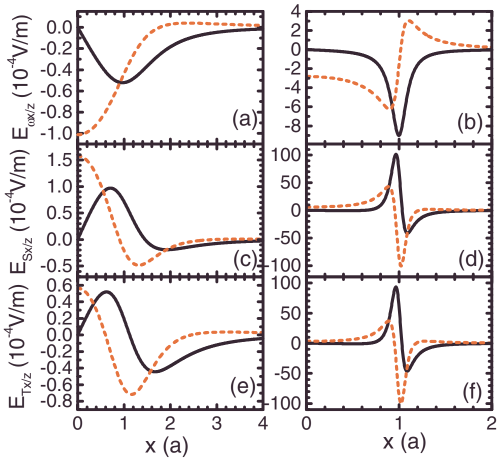

FIG. 18. (Color online) The electric-field strengths [(a) and (b)] $E_{\omega x/z}$ [(c) and (d)] $E_{Sx/z}$ and [(e) and (f)] $E_{Tx/z}$ vs the position x along the horizontal dashed lines in Fig. 17 with [(a), (b), and (c)] $z=a$ and [(b), (d), and (f)] $z=0.1a$. The parameters are $\alpha_R=3\times10^{-11}$ eV m, $\Phi_0=0$, the ring radius $a=50$ nm, and only the lowest level $n=0$ occupied. The solid curve and the dotted curve correspond to $E_{\omega/S/T,x}$ and $E_{\omega/S/T,z}$, respectively.

and 18(f)], $\mathbf{E}_S$ is much larger than $\mathbf{E}_\omega$, and $\mathbf{E}_S$ is dominant in $\mathbf{E}_T$. It is worth mentioning that the total electric field $\mathbf{E}_T$ can reach 10^{-2} V/m at the point $(a,0,0.1a)$, which is $0.1a=5$ nm over the ring $(a,0,0)$ [see Fig. 18(f)]. Also, let us estimate the electric potential difference due to $\mathbf{E}_T$; this potential difference between two points $(a,0,0.01a)$ and $(a,0,0.01a)$ is about 1 nV. Although this potential value is very small, it is measurable in the present technology.[24,32]

IX. SPIN-SPIN INTERACTION AND CONSERVED PERSISTENT SPIN CURRENT

In the previous sections, a single spin picture is adapted; i.e., there is no spin-spin interaction. We find that, in general, the spin current is not conserved. In this section, we demonstrate that if one includes a strong spin-spin interaction, the spin current will be conserved. In this case, if a spin precesses, the response of other spins will be against it; i.e., they will precess in opposite directions. As a result, the total spin precession is zero everywhere, and therefore the spin current is automatically conserved by using the conventional definition.[27,59] To account for the spin-spin interaction, the Hamiltonian H is

$$H=\sum_i H_0(\mathbf{r}_i)+\sum_{i,j} J(\mathbf{r}_i,\mathbf{r}_j)\mathbf{S}(\mathbf{r}_i)\cdot\mathbf{S}(\mathbf{r}_j), \tag{39}$$

where $H_0(\mathbf{r})$ is the one-body Hamiltonian [e.g., the Hamiltonian in Eq. (1) for the ring device] and the second term is the spin-spin interaction. Usually, it is very difficult to solve this Hamiltonian because of the many-body interaction, $\sum_{i,j} J(\mathbf{r}_i,\mathbf{r}_j)\mathbf{S}(\mathbf{r}_i)\cdot\mathbf{S}(\mathbf{r}_j)$. By introducing an induced self-consistent field $\vec{H}_1$ that could be due to spin-spin interaction,[27] the many-body Hamiltonian H in Eq. (39) reduces into the one-body form,

$$H(\mathbf{r})=H_0(\mathbf{r})+\hat{\sigma}\cdot\vec{H}_1(\mathbf{r}). \tag{40}$$

Now, the Hamiltonian of Eq. (40) is easily solved, and the linear spin current and the spin torque all depend on $\vec{H}_1$. The induced self-consistent field $\vec{H}_1$ in Eq. (40) is determined by requiring that the spin torque (or the angular spin current) is zero or $\mathbf{I}_w(\vec{H}_1)=0$ for any $\mathbf{r}$. Once $\vec{H}_1$ is solved, the persistent spin current is automatically conserved by using the conventional definition.

Now, we apply this method to the ring without the normal region and set SOI to a constant with $\alpha_R(\varphi)=\alpha_R$. We start with the following Hamiltonian:

$$H=H_0+\sigma_r H_1, \tag{41}$$

where H_0 is the original Hamiltonian [see Eq. (1)] for a ring with full SOI. The reason that we choose the induced field as $\sigma_r H_1$ is because we know I_z is conserved and there is a torque along $\hat{e}_\varphi$. So, a self-consistent induced magnetic field is needed (a term $\sigma_r H_1$) to balance the torque. The eigenfunction of the new Hamiltonian is the same as Eq. (25) given by

$$\Psi_n(\varphi)=\begin{pmatrix}\cos(\theta/2)e^{in\varphi}\\ -\sin(\theta/2)e^{i(n+1)\varphi}\end{pmatrix}, \tag{42}$$

where the eigenvalue looks the same as before

$$E_n=E_a[n^2+(n+1/2)(1-1/\cos\theta)], \tag{43}$$

but θ takes a different value,

$$\tan\theta=\frac{\alpha}{aE_a}+\frac{H_1}{E_a(n+1/2)}. \tag{44}$$

Note that if the self-consistent field H_1 is zero, $\tan\theta$ recovers the noninteracting case. Using the conventional definition, the persistent spin current with polarization in three directions are easily calculated and found to be

$$I_{Sx}=-E_aF_1(\theta)\sin\theta\cos\varphi,$$
$$I_{Sy}=-E_aF_1(\theta)\sin\theta\sin\varphi, \tag{45}$$
$$I_{Sz}=-E_a[n+1/2-1/(2\cos\theta)]\cos\theta,$$

where

$$F_1(\theta)=[n+1/2-\alpha/(2aE_a\sin\theta)]/2\pi. \tag{46}$$

If $H_1=0$, $F_1(\theta)=[n+1/2-1/(2\cos\theta)]/2\pi=F(\theta)$, which is the previous result without the spin-spin torque interaction. Now, we calculate the total torque. From a continuity equation, the torque is just the angular spin current $\mathbf{I}_\omega=\mathrm{Re}[\Psi^\dagger\hat{\vec{\omega}}\times\hat{s}\Psi]$, with $\hat{\vec{\omega}}=H_1\vec{e}_r-(i\alpha/a)\partial/\partial\varphi\vec{e}_\varphi\times\vec{e}_z$. It is easy to show that $\mathbf{I}_\omega$ has only $\vec{e}_\varphi$ component,

$$I_{\omega\varphi}=H_1\cos\theta+\frac{\alpha}{a}(n\cos\theta-\sin^2\theta/2). \tag{47}$$

Note that the purpose of introducing the self-consistent field H_1 is to make sure that the spin current is conserved or the torque vanishes, as we have discussed earlier. Setting the torque $I_{\omega\varphi}$ to zero, we obtain the second equation that determines the self-consistent field H_1,

$$H_1 = -\frac{\alpha}{a\cos\theta}(n\cos\theta - \sin^2\theta/2) = -\frac{\alpha}{a}[n + 1/2 - 1/(2\cos\theta)]. \quad (48)$$

Once H_1 is solved from Eqs. (44) and (48), the spin current will be conserved following the continuity equation. Plugging Eq. (48) into Eq. (44), we find

$$(n + 1/2)\sin\theta = \frac{\alpha}{2aE_a}. \quad (49)$$

This means that $F_1(\theta)$ defined in Eq. (46) is zero, and the conserved persistent spin current is nonzero only for the spin polarization along the z direction,

$$I_{Sz} = -E_a[n + 1/2 - 1/(2\cos\theta)]\cos\theta, \quad (50)$$

where θ is determined by Eq. (49).

We wish to point out that even for the hybrid ring, the above approach can be used, and the persistent spin current is also conserved in the presence of the spin-spin interaction. The only difference is that we have to introduce three self-consistent fields, H_i, $i=1,2,3$. From the energy dispersion relation, we have the relationship between θ and H_i. By requiring the torque along each direction to be zero, we obtain three additional equations. These four equations will determine H_i and θ. This, in turn, gives the displacement spin current and hence the conserved persistent spin current.

X. CONCLUSION

In summary, we have investigated two closely related subjects: (a) the prediction of a pure persistent spin current in an equilibrium mesoscopic device with solely SOI and (b) the issues concerning the definition of the spin current. Through the physical arguments and physical pictures from four different aspects, the analytic calculation results of a SOI-normal hybrid ring, as well as the discussion of the sharp interface between the normal and SOI parts, we demonstrated that the persistent spin current indeed exists in the equilibrium device with a SOI alone. In particular, we emphasize that this persistent spin current is an analog of the persistent charge current in the mesoscopic ring threaded by a magnetic flux, and it describes the real spin motion and is experimentally measurable.

After showing the existence of the persistent spin current, we investigate the definition of the spin current. We point that (i) the nonzero spin current in the equilibrium SOI's device is the persistent spin current; (ii) in general, the spin current is not conserved; and (iii) the Onsager relation is violated for the spin transport, and, in particular, it cannot be recovered through modification of the definition of the spin current. So, these three flaws, the nonzero spin current in the equilibrium case, the nonconserved spin current, and the violation of the Onsager relation, of the conventional definition of the spin current are intrinsic properties of spin transport. In particular, the conventional definition, $\mathbf{I}_S = \mathrm{Re}\,\Psi^\dagger \hat{v}\hat{s}\Psi$, possesses a very clear physical picture, and is capable of describing the spin motion. So, we draw the conclusion that the conventional definition of the spin current makes physical sense and does not need modification.

In addition, a number of problems have also been discussed. The relation between the persistent spin current and transport spin current is discussed, and we find that they are indistinguishable in the coherent part of the device. We calculate the persistent linear and angular spin currents in the SOI's region of the hybrid ring, and the results show that the persistent spin current still exists in the SOI's region, even when the SOI covers the whole ring. The measurement issue of the persistent spin current is also discussed; we suggest that the persistent spin current can be observed by detecting its induced electric field. In the presence of a spin-spin interaction in the ring, we find that the persistent spin current using the conventional definition is automatically conserved.

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APPENDIX

In this appendix, we analyze the motion of the spin, which relates to the persistent spin current in the equilibrium. For simplicity, we consider the constant SOI case, and the eigen-wave-function has been solved in Eq. (25). From this wave function, the spin $\vec{S} = \langle\Psi_n|\hat{s}|\Psi_n\rangle = \frac{\hbar}{2}(-\sin\theta\cos\varphi, -\sin\theta\sin\varphi, \cos\theta)$. This spin vector is in the $\vec{e}_z$-$\vec{e}_r$ plane, and its angle with the $\vec{e}_z$ (i.e., z) axis is $-\theta$. Then, the spin motion can also be obtained straightforwardly by solving the velocity and its angular velocity $\vec{\omega}$. The direction of the spin (translational) motion is counterclockwise, while precessing with $\vec{\omega}$ in the perpendicular direction of $\vec{S}$, so that the spin is in the $\vec{e}_z$-$\vec{e}_r$ plane all along. The element of the spin in the x-y plane and its motion are shown as in Fig. 12(b). On the other hand, for the time-reversal state $T\Psi_n$, the spin direction, its motion direction, and the precession direction all reversed, as shown in the clockwise arrow in Fig. 12(b). However, the spin current of $T\Psi_n$ is completely the same with that of Ψ_n. Therefore, the persistent spin current indeed describes the real motion of the spin.

In fact, besides the ring geometry, the device can also have other shapes. For example, we have analyzed the spin motion and the persistent spin current in the quasi-one-dimensional equilibrium quantum wire.[36] Similar conclusions can be drawn.

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[42] In fact, the boundary condition of the flux can be deduced from the Schrödinger equation by taking the integral $\int_{0_-}^{0^+} H(\varphi)\Psi(\varphi)d\varphi=\int_{0_-}^{0^+} E\Psi(\varphi)d\varphi$.

[43] Note that the eigenlevel of $n=0$ is twofold degenerate, and this level does not split for nonzero α_R.

[44] For a continuous wave function $\Psi(\varphi)$, its normalization equation is $\int_0^{2\pi}\Psi^{\dagger}(\varphi)\Psi(\varphi)ad\varphi=1$. For a discrete wave function Ψ_j, its normalization equation is $\Sigma_j\Psi_j^{\dagger}\Psi_j a\Delta\varphi=1$.

[45] Because of the twofold degeneracy for each level E_n, we have multiplied the factor 2 in the plotting of I_{Si}^n, i.e., $I_{Si}^n=2\,\mathrm{Re}\,\Psi_n^{\dagger}\hat{v}\hat{\sigma}_i\Psi_n$ and $I_{Si}=\Sigma_n I_{Si}^n f(E_n)$.

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[53] By the way, we emphasize that the implications of the angular spin current and the spin torque are different, although they are completely the same in terms of numerical value. The angular spin current describes the spin motion or the spin flow, but the spin torque expresses the action on the spin. This is similar to the relation between the force $\vec{F}$ and the acceleration $\vec{a}$. If we take the electron mass $m=1$ as the unit, the value of the force equals that of the acceleration, but their physical meanings are different.

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[59] When the total spin precession is everywhere zero, the angular spin current $\mathbf{I}_{\omega}=0$, and the spin continuity equation becomes $\nabla\cdot\mathbf{I}_{Si}+dS_i/dt=0$, where S_i is the spin accumulation and is zero in the present equilibrium case. Note that this continuity equation for every spin element $i=x,y,z$ is completely the same as the charge continuity equation. Therefore, the conserved spin current can be derived from Noether's theorem, similar to the conserved charge current.