# Disorder-enhanced transmission of a quantum mechanical particle through a disordered tunneling barrier in one dimension: Exact calculation based on the invariant imbedding method

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(Received 12 September 2007; revised manuscript received 4 December 2007; published 17 January 2008)

We revisit the problem of disorder-enhanced tunneling transmission of a quantum mechanical particle through a disordered tunneling barrier in one dimension. Using the invariant imbedding theory of wave propagation generalized to randomly stratified media, we calculate the disorder-averaged logarithmic transmittance in the thick barrier limit and the disorder-averaged transmittance in a numerically exact manner. We confirm that the tunneling decay length obtained from the mean logarithmic transmittance behaves nonmonotonically as a function of the disorder strength and takes its maximum value at some finite value of the disorder parameter. We find that this nonmonotonic dependence persists in the presence of weak inelastic scattering inside the tunneling barrier. When the system size is larger than some critical value, which is somewhat smaller than the wavelength of the incident matter wave, we observe that the disorder-averaged transmittance also shows a similar nonmonotonic dependence on the disorder strength. In other words, weak disorder enhances the transmission, while strong disorder suppresses it. When the system size is smaller than the critical value, the disorder-averaged transmittance decreases monotonically as the disorder strength increases.

DOI: 10.1103/PhysRevB.77.024203 PACS number(s): 42.25.Dd, 73.20.Fz, 73.23.−b, 73.40.Gk

## I. INTRODUCTION

Tunneling is one of the most basic and important quantum mechanical processes. It occurs in a wide variety of natural phenomena including thermonuclear reactions in stars and radioactive nuclear decays. Various kinds of tunnel junctions and devices based on electron or atom tunneling, which include Esaki tunnel diodes, Josephson junctions, and scanning tunneling microscopes among many others, have played revolutionary roles in the development of science and technology. They continue to be very important areas of research both in fundamental sciences and in applications.[1–6]

Tunneling of quantum mechanical particles occurs due to their wavelike nature. Therefore, similar phenomena can occur in all kinds of classical waves including electromagnetic waves,[7,8] plasma waves,[9] and water waves.[10] A well-known example from optics is the tunneling of electromagnetic waves in frustrated total internal reflection situations.[8] The so-called transparent metal structure consisting of alternating metal and dielectric layers exploits the tunneling of electromagnetic waves through the metal layers, too.[11] Tunneling is also a very important process in the radio frequency heating of inhomogeneous space and laboratory plasmas by external electromagnetic waves.[9,12,13]

In realistic situations, tunneling barriers contain various kinds of imperfections. For example, the electrode-barrier interfaces of a tunnel junction may not be smooth and have a substantial degree of surface roughness. Furthermore, the barrier itself may contain impurities, defects, and other kinds of structural disorder. In these cases, the barrier parameters become spatially fluctuating quantities. The influence of spatial disorder on the properties of tunnel junctions has been a subject of active experimental and theoretical research.[1,14–18]

A peculiar disorder-induced phenomenon occurring when a wave or a quantum mechanical particle passes through a disordered one-dimensional tunneling barrier was reported for the first time by Freilikher *et al.* in 1996.[19] Using a diagrammatic perturbation theory, they found a surprising result that the disorder-averaged tunneling transmittance behaved nonmonotonically as a function of the disorder strength. Contrary to the naive expectation, sufficiently weak disorder was shown to enhance the tunneling transmittance, whereas strong disorder suppressed it.

More recently, Luck reconsidered this problem and pointed out that previous results on the theory of one-dimensional localization could be used to derive an exact analytical formula for the tunneling decay length in the case of a Gaussian white-noise potential.[20] He also obtained the behavior of various moments of the transmittance in the weak and strong disorder limits, when the system size is sufficiently large.

Due to the mathematical difficulty of calculating the exact disorder-averaged transmittance in disordered systems, previous works on the tunneling transmittance either used approximate methods or considered only some limiting cases. Based on the invariant imbedding theory of wave propagation,[21–25] one of the authors has developed an efficient numerical method of calculating the disorder-averaged transmittance and reflectance of waves propagating in randomly stratified media exactly.[26] This method was used successfully in calculating the probability distributions of the reflectance and the phase of the reflection coefficient of waves propagating in random media.

In this paper, we apply the method developed in Ref. 26 to the disordered tunneling barrier problem and calculate the tunneling decay length and the disorder-averaged transmittance exactly. Our result on the tunneling decay length agrees perfectly with the analytical formula given in Ref. 20 and it demonstrates that our method is, indeed, exact. Furthermore, we calculate the decay length in a more general case where the particle experiences weak inelastic scattering inside the tunneling barrier. We find that the nonmonotonic dependence of the decay length on the strength of disorder survives in the presence of weak inelastic scattering.

We also present the invariant imbedding equations which are used in the exact calculation of the disorder-averaged transmittance and its moments, and the results of our numerical calculation. Our results are obtained for systems of finite size and do not assume weak or strong disorder limits. We find that when the system size is larger than a certain critical value, which is somewhat smaller than the wavelength of the incident wave, the average transmittance behaves nonmonotonically and takes a maximum value at some finite value of the disorder parameter. When the system size is smaller than the critical value, the average transmittance decreases monotonically as the disorder parameter increases.

In Sec. II, we describe our invariant imbedding method and present the invariant imbedding equations, which are used in the numerical calculation. In Sec. III, we show our results on the tunneling decay length. In Sec. IV, we present the results on the disorder-averaged transmittance. Finally, in Sec. V, we discuss the experimental ramifications of our theory and conclude the paper.

## II. INVARIANT IMBEDDING METHOD

We are interested in the tunneling transmission of a quantum mechanical particle through a disordered tunneling barrier in one dimension, which is described by the Schrödinger equation

$$\frac{d^2\psi}{dx^2} + k^2\left[1 - \frac{V(x)}{E}\right]\psi = 0, \qquad (1)$$

where $E$ $(=\hbar^2k^2/2m_0)$ is the energy of the incoming particle of mass $m_0$ and $V(x)$ is the potential. We assume that the potential is equal to 0 for $x<0$ and $x>L$, and $V(x)=V_0(x)+\delta V(x)-i\Gamma$ for $0\leq x\leq L$, where $V_0(x)$ is a positive real function and $\Gamma$ is a positive real constant. We suppose $\delta V(x)$ to be a Gaussian random function with zero mean and a white-noise spectrum:

$$\langle \delta V(x)\delta V(x')\rangle = 2D(x)\delta(x-x'), \quad \langle \delta V(x)\rangle = 0, \qquad (2)$$

where $\langle\cdots\rangle$ denotes averaging over disorder and the positive function $D(x)$ is a measure of the strength of nonuniform randomness. Tunneling occurs when the particle energy $E$ $(>0)$ is smaller than $V_0(x)$. The constant $\Gamma$ represents the decrease of the number of particles with energy $E$ due to inelastic scattering inside the barrier.

As mentioned in the Introduction, tunneling can occur in all kinds of classical waves. For instance, $s$-polarized plane electromagnetic waves of vacuum wave number $k_0$, which are propagating in the $xy$ plane and incident from a uniform region with high refractive index $n_1$ $(=\sqrt{\epsilon_1})$ onto a stratified dielectric slab at an incident angle $\theta$, satisfy the wave equation

$$\frac{d^2\mathcal{E}}{dx^2} + (n_1k_0\cos\theta)^2[1+\tilde{\epsilon}(x)]\mathcal{E} = 0,$$

$$\tilde{\epsilon}(x) = \frac{\epsilon(x)-\epsilon_1}{\epsilon_1\cos^2\theta}, \qquad (3)$$

where $\mathcal{E}$ is the electric field amplitude and $\epsilon(x)$ is the dielectric permittivity of the slab. This equation has precisely the same form as Eq. (1) if we identify $n_1k_0\cos\theta$ and $\tilde{\epsilon}(x)$ with $k$ and $-V(x)/E$, respectively. Let us suppose that $\epsilon(x)$ is given by $\epsilon(x)=\epsilon_R+i\epsilon_I+\delta\epsilon(x)$, where $\delta\epsilon(x)$ is a Gaussian random function satisfying

$$\langle \delta\epsilon(x)\delta\epsilon(x')\rangle = 2\eta(x)\delta(x-x'), \quad \langle \delta\epsilon(x)\rangle = 0. \qquad (4)$$

Tunneling of electromagnetic waves occurs if $0<\epsilon_R<\epsilon_1$ and $(\epsilon_R-\epsilon_1)/(\epsilon_1\cos^2\theta)<-1$, which are equivalent to the condition for total internal reflection, $\sin\theta>\sqrt{\epsilon_R/\epsilon_1}$. The imaginary part of the dielectric permittivity, $\epsilon_I$, describes the absorption of wave energy inside the slab and is related to the inelastic scattering parameter $\Gamma$ in the quantum case by $\epsilon_I/\epsilon_1\cos^2\theta \leftrightarrow \Gamma/E$. The quantity representing the strength of randomness, $\eta(x)$, is related to $D(x)$ by $\eta(x)/\epsilon_1^2\cos^4\theta \leftrightarrow D(x)/E^2$.

Another example where tunneling of a classical wave is described by Eq. (1) is the propagation of circularly polarized electromagnetic waves in a direction parallel to the external magnetic field, $\mathbf{B}=B_0\hat{\mathbf{x}}$, in a magnetized electron plasma slab.[27] The electric field amplitude $\mathcal{E}_+$ $(\mathcal{E}_-)$ for right (left) circularly polarized waves propagating in the negative $x$ direction satisfies

$$\frac{d^2\mathcal{E}_\pm}{dx^2} + k_0^2\left[1 - \frac{\omega_p^2}{\omega(\omega\mp\omega_c+i\nu)}\right]\mathcal{E}_\pm = 0, \qquad (5)$$

where $\omega_p$ $(=[4\pi e^2 n_e(x)/m_e]^{1/2})$ is the electron plasma frequency and $\omega_c$ $(=eB_0/m_ec)$ is the electron cyclotron frequency. $n_e(x)$ is the electron number density and $\nu$ is the collision frequency. From Eq. (5), it is easy to see that in the frequency range, $\omega_c<\omega<\omega_-$, where $\omega_-=[(\omega_c^2+4\omega_p^2)^{1/2}-\omega_c]/2$, both right and left circularly polarized waves cannot propagate inside the plasma and are transmitted only by tunneling. We note that the electron density plays the role of the potential in the present case. Therefore, if the density contains a component which is random in $x$, Eq. (5) will be completely equivalent to Eq. (1).

We assume that the particle, which is described by a plane wave of unit magnitude $\psi(x)=e^{ik(L-x)}$, is incident onto the barrier from the region where $x>L$ and transmitted to the region where $x<0$. The quantities of main interest are the complex reflection and transmission coefficients, $r=r(L)$ and $t=t(L)$, defined by the wave functions outside the medium:

$$\psi(x) = \begin{cases} e^{ik(L-x)} + r(L)e^{ik(x-L)}, & x>L \\ t(L)e^{-ikx}, & x<0. \end{cases} \qquad (6)$$

Using the invariant imbedding method, we derive exact differential equations satisfied by $r$ and $t$:

$$\frac{dr(l)}{dl} = 2ikr(l) + \frac{ik}{2}[-v_0(l)+i\gamma+\delta v(l)][1+r(l)]^2,$$

$$\frac{dt(l)}{dl} = ikt(l) + \frac{ik}{2}[-v_0(l) + i\gamma + \delta v(l)][1 + r(l)]t(l), \quad (7)$$

where we have defined

$$v_0(l) = \frac{V_0(l)}{E}, \quad \gamma = \frac{\Gamma}{E}, \quad \delta v(l) = -\frac{\delta V(l)}{E}. \quad (8)$$

The random function $\delta v(l)$ satisfies

$$\langle \delta v(l) \delta v(l') \rangle = \frac{2D(l)}{E^2}\delta(l-l'), \quad \langle \delta v(l) \rangle = 0. \quad (9)$$

The invariant imbedding equations [Eq. (7)] are integrated from $l=0$ to $l=L$ using the initial conditions for $r$ and $t$:

$$r(0) = 0, \quad t(0) = 1. \quad (10)$$

We will use Eqs. (7), (9), and (10) in calculating the exact disorder averages of various physical quantities consisted of $r$ ($\equiv \sqrt{R}e^{i\theta}$) and $t$ ($\equiv \sqrt{T}e^{i\phi}$), where the reflectance $R=|r|^2$ and the transmittance $T=|t|^2$ as well as the phases $\theta$ and $\phi$ are functions of $L$. In particular, we are interested in calculating the tunneling decay length $\xi$ of the matter wave defined by

$$\lim_{L\to\infty} \langle \ln T \rangle = -L/\xi \quad (11)$$

and the moments $\langle T^m R^n \rangle$ as a function of $L$ for various values of $m$ and $n$.

An infinite number of coupled nonrandom differential equations satisfied by the moments $\langle T^m R^n \rangle$ are obtained using Eqs. (7) and (9) and Novikov's formula.[28] It turns out that in order to compute $\langle T^m R^n \rangle = \langle T^m r^n r^{*n} \rangle$ for a non-negative integer $n$, one needs to compute the moments $Z_{mn\tilde{n}} \equiv \langle T^m r^n r^{*\tilde{n}} \rangle$ for all non-negative integers $n$ and $\tilde{n}$. In other words, the moments $Z_{mn\tilde{n}}$ with $n=\tilde{n}$ are coupled to $Z_{mn\tilde{n}}$ with $n \neq \tilde{n}$. The nonrandom differential equation satisfied by $Z_{mn\tilde{n}}$ has the form

$$\begin{aligned}
\frac{1}{k}\frac{d}{dl}Z_{mn\tilde{n}} = {} & \{i[2 - v_0(l)](n-\tilde{n}) - \gamma(m+n+\tilde{n}) + g(l)[-m(n+\tilde{n}+1) + 4n\tilde{n} - 3n^2 - 3\tilde{n}^2]\}Z_{mn\tilde{n}} \\
& - (m+n)\left\{g(l)[2(n-\tilde{n})+1] + i\frac{v_0(l)}{2} + \frac{\gamma}{2}\right\}Z_{m,n+1,\tilde{n}} + (m+\tilde{n})\left\{g(l)[2(n-\tilde{n})-1] + i\frac{v_0(l)}{2} - \frac{\gamma}{2}\right\}Z_{mn,\tilde{n}+1} \\
& - n\left\{g(l)[2(n-\tilde{n})-1] + i\frac{v_0(l)}{2} + \frac{\gamma}{2}\right\}Z_{m,n-1,\tilde{n}} + \tilde{n}\left\{g(l)[2(n-\tilde{n})+1] + i\frac{v_0(l)}{2} - \frac{\gamma}{2}\right\}Z_{mn,\tilde{n}-1} \\
& + g(l)(m+n)(m+\tilde{n})Z_{m,n+1,\tilde{n}+1} + g(l)n\tilde{n}Z_{m,n-1,\tilde{n}-1} + g(l)(m+n)\tilde{n}Z_{m,n+1,\tilde{n}-1} + g(l)n(m+\tilde{n})Z_{m,n-1,\tilde{n}+1} \\
& - \frac{g(l)}{2}(m+n)(m+n+1)Z_{m,n+2,\tilde{n}} - \frac{g(l)}{2}(m+\tilde{n})(m+\tilde{n}+1)Z_{mn,\tilde{n}+2} - \frac{g(l)}{2}n(n-1)Z_{m,n-2,\tilde{n}} - \frac{g(l)}{2}\tilde{n}(\tilde{n}-1)Z_{mn,\tilde{n}-2},
\end{aligned} \quad (12)$$

where $g(l) \equiv D(l)k/2E^2$. For positive integers $n$ and $\tilde{n}$ and for an arbitrary real number $m$, $Z_{mn\tilde{n}}$ satisfies the conditions

$$Z_{mn\tilde{n}}(l=0) = Z_{mn0}(l=0) = Z_{m0\tilde{n}}(l=0) = 0,$$

$$Z_{m00}(l=0) = 1,$$

$$Z_{000}(l) = 1. \quad (13)$$

We will solve Eq. (12) numerically using the truncation method developed in Ref. 26. In disordered systems, we find that the magnitude of the moment $Z_{mn\tilde{n}}$ decays rapidly as either $n$ or $\tilde{n}$ increases. We assume $Z_{mn\tilde{n}}=0$ for either $n$ or $\tilde{n}$ greater than some large positive integer $N$ and solve the *finite* number $[=(N+1)^2]$ of coupled ordinary differential equations numerically for given values of $m$, $\gamma$, and $kL$ and for given functions $v_0(l)$ and $g(l)$. We increase the cutoff $N$, repeat a similar calculation, and then compare the newly obtained $Z_{mn\tilde{n}}$ with the value of the previous step. If there is no change in the values of $Z_{mn\tilde{n}}$ within an allowed numerical error, we conclude that we have obtained the exact solution of $Z_{mn\tilde{n}}$.

In order to compute the tunneling decay length as defined by Eq. (11), we need to compute the mean logarithmic transmittance $\langle \ln T \rangle$ in the $l \to \infty$ limit. The nonrandom differential equation satisfied by $\langle \ln T \rangle$ is obtained using Eq. (7) and Novikov's formula in a straightforward manner:

$$\begin{aligned}
\frac{1}{k}\frac{d}{dl}\langle \ln T \rangle = {} & -\gamma - g(l) - \mathrm{Re}\{[iv_0(l) + \gamma + 2g(l)]Z_{010}(l) \\
& + g(l)Z_{020}(l)\}.
\end{aligned} \quad (14)$$

### III. TUNNELING DECAY LENGTH

In the present paper, we restrict our attention to the cases where $v_0(l)$ and $g(l)$ are constants independent of $l$. Then the tunneling decay length $\xi$ is given by

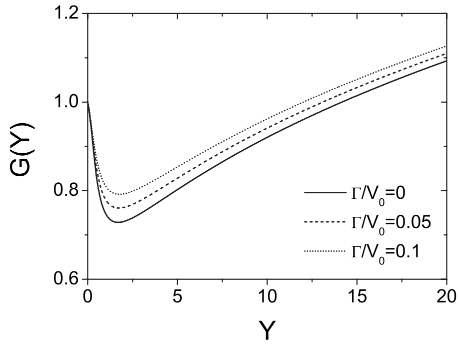


FIG. 1. $G(Y)$ ($=1/2sk\xi$) plotted vs $Y$ ($=2g/s^3$), where $s=\sqrt{v_0-1}$, for $\Gamma/V_0=0$, 0.05, 0.1 and $v_0=2$. The minima of $G(Y)$ (equivalently, the maxima of $\xi$) for $\Gamma/V_0=0$, 0.05, and 0.1 occur at $Y=1.695$, 1,732, and 1.769, respectively.

$$\frac{1}{k\xi}=\gamma+g+\mathrm{Re}[(iv_0+\gamma+2g)Z_{010}(l\to\infty)+gZ_{020}(l\to\infty)]. \tag{15}$$

The values of $Z_{010}(l\to\infty)$ and $Z_{020}(l\to\infty)$ are obtained by solving Eq. (12) numerically for $m=0$.

In Fig. 1, we plot the quantity $G(Y)$ ($\equiv 1/2sk\xi$) as a function of $Y$ ($\equiv 2g/s^3$), where $s=\sqrt{v_0-1}$, for $\Gamma/V_0=0$, 0.05, and 0.1 and $v_0=2$. $G(Y)$ is inversely proportional to $\xi$, and $Y$ is proportional to the disorder strength. In the case where the parameter $\Gamma=0$, the function $G(Y)$ is universal in the sense that it does not depend on the value of $v_0$ or $s$, and agrees perfectly with the analytical result obtained in Ref. 20:

$$G(Y,\Gamma=0)=Y^{1/3}F^R(Y^{-2/3}),$$

$$F^R(X)=\frac{\mathrm{Ai}(X)\mathrm{Ai}'(X)+\mathrm{Bi}(X)\mathrm{Bi}'(X)}{\mathrm{Ai}(X)^2+\mathrm{Bi}(X)^2}, \tag{16}$$

where Ai($X$) and Bi($X$) are the Airy functions. This universality, however, does not hold in the $\Gamma\neq 0$ case.

In Fig. 1, we notice that the tunneling decay length takes its maximum value at some nonzero value of the disorder

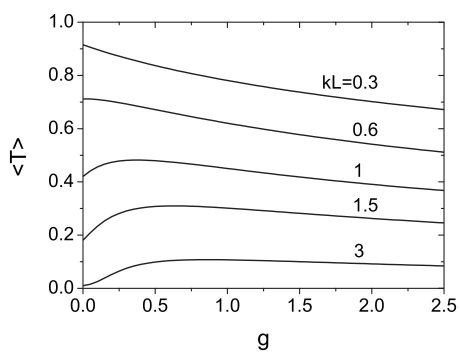


FIG. 2. Disorder-averaged transmittance $\langle T\rangle$ plotted vs the disorder parameter $g$ for $v_0=2$, $\gamma=0$, and $kL=0.3$, 0.6, 1, 1.5, and 3. When $kL=0.3$, $\langle T\rangle$ decreases monotonically as $g$ increases. When $kL$ is larger than about 0.58, $\langle T\rangle$ behaves nonmonotonically and takes its maximum value at $g_m$.

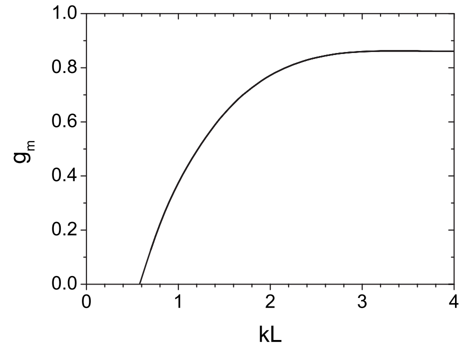


FIG. 3. Dependence of $g_m$ on $kL$ when $v_0=2$ and $\gamma=0$. Above the critical value of $kL$, which is about 0.58, $g_m$ varies continuously as $kL$ increases.

parameter $Y$, in both the $\Gamma=0$ and $\Gamma\neq 0$ cases. In other words, the nonmonotonic behavior of the decay length persists in the presence of weak inelastic scattering. The $Y$ value at which $\xi$ takes the maximum increases slowly as $\Gamma$ increases. For $\Gamma/V_0=0$, 0.05, and 0.1, the maxima occur when $Y=1.695$, 1.732, and 1.769, respectively. As expected, for a fixed $Y$, $\xi$ decreases as the strength of inelastic scattering increases.

## IV. DISORDER-AVERAGED TRANSMITTANCE

We have calculated the disorder-averaged transmittance $\langle T\rangle$ as a function of the disorder parameter $g$ for fixed values of $kL$, $v_0$, and $\gamma$, using Eq. (12) for $m=1$. In Fig. 2, we show our results for several values of $kL$ when $v_0=2$ and $\gamma=0$. We find that for small values of $kL$, $\langle T\rangle$ is a monotonically decreasing function of disorder, whereas for large values of $kL$, it takes the maximum value at a nonzero value of $g$, which we call $g_m$. In other words, the disorder-averaged transmittance is enhanced as disorder increases in the weak disorder regime, while it is suppressed as disorder increases in the strong disorder regime.

In the nonmonotonic regime, the value of $g_m$, at which $\langle T\rangle$ takes the maximum, varies continuously as $kL$ increases, as shown in Figs. 3–5, which correspond to the cases with $v_0=2$, 1.5, and 3, respectively. The parameter $\gamma$ is set to zero. We find that there is a critical value of $kL$ below which the

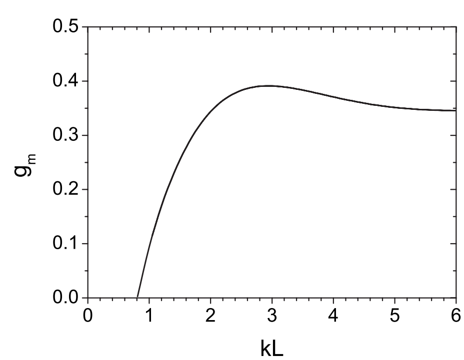


FIG. 4. Dependence of $g_m$ on $kL$ when $v_0=1.5$ and $\gamma=0$.

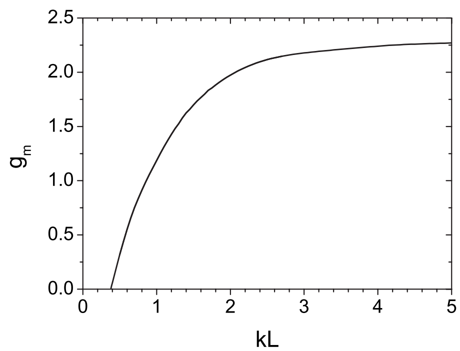


FIG. 5. Dependence of $g_m$ on $kL$ when $v_0=3$ and $\gamma=0$.

nonmonotonic behavior does not occur and $g_m=0$. When $v_0=2$, the critical value is about 0.58. This corresponds to the system size approximately equal to $0.09\lambda$, where $\lambda$ is the wavelength of the incident wave. The critical value of $kL$ decreases as $v_0$ increases.

In general, the dependence of $g_m$ on $kL$ is nonmonotonic. We find that as $kL$ increases, $g_m$ converges slowly to the value of $g$ where the tunneling decay length takes the maximum. Since the maximum of $\xi$ occurs at $Y=2g/s^3\approx 1.695$ in the nonabsorbing case, we expect $g_m$ to converge to 0.848, 0.3, and 2.397, respectively as $kL\to\infty$ in Figs. 3–5.

## V. DISCUSSION AND CONCLUSION

Although there have been many studies on the influence of impurities and structural disorder on the properties of tunnel junctions, there exists no direct experimental evidence of the disorder-enhanced tunneling transmission discussed in this paper. The present theory assumes that the random potential depends on only one spatial coordinate. Since the random potential in real tunnel junctions is usually two- or three-dimensional, it is not possible to compare our theory directly with the experiments on tunnel junctions, though we believe the disorder-enhanced tunneling transmission will occur in the presence of two- or three-dimensional disorder, too. Another experimental difficulty is to find a method to change the strength of weak disorder in a systematic and continuous manner.

It appears that the effect studied here can be observed more easily in the case of classical waves. For example, one can fabricate a binary randomly layered medium composed of alternating layers of two different dielectrics and of random thicknesses[29] and study the tunneling of electromagnetic waves in the total internal reflection situation described by Eq. (3). It is also relatively easy to study the tunneling of electromagnetic waves through a magnetized plasma slab, which is described by Eq. (5). Spatial disorder, in this case, is introduced by thermal fluctuations of the plasma density and there are many situations where the disorder can be considered to be one-dimensional.[30] It is possible to control the strength of disorder by simply changing the plasma temperature.

In our opinion, the easiest case to observe the disorder-enhanced tunneling transmission is provided by shallow water waves. In the linear case, the propagation of shallow water waves is governed by the same wave equation as that for $p$-polarized electromagnetic waves and the role of the dielectric permittivity is played by the inverse water depth.[31,32] Shallow water waves incident obliquely on a region where the water depth increases suddenly can tunnel through the region.[10] Let us suppose that the bottom topography in this region has one-dimensional randomness. Then the situation becomes very similar to that studied so far. Experiments on shallow water waves can be performed readily using a wave tank with an artificially designed bottom shape. The strength of disorder can be controlled very easily by changing the average water depth.

In summary, we reconsidered the problem of disorder-enhanced transmission of a quantum mechanical particle through a disordered tunneling barrier in one dimension. Using the invariant imbedding theory of wave propagation generalized to randomly stratified media, we have calculated the tunneling decay length and the disorder-averaged tunneling transmittance in a numerically exact manner. We have confirmed the previous result that the tunneling decay length behaves nonmonotonically as a function of the disorder strength. In addition, we have found that this nonmonotonic dependence persists in the presence of weak inelastic scattering inside the tunneling barrier. When the system size is larger than some critical value, we have found that the disorder-averaged transmittance also shows a similar nonmonotonic dependence on the disorder strength. When the system size is smaller than the critical value, the average transmittance decreases monotonically as the disorder strength increases.

The method presented in this paper can be applied easily to the exact calculation of other interesting quantities such as $\langle T^{-1}\rangle$, $\langle T^{-2}\rangle$, and $\langle T^{2}\rangle$. It can also be used to study more complicated cases where $V_0(l)$ and $D(l)$ are functions of $l$, instead of being constants. A straightforward generalization of our method using Shapiro and Loginov's formula will allow one to study the case where the random potential is short-range correlated with a finite correlation length.[33] These generalizations will be considered in future publications.

## ACKNOWLEDGMENTS

This work has been supported by the Korea Science and Engineering Foundation grant (Grant No. R0A-2007-000-20113-0) funded by the Korean Government (MOST).

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