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Nonequilibrium interacting electrons in a ferromagnet

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The dynamics of the magnetization in ferromagnets is examined in the presence of transport electrons, allowing the latter to interact. It is found that the existence of inhomogeneities such as domain wall (DW) structures leads to changes that affect the dynamical structure of the equations of motion for the magnetization. Only in the limit of uniform magnetizations or sufficiently wide DWs do the equations of motion maintain the form they have in the noninteracting case. In this limit, results like spin torques, the Gilbert parameter, and DW velocities become renormalized. However, the length scale that defines such a limit depends on the strength of the interaction. It is shown that if large ferromagnetic fluctuations exist in the metallic band, then the range for which conformity with the noninteracting case holds extends to the limit of arbitrarily narrow DWs.

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Increasing possibilities for spin-selective transport of electrons in different materials and their promising physical and technological consequences have yielded a broad variety of research on electronic motion in the presence of a ferromagnetic background. Interest ranges from semiconductors to transition metal ferromagnets.[1–7]

A question under current attention is how electrons flowing in a ferromagnetic medium should modify the dynamics of its order parameter $\mathbf{M}(\mathbf{r},t)$. In recent years, a typical route to handle this problem has consisted of treating conduction and valence electrons distinctively, the former assumed to belong to an *s* band, while the latter being in a filled *d* band responsible for the local moments that give rise to $\mathbf{M}$.

One often uses some equation of motion [such as, e.g., the Landau-Lifschitz-Gilbert[8] (LLG) equation] to treat $\mathbf{M}$ as a classical field, while a kinetic transport equation is used for the dynamics of the conduction *s* electrons, usually in the limit of long length scales where semiclassical equations are valid for the electronic spin.[9] Coupling of these equations is achieved by including an *s*-*d* channel through a Kasuya interaction operator[10,11] whose form is familiar, e.g., in the Kondo physics of dilute magnetic alloys.[12] In transition metal ferromagnets this approach has been used in attempts to draw an adequate picture to understand experiments.[13–16] Recently discussed issues include the role of the so-called nonadiabatic spin torque that the electronic current produces on a domain wall (DW), affecting its motion and stability, and the contribution of the conduction electrons for the damping of $\mathbf{M}$.[17,18]

Underlying the theories used to treat these phenomena we may identify two assumptions (among many others): first, that the ferromagnetic state is a broken symmetry of the interacting electrons within the *d* band, and second, that the *s*-band interelectronic interaction (*s*-*s*) leads only to trivial renormalizations or else that the *s*-*s* interaction is sufficiently weak not to produce any measurable effects on $\mathbf{M}$. Thus while the interaction is considered to be sufficiently important in the *d* band to drive its electrons into a ferromagnetic state, the *s* band is taken as an essentially noninteracting-electron metallic band.

In this article we relax the second of these assumptions and investigate some preliminary general consequences that are expected to be seen on the dynamics of the magnetization. As we will see, the interactions may affect the dynamical structure of the equations of motion. In the limit of uniform magnetizations or sufficiently wide DWs, treating the *s* electrons as noninteracting is justified with the proper renormalizations. We show that in the particular limit for which large ferromagnetic fluctuations exist in the *s* band, decoupling of the equations of motion in a local frame is possible for narrow DWs. Consequently, not only the length for which the "wide-DW" approximations become valid is lowered, but the equations of motion acquire the form they have for noninteracting *s* electrons.

For these purposes, we take the usual algorithm described in the third paragraph and modify it to include the *s*-*s* interactions. This is done by using Fermi liquid theory[9,19,20] and the results should hold provided the conduction electrons are normal (no assumption is required to whether the interactions are weak or not).

The transport equation for the *s*-electron distribution can be written in a compact form[9]

$$\partial_t \mathcal{N} + \{\mathcal{N},\mathcal{H}\} + \frac{i}{\hbar}[\mathcal{H},\mathcal{N}] = I[\mathcal{N}], \tag{1}$$

where all quantities are 2×2 matrices in spin space, $\mathcal{N}_{\mathbf{p}}(\mathbf{r},t)$ is the full distribution function for the *s* electrons, $\mathcal{H}_{\mathbf{p}}(\mathbf{r},t)$ is the effective Hamiltonian, and $I[\mathcal{N}]$ is a collision integral. Curly Poisson and straight commutator brackets are also implied in the notation. The commutator is kept with respect to the spin matrices as a consequence of using a local quantization axis. These functions must be understood as averages in the state of the system, which satisfy quantum mechanical equations of motion that reduce to the above semiclassical equation in the long-wavelength limit.[21]

Inclusion of the *s*-*d* coupling to the order parameter of the ferromagnetic band is achieved by using an exchange interaction besides the Fermi liquid Hamilitonain $\mathcal{H}_{FL}$, $\mathcal{H}=\mathcal{H}_{FL}+\mathcal{H}_{sd}$, where $\mathcal{H}_{sd}=SJ_{ex}\sigma\cdot\mathbf{M}(\mathbf{r},t)/M_s$, S is the projection of the local spin for the *d* electrons, M_s is the saturation magnetization, and σ are the usual Pauli matrices.[17] Note that the structure of the transport equation in spin space carries a commutator even in the absence of the *s*-*d* ex-

change coupling, which has long been known;[9] however, the intrinsic s-electron commutator vanishes if these electrons are noninteracting.

To write equations of motion from the transport equation, we trace Eq. (1) to obtain equations for the local spin density of the s electrons, $\mathbf{m}=\mathrm{tr}[\sigma\mathcal{N}]$, and the spin current tensor, $\boldsymbol{J}_i \equiv \mathrm{tr}[\sigma(\partial\mathcal{H}/\partial p_i)\mathcal{N}]$.

Before writing these equations we separate, as in Ref. 17, the local spin density in terms of an adiabatic component in the direction of $\mathbf{M}$ and a transverse (nonadiabatic) component $\delta\mathbf{m}$,

$$\mathbf{m} = \frac{n_0'}{M_s}\mathbf{M} + \delta\mathbf{m}, \tag{2}$$

where $n_0'=n_0/(1+F_0^a)$, n_0 is the local equilibrium spin density, and F_ℓ^a is the usual notation for the standard Landau antisymmetric dimensionless parameters.

The spin current tensor can similarly be split into two contributions,

$$\boldsymbol{J}_i = -\frac{\mu_B P'}{eM_s} j_{e_i}\mathbf{M} + \boldsymbol{j}_{\sigma_i}, \tag{3}$$

where $\boldsymbol{j}_e$ is the electric current density, e is the electron's charge, μ_B is the Bohr magneton, and $P'=P/(1+F_0^a)$ with the current's spin polarization given by P.

We note that the adiabatic responses to $\mathbf{M}$ are renormalized by the interaction in exactly the same way as the Pauli susceptibility in a paramagnet. This comes from the s-d coupling that effectively adds a local contribution

$$\mathbf{h}_\mathbf{p}^0 = \frac{SJ_{ex}}{2M_s}\mathbf{M} \tag{4}$$

to the molecular field, $\mathbf{h}_\mathbf{p}=\mathbf{h}_\mathbf{p}^0+\mathbf{h}_{\mathbf{p}FL}$, acting on the Fermi liquid quasiparticles.[20] Formally, the local field $\boldsymbol{S}=-S\mathbf{M}/M_s$ plays the role of a "magnetic field" in the equations of motion of a uncharged Fermi liquid.

The equation of motion for the local spin density becomes

$$\partial_t\mathbf{m} + \partial_i\boldsymbol{J}_i = -\frac{1}{\tau_{ex}M_s}\mathbf{M}\times\delta\mathbf{m} - \frac{\delta\mathbf{m}}{\tau_{sf}}, \tag{5}$$

where summation is implied over repeated indices. Equation (5) is the continuity equation for the s spins modified by keeping the part of the collision integral that does not conserve spin (here we use a relaxation time approximation with τ_{sf} for scattering processes other than the ones mediated by the s-s interaction). The term with $\tau_{ex}=\hbar/SJ_{ex}$ comes from the s-d Hamiltonian that affects the molecular field through $\mathbf{h}_\mathbf{p}^0$ given by Eq. (4).

For the spin current, we obtain

$$\partial_t\boldsymbol{j}_{\sigma_i} + c_s^2\partial_i\delta\mathbf{m} = \left[\frac{\hbar\mathbf{M}}{\tau_{ex}M_s} + \frac{2\mathbf{m}}{\hbar N(0)}\left(F_0^a - \frac{F_1^a}{3}\right)\right]\times\boldsymbol{j}_{\sigma_i} - \frac{\boldsymbol{j}_{\sigma_i}}{\tau_{sf}'}, \tag{6}$$

with $c_s^2=(v_F^2/3)(1+F_0^a)(1+F_1^a/3)$, $N(0)$ the density of states at the s-band unpolarized Fermi surface, and $\tau_{sf}'^{-1}=(1+F_1^a/3)\tau_{FL}^{-1}+\tau_{sf}^{-1}$. The fact that $\tau_{sf}'\neq\tau_{sf}$ results from the internal spin diffusion in the Fermi liquid, for which spin current is not a conserved quantity. Such a difference exists even in the absence of interaction (as is easily seen by putting $F_\ell=0$) although this has not been always considered.

Now, with the usual assumption that the s-electron dynamics is much faster than that of $\mathbf{M}$, we look for the solution of Eq. (6) after the transient part has practically vanished, which occurs after a few relaxation times. At these times, the quasisteady solution consists of the precession of the spin current about $\mathbf{M}$, so that Eq. (6) becomes

$$g\mathbf{m}\times\boldsymbol{j}_{\sigma_i} + \boldsymbol{j}_{\sigma_i} = -D_s\partial_i\delta\mathbf{m}, \tag{7}$$

where $g=2\tau_{sf}'(F_0^a-F_1^a/3)/\hbar N(0)$ measures the strength of the interaction and $D_s=v_F^2\tau_{sf}'(1+F_0^a)/3$ is the spin diffusion coefficient.

In Eq. (7) the differences between interacting and noninteracting s electrons become apparent. By setting the interaction to zero ($F_\ell=0$) one recovers Fick's relation between spin current and spin density. The additional term on the left side originates from the precession of the s-electron spin current about the local spin density, the same phenomenon that causes the long-known Leggett-Rice effect in normal ^{3}He at very low temperatures.[22,23]

Combining Eqs. (7) and (5), we obtain

$$D_s'\nabla^2\delta\mathbf{m} - \frac{1}{\tau_{ex}M_s}\delta\mathbf{m}\times\mathbf{M} - \frac{\delta\mathbf{m}}{\tau_{sf}} + \boldsymbol{\Delta} = \frac{n_0'}{M_s}\partial_t\mathbf{M} - \frac{\mu_B P'}{eM_s}(\boldsymbol{j}_e\cdot\boldsymbol{\nabla})\mathbf{M}, \tag{8}$$

where $D_s'=D_s/(1+g^2n_0')$ and

$$-\frac{\boldsymbol{\Delta}}{D_s'} = \frac{gn_0'}{M_s}\partial_i(\mathbf{M}\times\partial_i\delta\mathbf{m}) + \left(\frac{gn_0'}{M_s}\right)^2\partial_i[\mathbf{M}(\mathbf{M}\cdot\partial_i\delta\mathbf{m})]. \tag{9}$$

Equation (8) is the central result of this article. Its form suggests that since the interaction between s electrons is in general not weak, the use of an independent-electron approach in the s band, as has been common in the literature for s-d-coupling models, is not correct in principle. Regardless of the equation of motion satisfied by $\mathbf{M}$, one sees by a careful examination of Eqs. (8) and (9) that the dynamics obtained for $\mathbf{M}$ in the absence of an s-s interaction will be in general rather different from that in its presence.

To illustrate how the s-s interaction may affect the dynamics of $\mathbf{M}$, we shortly indicate how the result given by Eq. (8) changes the widely used LLG equation,[8] without questioning its limits of validity,[24] since this specific choice does not change the general conclusions we want to draw in what follows.

The interaction of the spin density $\delta\mathbf{m}$ with $\mathbf{M}$ brings an additional torque to the LLG equation, $\mathbf{T}=(1/\tau_{ex}M_s)\delta\mathbf{m}\times\mathbf{M}$. From this torque $\delta\mathbf{m}$ can be taken as a function $\mathbf{M}$ and used in Eq. (8) to give an equation for the dynamics of the order parameter $\mathbf{M}$. A more detailed calculation of how the dynamics of $\mathbf{M}$ changes with the s-s interaction will be shown elsewhere, but it is clear, by simple

inspection of Eqs. (8) and (9), that the result obtained will be two quite different equations for interacting and noninteracting s electrons.

In the limit of vanishing s-s interaction these expressions give known results for the equations of motion.[17] The additional dynamical structure in the interacting case comes from the term $\mathbf{\Delta}$ given in Eq. (9). Besides this additional contribution, there are also renormalizations of the coefficients in the other terms. We emphasize that our result is exact, apart from the usual assumptions of Fermi liquid theory and the general hypotheses that are used to validate the s-d model.

We see that in the limit of strong interactions $\mathbf{\Delta}$ may introduce changes that lead to different dynamics. As a matter of fact, the strongly interacting limit is somewhat difficult to evaluate, since while $n_0'=n_0/(1+F_0^a)$ becomes negligible in the limit of large F_ℓ^a's, the exact behavior of the product gn_0' that governs $\mathbf{\Delta}$ is difficult to predict without explicit knowledge of the Landau parameters and also the effective mass that enters $N(0)$. One can, nonetheless, keep the values of the Landau parameters finite and $\neq 0$ to study the overall differences that $\mathbf{\Delta}$ is expected to produce.

It is immediately noted, however, that $\mathbf{\Delta}$ is only different from zero if the gradients of $\mathbf{M}$ are not zero—i.e., in the presence of inhomogeneities. For uniform or nearly uniform magnetizations, one sees that the sole effect of the interaction is to renormalize the equations of motion through the coefficients P' and n_0'.

In what follows, we will avoid the more complicated task of solving the full equation for a finite $\mathbf{\Delta}$ [Eq. (8)] and rather concentrate on presenting a precise meaning for the limit of nearly uniform magnetization when s-s interactions are not zero.

First we assume that in this limit, the DW width (or, equivalently, the characteristic average width of the inhomogeneity) is very large compared to any diffusion length of the problem, so that $\mathbf{M}$ varies slowly in space. Under this assumption, decoupling of Eq. (8) is possible in a local frame in a similar way that it is accomplished in the noninteracting case. Taking the z axis of such a frame parallel to $\mathbf{M}$ yields the equation

$$\nabla^2 \delta \mathrm{m}_- - \frac{1}{\lambda^2}\delta \mathrm{m}_- = f_-(\mathbf{r}), \tag{10}$$

where $\delta \mathrm{m}_- = \delta \mathrm{m}_x - i\,\delta \mathrm{m}_y$, $f_- = f_x - i f_y$,

$$\mathbf{f}(\mathbf{r}) = \frac{1}{1+ign_0'}\left[\frac{n_0'}{M_s}\partial_t \mathbf{M} - \frac{\mu_B P'}{eM_s}(\boldsymbol{j}_e\cdot\boldsymbol{\nabla})\mathbf{M}\right], \tag{11}$$

and $\lambda^2 = D_s'(1+ign_0')/(\tau_{sf}^{-1}+i\tau_{ex}^{-1})$. Equation (10) admits a standard Green's function solution

$$\delta \mathrm{m}_- = \frac{1}{4\pi}\int d\mathbf{r}\frac{e^{-|\mathbf{r}-\mathbf{r}|/\lambda}}{|\mathbf{r}-\mathbf{r}'|}f_-(\mathbf{r}'). \tag{12}$$

Let the typical scale over which $\mathbf{f}(\mathbf{r})$ varies be w. It is clear from Eq. (11) that w is given by the average distance over which $\mathbf{M}$ varies in space. For DW structures, w is given by the average DW width. If $w \gg |\lambda|$, then we can take $f_-(\mathbf{r})$ out of the integral in Eq. (12), to obtain $-\delta \mathrm{m}_-/\lambda^2 = f_-(\mathbf{r})$ and, from Eq. (10), $\nabla^2 \delta \mathbf{m} = 0$ and $\mathbf{\Delta} = 0$.

Equation (8) then acquires the same form as the one used for noninteracting s electrons, and the torque and other quantities can be determined by simply replacing the renormalized parameters in the results earlier obtained in the absence of an s-s interaction.[17,18]

The condition $w \gg |\lambda|$ that validates this "wide-DW" approximation can be explicitly written as

$$w \gg \sqrt{\frac{v_F^2 \tau_{sf}' \tau (1+F_0^a)}{3\sqrt{1+(gn_0')^2}}} = |\lambda|, \tag{13}$$

where $\tau^{-2} = \tau_{sf}^{-2} + \tau_{ex}^{-2}$.

We see that the characteristic length for this limit to be set depends on the strength of the interaction. Again, the simple limit of very large interactions is difficult to evaluate since one does not in general know how the Landau parameters and the effective mass are exactly related.

However, other strongly correlated limits are approached when the s electrons are driven toward some instability. The general Pomeranchuk instability[20] is set when $F_\ell^a/(2\ell+1) \to -1_+$. There are other correlation-driven instabilities whose onsets could be studied, like the metal-insulator transition or the spin-density-wave (SDW) instability.[25]

We will focus here on the Stoner instability that is approached when $F_0^a \to -1_+$. The onset of such an instability is characterized by the presence of large ferromagnetic fluctuations in the system of s electrons. If one thinks of a "weakly interacting" s band, it may appear unphysical to assume the s electrons are in such a regime. However, we just showed that no physical "weakly interacting" scenario exists in a nontrivial solution (F_ℓ's not all equal to zero). The electrons are rigorously undistinguishable; s-d models separate them into two groups with hopes that this will be an effective description. It is quite compelling, then, that the strong fluctuations that are present in the d-electron part of the wave function will be also shown in the s-electron part. Such a regime is analogous to the so-called "nearly ferromanetic Fermi liquid," a regime that, e.g., normal ^{3}He undergoes as its temperature is lowered at room pressure.[20]

In the presence of large ferromagnetic fluctuations, Eq. (13) becomes

$$w \gg \sqrt{\frac{v_F^2 \tau_{sf}' \tau}{3gn_0}(1+F_0^a)} \to 0, \tag{14}$$

and the effective length defining the "wide-DW" limit is arbitrarily lowered. It is clear, however, that a lower limit exists for the usual approximations to be valid (for example, the semiclassical equation of motion is valid for long wavelengths); however, these scales are usually much less than the typical diffusion paths that relate to $|\lambda|$ in the limit of vanishing s-s interaction.

As examples, the renormalizations of the gyromagnetic ratio and Gilbert parameter in the LLG equation in the regime of strong ferromagnetic fluctuations are given by

$$\gamma' = r\gamma \quad \text{and} \quad \alpha' = r\alpha + \frac{\tau_{ex}}{\tau_{sf}}, \tag{15}$$

where $r = M_s(1+F_0^a)(1+\tau_{ex}^2/\tau_{sf}^2)/n_0$.

In closing, we considered the effect of the interelectronic (s-s) interaction in an s band coupled to a ferromagnetic d band and found that structural changes in the dynamics of the order parameter **M** may occur.

The theory correctly reproduces the noninteracting limit that has been used as the standard approach for electrons flowing in ferromagnetic metals, which include transition metals and their alloys. We saw that, in the general case, the only safe assumption to boldly ignore the s-s interaction would be that its effective amplitude is too small, which does not appear to be a plausible demand for these systems.

In the particular regime of large ferromagnetic fluctuations in the s band, the inclusion of the s-s interaction is shown to produce two main effects: (i) rescaling the equations of motion and (ii) extending the "wide-inhomogeneity" limit to arbitrarily narrow domain walls.

This latter effect may be physically understood by noticing that in such a regime of large ferromagnetic fluctuations the spin density (**m**) in the s band becomes highly susceptible to changes in the magnetization (**M**) (analogously to what happens in a usual paramagnet at the onset of a ferromagnetic instability). The closer the instability is, the stronger the tendency of **m** to track **M**, so that the length scales within which **M** changes must be narrower in order for the dynamics of **m** to be affected by them. This is appreciated only if the interactions are properly accounted for.

The fact that early approaches that do not consider the s-s channel work reasonably well in explaining some observed properties[17,18] may be suggestive that the s-band electrons in the theoretical description of these materials must be considered as a strongly correlated state with large ferromagnetic fluctuations.

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