Statistical analysis for current fluctuations in a disordered quantum pump

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A statistical analysis for pumped current fluctuations in a disordered two-dimensional quantum pump is investigated by a Green's function approach. In the diffusive regime where dc conductance shows familiar universal conductance fluctuations (UCFs), the pumped current is also found to fluctuate with a generic behavior, which is independent of impurity strength. However, different from UCF, the amplitude of current fluctuations shows a strong dependence on impurity density. As the impurity density increases, the system goes from metallic regime to diffusive regime. The distribution of pumped current is found to cross over from non-Gaussian-like to Gaussian-like with the variation of impurity density.

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I. INTRODUCTION

Quantum pump is a device that drives a dc at zero bias by periodically changing system parameters.[1,2] Using the Coulomb blockade,[3] photovoltaic effects,[4] and photon-assisted tunneling,[5] a variety of pumps have been proposed.[6–11] Some of them have been realized experimentally.[12–14] Relatively, less attention has been paid to the sample-to-sample statistical properties of pumped current. In a disordered conductor, impurities can greatly influence the pumping of a dc which can vary from sample to sample. A meaningful approach is to examine a statistical ensemble of samples to cover as many microscopic details as possible and to obtain an average physical picture. In experimental measurements, the statistical approach to quantum transport has been widely used by carrying out thousands of measurements[15,16] and can, in principle, be carried out for the disordered quantum pumps. Theoretically, there have been some analytic treatments on this issue.[2,17,18] The mesoscopic mechanism of pumped charge has been regarded as random sample specific quantities and determined by quantum interference.[17,18] The distribution of pumped current for a chaotic quantum dot has also been studied.[2] It was found that when the number of incoming channels is large, the distribution is Gaussian. Numerically, one can do sample-to-sample statistical analysis by investigating pumped current of many samples. It is the purpose of this paper to give a detailed analysis on the sample-to-sample statistical properties of pumped current in a two-dimensional disordered quantum pump.

The detailed behavior of pumped current I_p in a disordered conductor depends on several characteristic length scales. We restrict our discussion to the mesoscopic regime $L \leq l_\phi$, where L is the size of the conductor and l_ϕ the phase coherence length. The disorder can be described by the elastic mean free path l and by the localization length ξ. Ballistic and strong localized behavior occur for $L<l$ and $L>\xi$, respectively. As the analytic method is suitable to cope with disorder in diffusive regime, we focus on the diffusive regime defined by $l<L<\xi$. Similar to the conclusion of Ref. 18, our numerical results show that the fluctuation of pumped current rms(I_p) is a sample dependent property which depends strongly on the impurity density. However, we do find some universal feature of the fluctuation of pumped current; i.e., for a fixed impurity density, the fluctuation is independent of impurity strength. From the data of distribution of pumped current, we find that when the number of impurities N is small, the distribution is asymmetric because the system is still in the metallic regime. Increasing N, the system crossover to the diffusive regime and the distribution becomes Gaussian. Further increasing N for large impurity strength $\gamma=50$, the distribution is non-Gaussian but more or less symmetric presumably because the system enters the insulating regime.

II. THEORETICAL FORMULATION

Generally, there are different statistical methods to deal with impurity effects. A commonly used one is the disordered Anderson model which is a discrete model with tight-binding approximation. It can deal with disorders in all regimes. However, it becomes more difficult to cope with large systems with atoms (lattice sites) up to a few thousands due to the capacity of computer. Another method is to use analytical method to calculate the Green's function in real space using the Dyson equation and then average the pumped current numerically using the obtained Green's function. It is based on a continuous model and usually used in transport in diffusive regime. It can cope with large systems with large number of atoms. In this paper, we use the analytic method.

To be specific, we consider a two-dimensional (2D) diffusive conductor with N-impurity scattering potential $V_I(\mathbf{r}) = \sum_i \gamma_i \delta(\mathbf{r}-\mathbf{r}_i)$, where γ_i is the impurity strength of the ith impurity located at position $\mathbf{r}_i$. By adding two pumping gate voltages inside the scattering region (see Fig. 1 for schematic plot), an average dc will be generated at zero external bias. To analyze parametric quantum pumped current for this conductor, we make use of the scattering theory developed by Brouwer,[2] where instant scattering approximation is employed by assuming that the variation of system parameters is very slow compared with the characteristic time scale of

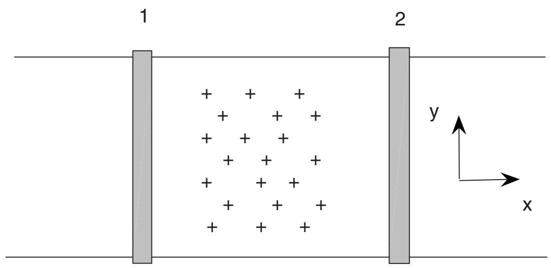

FIG. 1. Schematic picture of the model. Shadowed bars represent pumping gates X_1 and X_2; crosses represent random scatters.

electron traversing the scattering regime. This is so-called the adiabatic regime, and the pumped current is expanded up to the first order in pumping frequency [see Eq. (1) below]. In order to pump out a current in the adiabatic regime, we need simultaneous variation of two pumping parameters $X_1(t)=X_1\sin(\omega t)$ and $X_2(t)=X_2\sin(\omega t+\phi)$, with ϕ the phase difference between two pumping potentials. In a period of pumping cycle, the average current flowing through lead α due to variation of parameters X_1 and X_2 is given by[2]

$$I_\alpha=\frac{q\omega}{2\pi}\int_0^\tau dt\left[\frac{dN_\alpha}{dX_1}\frac{dX_1}{dt}+\frac{dN_\alpha}{dX_2}\frac{dX_2}{dt}\right], \quad (1)$$

where ω is the pumping frequency and $\tau=2\pi/\omega$ is the period of pumping cycle. The quantity dN_α/dX is the partial density of states in the configurational space describing the density of states of electrons emitted through lead α. It is called emissivity[19] in the scattering theory and is determined by the following expression:

$$\frac{dN_\alpha}{dX_i}=\sum_n\int\frac{dE}{2\pi}(-\partial_E f)\sum_{\beta m}\mathrm{Im}\frac{\partial S_{\alpha n\beta m}}{\partial X_i}S^*_{\alpha n\beta m}, \quad (2)$$

where f is the Fermi distribution function and $S_{\alpha n\beta m}$ is the scattering matrix with an electron incoming from lead β in mode m and emitting through mode n in lead α. The scattering matrix $S_{\alpha n\beta m}$ is related to the Green's function from the Fisher-Lee relation,[20]

$$S_{\alpha n\beta m}=-\delta_{x_\alpha x_\beta}\delta_{nm}+2i\sqrt{k_nk_m}G_{\alpha n\beta m}, \quad (3)$$

where we have set $(\hbar=1)$ and defined

$$G_{\alpha n\beta m}=\int\int\chi_n(y_\alpha)[G(\mathbf{r}_\alpha;\mathbf{r}_\beta)]\chi_m(y_\beta)dy_\alpha dy_\beta, \quad (4)$$

where $\chi_n(y_\alpha)$ is the transverse wave function of the nth mode and $\mathbf{r}_\alpha$ labels the site on the interface between scattering region and lead α. Now, we proceed to calculate the Green's function for the 2D conductor with N-impurity scattering potential $V_I(\mathbf{r})=\Sigma_i\gamma\delta(\mathbf{r}-\mathbf{r}_i)$. For simplicity, we have fixed $\gamma_i=\gamma$ and assumed that the randomness comes from the positional disorder of the impurities with equal strength. The N-impurity Green's function $G_N(\mathbf{r}_1;\mathbf{r}_2)$ can be calculated by iterating the Dyson equation,[21]

$$G_N(\mathbf{r}_1;\mathbf{r}_2)=G^0(\mathbf{r}_1;\mathbf{r}_2)+\gamma\sum_{i=1}^N G^0(\mathbf{r}_1;\mathbf{r}_i)G_N(\mathbf{r}_i;\mathbf{r}_2), \quad (5)$$

where $G^0(\mathbf{r}_1;\mathbf{r}_2)$ is simply the Green's function for an infinitely long quasi-one-dimensional ballistic quantum wire with no impurity,[22]

$$G^0(\mathbf{r}_1;\mathbf{r}_2)=\sum_{n=1}^{N_m}\chi_n(y_1)\chi_n(y_2)\frac{e^{ik_n|x_1-x_2|}}{2ik_n}, \quad (6)$$

where N_m is the total number of modes in the quantum wire and k_n the corresponding momentum of the electron. By solving the Dyson equation, the N-impurity Green's function can be obtained,[23]

$$G_N(\mathbf{r}_1;\mathbf{r}_2)=G^0(\mathbf{r}_1;\mathbf{r}_2)+\gamma\sum_{i=1}^N G^0(\mathbf{r}_1;\mathbf{r}_i)M_{ij}G^0(\mathbf{r}_j;\mathbf{r}_2), \quad (7)$$

where $M_{ij}=[\delta_{ij}-\gamma\eta_{ij}]^{-1}$ and $\eta_{ij}=G^0(\mathbf{r}_i;\mathbf{r}_j)$. Similar expression can be obtained for the Green's function G in the presence of two pumping potentials X_1 and X_2 placed at two boundaries of the scattering region. Here, $X_1(t)=V_{1p}\sin(\omega t)\delta(x_1+a)$ and $X_2(t)=V_{2p}\sin(\omega t+\phi)\delta(x_2-a)$, where V_{1p} and V_{2p} are the pumping amplitudes. For small pumping amplitude, we expand the Green's function G up to the linear order in V_{1p} and V_{2p},

$$G(\mathbf{r}_\alpha;\mathbf{r}_\beta)=G_N(\mathbf{r}_\alpha;\mathbf{r}_\beta)+V_{1p}\sin(\omega t)G_N(\mathbf{r}_\alpha;\mathbf{r}_1)G_N(\mathbf{r}_1;\mathbf{r}_\beta)+V_{2p}\sin(\omega t+\phi)G_N(\mathbf{r}_\alpha;\mathbf{r}_2)G_N(\mathbf{r}_2;\mathbf{r}_\beta). \quad (8)$$

From the Fisher-Lee relation and Eq. (8), we obtain the derivative of scattering matrix at small pumping potential $(X_i=V_{ip})$,

$$\frac{\partial S_{\alpha n\beta m}}{\partial X_i}=2i\sqrt{k_nk_m}\sum_{k=1}^{N_m}G_{\alpha nik}G_{ik\beta m}. \quad (9)$$

In the limit of small pumping amplitude, the lowest order contribution to the pumped current from lead 1 is of bilinear form,[2]

$$I_1=\frac{q\omega}{2\pi}V_{1p}V_{2p}\sin(\phi)\left[\frac{\partial}{\partial X_1}\frac{dN_1}{dX_2}-\frac{\partial}{\partial X_2}\frac{dN_1}{dX_1}\right]. \quad (10)$$

Combining Eq. (2) with Eq. (9), Eq. (10) can be written as

$$I_1=\frac{q\omega}{2\pi}V_{1p}V_{2p}\sin(\phi)\sum_n\sum_m \mathrm{Im}(4k_nk_m)\times\sum_{jk}(G_{1n2j}G_{2j1m}G^*_{1n1k}G^*_{1k1m}+G_{1n2j}G_{2j2m}G^*_{1n1k}G^*_{1k2m}-G_{1n1j}G_{1j1m}G^*_{1n2k}G^*_{2k1m}-G_{1n1j}G_{1j2m}G^*_{1n2k}G^*_{2k2m}). \quad (11)$$

With this expression, we can investigate the sample-to-sample statistical properties of the current fluctuation in a disordered charge pump. For a given impurity configuration $\{\mathbf{r}_i\}$ generated randomly and distributed uniformly, we can evaluate the quantity M_{ij} by direct matrix inversion. The rest of the expressions are calculated accordingly once M_{ij} is

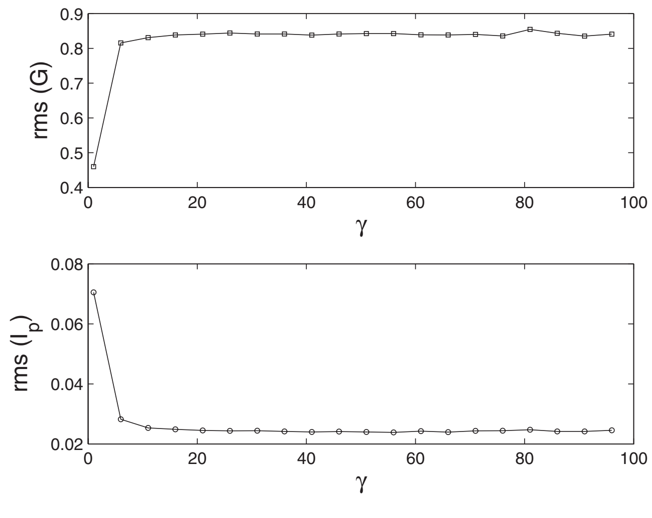

FIG. 2. The conductance fluctuations and current fluctuations as a function of impurity strength at impurity number $N=100$. Over 10 000 configurations were averaged. The unit for conductance is e^2/h and the unit for current is 9.76 pA.

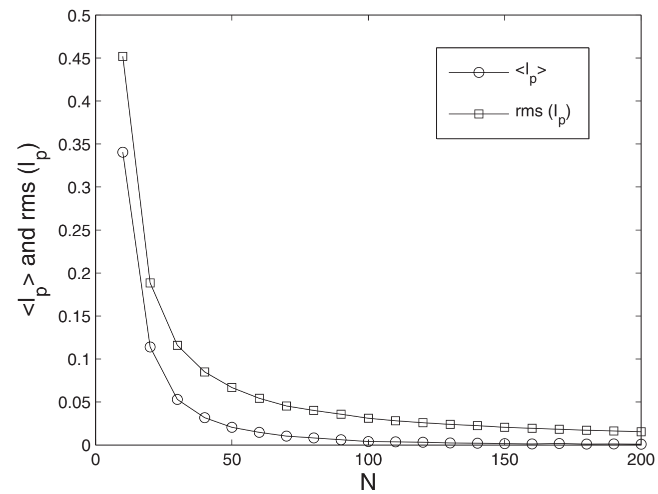

FIG. 3. The averaged pumped current and its fluctuations as a function of impurity number N at fixed impurity strength $\gamma=50$. Over 10 000 configurations were averaged for each N.

known. The transverse modes $\chi_n(y)$ are the usual sine functions, and we have set the incoming electron energy so that there are 20 transverse modes in the conductor whose contributions are summed. The sample-to-sample statistical analysis is carried out by average many independent impurity configurations for each N. The amplitude of sample-to-sample fluctuations is defined as $\mathrm{rms}(A)=\sqrt{\langle A^2\rangle-\langle A\rangle^2}$, where A can be pumped current I_p or conductance G.

III. NUMERICAL RESULTS

In the numerical calculation, the length of the conductor is fixed at a and its width is a, with $a=0.53$ μm. The energy unit is chosen as 4 μeV,[24] corresponding to frequency unit $\omega=6.1\times10^9$ Hz. The incoming energy is chosen as $E_F=(20\pi)^2+4$. For this incoming energy, the number of incoming channels is 20. We have chosen the pumping frequency $\omega=0.1$ and the pumping amplitude $V_{1p}=V_{2p}=40$.[25] With the choice of these parameters, the unit for charge current becomes 9.76 pA. In the absence of pumping parameters, our diffusive conductance should exhibit universal conductance fluctuation (UCF) behavior in the presence of disorders. In Fig. 2 (upper panel) we plot the conductance fluctuation versus the impurity strength γ while fixing the number of impurity to be $N=100$. Indeed, for large impurity strength γ, a UCF behavior independent of γ is observed with the UCF value close to the expected value of $0.86e^2/h$ for 2D systems. Figure 2 (lower panels) plots the fluctuation of pumped current versus impurity strength γ at $N=100$. In the calculation, 10 000 impurity configurations were averaged for each γ. Similar to the case of conductance fluctuations, the fluctuation of pumped current $\mathrm{rms}(I_p)$ is independent of γ for large γ, showing certain degree of universal behavior. However, our numerical results indicate that this $\mathrm{rms}(I_p)$ depends on the number of impurities in the scattering region. Figure 3 depicts the averaged pumped current and the amplitude of its fluctuations versus the number of impurities N at the fixed impurity strength γ. The following observations are in order. (1) The average pumped current and its fluctuation strongly depend on the impurity density. The larger the impurity density, the smaller they are. (2) The averaged pumped current falls off exponentially with increasing impurity density, indicating the system crossover from metallic regime to the diffusive regime. As we see from Fig. 2, the conductance fluctuation increases with the increasing impurity strength and saturates to the UCF value. The behavior of $\mathrm{rms}(I_p)$ is opposite. When disorder is absent, there is no fluctuation. It increases quickly at weak disorder and then decreases upon further increasing the disorder strength. Similar behavior is observed in Fig. 3 where the impurity density plays similar role of impurity strength. In particular, the $\mathrm{rms}(I_p)$ also decay exponentially with increasing impurity density when crossover from metallic regime to diffusive regime. Finally, we examine the distribution function $P(I_p)$ obtained from numerical analysis for different N values where 10 000 impurity configurations were averaged for each N. We focus on the regime where the dc conductance shows the signature of UCF. We first investigate the case of small γ where large $\mathrm{rms}(I_p)$ occurs (see the lower panel of Fig. 2). From Fig. 4, we see that for large impurity density N, the distribution of pumped current shows a Gaussian distribution, while for small N, the distribution is asymmetric. This is because at small impurity density, the system is still in the metallic regime and it enters the diffusive regime upon increasing the impurity density. Now, we look at the case of large γ. Figure 5 depicts the distribution of pumped current for different N. We see that when the impurity density is small ($N=10$), the distribution is asymmetric and non-Gaussian, again indicating that the system is still in the metallic regime. As N increases to $N=30$, the system starts to crossover to the diffusive regime and the distribution becomes more symmetric and approaches to a Gaussian distribution. For larger $N=100$, the distribution is a Gaussian showing the signature of diffusive regime. For even larger $N=200$ (now, $\gamma=50$), the

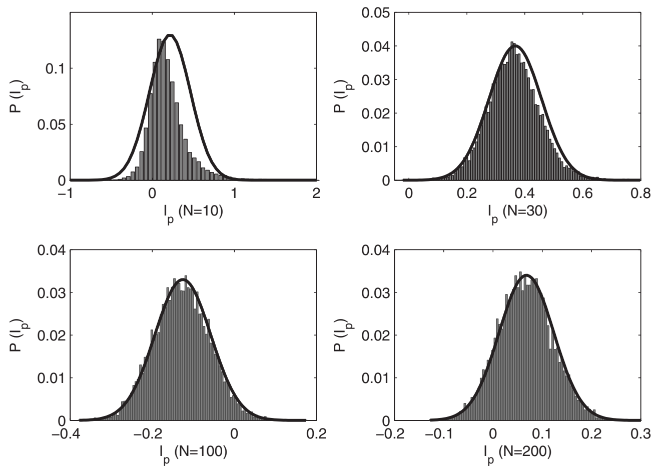

FIG. 4. The distribution function $P(I_p)$ for impurity number $N=10$, 30, 100, and 200. Over 10 000 configurations were averaged for each N. Impurity strength is chosen as $\gamma=1$. The solid line is the Gaussian fit for the distribution data.

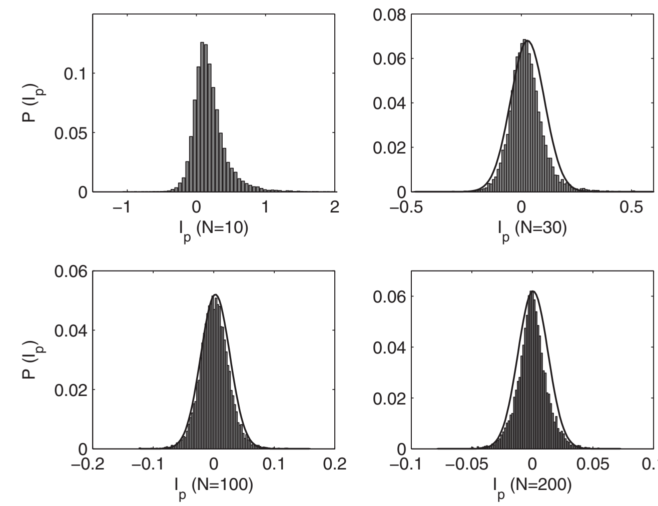

FIG. 5. The distribution function $P(I_p)$ for impurity number $N=10$, 30, 100, and 200. Over 10 000 configurations were averaged for each N. Impurity strength is chosen as $\gamma=50$.

system becomes an insulator and the distribution becomes a symmetric non-Gaussian. From random matrix theory, Brouwer[2] found that for a chaotic quantum dot, the distribution is Gaussian when the number of incoming channels $N_m \gg 1$. Our result gives a Gaussian distribution at large $N_m=20$ for large impurity density N that is consistent with quantum dot result of Brouwer.

In summary, we have analyzed the sample-to-sample fluctuations of pumped current for a two-dimensional disordered mesoscopic conductor. Our analysis was based on Brouwer's scattering theory for parametric pumping. A Green's function technique was used to deal with multi-impurity scattering. In the diffusive regime where the dc conductance shows the familiar UCF, the averaged current in a pumping cycle also fluctuates with a generic behavior, which is independent of impurity strength. However, the pumped current depends strongly on the density of impurity, which is different from the familiar UCF in the mesoscopic transport regime. This observation agrees with the conclusion drawn from the analytic approach.[18] As the system crossover from metallic to diffusive regimes, the distribution of pumped current is found to crossover from non-Gaussian-like to Gaussian-like with variation of impurity density.

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[25] The energy range between the incoming energy of 20 modes and its neighbor energy of 21 modes is about 400; therefore, $V_{1p}=V_{1p}=40$ can be considered as good perturbations.