Percolation model for the superconductor-insulator transition in granular films

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We study the temperature dependence of the superconductor-insulator transition in granular superconductors. Empirically, these systems are characterized by very broad resistance tails, which depend exponentially on the temperature, and the normal state resistance. We model these systems by two-dimensional random resistor percolation networks in which the resistance between two grains is governed either by Josephson junction coupling (Cooper pair's tunneling) or by quasiparticle tunneling. Our numerical simulations as well as an effective medium evaluation explain the experimental results over a wide range of temperatures and resistances. Using effective medium approximation we find an analytical expression for the effective resistance of the system and the value of the critical resistance separating conducting from insulating branches.

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I. INTRODUCTION

The disorder driven superconductor-insulator transition (SIT) in thin films has gained revived attention over the past few years mainly because of the possibility that it represents a quantum phase transition.[1–7] Disordered superconductors can be categorically divided into two groups, granular and homogeneous.[8] Experimentally, homogeneous samples are found to exhibit sharp superconducting transitions and the crossover between insulating to superconducting behavior seems to occur at sheet resistance $R \cong h/4e^2 \approx 6.5\ \mathrm{k}\Omega$.[1] On the other hand, granular films are characterized by very broad tails in the resistance $R(T)$ and the transition between the insulator and the superconductor phases seems to be much less universal.[2–7,9–11] In these samples it was found that the temperature dependence of the sheet resistance, $R(T)$, below the critical temperature, T_c, can be described by an inverse Arrhenius law as follows:

$$R(T) \sim \exp(T/T_0), \tag{1}$$

where T_0 is a constant.[5] The $R(T)$ curves following Eq. (1) have been observed for a large variety of granular superconductors and over a wide range of temperatures (down to temperatures below $T_c/10$). A typical example of the granular film's microstructure is shown in Fig. 1 and a typical example for a set of measurements[7] of $R(T)$ for quench condensed Pb granular films is shown in Fig. 2(a). The latter figure shows that the considered system can be driven through the SIT as a function of the film's *mean thickness*.

It is important to distinguish between local and global superconductivities. When cooling the sample, each grain of the array becomes separately and independently superconducting at the same critical temperature T_c (approximately equal to Bardeen-Cooper-Schrieffer's critical temperature, T_c, in bulk samples), for both insulating and superconducting sides of the SIT. This is in contrast to high-T_c granular superconductors in which the grains overcome to the superconducting state over some wide temperature range.[12] Therefore, the global superconductivity and SIT in the granular film is a result of a competition between Josephson coupling and phase fluctuations. There are two types of these fluctuations: thermal and quantum. The latter are due to the charging energy, which pins the charge, which is quantum conjugate to the phase. Thus, via the uncertainty principle, the phase fluctuates.

Each grain is described by a condensate wave function (a complex superconducting order parameter[13,14]) $\Psi_i = \sqrt{\rho}\exp(i\phi_i)$. The density ρ of Cooper pairs on each grain is assumed to be a constant and only the phases ϕ_i are allowed to fluctuate. To describe these fluctuations, one typically employs a Hamiltonian[15–17]

$$\hat{H} = \frac{1}{2}\sum_{ij} E_{ij}^{(c)} n_i n_j + \sum_{ij} V_{ij} + \frac{E_J}{2}\sum_{i,j=i\pm1}[1-\cos(\phi_i-\phi_j)] + \sum_{ij}(\phi_i-\phi_j)X_{ij} + \sum_{ij} h_{ij}(X_{ij}), \tag{2}$$

where the first term is the charging energy, the third term

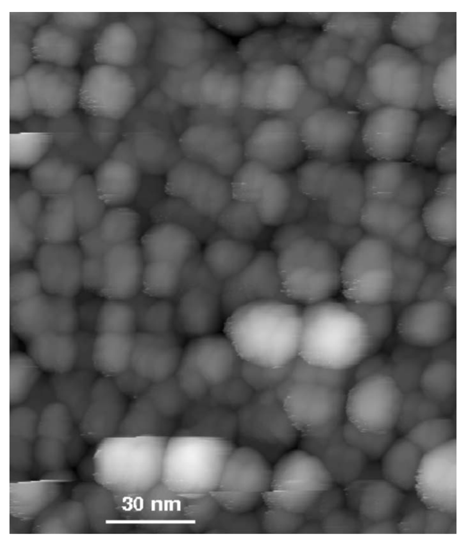

FIG. 1. A typical atomic force micrography photography (reproduced from Ref. 11) of Pb grains deposited on the insulating SiO_2 plate. Each grain is individually superconducting with its own phase. The mean size of the grain is of the order of 20 nm, while the intergrain distance is 1 order smaller, i.e., is about 2 nm.

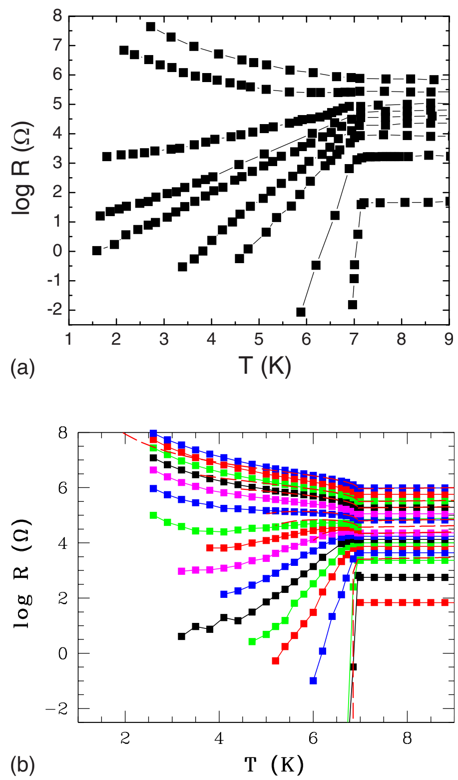

FIG. 2. (Color online) (a) Experimental plots of $\log_{10} R$ the sheet resistance versus temperature, T, for quench condensed Pb films (Ref. 7) with grain sizes of the order $\sim 5-10$ nm, having different intergrain coupling. The higher curves correspond to the thinner films, i.e., to films with larger intergrain distance and, therefore, to the stronger disorder which is determined by κ. (b) Theoretical plots of SIT: $\log_{10} R$ vs T for the systems with different disorder strengths [see Eq. (7)], $\kappa = 20, 19, 18, 17, 16, 15, 14.5, 14, 13.5, 13, 12, 11, 10, 9$ (from top to bottom, respectively). Squares are the results of our numerical simulations, while the dashed lines are the plots of our analytical prediction (12). The sample size in simulations is 40×40 resistors, $T_c = 7$, $\alpha = 3.25 \times 10^3$, $z = 10$, and $R_0 = T$ (in dimensionless units as is defined in the text). The coefficient $\beta(r_0/d)E_c^{(0)}/k_B$ [see Eq. (8)] was taken in our calculations to be equal to 0.5 K.

describes the Josephson coupling, and the operator V_{ij} is needed for the description of the one-electron hopping conductivity and the derivation of the Josephson coupling energy in the presence of the charging energy (for details, see Refs. 15 and 16). n_i is the deviation from the average number of electrons in the ith grain (an operator which is conjugate to the relative phase $\phi = \phi_i - \phi_j$ so that $n = -i2\partial/\partial\phi$ and satisfying the commutation relation $[\phi_i, q_i] = 2ei$ or $[\hat{\phi}_i, \hat{n}_j] = i\delta_{ij}$). The charging energy $E_{ij}^{(c)}$ includes both the short-range (diagonal) and the off-diagonal parts. A mean-field approximation can replace the third term in Eq. (2) by $-2zE_J\langle\cos\phi\rangle\Sigma_i \cos\phi_i$, where $\langle\cos\phi\rangle$ is the average order parameter (for details, see Ref. 15). The last two terms in the Hamiltonian (2) represent the effects of dissipation in the junctions by coupling the phase difference across the junction to a heat bath and the Hamiltonian of the heat bath, respectively.[16] X_{ij} is a collective coordinate of the heat bath describing the coupling associated with the shunt resistor across the junction $\langle ij \rangle$.

By neglecting the dissipation terms in Eq. (2), it is possible to obtain some expressions for determination of the critical value of the Josephson coupling energy, below which the superconducting bond becomes disconnected, as a result of the combined effect of the thermal and the charging energy. Finally, Ref. 15 obtains a phase diagram (in terms of E_c/E_J, where E_c is the energy of Coulomb blockade) and a condition when two neighboring grains become "phase locked:" $(E_c/E_J)^{1/2} \coth(E_c E_J / k_B^2 T^2)^{1/2} = 1$.

We also propose a simplified version of phase dynamics (see Refs. 18 and 19 and results of de Gennes in Ref. 20 derived in the mean-field approach). The criterion for the two neighboring grains to become phase locked[15,18] is

$$zE_J \geq k_B T + E_c, \tag{3}$$

where z is a parameter of the order of the number of the nearest-neighboring grains.[18,19] That is, Eq. (3) conveniently puts thermal and quantum fluctuations on equal footing and just clarifies that if E_J is larger than $k_B T + E_c$, the phase is locked, while in the opposite case it is not locked.

In this paper we present a model based on percolation to account for the observed temperature dependence of the resistance in granular superconductors.[21] Our percolation approach is based on a two-dimensional (2D) random disordered array of grains; each neighboring pair represents a superconducting junction in which transport can be achieved either by Josephson tunneling or by quasiparticle tunneling, depending on the intergrain coupling and temperature.[19] Our numerical simulations of such a system exhibit an exponential-like dependence of $R(T)$ over a large range of temperatures which is in good agreement with recent experiments. We find that the critical resistance that separates insulating from superconducting branches depends on the distribution of disorder and on the nature of the percolation network of the current trajectories.[22,23]

The remainder of this paper is arranged as follows. In Sec. II, we describe our model and numerical scheme of simulations as well as results of simulations. In Sec. III, we present analytical evaluation of SIT in the framework of effective medium approximation (EMA), followed by a brief discussion in Sec. IV.

II. MODEL AND NUMERICAL SIMULATIONS

When the film is enough thin, it breaks into a set of spatially separated grains. The thinner is the film, the larger is the mean distance between the grains. Therefore, the different thickness of the film can be represented in our percolation model by different distributions of intergrain mean distances.

If two grains are sufficiently decoupled, the resistance between two neighboring sites, i and j, is given by[24–26]

$$R_{ij} = R_0 \exp(r_{ij}/r_0 + \varepsilon_{ij}/k_B T), \tag{4}$$

where $R_0 = Tk_B/(e^2\gamma_{ij}^0)$, γ_{ij}^0 is a rate constant related to the electron-phonon interaction[26] [usually of the order 10^{12} s^{-1}, enabling us to assume in our further consideration that $k_B/e^2\gamma_{ij}^0 \simeq 1$ and $R_0 = T$ (in dimensionless units)], r_{ij} is the distance between the two sites, r_0 is the scale over which the wave function decays outside the grain, and $\varepsilon_{ij} = (|E_i| + |E_j| + |E_i - E_j|)/2$ is the zero field activation energy, which can be determined from physical principals. In the case of superconducting grains this is a nontrivial problem, but in general ε_{ij} is related to the superconducting gap $\Delta_{ij}(T)$ and Coulomb energy $E_c \sim e^2/2C$ (where C is capacitance and e is the electron charge):[19,27] $\varepsilon_{ij} = \Delta_{ij}(T) + E_{c,ij}$. Since the grains are assumed to be large enough to sustain bulk superconductivity, we assume $\Delta(T)$ is the same for all grains.

It is known that dissipation tends to suppress the quantum fluctuations.[16,28,29] Consequently, there can be some changes in the phase diagram to take place in the presence of dissipation in the junctions.[30] Therefore, in general, Eq. (3) should be modified accordingly. However, for simplicity, we do not consider this effect in our paper.

According to Ambegaokar-Baratoff formula, the Josephson coupling constant in Eqs. (2) and (3) can be written as[19,31–33]

$$E_J = \alpha[\Delta(T)/R_{ij}^{(N)}]\tanh[\Delta(T)/2k_BT], \qquad (5)$$

where $\alpha = \pi\hbar/4e^2 \cong 6.5/2 = 3.25$ kΩ and $R_{ij}^{(N)} = R_{ij}\exp[-\Delta(T)/k_BT]$ is the local normal resistance (local resistance at $\Delta = 0$) between the grains. The Josephson energy E_J is related to the Josephson current J_J as follows: $E_J = (\hbar/2e)J_J$.

To perform numerical simulations of this model, we assume that the random distance between grains is $r_{ij} = 2\bar{l}\cdot\xi_{ij}$, where ξ_{ij} is a random number taken from a uniform distribution in the range (0,1), i.e., $0 \le \xi_{ij} \le 1$, and $\bar{l}$ is the mean distance between metallic grains.[34] Therefore the term r_{ij}/r_0 can be expressed as $\kappa\xi_{ij}$, where $\kappa \equiv 2\bar{l}/r_0$ can be interpreted as the dimensionless mean hopping distance or as the degree of disorder[35] (the lower density of the deposited grains represents larger κ). Similarly, the charging energy, E_c, can be expressed through the same factor $\kappa\xi_{ij}$ as follows:

$$E_c = \beta\frac{2e^2}{4\pi\epsilon_0\epsilon d}\frac{r_{ij}}{d} \equiv \beta\left(\frac{r_0}{d}\right)E_c^{(0)}(\kappa\xi_{ij}), \qquad (6)$$

where d and ϵ are the mean size of the grains and the dielectric constant, of the substrate, respectively.[18,36] The value $E_c^{(0)} = 2e^2/(4\pi\epsilon_0\epsilon d)$ is a mean charging energy of a single grain, and $\beta \sim 0.1$ is the effective coefficient which was invoked as a result of the influence of the surrounding grains.[18,37]

Finally we can rewrite expression (4) for the local net resistor mimicking the local hopping resistance between the grains in the convenient form as follows:

$$R_{ij} = R_0\exp[\tilde{\kappa}\xi_{ij} + \Delta(T)/k_BT], \qquad (7)$$

where

$$\tilde{\kappa} \equiv \kappa[1 + \beta(r_0/d)E_c^{(0)}/k_BT]. \qquad (8)$$

The superconducting gap $\Delta(T)$ is the solution of the integral equation[13,31] $\ln(\Delta(0)/\Delta) = 2I[\Delta(T)/T]$, where $I(u) \equiv \int_0^\infty[\sqrt{x^2+u^2}(\exp\sqrt{x^2+u^2}+1)]^{-1}dx$. For temperatures near the critical value $T \sim T_c$, the gap $\Delta(T)$ can be approximated by the analytical form[13,38] $\Delta(T) = 3.06k_BT_c(1 - T/T_c)^{1/2}$.

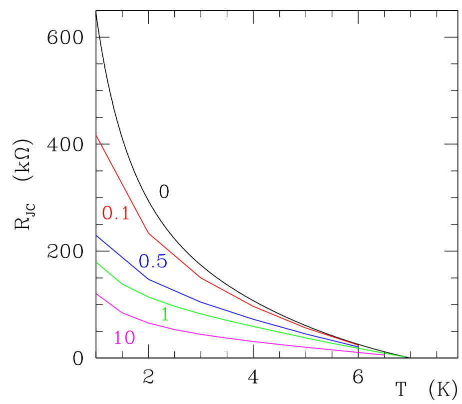

FIG. 3. (Color online) Critical resistance R_{JC} [self-consistency solution of Eq. (9)] vs temperature, T for different charging energy $\beta(r_0/d)E_c/k_B = 0, 0.1, 0.5, 1, 10$ K (from top to bottom).

Since the Coulomb energy can be expressed through the parameter κ [see Eq. (6)], we can write a self-consistency equation for the upper value of $R_{ij}^{(N)}$ for which Eq. (3) is fulfilled,

$$R_{JC} = \frac{z\alpha[\Delta(T)/k_BT]\tanh[\Delta(T)/2k_BT]}{1 + \dfrac{\beta(r_0/d)E_c^{(0)}/k_BT}{1 + \beta(r_0/d)E_c^{(0)}/k_BT}\ln(R_{JC}/R_0)}. \qquad (9)$$

Here R_{JC} is the local critical parameter where the subscripts ij have been omitted for simplicity. When $R_{ij}(\Delta = 0) \le R_{JC}$ the neighboring ith and jth grains are Josephson coupled. Here we have used the relation $\kappa\xi_{ij} = \ln(R_{ij}^{(N)}/R_0)/[1 + \beta(r_0/d)E_c^{(0)}/k_BT]$. The solution of Eq. (9) (i.e., the dependence of the critical resistance R_{JC} on T) for different $\beta(r_0/d)E_c^{(0)}$ is shown in Fig. 3. In the case of small E_c (when $E_J/E_c \to \infty$, i.e., in the case of classical SIT), inequality (3) can be reduced to a simple intrinsic condition:[19,32,33] $R_{ij}^{(N)} \le z\alpha[\Delta(T)/k_BT]\tanh[\Delta(T)/(2k_BT)]$.

Next we aim to evaluate the total resistance of the network system. Our numerical simulations were performed considering a 2D bond-percolating resistor network where R_{ij} of each resistor is zero if Eq. (3) is fulfilled, otherwise it is given by Eq. (7). We solve the obtained system of linear Kirchhoff equations[34,39] and calculate the total effective resistance, $R(T)$, of the 2D network. The results are shown in Fig. 2(b). These results are in good agreement with the experimental data shown in Fig. 2(a).[40] From Fig. 2 we can also see that the SIT is a result of an interplay between quasiparticle tunneling, which tends to turn the curves up [i.e., to increase resistance, R, with decreasing the temperature, T, see Eq. (7)] and Josephson coupling mechanism, which tend to turn curves down [i.e., to decrease the resistance, R, due to increase the total number of the Josephson junctions, which is proportional to superconducting gap $\Delta(T)$, see Eqs. (5)–(9)]. To qualitatively understand the behavior obtained in our simulations [which are very similar to the experiments (Fig. 2)], we describe the percolation mechanism leading to

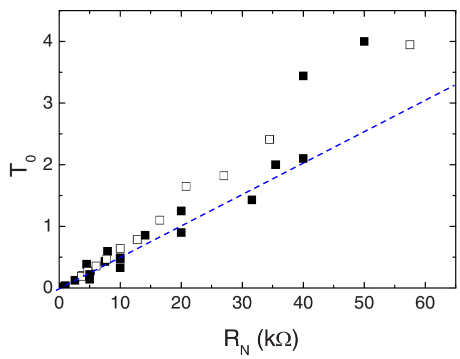

FIG. 4. (Color online) The inverse slopes of the $R(T)$ tails [T_0 of Eq. (1)] as a function of the normal state resistance R_N (Ref. 5) for several sets of Pb granular films measurements (Ref. 5) (full squares) and our simulation results (empty squares). The dashed line is our analytical prediction, Eq. (15).

this behavior. For large κ (strong disorder), the current flows along a single path of the percolation cluster which is the path with minimal total resistance[41] and the total resistance of the path is determined by few critical (red bonds)[42] resistors.[41,43] The tail decrease of R in Fig. 2 can be understood since when T decreases (below T_c), R_{JC} increases and more critical resistors become superconductors.

It is seen that an inverse Arrhenius dependence of $R(T)$ is obtained over a wide range of temperatures. Our agreement with experiments is further demonstrated in Fig. 4 which shows the dependence of T_0 [the slope of the tails, see Eq. (1)] as a function of the normal state resistance (resistance at $T=T_c$) of the sample, R_N, for both experimental and simulation results.

We conclude this section by estimation of the charging energy in SIT's experiments[1–6,9] and its role in our calculations. A simple estimation (using the formula $e^2/2C$ and typical grain's sizes $\sim$20 nm) gives the values of the order hundreds of Kelvin which is much larger than the critical temperature $T_c\sim 7$ K, and makes it not relevant to the SIT [does not fulfill condition (3)]. This problem is already discussed in literature. For example, in Ref. 44 it was assumed that the dielectric constant is larger than 10 leading to islands of grains. According to Abeles *et al.*,[36] to estimate the charging energy is not a trivial problem because of its complicated dependence on the grain size, separation, and the dielectric constant of the oxide coating. Using the results of Ref. 36, we presented the charging energy in Eqs. (6)–(9) as a product of the value usually used for Coulomb blockade estimation $E_c^{(0)}$ by a prefactor $\beta(r_0/d)$, which describes the features of the sample microstructure. So, even if $E_c^{(0)}$ can be of the order $\sim 10-10^2$, the prefactor $\beta(r_0/d)$ is of the order $\sim 10^{-3}-10^{-2}$ and, therefore, their product $\beta(r_0/d)E_c^{(0)}/k_B$ written in Eq. (9) should be in the range $10^{-2}-1$ K. The curves shown in Fig. 2(b) are calculated for the case $\beta(r_0/d)E_c^{(0)}/k_B=0.5$ K. From Eq. (9) it is clear (see also Fig. 3) that the charging energy decreases the value R_{JC} that suppresses the SIT. As a result the critical resistance (separating the metal-like branches from the insulatinglike) R_{crit} appears at a smaller value compared to the case $E_c=0$. In order to bring it back to consistency with the experimental data [see Fig. 2(a)] one needs, e.g., to increase the value of the neighbors z, which requires special justification.

III. EFFECTIVE MEDIUM APPROXIMATION

For further understanding of this complex transition, we have also calculated the total effective resistance, R, of such a network using the symmetric self-consistency EMA.[34,45] The effective resistance R of the random conductance network [the local resistivities, ρ, of which are distributed continuously according to some distribution function $f(\rho)$] can be found as a solution of the integral equation

$$\int f(\rho)\left(\frac{\rho-R}{a\rho+R}\right)d\rho=0. \tag{10}$$

If ξ in Eq. (7) is uniformly distributed between 0 and 1, then $f(\rho)=1/\kappa\rho$, and ρ is varied in the range $R_0e^{\Delta/k_BT}\leqslant\rho\leqslant R_0e^{\Delta/k_BT+\kappa}$ [see Eq. (7)],

$$\int_{R_0e^{\Delta/k_BT}}^{R_0e^{\tilde{\kappa}+\Delta/k_BT}}\theta(\rho e^{-\Delta/k_BT}-R_{JC})\frac{1}{\kappa\rho}\left(\frac{\rho-R}{a\rho+R}\right)d\rho - \int_{R_0e^{\Delta/k_BT}}^{R_0e^{\tilde{\kappa}+\Delta/k_BT}}\theta(R_{JC}-\rho e^{-\Delta/k_BT})\frac{1}{\kappa\rho}d\rho=0, \tag{11}$$

where R_{JC} is a critical resistance value determined by Eq. (9), $a=z/2-1$, and z is the number of bonds at each node of the network. Here we split the integral into two parts in order to take the Josephson coupling into account in accordance with condition (9), and have used the fact that $\rho|_{\Delta=0}=\rho\exp[-\Delta(T)/k_BT]$ [see Eq. (7)]. In the first integral we calculate the cases in which ρ is larger than necessary for the Josephson coupling $\rho>R_{JC}$ [i.e., when $\theta(\rho-R_{JC})=1$, where θ is the Heaviside function]. In the second integral we consider the opposite situation, i.e., when $\theta(R_{JC}-\rho)=1$. In this case Josephson coupling exists and ρ in the brackets should taken as zero ($\rho\to 0$).

From Eq. (11) we can find the solution (for $R_0e^{\Delta/k_BT}<R_{JC}\leqslant R_0e^{\Delta/k_BT+\kappa}$)

$$R=\frac{1-p_c}{p_c}\frac{(R_0e^{\tilde{\kappa}p_c}-R_{JC})e^{\Delta/k_BT}}{1-e^{-\tilde{\kappa}(1-p_c)}}, \tag{12}$$

where R_{JC} is given by Eq. (9), and $p_c=1/(1+a)=2/z$ (see also Ref. 34).

Equation (12) can be understood qualitatively as follows: The total resistance of the system at $T>T_c$ [when $\Delta(T)=0$] and for large κ is equal to $R_0e^{p_c\kappa+[p_c\beta E_c+\Delta(T)]/k_BT}$ (when $z=4$, i.e., $p_c=0.5$). As discussed above, if the system is strongly disordered, then its total resistance is determined by few resistors[42,43] along a path on the spanning cluster.[26,34,35] At $T<T_C$ some of the grains along this path have $R_{ij}(\Delta=0)$ smaller than R_{JC} and, according to Eq. (9), these will be in the superconducting state. Therefore this resistance, proportional to R_{JC}, should be subtracted from the total resistance: $R=(R_0e^{p_c\tilde{\kappa}}-R_{JC})e^{\Delta/k_BT}$.

We expand Eq. (12) linearly near the critical temperature ($T\sim T_c$), from which we get an expression for R_e linear in terms (T_c-T) as follows:

$$R \simeq R_0 e^{\Delta(T)/k_B T + p_c \kappa} - (3.06^2 z\alpha/2T_c)(T_c - T). \quad (13)$$

In the same approximation we get

$$\ln[R/R(T_c)] \simeq -(3.06^2 z\alpha/2T_c) e^{-p_c \kappa}(T_c - T), \quad (14)$$

where $R_N \equiv R(T_c) = R_0 \exp(p_c \kappa)$ is the system resistance at $T = T_c$. Note that the inverse Arrhenius law (1) follows immediately from the latter expression with

$$T_0 = (2T_c/3.06^2 z\alpha) R_N, \quad (15)$$

shown in Fig. 4 in comparison to experimental and numerical results. It should be noted that the system with exponential disorder (4) behaves differently than the system with two levels of resistivity (e.g., $\rho = 0$ and $\rho = 1$).

The analytical results can be also used in order to find the critical value of the effective resistance R_{cr} separating the metal-like ($\partial R/\partial T \geqslant 0$) and insulatorlike ($\partial R/\partial T < 0$) behaviors. By taking the derivative and solving the equation $\partial R/\partial T = 0$, we get a self-consistency equation, which determines the critical value separating metal-like behavior (at $R \leqslant R_{cr}$) from insulatorlike (at $R > R_{cr}$). The value of R_{cr} [as well as R, see Eq. (12)] depends on two main factors: number of neighboring grains z and charging energy $\mathcal{E}_c$. For small $\mathcal{E}_c$ we get a simple expression

$$R_{cr} = z\alpha(1 - p_c)/p_c. \quad (16)$$

IV. SUMMARY

In summary we have modeled and studied the temperature dependence of the superconductor-insulator transition in granular superconductors. Our numerical simulations explain well the experimental results over a wide range of temperatures and resistances. Calculations of effective medium approximation also show excellent agreement with the experiments for temperatures close to T_c. These calculations also enable us to determine the critical resistance value, separating the superconducting and insulating branches.

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[1] A. M. Goldman and N. Markovic, Phys. Today **51**(11), 39 (1998) and references therein.
[2] R. C. Dynes, J. P. Garno, and J. M. Rowell, Phys. Rev. Lett. **40**, 479 (1978).
[3] R. P. Barber, Jr. and R. C. Dynes, Phys. Rev. B **48**, R10618 (1993).
[4] L. Merchant, J. Ostrick, R. P. Barber, Jr., and R. C. Dynes, Phys. Rev. B **63**, 134508 (2001).
[5] A. Frydman, O. Naaman, and R. C. Dynes, Phys. Rev. B **66**, 052509 (2002); A. Frydman, Physica C **391**, 189 (2003).
[6] S. R. Khan, E. M. Pedersen, B. Kain, A. J. Jordan, and R. P. Barber, Jr., Phys. Rev. B **61**, 5909 (2000).
[7] S. Y. Hsu and J. M. Valles, Phys. Rev. B **48**, 4164 (1993).
[8] Though some granularity can exist even in "homogeneous" superconductors.
[9] H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman, Phys. Rev. B **40**, 182 (1989); D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. **62**, 2180 (1989).
[10] S. Bose, P. Raychaudhuri, R. Banerjee, P. Vasa, and P. Ayyub, Phys. Rev. Lett. **95**, 147003 (2005); E. Chow, P. Delsing, and D. B. Haviland, *ibid.* **81**, 204 (1998).
[11] A. Frydman and R. C. Dynes, Philos. Mag. B **81**, 1153 (2001).
[12] L. Burlachkov, E. Mogilko, Y. Schlesinger, Y. M. Strelniker, and S. Havlin, Phys. Rev. B **67**, 104509 (2003); Y. M. Strelniker, D. J. Bergman, S. Havlin, E. Mogilko, L. Burlachkov, and Y. Schlesinger, Physica A **330**, 291 (2003); Y. M. Strelniker, S. Havlin, and A. Frydman, Physica B **394**, 368 (2007); E. Mogilko, L. Burlachkov, Y. M. Strelniker, Y. Schlesinger, and S. Havlin, *ibid.* **329**, 1500 (2003); E. Mogilko, Y. M. Strelniker, L. Burlachkov, Y. Schlesinger, and S. Havlin, Physica C **372**, 960 (2002).
[13] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon, Oxford, 1980), Pt. 2.
[14] H.-K. Janssen and O. Stenull, Phys. Rev. E **67**, 046115 (2003).
[15] E. Simanek, Solid State Commun. **31**, 419 (1979); Phys. Rev. B **25**, 237 (1982).
[16] S. Kim and M. Y. Choi, Phys. Rev. B **42**, 80 (1990).
[17] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[18] G. Deutscher, O. Entin-Wohlman, S. Fishman, and Y. Shapira, Phys. Rev. B **21**, 5041 (1980); O. Entin-Wohlman, A. Kapitulnik, and Y. Shapira, *ibid.* **24**, 6464 (1981).
[19] B. I. Belevtsev, Yu. F. Komnik, and A. V. Fomin, J. Low Temp. Phys. **69**, 401 (1987); K. B. Efetov, Zh. Eksp. Teor. Fiz. **78**, 2017 (1980) [Sov. Phys. JETP **51**, 1015 (1980)]; B. I. Belevtsev, Usp. Fiz. Nauk **160**, 65 (1990) [Sov. Phys. Usp. **33**, 36 (1990)]; G. Mancini and R. Natali, Phys. Status Solidi B **242**, 632 (2005); S. M. Chudinov, R. Ferretti, S. Fusari, G. Mancini, and S. Stizz, Phys. Rev. B **62**, 12516 (2000); Y. Imry and M. Strongin, *ibid.* **24**, 6353 (1981); D. M. Wood and D. Stroud, *ibid.* **25**, 1600 (1982); D. R. Bowman and D. Stroud, Phys. Rev. Lett. **52**, 299 (1984); K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979); G. Schon and A. D. Zaikin, Phys. Rep. **198**, 237 (1990).
[20] T. Worthington, L. Lindenfeld, and G. Deutscher, Phys. Rev. Lett. **41**, 316 (1978); see also B. Giovannini and L. Weiss, Helv. Phys. Acta **51**, 76 (1978).
[21] For other different approaches, see M. V. Feigel'man and A. I. Larkin, Chem. Phys. **235**, 107 (1998); V. M. Galitski and A. I. Larkin, Phys. Rev. Lett. **87**, 087001 (2001); M. V. Feigel'man, A. S. Ioselevich, and M. A. Skvortsov, *ibid.* **93**, 136403 (2004); Y. Dubi, Y. Meir, and Y. Avishai, Phys. Rev. B **73**, 054509 (2006).

[22] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992).
[23] *Fractals and Disordered Systems*, edited by A. Bunde and S. Havlin (Springer-Verlag, Berlin, 1996).
[24] A. Miller and E. Abrahams, Phys. Rev. **120**, 745 (1960).
[25] V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).
[26] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, New York, 1984).
[27] V. F. Gantmakher, *Electrons and Disorder in Solids* (Oxford University Press, New York, 2005).
[28] T. J. Shaw, M. J. Ferrari, L. L. Sohn, D.-H. Lee, M. Tinkham, and J. Clarke, Phys. Rev. Lett. **76**, 2551 (1996).
[29] N. Mason and A. Kapitulnik, Phys. Rev. Lett. **82**, 5341 (1999); L. Capriotti, A. Cuccoli, A. Fubini, V. Tognetti, and R. Vaia, *ibid.* **94**, 157001 (2005); G. Refael, E. Demler, Y. Oreg, and D. S. Fisher, Phys. Rev. B **68**, 214515 (2003).
[30] E. Simanek, Phys. Rev. B **32**, R500 (1985); E. Simanek and R. Brown, *ibid.* **34**, R3495 (1986).
[31] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957); B. D. Josephson, Adv. Phys. **14**, 419 (1965).
[32] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); **11**, 104 (1963).
[33] T. C. Choy and A. M. Stoneham, J. Phys.: Condens. Matter **2**, 939 (1990).
[34] Y. M. Strelniker, S. Havlin, R. Berkovits, and A. Frydman, Phys. Rev. E **72**, 016121 (2005); Y. M. Strelniker, R. Berkovits, A. Frydman, and S. Havlin, *ibid.* **69**, 065105(R) (2004); Y. M. Strelniker, Phys. Rev. B **73**, 153407 (2006); Y. M. Strelniker, S. Havlin, R. Berkovits, and A. Frydman, J. Appl. Phys. **99**, 08P905 (2006).
[35] T. Kalisky, L. A. Braunstein, S. V. Buldyrev, S. Havlin, and H. E. Stanley, Phys. Rev. E **72**, 025102(R) (2005).
[36] B. Abeles, P. Sheng, M. Coutts, and Y. Arie, Adv. Phys. **24**, 407 (1975); B. Abeles, Phys. Rev. B **15**, 2828 (1977); W. Wu and P. W. Adams, Phys. Rev. Lett. **73**, 1412 (1994).
[37] In this case, the charging energy of each grain was assumed to reduce due to the fact that each grain participates in several junctions and the charge transferred is spread over all neighboring grains, leading to an activation energy of $0.1E_c$. See, e. g., Refs. 15 and 18.
[38] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[39] A. K. Sarychev, D. J. Bergman, and Y. M. Strelniker, Phys. Rev. B **48**, 3145 (1993); Europhys. Lett. **21**, 851 (1993).
[40] The results of the numerical simulation can be made to be even more similar to the experimental if we assume that ϵ_{ij} is randomly distributed in the range $0 \leqslant \epsilon_{ij} \leqslant \Delta(T)$. This can be due to the randomization of the grain Fermi energies [see J. Zhang and B. I. Shklovskii, Phys. Rev. B **70**, 115317 (2004).] However, in order not involve any additional fitting parameter, we did not assume this effect.
[41] Z. Wu, E. Lopez, S. V. Buldyrev, L. A. Braunstein, S. Havlin, and H. E. Stanley, Phys. Rev. E **71**, 045101(R) (2005).
[42] A. Coniglio, J. Phys. A **15**, 3829 (1982).
[43] Y. Chen, E. Lopez, S. Havlin, and H. E. Stanley, Phys. Rev. Lett. **96**, 068702 (1992).
[44] K. L. Ekinci and J. M. Valles, Jr., Phys. Rev. Lett. **82**, 1518 (1999).
[45] R. Juretschke, R. Landauer, and J. A. Swanson, J. Appl. Phys. **27**, 838 (1956); D. A. G. Bruggeman, Ann. Phys. **24**, 636 (1935); S. Kirkpatrick, Rev. Mod. Phys. **45**, 574 (1973); D. J. Bergman and Y. M. Strelniker, Phys. Rev. B **60**, 13016 (1999); Phys. Rev. Lett. **80**, 857 (1998); Y. M. Strelniker and D. J. Bergman, Phys. Rev. B **61**, 6288 (2000); Y. M. Strelniker and D. J. Bergman, *ibid.* **67**, 184416 (2003).